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IMPACT OF ION PRESSURE ANISOTROPY IN COLLISIONAL QUANTUM MAGNETO-PLASMA WITH HEAVY AND LIGHT IONS

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We have examined collisional degenerate plasma composed of charged state of heavy positive ion and light positive as wel as negative ion. Employing the reductive perturbation method, we derived the damped Korteweg-de Vries-Burgers (dKdV-B) equation and by using its standard solution we analyze the characteristics of the solitary-shock profile under varying parameters. Furthermore, with the application of planar dynamical systems bifurcation theory, the phase portraits have been analyzed. This dynamical system analysis allowed us to extract important information on the stability of these structures as represented by the dKdV-B equation.

Keywords: *dKdV-B Equation; Quantum Plasma; Dynamical System; Reductive perturbation method; Pressure Anisotropy*

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1. INTRODUCTION

The physics of quantum plasmas comprising both positive and negative ions, particularly multi-ion plasmas have garnered significant attention recently due to its obligatory presence from laboratory to astrophysical plasma environments [1–4]. The constituents of the degenerate quantum plasma includes electrons, heavy ions with positive charges, and light ions [5]. Electrons, positive ions, and negative ions are all present in negative ion plasma. A portion of the electrons in these kinds of plasmas are bound to negative ions. Negative ion plasma can be created in a laboratory and is a naturally occurring phenomenon in space and astrophysical surroundings. Examples of plasma systems containing negative ions are plasma processing reactors, the Sun's photosphere, the D region of the ionosphere, and neutral beam sources. Both theoretical and practical research has shown that the presence of these negative ions dramatically changes a number of distinctive plasma phenomena. Electronic behavior and the plasma potential are altered by negative ions. It is also commonly known that negative ions exist in the comet Halley's comet [6] and the Earth's ionosphere [7]. Positive-negative ion plasmas have also been discovered to exist in a variety of settings, including neutral beam sources [8], low-temperature laboratory studies [9], reactors for plasma processing [10], etc. Numerous authors [4, 11–14] used positively charged heavy and light ions in quantum plasmas to study nonlinear waves. Akhtar and Hussain [15] investigated ion acoustic shock waves in degenerate plasma with negative ions, They found that quantum parameters, temperature of positive and negative ions have significant impact on shock wave structure in negative ion degenerate plasma. Hussain and Akhtar [16] studied collisional effects in negative ion plasmas in the presence of degenerate electrons. Mohsenpour et al [17] studied ion acoustic solitons in negative ion degenarate plasma. they found that negative ion parameters have influence on width and amplitude of the soliton.

In the presence of elevated magnetic fields, the plasma ion pressure exhibits anisotropic behavior, and the plasma behaves differently in parallel and perpendicular directions relative to the external magnetic field [18]. So, to consider the effect of ionic pressure anisotropy pressure i.e., the parallel ($P_{\|}$) and perpendicular ($P_{\perp}$) ion pressure become very important. Numerous studies have been reported on the impact of pressure anisotropy on the propagation of solitary and shock waves in different plasma regimes [19, 20]. For example, Almas et al. [21] investigated the properties of ion-acoustic solitary waves composed of anisotropic pressure of electron-positron-ion (e-p-i) plasma and found that the characteristics of such waves are more sensitive to parallel ion pressure than perpendicular ion pressure. Khalid et al. [22] also studied the propagation of ion-acoustic electrostatic waves in a magnetized electron-ion plasma with pressure anisotropy. Mahmood et al. [23] studied the properties of non-linear electrostatic structure in anisotropic pressure plasma and found that only the width of the soliton depends on the perpendicular pressure ($P_{\perp}$), however, an increase in the parallel pressure ($P_{\|}$) decreases both the amplitude as well as the width of the soliton. Manesh et al. [24] studied the properties of solitary waves in an anisotropic plasma with lighter and heavier ions and found that the light ion's pressure anisotropy determines the polarity of solitary waves, and it is rarefactive for anisotropic lighter ion whereas compressive for the isotropic lighter ion. Khan et al. [25] studied the properties of soliton and cnoidal wave in an anisotropic superthermal electron-positron-ion plasma and found that the wavelength of the cnoidal wave structure is reduced on increasing the parallel and perpendicular

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anisotropy of ion. Khalid and Rahman [26] studied the ion pressure anisotropy of the ion acoustic non-linear periodic waves in a magnetized plasma. They reported that the increase of parallel pressure of ions decreases the amplitude and width of the ion-acoustic periodic waves and the ion-acoustic waves behave differently than ion-acoustic periodic (cnoidal) waves in anisotropic plasmas.

Apart from classical plasmas, the effect of pressure anisotropy has been widely investigated in dense quantum magnetized plasmas. For example, Bordbar and Karami [27] studied the structural properties of an anisotropic dense neutron star and analyzed the compactness, redshift, etc. of such a dense matter as a function strong magnetic field of the order of 10^{17} Gauss which creates the anisotropy. Patidar and Sharma [28] explored the MHD wave modes in anisotropic relativistic degenerate plasma and found fast and slow wave modes propagating under the combined influence of various forces such as pressure anisotropy, exchange potential, Bohm force, and magnetic field. Irfan et al. [29] observed a strong modification of amplitude and width of weakly nonlinear ion-acoustic waves considering the pressure anisotropy of positive ions and electron trapping effects in a dense quantum magneto-plasma. Moreover, in the non-relativistic and ultra-relativistic regimes, the anisotropic ion pressure also affects the stability of solitary waves.

Various nonlinear waves, such as shock, solitary, rogue, etc., present in our environment are addressed using different mathematical nonlinear equations like Zakharov-Kuznetsov Burger (ZKB) equation, Korteweg-de-Vries (KdV) equation, Non-linear Schrodinger (NLSE) equation, Burgers equation etc [30–33]. In plasma, damping of various types of nonlinear waves can occur due to collisions between plasma species, elevated temperature of the inertial providing fluid, fluid viscosity, nonlinear Landau damping, among other factors. Most of the natural systems are not in perfect equilibrium, nearly all plasma waves experience some degree of damping [34–46]. The propagation of nonlinear waves is significantly influenced by particle collisions. Findings revealed that the effect of collision between charged particles may have a substantial impact on the wave's characteristics [47, 48].

Phase plane analysis is an effective technique for exploring the qualitative behavior of dynamical systems, a graphical approach specifically designed for examining second-order systems concerning their initial condition. Geometrically, in a phase plane, the trajectory of a dynamical system for a given initial condition is represented by a curve or point. Additionally, this technique allows us to get information about the stability of the system and gain further insight into the existence of solutions [49]. The significance of phase plane analysis in understanding the qualitative solutions of plasma systems is commonly acknowledged and used by researchers [50–53]. Recently, in various plasma systems, researchers have examined the bifurcation features of small-amplitude nonlinear waves within the framework of equations such as the Burgers equation [54], ZK equation [55], etc. [56, 57]

The objective of the present paper is to study the solitary-shock wave propagation in collisional quantum magneto-plasma considering the ionic pressure anisotropy as well as anisotropic viscosities. The damped Korteweg-de Vries Burger (dKdV-B) equation is derived using the RPT to study the shock wave nature in such plasma. These plasmas are believed to exist in white dwarfs and neutron stars. The results obtained here may be useful for laboratory as well as space astrophysical plasma environments wherein such plasma environments are prevalent.The manuscript is arranged as follows, Section 2 contains the detail theoretical formulation, Section 3 contains methodologies as well as detail derivations of dKdV-B Equation, Section 4 contains the results and discussion part, Section 5 contains the Dynamical system analysis and overall conclusion is presented in Section 6.

2. THEORETICAL FORMULATION

We consider a collisional plasma composed of charged state of heavy positive ion and light positive as wel as negative ion.The normalized set of governing equations is given by [58]:

$$\frac{\partial N_{ln,lp}}{\partial T}+\frac{\partial(N_{ln,lp}V_{ln,lpx})}{\partial x}+\frac{\partial(N_{ln,lp}V_{ln,lpy})}{\partial y}+\frac{\partial(N_{ln,lp}V_{ln,lpz})}{\partial z}=0 \tag{1}$$

$$\frac{\partial V_{lnx}}{\partial T}+\left(V_{lnx}\frac{\partial}{\partial x}+V_{lny}\frac{\partial}{\partial y}+V_{lnz}\frac{\partial}{\partial z}\right)V_{lnx}=\vartheta\frac{\partial\Phi}{\partial x}+\eta_{ln\parallel}\frac{\partial^2 V_{lnx}}{\partial x^2}-P_{ln\parallel}N_{ln}\frac{\partial N_{ln}}{\partial x}-\nu_{ln}V_{lnx} \tag{2}$$

$$\frac{\partial V_{lny}}{\partial T}+\left(V_{lnx}\frac{\partial}{\partial x}+V_{lny}\frac{\partial}{\partial y}+V_{lnz}\frac{\partial}{\partial z}\right)V_{lny}=\vartheta\frac{\partial\Phi}{\partial y}+\eta_{ln\perp}\frac{\partial^2 V_{lny}}{\partial y^2}+V_{lnz}\Omega_{ln}-P_{ln\perp}\frac{1}{N_{ln}}\frac{\partial N_{ln}}{\partial y}-\nu_{ln}V_{lny} \tag{3}$$

$$\frac{\partial V_{lnz}}{\partial T}+\left(V_{lnx}\frac{\partial}{\partial x}+V_{lny}\frac{\partial}{\partial y}+V_{lnz}\frac{\partial}{\partial z}\right)V_{lnz}=\vartheta\frac{\partial\Phi}{\partial z}+\eta_{ln\perp}\frac{\partial^2 V_{lnz}}{\partial z^2}+V_{lny}\Omega_{ln}-P_{ln\perp}\frac{1}{N_{ln}}\frac{\partial N_{ln}}{\partial z}-\nu_{ln}V_{lnz} \tag{4}$$

$$\frac{\partial V_{lpx}}{\partial T}+\left(V_{lpx}\frac{\partial}{\partial x}+V_{lpy}\frac{\partial}{\partial y}+V_{lpz}\frac{\partial}{\partial z}\right)V_{lpx}=-\frac{\partial\Phi}{\partial x}+\eta_{lp\parallel}\frac{\partial^2 V_{lpx}}{\partial x^2}-P_{lp\parallel}N_{lp}\frac{\partial N_{lp}}{\partial x}-\nu_{lp}V_{lpx} \tag{5}$$

$$\frac{\partial V_{lpy}}{\partial T}+\left(V_{lpx}\frac{\partial}{\partial x}+V_{lpy}\frac{\partial}{\partial y}+V_{lpz}\frac{\partial}{\partial z}\right)V_{lpy}=-\frac{\partial\Phi}{\partial y}+\eta_{lp\perp}\frac{\partial^2 V_{lpy}}{\partial y^2}+V_{lpz}\Omega_{lp}-P_{lp\perp}\frac{1}{N_{lp}}\frac{\partial N_{lp}}{\partial y}-\nu_{lp}V_{lpy} \tag{6}$$

$$\frac{\partial V_{lpz}}{\partial T}+\left(V_{lpx}\frac{\partial}{\partial x}+V_{lpy}\frac{\partial}{\partial y}+V_{lpz}\frac{\partial}{\partial z}\right)V_{lpz}=-\frac{\partial \Phi}{\partial z}+\eta_{lp\perp}\frac{\partial^2 V_{lpz}}{\partial z^2}+V_{lpy}\Omega_{lp}-P_{lp\perp}\frac{1}{N_{lp}}\frac{\partial N_{lp}}{\partial z}-\nu_{lp}V_{lpz} \tag{7}$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=N_e\left[1+Z_{hp}\mu_{hp}+\alpha_p-\mu_{ln}\right]+N_{ln}\mu_{\mathrm{ln}}-Z_{hp}\mu_{hp}-N_{lp}-N_p\left[\alpha_e+\mu_{ln}-1-Z_{hp}\mu_{hp}\right] \tag{8}$$

Here $\Omega_{lp,ln}=\omega_{clp,ln}/\omega_{ph}, \vartheta=\frac{m_{\mathrm{lp}}Z_{\mathrm{ln}}}{Z_{\mathrm{lp}}m_{\mathrm{ln}}}, \upsilon_{ln(lp)}=\frac{\mu_{ln(lp)}}{m_{ln(lp)}n_{ln(lp)}}$ where $\mu_{ln(lp)}$ is the dynamic viscosity which is given by $\mu_{\mathrm{ln}(lp)}=2.21\times10^{-15}\frac{T_{ln(lp)}^{5/2}A_{\mathrm{ln}(lp)}^{1/2}}{Z_{\mathrm{ln}(lp)}^{4}\,\mathrm{ln}(lp)\Lambda}$; $\ln\Lambda$ is the logarithm of Coulomb parameter, $A_{ln(lp)}$ is the atomic weight of heavy positive ion, Z_{hp} is the charged state of heavy positive ions, $Z_{lp(ln)}$ is the charged state of light positive (negative) ions, ϕ is the normalized electrostatic potential, $\eta_{lp(ln)}$ is the normalized viscosity for light positive (negative) ion, $\nu_{lp(ln)}$ collisional frequency of light positive (negative) ion.$P_{lp(ln)\parallel}$ and $P_{lp(ln)\perp}$ are the parallel and perpendicular pressure of light positive (negative) ion. The pressure equations for the anisotropic and adiabatic system are given by Chew–Goldberger–Law popularly known as (CGL) or double adiabatic theory [59–61], according to which $\frac{d}{dt}\left(P_{i\parallel}B^2/n_i^3\right)=0$ and $\frac{d}{dt}\left(P_{i\perp}/n_iB\right)=0$. In the case of electrostatic waves in a magnetized plasma, the ambient magnetic field $B=B_0$ is constant with time, i.e. $\frac{d}{dt}(B)=0$ where B_0 is the magnetic field at equilibrium. Moreover, the normalized parallel and perpendicular ion pressures obtained from the CGL theory are given as $P_{i\parallel}=3P_{i\parallel0}/n_{i0}\varepsilon_{Fe}$ and $P_{i\perp}=P_{i\perp0}/n_{i0}\varepsilon_{Fe}$ where $P_{i\parallel0}=n_{i0}T_{i\parallel}$ and$P_{i\perp0}=n_{i0}T_{i\perp}$ are the equilibrium values of parallel and perpendicular pressure functions respectively, and n_{i0} is the unperturbed ion density. [18,60,62] The variations in the ambient magnetic field alter the ionic temperatures in parallel and perpendicular directions to the magnetic field, i.e., $T_{i\parallel}\propto B_0$ and $T_{i\perp}\propto\left(\frac{n_{i0}}{B_0}\right)^2$ respectively [62,63].

The other plasma parameters are normalized as follows:

$\Phi=\frac{\epsilon_{Fe}}{e},\ t=T\omega_p^{-1},\ x=X\times\lambda_{Fe},\ N_j=\frac{n_j}{n_{j0}},\ \lambda_{Fe}=\frac{C_s}{\omega_s},\ \varepsilon_{Fe}=\left(\frac{\hbar}{2m_e}\right)\left(\frac{3\pi^2}{n_{e0}}\right)^{\frac{2}{3}}$ Where, λ_{Fe} is the Thomas-Fermi length, C_s is the Fermi ion sound velocity, ω_{ph} is the plasma frequency, $m_{lp(ln)}$ is the mass of light negative (light positive) ions

3. DERIVATION OF DKDV-B EQUATION

To derive the evolution equation we employed the reductive perturbation technique. The stretched coordinates [18] used here are given by:

$$\begin{gathered}\xi=\varepsilon^{1/2}\left(I_xx+I_yy+I_zz-MT\right),\eta_{ln(lp)\parallel}=\epsilon^{\frac{1}{2}}\eta_{ln(lp)\parallel0},\\ \eta_{ln(lp)\perp}=\epsilon^{\frac{1}{2}}\eta_{ln(lp)\perp0},\nu_{ln(lp)}=\varepsilon^{3/2}\nu_{ln0(lp0)}\end{gathered} \tag{9}$$

Where, M is the phase velocity (Mach number) and ϵ is a small nonzero constant measuring the strength of dispersion. In terms of the expansion parameter ϵ, the physical variables in equations are expanded in a power series as

$$\left.\begin{aligned}N_j&=1+\epsilon N_j^{(1)}+\epsilon^2N_j^{(2)}+\epsilon^3N_j^{(3)}+\dots\\ V_{ix}&=\epsilon V_{ix}^{(1)}+\epsilon^2V_{ix}^{(2)}+\epsilon^3V_{ix}^{(3)}+\dots\\ V_{jy,z}&=\epsilon^{\frac{3}{2}}V_{jy,z}^{(1)}+\epsilon^2V_{jy,z}^{(2)}+\epsilon^{\frac{5}{2}}V_{jy,z}^{(3)}+\dots\\ \phi&=\epsilon\phi^{(1)}+\epsilon^2\phi^{(2)}+\epsilon^3\phi^{(3)}+\dots\end{aligned}\right\} \tag{10}$$

Substituting the above stretched coordinates from Eq.(8) and the respective expansions from Eq. (10) in the Eqs.(1)-(8), and then collecting the terms appearing in the lowest order of ϵ gives the following relations which gives the phase velocity as

$$M=\pm\sqrt{\frac{b\pm\sqrt{b^2-4ac}}{2a}} \tag{11}$$

Where
$a=\left(\mu_e\alpha_1-\mu_p\Upsilon_1\right)$
$b=I_x^2\left(\mu_e\alpha_1\left(P_{\mathrm{ln}\parallel}+P_{lp\parallel}\right)-(1-\vartheta\mu_{\mathrm{ln}})+\mu_p\Upsilon_1\left(P_{ln\parallel}+P_{lp\parallel}\right)\right)$
$c=I_x^4\left(\mu_e\alpha_1P_{ln\parallel}P_{lp\parallel}+P_{ln\parallel}-\vartheta\mu_{ln}P_{lp\parallel}-\mu_p\Upsilon_1P_{ln\parallel}P_{lp\parallel}\right)$

Using standard procedure we obtain the following equation

$$p\frac{\partial\phi^{(1)}}{\partial\tau}+q\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+r\frac{\partial^3\phi^{(1)}}{\partial\xi^3}-s\frac{\partial^2\phi^{(1)}}{\partial\xi^2}+t\phi^{(1)}=0 \tag{12}$$

Where

$$p = \left(\frac{\mu_{ln} 2\vartheta M I_x^2}{\left(M^2 - P_{ln\parallel} I_x^2\right)^2} + \frac{2 M I_x^2}{\left(M^2 - P_{lp\parallel} I_x^2\right)^2} \right)$$

$$q = \left(2\mu_p \Upsilon_2 - 2\mu_e \alpha_2 - \frac{\mu_{ln} \vartheta^2 M^2 I_x^2 \left(3 I_x^2 + P_{ln\parallel} \frac{I_x^4}{M^2}\right)}{\left(M^2 - P_{ln\parallel} I_x^2\right)^3} + \frac{M^2 I_x^2 \left(3 I_x^2 + P_{lp\parallel} \frac{I_x^4}{M^2}\right)}{\left(M^2 - P_{lp\parallel} I_x^2\right)^3} \right)$$

$$r = \left(1 - \left(\left(I_x^2 P_{ln\perp} + 1\right) \frac{\left(1 - I_x^2\right) \mu_{ln} \vartheta M^2}{\left(M^2 - P_{ln\parallel} I_x^2\right)^2 \Omega_{ln}^2} + \left(I_x^2 P_{lp\perp} + 1\right) \frac{\left(1 - I_x^2\right) M^2}{\left(M^2 - P_{lp\parallel} I_x^2\right)^2 \Omega_{ln}^2} \right) \right)$$

$$s = \left(\frac{\vartheta \mu_{ln} M \eta_{ln\parallel 0} I_x^4}{\left(M^2 - P_{ln\parallel} I_x^2\right)^2} + \frac{M \eta_{lp\parallel 0} I_x^4}{\left(M^2 - P_{lp\parallel} I_x^2\right)^2} \right)$$

$$t = \frac{\vartheta \mu_{ln} I_x^2 M \nu_{ln}}{\left(M^2 - I_x^2 P_{ln\parallel}\right)} + \frac{M \nu_{lp} I_x^2}{\left(M^2 - I_x^2 P_{lp\parallel}\right)}$$

Finally, the dKdV-B equation can be extracted as

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} - C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + D\phi^{(1)} = 0 \tag{13}$$

Where

$$A = \frac{q}{p}, \; B = \frac{r}{p}, \; C = \frac{s}{p}, \; D = \frac{t}{p}$$

To obtain the solution of Eq. (13), the authors consider the new variable $\chi = \xi - U\tau$ where χ is the transformed coordinate with respect to a frame moving with velocity U. By taking $\phi^{(1)} = \phi$, Eq.(13) becomes

$$-U \frac{d\phi}{d\chi} + A\phi \frac{d\phi}{d\chi} + B \frac{d^3\phi}{d\chi^3} - C \frac{d^2\phi}{d\chi^2} + D\phi = 0 \tag{14}$$

Now, employing the method used in [64] results the solution of equation(14) as

$$\phi = \frac{2(144BC + 12CU + BD + 2UD)}{6A(4C - D)} + \left(\frac{2(-6C - D)}{5A} \right) \tanh(\chi) - \frac{12B}{A} \tanh^2(\chi) \tag{15}$$

4. RESULTS AND DISCUSSION

We have obtained the asymptotic solution of the dKdV-B equation in equ.(15). Now we find that the solution has got a solitary wave structure composed with a shock. Additionally due to damping the shock amplitude is affected and there is a constant part. We now take the parameters and study their effect on the nature and properties of these solitary-shock profiles. The tuning parameters are: Charged state of heavy positive ions (Z_{hp}), viscosity for light positive ion in parallel direction ($\eta_{lp\parallel 0}$), viscosity for light negative ion in parallel direction ($\eta_{ln\parallel 0}$), parallel pressure of light positive ion ($P_{lp\parallel}$), parallel pressure of light negative ion ($P_{ln\parallel}$) and perpendicular pressure of light positive ion ($P_{lp\perp}$), perpendicular pressure of light negative ion ($P_{ln\perp}$) respectively.

Now by taking combinations of other parameters and tuning one we obtain a series of figures which we discuss below.

In Figure 1 we take different pressure combination of light negative and positive ions. We plot curves for different charge density of heavy positive ion (Z_{hp}). Both the light negative and positive ions in subfigure (i) have different parallel and perpendicular pressure values, indicating that they are anisotropic. Light negative ions are anisotropic in subfigure (ii), but light positive ions are isotropic. Light positive ions are anisotropic in subfigure (iii), but light negative ions are isotropic. Comparing sub Figure (i),(ii) and (iii) we conclude that as the perpendicular pressure component for light positive ion ($P_{lp\perp}$) causes the overall value of the solitary structure gets an upward lift. In subfigure(i) the left base of the potential profile was at 0.3, the peak is 0.4 and the right base at 0.365. However as perpendicular pressure component for light positive ion ($P_{lp\perp}$) increases for 0.2 to 0.5 in subfig(ii) the left base jumps to 0.48 and the peak at 0.6 as the right base at 0.55. Additionally the separation between plots for different Z_{hp} gets widened. Now comparing subfig (i) and (ii) we see that as $P_{ln\perp}$ increases from 0.2 to 0.5 an upward potential shift is there but not as prominant as for a change in $P_{lp\perp}$. It may be inferred that pressure anisotropy due to light negative ion is causing a slight increase in the potential compared to that of positive light ion, which can be attributed to the repulsion of electrons by the negative light ions causing a lesser shift of potential profile. The light positive ion on the other hand exerts more nonlinear effects when compared to their negative counterparts.

In Figure 2, the light positive and light negative ions are anisotropic ($P_{lp\parallel} > P_{lp\perp}$) and isotropic respectively, in subfigure (i). In subfigure (ii), both ions are isotropic while in the subfigure (iii), both ions are anisotropic ($P_{lp\parallel} > P_{lp\perp}, P_{ln\parallel} < P_{ln\perp}$). Comparing subfigures (i) and (ii) we see that as $P_{lp\perp}$ increases from 0.2 to 0.5 the base level of the shock jumps abruptly manifold from -0.04 to 0.04 (approx). Also the relative value of the potential (ϕ) for different values of Z_{hp} also changes. A deep introspection shows that when for $\chi < 0$ the green curve was below the red, it suddenly comes

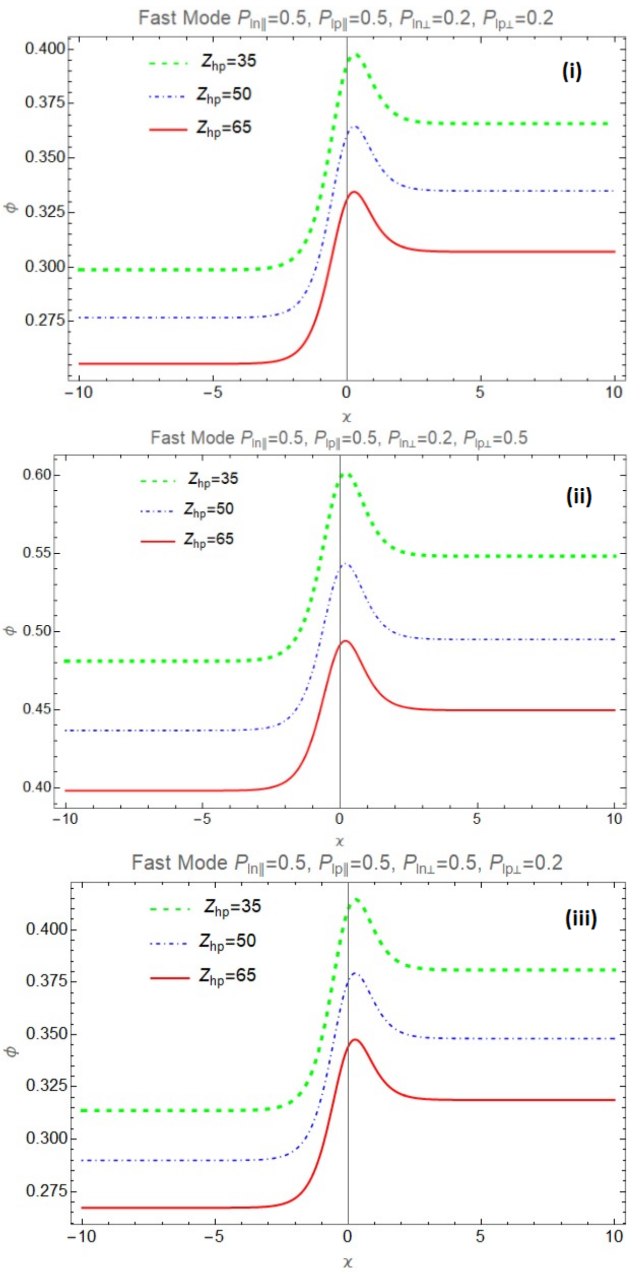

Figure 1. Variation of solitary-shock wave potential profile with different pressure combination of ions for different values of heavy positive ions

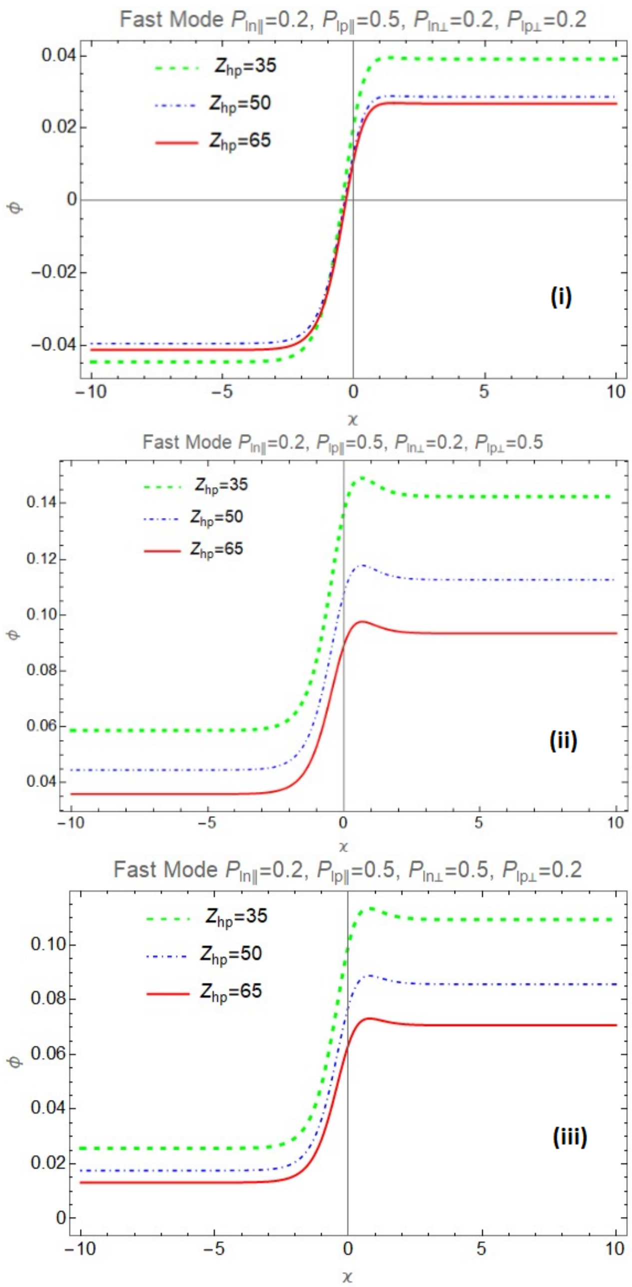

Figure 2. Variation of solitary-shock wave potential profile with different pressure combination of ions for different values of heavy positive ions

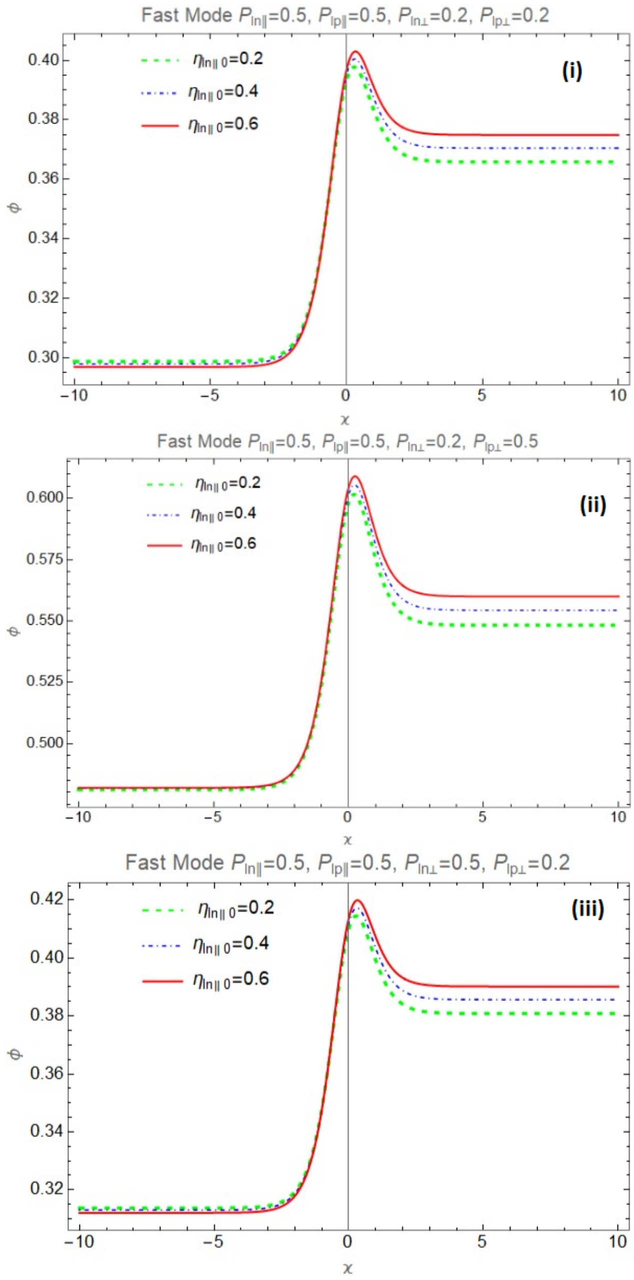

Figure 3. Variation of solitary-shock wave potential profile with different pressure combination of ions for different values of viscosity(parallel) of light negative ions

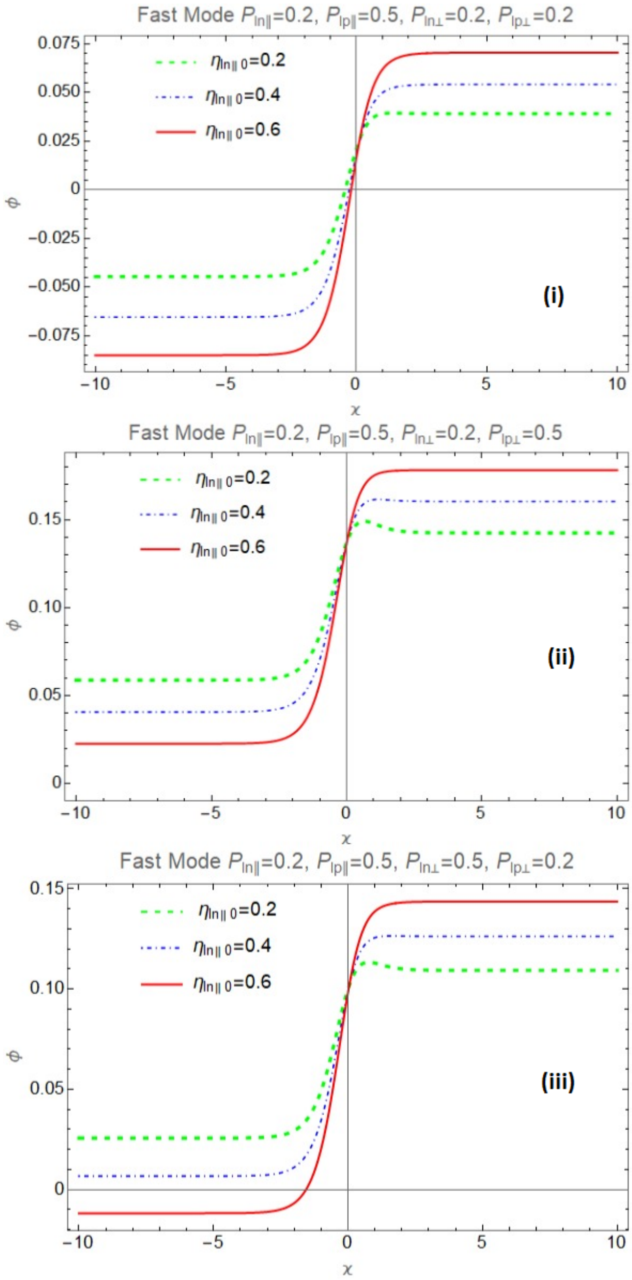

Figure 4. Variation of solitary-shock wave potential profile with different pressure combination of ions for different values of viscosity(parallel) of light negative ions

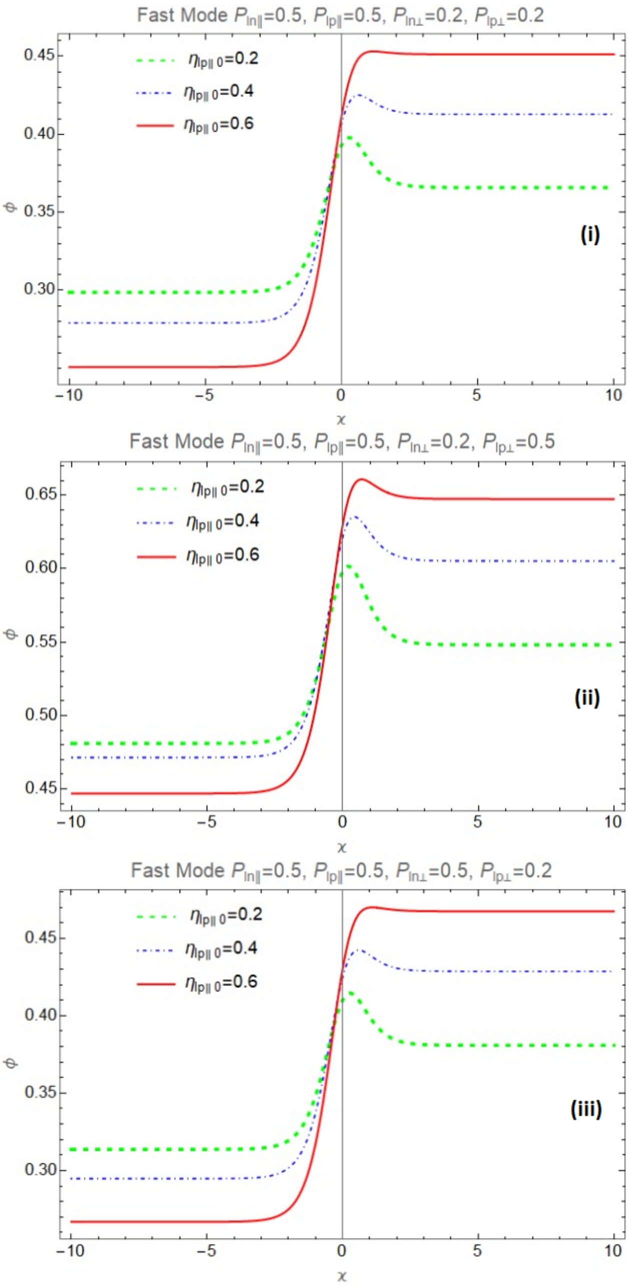

Figure 5. Variation of solitary-shock wave potential profile with different pressure combination of ions for different values of viscosity(parallel) of light positive ions

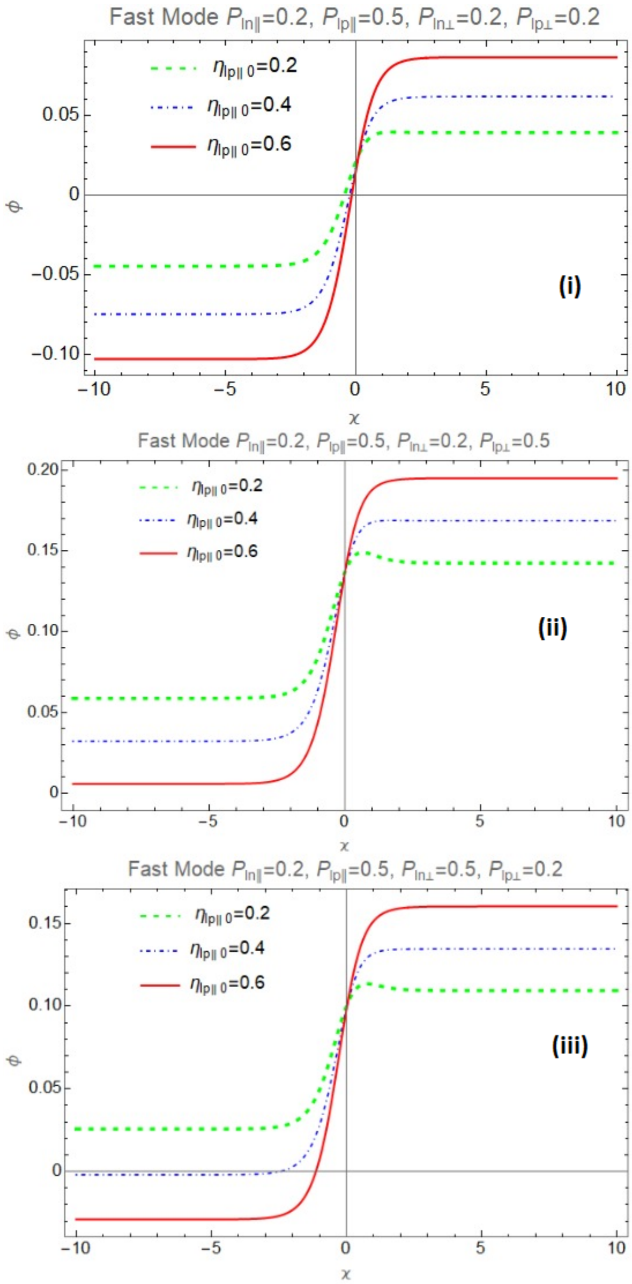

Figure 6. Variation of solitary-shock wave potential profile with different pressure combination of ions for different values of viscosity(parallel) of light positive ions

up above the red curve. Similar but less prominant nature is shown in subfigure (iii) when the perpendicular component of pressure changes but reversed for negative and positvie light ions. It is to be also noted that solitary nature is visible only when both the ions are either anisotropy or isotropic((ii) and (iii)).

In Figures 3 and 4 the effects of viscosity due to light negative ions has been studied. Figure 3 shows that for similar values of parallel pressure for light negative and positive ions the left branch of the stationary profile does not show much difference when as the right part shows certain difference in potential values, the only interesting feature is that higher the value of $\eta_{ln\|}$, higher is the damping effect due to viscosity. Such a higher energy loss balances the dispersive effects. Such a thing can be witnessed by comparing sub figure(i) with (ii) and (iii) respectively. Likewise in fig4 we see that the solitary nature is almost non existant. The Shock profile shows dependence on viscosity co-efficient. The pressure anisotropy are instrumental in the shifting of the potential profiles. The effect of (parallel)viscosity coefficient for light positive ion is depicted in Figures 5 and 6. In this case also the nature is similar to Figures 3 and 4. The only difference is that the solitary nature is far less prominant which implies to the fact that non linear effects are not much prominant. The other features are similar to these Figures 3 and 4 and therefore no additional explanation required.

5. DYNAMICAL SYSTEM ANALYSIS

Dynamical systems equations and phase portraits play a pivotal role in understanding and analyzing the behavior of plasma in physics. Plasma, often termed as the fourth state of matter, exhibits complex dynamics influenced by electromagnetic fields and particle interactions. Dynamical systems equations provide a mathematical framework to model these intricate dynamics, allowing scientists to predict and interpret the behavior of plasma systems. By formulating differential equations that describe the evolution of plasma parameters such as density, temperature, and velocity, researchers can gain insights into phenomena like plasma instabilities, turbulence, and wave propagation. Phase portraits, a visual representation of dynamical systems, offer a powerful tool to analyze the solutions of these equations. They provide a comprehensive overview of the system's behavior by plotting the trajectories of different plasma states in a multi-dimensional space. By examining the topology and stability of these trajectories, scientists can discern crucial information about the underlying dynamics, identifying equilibrium points, periodic orbits, and attractors.

This understanding is invaluable for optimizing plasma confinement in fusion reactors, developing plasma-based technologies like plasma thrusters for spacecraft propulsion, and advancing our knowledge of fundamental plasma physics phenomena. In essence, dynamical systems equations and phase portraits serve as indispensable tools for unraveling the complexities of plasma physics, driving progress in both theoretical understanding and practical applications.

In order to obtain the dynamical system equation we apply the transformation of the space and variable as $\chi = \xi - U\tau$ and finally obtain the transformed equation as

$$\begin{aligned} \frac{d\phi}{d\chi} &= z_1 \\ \frac{dz_1}{d\chi} &= z_2 \\ \frac{dz_2}{d\chi} &= \frac{Uz_1 - A\phi z_1 + Cz_2 - D\phi}{B} \end{aligned} \tag{16}$$

In the subsequent figure plots the projections of z_1, z_2, ϕ in mutual planes. the horizontal lines are for a given values of perpendicular. In Figure 7 we alter the values of heavy positive ion charge density which varies as 35,50 and 65. The other perpendicular are given as: $\eta_{ln\|0} = 0.2, \eta_{lp\|0} = 0.2, P_{ln\|} = 0.5, P_{lp\|} = 0.5, P_{ln\perp} = 0.2, P_{ln\perp} = 0.2$

We see that all the $z_1 - \phi, z_2 - \phi, z_1 - z_2$ plots shows the damping effects by the inward spiral motion. Damping here happens due to two factors, viscosity and collision. By thoroughly studying the figures we see that the $\phi - z_2$ plot has an inclined axis with negative slope. It can be interpretated as the spatial division of the electric field has a negative slope which implies that it is asymptotically stable. There is not much change in nature of curve in Figure 8 and Figure 9.

Figure 10 shows that effect of pressure anisotropy of light positive ions in the perpendicular direction. Hence there are great changes with change in parameter $P_{lp\perp}$, the spirality changes. Similar effects shows in Figure 11, when $P_{ln\perp}$ changes, i.e. the perpendicular component of pressure light negative ions changes. The effect of collision is shown in Figure 12. Collision plays an significant role in damping effects as the value of collision parameter increses the spirals are spread out along the axis. Such a nature is due to the dissipation of energy through collision. This is relatable non-radiative dissipation spectroscopy and thermal physics.

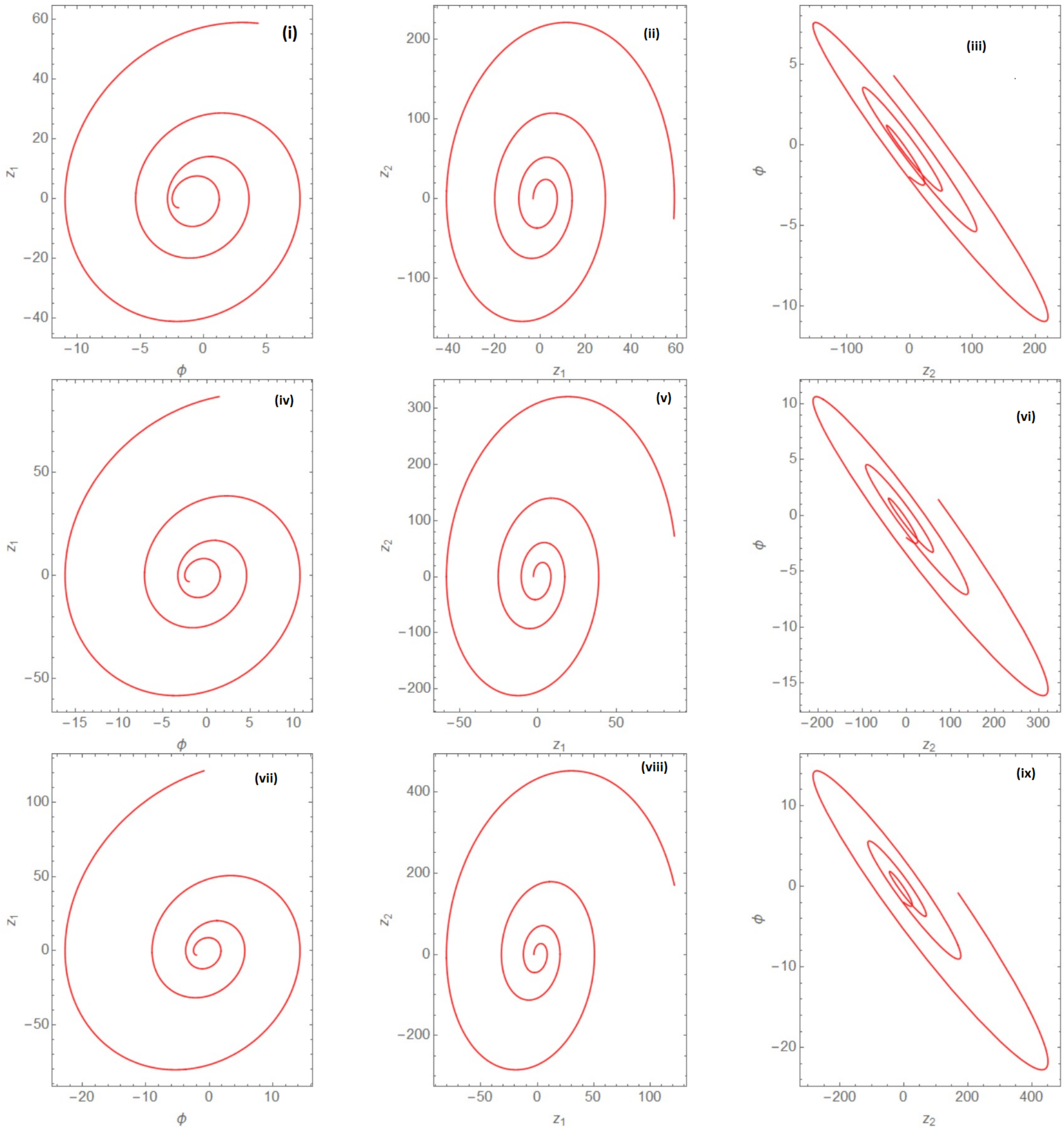

Figure 7. Phase Portrait for system (16) for $\eta_{ln\|0} = 0.2, \eta_{lp\|0} = 0.2, P_{ln\|} = 0.5, P_{lp\|} = 0.5, P_{ln\perp} = 0.2, P_{ln\perp} = 0.2$(i),(ii),(iii) $Z_{hp} = 35$ (vi),(v),(vi) $Z_{hp} = 50$,(vii),(viii),(ix) $Z_{hp} = 50$

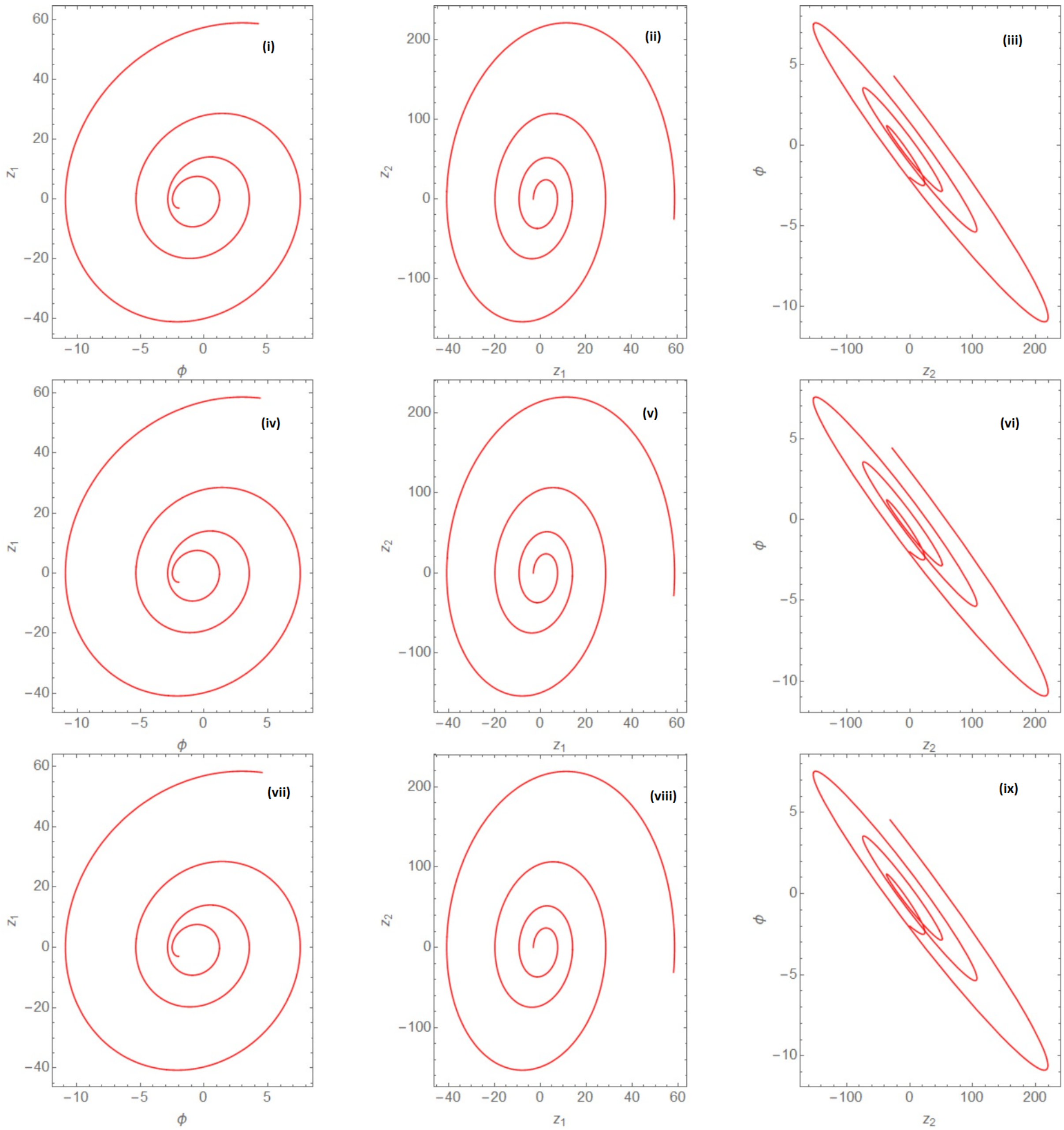

Figure 8. Phase Portrait for system (16) for $Z_{hp} = 35, \eta_{lp\|0} = 0.2, P_{ln\|} = 0.5, P_{lp\|} = 0.5, P_{ln\perp} = 0.2, P_{ln\perp} = 0.2$(i),(ii),(iii) $\eta_{ln\|0} = 0.2$ (vi),(v),(vi) $\eta_{ln\|0} = 0.4$,(vii),(viii),(ix) $\eta_{ln\|0} = 0.6$

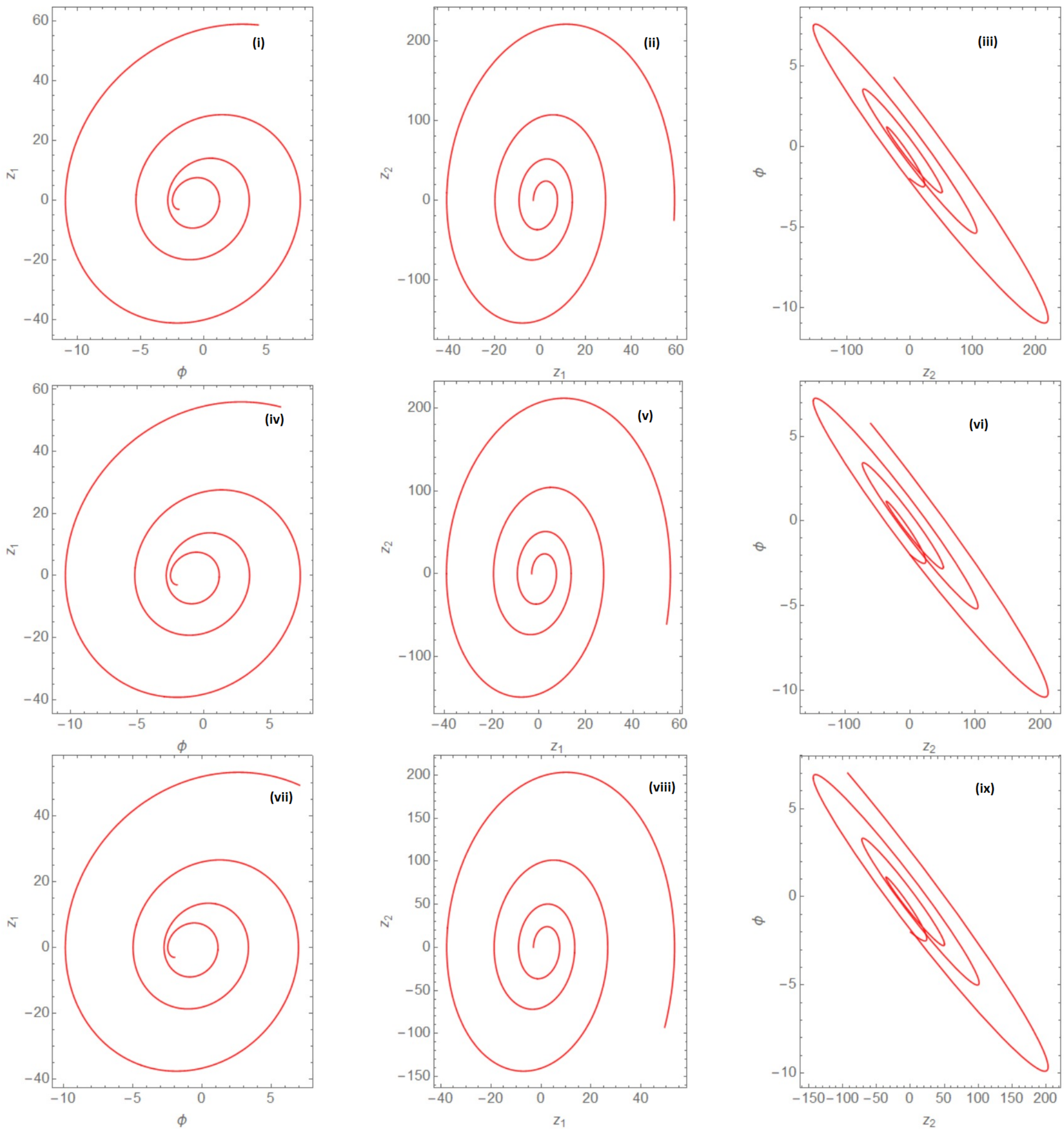

Figure 9. Phase Portrait for system (16) for $Z_{hp} = 35, \eta_{ln\|0} = 0.2, P_{ln\|} = 0.5, P_{lp\|} = 0.5, P_{ln\perp} = 0.2, P_{ln\perp} = 0.2$(i),(ii),(iii) $\eta_{lp\|0} = 0.2$ (vi),(v),(vi) $\eta_{lp\|0} = 0.4$,(vii),(viii),(ix) $\eta_{lp\|0} = 0.6$

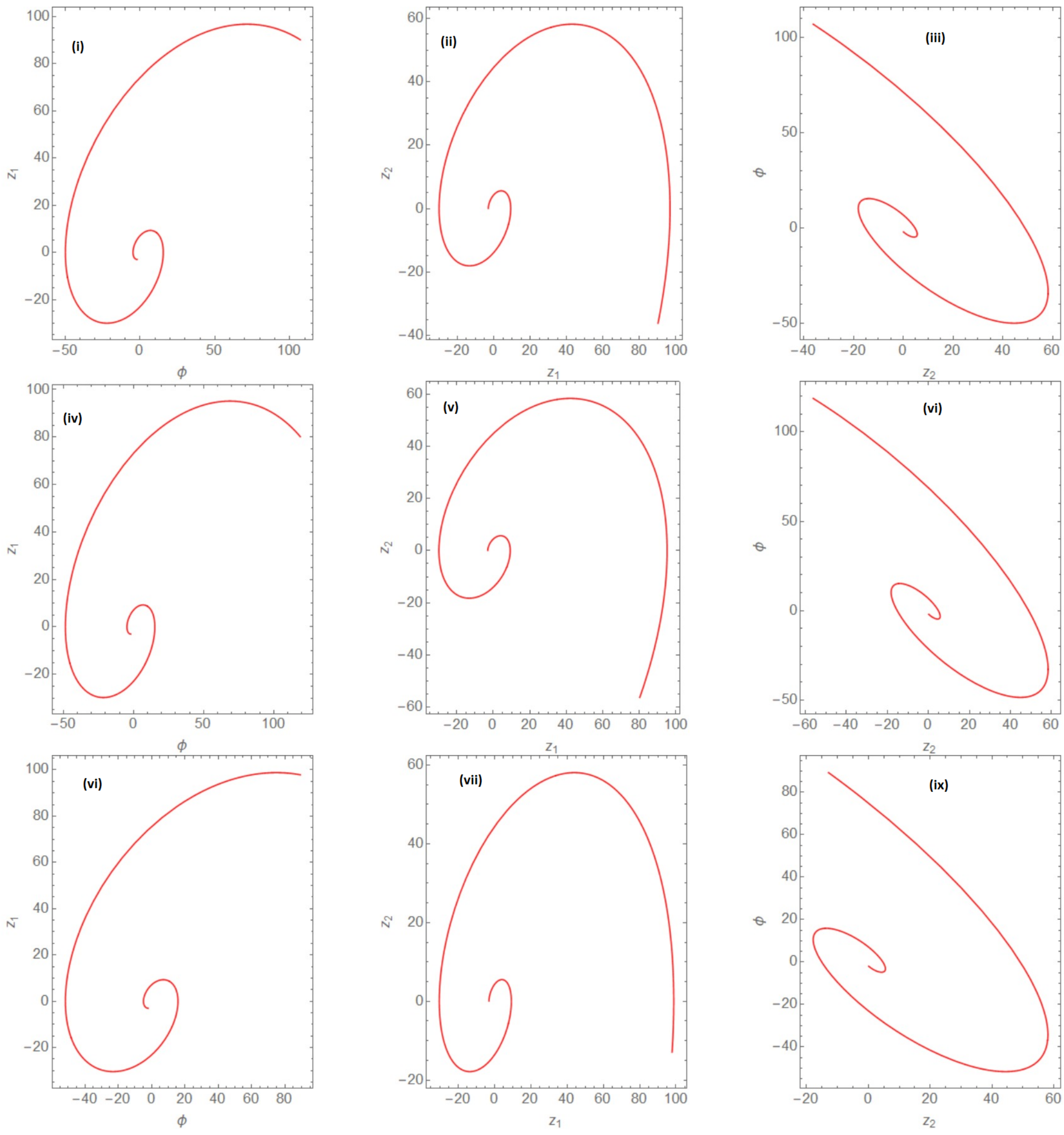

Figure 10. Phase Portrait for system (16) for $Z_{hp} = 35, \eta_{ln\|0} = 0.2, \eta_{lp\|0} = 0.2, P_{ln\|} = 0.5, P_{lp\|} = 0.5, P_{ln\perp} = 0.2$(i),(ii),(iii) $P_{lp\perp} = 0.5$ (vi),(v),(vi) $P_{lp\perp} = 0.2$,(vii),(viii),(ix) $P_{lp\perp} = 0.9$

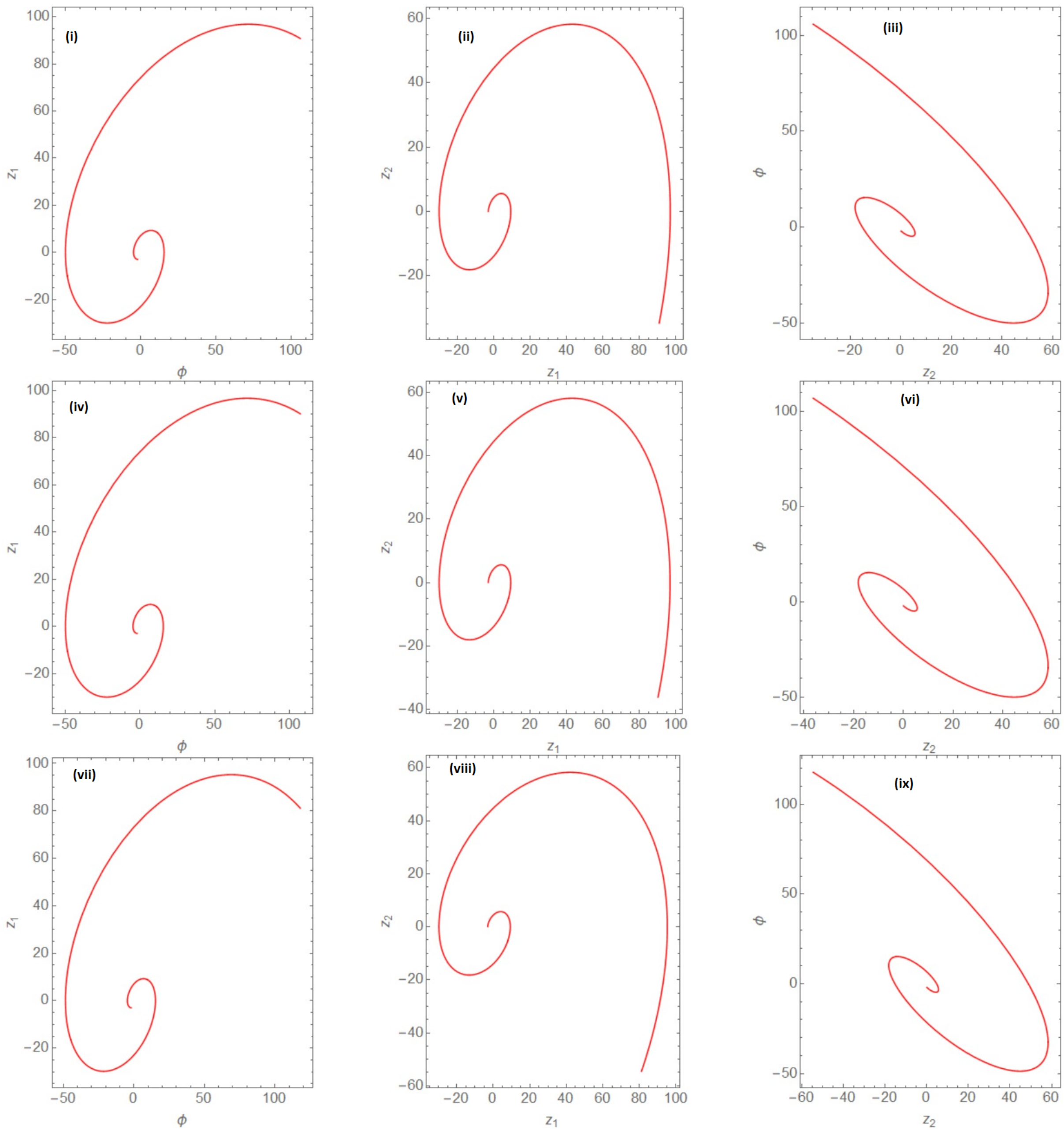

Figure 11. Phase Portrait for system (16) for $Z_{hp} = 35, \eta_{ln\|0} = 0.2, \eta_{lp\|0} = 0.2, P_{ln\|} = 0.5, P_{lp\|} = 0.5, P_{ln\perp} = 0.2$(i),(ii),(iii) $P_{ln\perp} = 0.5$ (vi),(v),(vi) for $P_{ln\perp} = 0.2$,(vii),(viii),(ix) $P_{ln\perp} = 0.9$

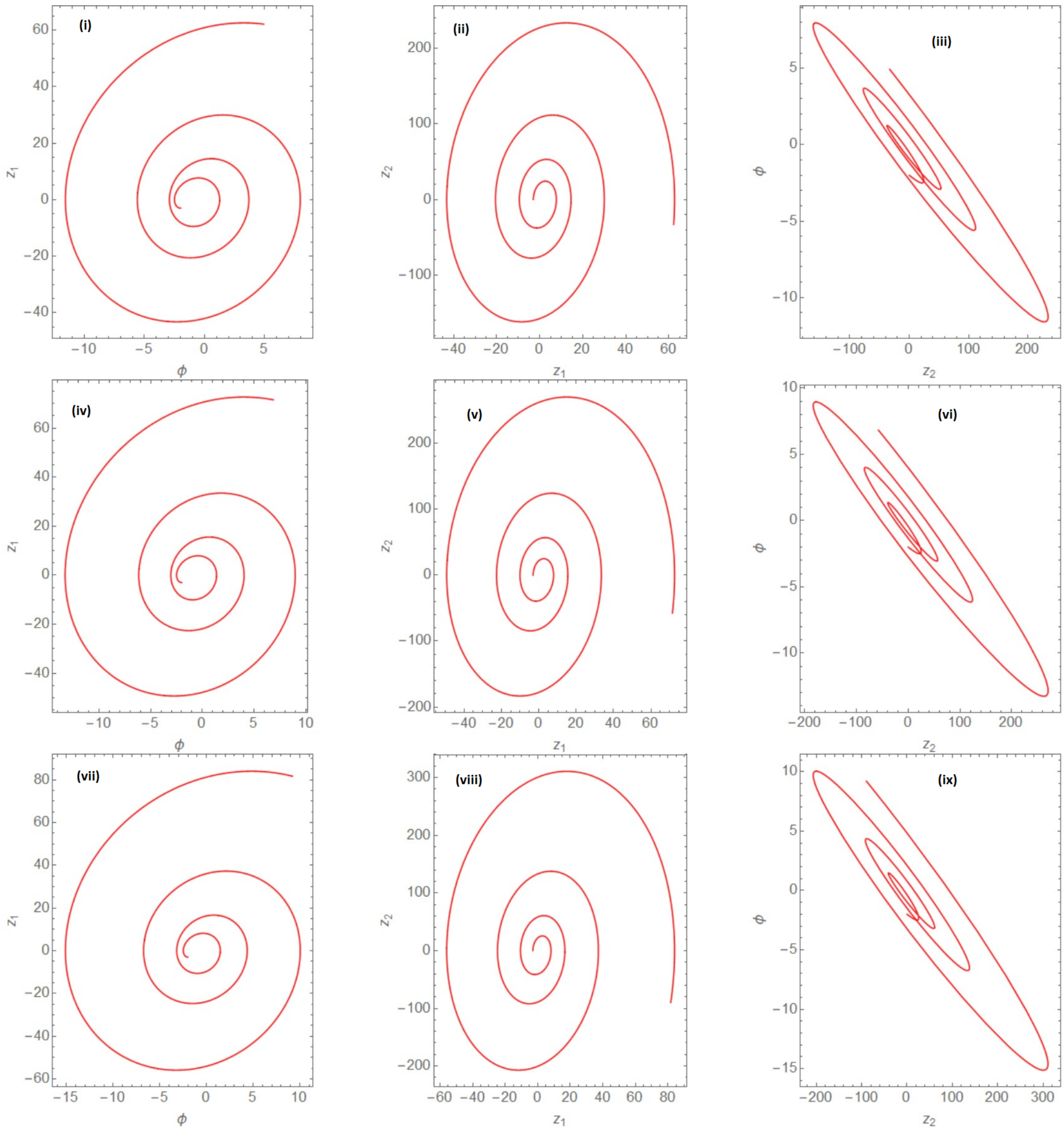

Figure 12. Phase Portrait for system (16) for $\eta_{ln\|0} = 0.2, \eta_{lp\|0} = 0.2, P_{ln\|} = 0.5, P_{lp\|} = 0.5, P_{ln\perp} = 0.2, P_{ln\perp} = 0.2$(i),(ii),(iii) $\nu_{ln} = 0.1, \nu_{lp} = 0.1$ (vi),(v),(vi)for $\nu_{ln} = 0.3, \nu_{lp} = 0.3$,(vii),(viii),(ix) $\nu_{ln} = 0.5, \nu_{lp} = 0.5$

6. CONCLUSION

The features of ion-acoustic solitary-shock wave propagating in a collisional plasma under the influence of ionic pressure anisotropy and viscosity is studied with the help of dKdV-B equation and by using standard procedure we obtained the analytic solution. The effect of heavy positive ion, viscosity of light positive ion and light negative ion as well as pressure anisotropy of light positive ion and light negative ion are investigated. Subsequently, we converted our evolutionary equation into a three dimensional dynamical system to perform phase plane analysis. The findings presented here may have implications in both laboratory as well as astrophysical plasma environment

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ВПЛИВ АНІЗОТРОПІЇ ІОННОГО ТИСКУ В КВАНТОВІЙ КВАНТОВІЙ МАГНІТОПЛАЗМІ З ВАЖКИМИ ТА ЛЕГКИМИ ІОНАМИ

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Ми розглянули вироджену плазму зіткнення, що складається із зарядженого стану важкого позитивного іона та легкого позитивного, а також негативного іона. Використовуючи метод відновних збурень, ми вивели рівняння Кортевега-де Фріза-Бюргерса (dKdV-B) і, використовуючи його стандартне рішення, аналізуємо характеристики профілю одиночного удару за різних параметрів. Крім того, із застосуванням теорії біфуркацій планарних динамічних систем проаналізовано фазові портрети. Цей аналіз динамічної системи дозволив нам отримати важливу інформацію про стабільність цих структур, представлену рівнянням dKdV-B.

Ключові слова: *рівняння dKdV-B; квантова плазма; анізотропія тиску; аналіз фазової площини; динамічна система*