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FIVE-DIMENSIONAL STRANGE QUARK BIANCHI TYPE-I COSMOLOGICAL MODEL IN THE FRAMEWORK OF SAEZ-BALLESTER THEORY OF GRAVITY

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In this paper, we have constructed a five-dimensional Bianchi type I cosmological model with strange quark matter in the context of Saez-Ballester theory of gravitation. We have discussed a five-dimensional cosmological model by using the special rule of variation for the Hubble's parameter in the shape of $H = Da^{-1}$ and the equation of state for strange quark matter. Two different models for $n \neq 0$ and $n = 0$ has been discussed. Furthermore, the accelerated expansion of the universe has been discussed by using different physical parameters along with their graphical representations.
Keywords: *Bianchi Type-I; Strange Quark Matter; Saez and Ballester Theory*
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1. INTRODUCTION

In recent years, analysis of diverse observational data has revealed that the universe is experiencing a rapid expansion. This phenomenon has sparked significant interest in formulating cosmological models within different gravitational theories. The general theory of relativity (GTR) offers a mathematically precise and physically robust explanation of gravity, serving as a foundation for developing cosmological models of the universe. But, describing the present state of the universe requires more than what is GTR explained. As a result, numerous efforts have been undertaken to modify Einstein's GTR, incorporating alternative and modified theories of gravitation. Hence, recently, researchers have shown significant interest in formulating cosmological models by employing alternative theories of gravitation such as Lyra Geometry, Brans Dicke Theory, Barber's first, second self-creation theory and Saez - Ballester theory [1-4].

Saez and Ballester [4] have formulated a theory in which the metric is interconnected with a dimensionless scalar field φ, and this coupling of φ provides a satisfactory explanation for weak fields. The scalar-tensor theory contributes to resolving issues within non-flat Friedmann-Robertson-Walker (FRW) cosmologies. Bali and Chandnani [5-6], studied the cosmological model of Bianchi type-I, considering a time-varying gauge function β to accommodate a perfect fluid distribution and string dust magnetized in the context of Lyra geometry.

Many authors have explored cosmological models using Saez and Ballester's scalar-tensor theory of gravitation. The Bianchi type- I, III, V, VI_0, and Kantowski-Sachs type models were examined within a scalar tensor theory by Singh and Agrawal [7]. Ram and Singh [8-9] have investigated a metric that exhibits spatially homogeneity, local rotational symmetry (LRS), and allows for a group of motions conforming to the Bianchi-I pattern on hypersurfaces with constant time t. Also, an investigation is conducted on a Robertson-Walker model of the universe that is both spatially homogeneous and isotropic, and possesses zero-curvature in the context of the Saez-Ballester scalar-tensor theory of gravity. Reddy [10] investigated Bianchi type-I metric together with cosmic string in a scalar – tensor theory of gravity. He observed that scalar field and the density are free from initial singularity and the universe is expanding with cosmic time. Mohanty and Sahu [11-12] have studied Bianchi type-VI_0 and Bianchi type-I cosmological models in the context of scalar- tensor theory of gravitation. Reddy, Subba Rao and Koteswara Rao [13] investigated exact solutions for a spatially homogeneous and LRS Bianchi type-I space time with negative constant deceleration parameters by employing special law of variations for Hubble parameter in the Saez-Ballester scalar tensor theory. Samanta *et al.* [14] discussed LRS Bianchi type-I cosmological models with bulk viscosity in the Saez-Ballester theory of gravitation and found that cosmic strings do not sustain when $\rho + \lambda = 0$, but they do sustain for Takabayasi and Geometric strings. Pawar and Agrawal [15] examined cosmological models with five dimensions within the Kaluza-Klein space-time in the framework Saez and Ballester theory of gravity. Mahurley *et al.* [16] focused on examination of cosmological models in scalar-tensor theory of gravitation. Specifically, the study explores spatially homogeneous anisotropic five-dimensional Bianchi type-I model with a perfect fluid.

Strange quark matter (SQM) in the influence of magnetic flux with five-dimensional Bianchi type-I cosmological model in Saez-Ballester theory has been considered. The SQM has been important and interesting topic in nuclear, astrophysics and cosmology due to its far-reaching theoretical significance and determine primitive magnetic fields. The

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SQM is possibly produced by energetic heavy-ion collision experiments [17], or exists in cosmic rays and in the interior of compact stars. A magnetic field has strong effects on the properties and stability of SQM [18,19]. The surface of a pulsar may exhibit a characteristic strength approaching approximately 10^{12} Gauss. In comparison, certain magnets can possess even higher magnetic fields, reaching surface values as extensive as 10^{13}–10^{15} Gauss. At present, our understanding of the genesis of these intense magnetic fields is lacking. The magneto hydrodynamic dynamo mechanism, in which a protoneutron star's revolving plasma creates a strong magnetic field, is a widely accepted theory. Many investigations have been carried out to explain the primordial magnetic field and quark-gluon matter in the early universe. In exploring the initial phase of the cosmos, examining the quark-gluon plasma proves to be a valuable approach. Following the Big Bang, the universe experienced a transition to quark-gluon plasma at the crucial temperature Tc ≡ 100–200 MeV, leading to the expulsion of quark matter (QM). Quark-gluon plasmas have many diverse implications for cosmology and astrophysics. The challenge in isolating a quark stems from the fact that quarks are never found in isolation; rather, they always exist in groups. There are mainly six types of quarks: charm (c), top (t), bottom (b), up (u), down (d) and strange (s) [20-23]. The suggestion that a theoretical concept known as strange quark matter (SQM) could exhibit complete stability at zero temperature and in β-equilibrium has sparked significant research and exploration.

The equation of state (EoS) of SQM is given by $p=\rho-4B_c/3$, where B_c is Bag constant and the difference between the energy density of the perturbative and non-perturbative QCD vacuum and *ρ, p* are the energy density and thermodynamic pressure of the QM, respectively. Fundamentally, this is the EoS of a gas of massless particles with corrections due to the QCD trace glitch and perturbative interactions. At the surface of the star as $p \to 0$, we have $\rho \to 4B_c$. The characteristic value of the bag constant is of the order *B* = 57MeV/fm3 ≈10^{15} g/cm3. Several researchers have explored SQM with General relativity (GR), scalar tensor theory and other modified theories of gravity [24-28].

Motivated by the above discussion and work done by Pawar et al. [28], here we consider five-dimensional Bianchi type-I cosmological model with strange quark matter the framework of Saez-Ballester theory of gravity. The paper organized as follows: Section 2 contains field equations of Saez and Ballesters theory. Section 3 deals with metric and field equations. In section 4 we have obtained the solutions of field equations. Section 5 deals with the model for $n\neq 0$ with some physical and kinematical parameter of the model. Again, Section 6 deals with the model for $n=0$ with some physical and kinematical parameter of the model. In Section 7 we have kept the graphical representation of dynamical parameters of Model-I and Model-II. Lastly, in section 8 is the discussion and conclusion are provided.

2. FIELD EQUATION OF SAEZ AND BALLESTER THEORY

The field equations given by Saez and Ballester (1985) for the combined scalar and tensor fields are

$$G_i^j-\omega\varphi^n\left(\varphi_{,i}\varphi^{,j}-\frac{1}{2}\delta_i^j\varphi_{,k}\varphi^{,k}\right)=-T_i^j . \tag{1}$$

The scalar field φ satisfies the equation

$$2\varphi^n\varphi^i_{\ ;i}+n\varphi^{n-1}\varphi_{,k}\varphi^{,k}=0 , \tag{2}$$

where, $G_i^j=R_i^j-\frac{1}{2}R\delta_i^j$ is the Einstein's tensors, ω and n are constants, T_i^j is stress energy-momentum tensor, comma and semicolon represents partial and covariant differentiation respectively.

The energy-momentum tensor for Strange Quark Matter is given by

$$T_i^j=\left(\rho+p+h^2\right)u^ju_i+\left(\frac{h^2}{2}-p\right)\delta_i^j-h_ih^j , \tag{3}$$

where, ρ is the density, p is the pressure, h^2 is the magnetic flux. The magnetic flux is considered in the *x*-direction with $h_iu^i=0$. The four velocity vectors are given by $u^i=(0,0,0,0,1)$ with $u_iu^i=1$.

3. THE METRIC AND FIELD EQUATIONS

The homogeneous five-dimensional Bianchi type-I metric is given as

$$ds^2=-dt^2+A^2dx^2+B^2(dy^2+dz^2)+C^2dm^2 , \tag{4}$$

where, *A, B,* and *C* are functions of cosmic time *t* only.
From equation (1) - (3) for the equation (4) we have obtained,

$$\frac{2\dot{A}\dot{B}}{AB}+\frac{2\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}^2}{B}+\frac{\omega}{2}\varphi^n\dot{\varphi}^2=-\left(\rho+\frac{3h^2}{2}\right), \tag{5}$$

$$\frac{2\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{2\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B}-\frac{\omega}{2}\varphi^n\dot{\varphi}^2=\left(p-\frac{h^2}{2}\right), \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{\omega}{2}\varphi^n\dot{\varphi}^2=\left(p-\frac{h^2}{2}\right), \tag{7}$$

$$\frac{\ddot{A}}{A}+\frac{2\ddot{B}}{B}+\frac{2\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B}-\frac{\omega}{2}\varphi^n\dot{\varphi}^2=\left(p-\frac{h^2}{2}\right), \tag{8}$$

$$\ddot{\varphi}+\dot{\varphi}\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{n}{2}\frac{\dot{\varphi}^2}{\varphi}=0\,. \tag{9}$$

The contracted covariant derivative of energy-momentum tensor vanishes. i. e. from the energy conservation equation we get

$$T^{ij}_{;j}=0\,, \tag{10}$$

which yield to

$$\dot{\rho}+\left(\rho+p+2h^2\right)\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0\,. \tag{11}$$

4. SOLUTIONS OF THE FIELD EQUATIONS

Einstein's field equations (5) - (9) are system of highly non-linear differential equations which contains five independent equations containing seven unknowns *A, B, C, p,* ρ, φ and h^2. To obtain the solution of system of equations we have to consider two additional conditions. Initially, we examine the variation law for the generalized Hubble's parameter in space-time (4), as provided by Berman (1983). This law yields a constant deceleration parameter. Cosmological models with a constant deceleration parameter have been examined by numerous authors. Kumar and Singh [29] have investigated Bianchi type-I models in general relativity, where a constant deceleration parameter was maintained. This was achieved by employing a specific law for the variation of Hubble's parameter, resulting in constant value of deceleration parameter.

The special law of variation for the Hubble's parameter given by Berman (1983) is expressed as

$$H=Da^{-n}=D(AB^2C)^{-\frac{n}{4}}, \tag{12}$$

where, $D>0$ and $n\geq 0$ are constants.
On solving equation (12), we get

$$a=(AB^2C)^{\frac{1}{4}}\,. \tag{13}$$

The dynamical parameters for metric (4) are given as follows:
The spatial volume is defined as

$$V=a^4=(AB^2C)\,, \tag{14}$$

The mean Hubble's parameter is given by

$$H=\frac{\dot{a}}{a}=\frac{1}{4}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \tag{15}$$

The Expansion Scalar θ is obtained by

$$\theta=4H\,, \tag{16}$$

The mean Anisotropy Parameter A_m is given by

$$A_m=\frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i}{H}-1\right)^2, \tag{17}$$

The Shear Scalar is

$$\sigma^2 = \frac{3}{2} A_m H^2 , \tag{18}$$

The deceleration parameter (q) is defined to be

$$q = -\frac{a\ddot{a}}{\dot{a}^2} , \tag{19}$$

Solving equation (15), we get

$$a = (nDt + c_1)^{\frac{1}{4}} , \ n \neq 0 , \tag{20}$$

$$a = c_2 e^{Dt} , \ n = 0 , \tag{21}$$

where, c_1 & c_2 are integrating constants.
From equations (5) to (8), the metric potentials are obtained as follows,

$$A = a_1 a e^{b_1 \int a^{-4} dt} , \tag{22}$$

$$B = a_2 a e^{b_2 \int a^{-4} dt} , \tag{23}$$

$$C = a_3 a e^{b_3 \int a^{-4} dt} , \tag{24}$$

where,

$$a_1 = k_1^{\frac{1}{2}} k_2^{\frac{1}{4}}, \ \ a_2 = k_1^{-\frac{1}{4}} k_3^{\frac{1}{4}}, \ \ a_3 = k_2^{-\frac{1}{4}} k_3^{-\frac{1}{2}} \ \text{ and } b_1 = \frac{2d_1 + d_2}{4}, \ \ b_2 = \frac{d_3 + d_1}{4}, \ \ b_3 = \frac{-d_2 - 2d_3}{4}$$

which satisfy the relations

$$a_1 a_2^2 a_3 = 1, \ \ b_1 + 2b_2 + b_3 = 0. \tag{25}$$

Equation (9) gives

$$\varphi(t) = \left[\frac{k_4(n+2)}{2} \int a^{-4} dt \right]^{2/n+2} , \tag{26}$$

here, k_4 is constant of integration.

5. MODEL I: when $n \neq 0$.

Using equation (20) in (22) - (24) and (26), we get the metric potentials and scalar field as follows:

$$A = a_1 (nDt + c_1)^{\frac{1}{n}} e^{b_1 \left[\frac{(nDt+c_1)^{\frac{n-4}{n}}}{D(n-4)} \right]} , \tag{27}$$

$$B = a_2 (nDt + c_1)^{\frac{1}{n}} e^{b_2 \left[\frac{(nDt+c_1)^{\frac{n-4}{n}}}{D(n-4)} \right]} , \tag{28}$$

$$C = a_3 (nDt + c_1)^{\frac{1}{n}} e^{b_3 \left[\frac{(nDt+c_1)^{\frac{n-4}{n}}}{D(n-4)} \right]} , \tag{29}$$

$$\varphi(t) = \left[\frac{k_4(n+2)}{2} \right]^{\frac{2}{n+2}} \left[nDt + c_1 \right]^{\frac{2(n-4)}{n(n+2)}} . \tag{30}$$

Equation (27) to (30) satisfy the equation of conservation of energy (11) and hence the metric in (4) can be expressed as

$$ds^2 = -dt^2 + \left(a_1 (nDt + c_1)^{\frac{1}{n}} e^{b_1 \left[\frac{(nDt+c_1)^{\frac{n-4}{n}}}{D(n-4)} \right]} \right)^2 dx^2 + \left(a_2 (nDt + c_1)^{\frac{1}{n}} e^{b_2 \left[\frac{(nDt+c_1)^{\frac{n-4}{n}}}{D(n-4)} \right]} \right)^2 (dy^2 + dz^2) + \left(a_3 (nDt + c_1)^{\frac{1}{n}} e^{b_3 \left[\frac{(nDt+c_1)^{\frac{n-4}{n}}}{D(n-4)} \right]} \right)^2 dm^2 \quad (31)$$

The EoS for SQM given by Pawar et al, [28] as

$$p = \frac{\rho - 4B_c}{3}, \quad (32)$$

where, p, ρ, B_c represents pressure, energy density and Bag constant respectively.
Using equation (27) - (30) in (5) and (8) with (32) we get, p, ρ, h^2 as

$$p = 2D^2[3n-6] - D^2[2(nDt+c_1)^{-2}] - Q_1(nDt+c_1)^{-\frac{8}{n}} - \frac{4}{3}B_c, \quad (33)$$

$$\rho = 3D^2[(n-2) + (nDt+c_1)^{-2}] - Q_2(nDt+c_1)^{-\frac{8}{n}} - \frac{4}{3}B_c, \quad (34)$$

$$h^2 = D^2[2(nDt+c_1)^{-2} - (3n-6)] - Q_3(nDt+c_1)^{-\frac{8}{n}} + \frac{4}{3}B_c, \quad (35)$$

where,

$$Q_1 = 2b_1^2 + \frac{19}{3}b_2^2 + \frac{13}{3}b_1b_2 + \frac{2}{3}b_2b_3 + \frac{1}{3}b_1b_3 + \frac{1}{6}\omega k_4^2.$$

$$Q_2 = \frac{3}{2}b_1^2 + 6b_2^2 + \frac{11}{2}b_1b_2 + 3b_2b_3 + \frac{3}{2}b_1b_3 + \frac{3}{2}\omega k_4^2.$$

$$Q_3 = b_1^2 + \frac{10}{3}b_2^2 + \frac{7}{3}b_1b_2 + \frac{2}{3}b_2b_3 + \frac{1}{3}b_1b_3 + \frac{2}{3}\omega k_4^2.$$

The directional Hubble's parameters $H_i (i = 1,2,3,4)$ in *x, y, z* and *m* direction are obtained as

$$H_i = D(nDt+c_1)^{-1} + b_i(nDt+c_1)^{-\frac{4}{n}}. \quad (36)$$

Using (15) we get,

$$H = D(nDt+c_1)^{-1}. \quad (37)$$

The Expansion Scalar θ using (16) is obtained as

$$\theta = 4D(nDt+c_1)^{-1}. \quad (38)$$

The mean Anisotropy parameter A_m given by (17) is

$$A_m = \frac{(nDt+c_1)^{\frac{2n-8}{n}}}{4D^2}. \quad (39)$$

Using (14), the spatial volume is given by

$$V = a^4 = (nDt+c_1)^{\frac{1}{n}}. \quad (40)$$

On solving (18), the Shear scalar is given by

$$\sigma^2 = \frac{3}{2}A_m H^2 = \frac{3}{8}(nDt+c_1)^{-\frac{8}{n}}(b_1^2 + 2b_2^2 + b_3^2). \quad (41)$$

From (19) and (20) we get,

$$q = (n-1)\,, \tag{42}$$

From the above findings, it is observed that the volume of the universe is zero at $t = -(c_1 / nD)$ and expansion scalar tends to infinity which indicates that the universe evolved with zero volume with infinite rate of expansion. As time *t* increases, the scale factors and spatial volume increases but expansion scalar decreases. Thus, the rate of expansion of universe slows down with increasing time. Also, as *t* tends to infinity the scalar field, pressure, density, magnetic flux, mean Hubble's Parameter, Shear scalar, mean anisotropic parameter tends to 0. Hence, the model initially shows an empty universe for large time *t*. Thus, the from equation (42) it is observed that the model representing accelerating expansion of the universe for non-zero values of $n<1$ and shows decelerating nature for $n>1$.

6. Model II: when $n=0$

Using equation (21) in (22) - (24) and (26), we get the metric potentials and scalar field as follows:

$$A = a_1 c_2 e^{\left[Dt - \frac{b_1}{4Dc_2^4} e^{-4Dt}\right]}\,, \tag{43}$$

$$B = a_2 c_2 e^{\left[Dt - \frac{b_2}{4Dc_2^4} e^{-4Dt}\right]}\,, \tag{44}$$

$$C = a_3 c_2 e^{\left[Dt - \frac{b_3}{4Dc_2^4} e^{-4Dt}\right]}\,, \tag{45}$$

$$\varphi(t) = \left[\frac{k_4(n+2)}{8Dc_2^4}\right]^{\frac{2}{n+2}} e^{\frac{-8Dt}{n+2}}\,. \tag{46}$$

Equation (43) - (46) satisfy the equation of conservation of energy (11) and hence, the metric in (4) can be expressed as

$$ds^2 = -dt^2 + \left(a_1 c_2 e^{\left[Dt - \frac{b_1}{4Dc_2^4} e^{-4Dt}\right]}\right)^2 dx^2 + \left(a_2 c_2 e^{\left[Dt - \frac{b_2}{4Dc_2^4} e^{-4Dt}\right]}\right)^2 (dy^2 + dz^2) + \left(a_3 c_2 e^{\left[Dt - \frac{b_3}{4Dc_2^4} e^{-4Dt}\right]}\right)^2 dm^2\,. \tag{47}$$

Using equation (43) - (46) in (5) and (8) with (32) we get the values of $p,\ \rho,\ h^2$ as

$$p = 2D^2 + Q_1 c_2^{-8} e^{-8Dt} + \frac{2}{3} B_c\,, \tag{48}$$

$$\rho = -\frac{2}{3} D^2 + Q_2 c_2^{-8} e^{-8Dt} - \frac{8}{9} B_c\,, \tag{49}$$

$$h^2 = -8D^2 - Q_3 c_2^{-8} e^{-8Dt} + \frac{4}{3} B_c\,, \tag{50}$$

where,

$$Q_4 = \frac{b_1^2}{2} + \frac{4}{3} b_2^2 + \frac{2}{3} b_1 b_2 - \frac{1}{3} b_2 b_3 - \frac{1}{6} b_1 b_3 - \frac{5}{6} \omega k_4^2\,,$$

$$Q_5 = \frac{2}{3} b_1^2 + \frac{11}{9} b_2^2 - \frac{2}{9} b_1 b_2 - \frac{14}{9} b_2 b_3 - \frac{7}{9} b_1 b_3 + \frac{1}{18} \omega k_4^2\,,$$

$$Q_6 = b_1^2 + \frac{10}{3} b_2^2 + \frac{8}{3} b_1 b_2 + \frac{2}{3} b_2 b_3 + \frac{1}{3} b_1 b_3 - \frac{2}{3} \omega k_4^2\,.$$

The directional Hubble's parameters $H_i (i = 1,2,3,4)$ in *x, y, z* and *m* direction are obtained as

$$H_i = D + b_i (c_2^{-4} e^{-4Dt})\,. \tag{51}$$

Using (15) the mean Hubble's parameter is given by

$$H = D\,. \tag{52}$$

Using (16), the Expansion scalar θ is obtained as

$$\theta = 4D. \tag{53}$$

Using (17), the anisotropy parameter A_m is given by

$$A_m = \frac{1}{4D^2}(b_1^2 + 2b_2^2 + b_3^2)c_2^{-8}e^{-4Dt}. \tag{54}$$

Using (14), the spatial volume is given by

$$V = a^4 = (c_2 e^{Dt})^4. \tag{55}$$

Using (18), the shear scalar is given by

$$\sigma^2 = \frac{3}{2}A_m H^2 = \frac{3}{8}c_2^{-8}e^{-8Dt}(b_1^2 + 2b_2^2 + b_3^2). \tag{56}$$

Using (19) and (21), the deceleration parameter turns out to be

$$q = -1. \tag{57}$$

The spatial volume, scale factors, scalar field, pressure, density, magnetic flux, and other kinematical parameters are all constant at $t = 0$. Hence, the universe begins with a constant volume and expands exponentially. As t increases, the scale factors and spatial volume increases and scalar field, pressure, density, magnetic flux, and other kinematical parameters decrease. The expansion scalar and deceleration parameter are constant and hence the universe is expanding and accelerating for $n = 0$.

7. GRAPHICAL REPRESENTATION OF DYNAMICAL PARAMETERS FOR THE MODEL-I AND MODEL-II

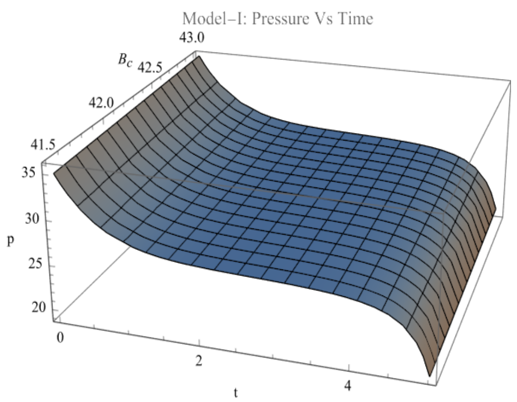

Figure 1. Pressure verses Time has been plotted by considering the values $D = 0.2, \quad n = -0.9, \; c_1 - 1, Q_1 = -10$

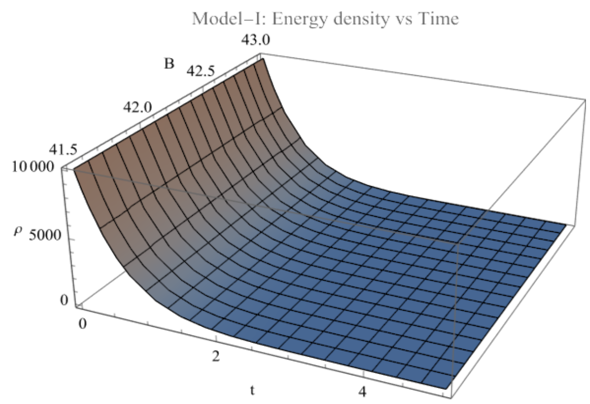

Figure 2. Energy density verses Time has been plotted by considering the values $D = 0.2, \quad n = -0.9, \quad c_1 - 1, \quad Q_2 = -10 \times 10^3$

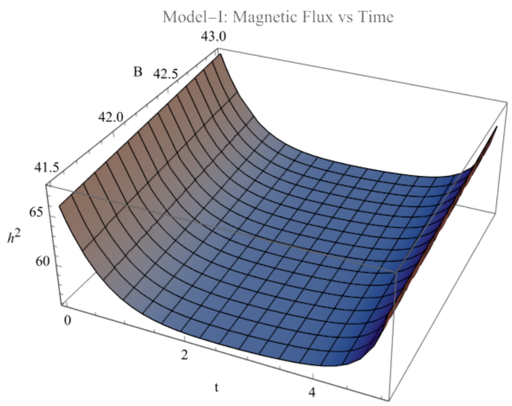

Figure 3. Magnetic flux verses Time has been plotted by considering the values $D = 0.2, n = -0.9, c_1 - 1, Q_3 = -10$

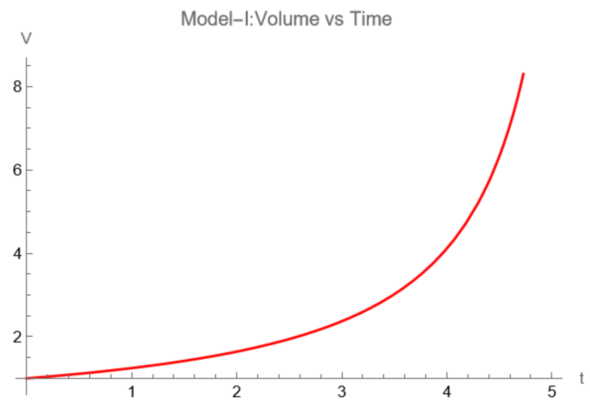

Figure 4. Volume verses time has been plotted by considering the values $D = 0.2, n = -0.9, c_1 - 1$

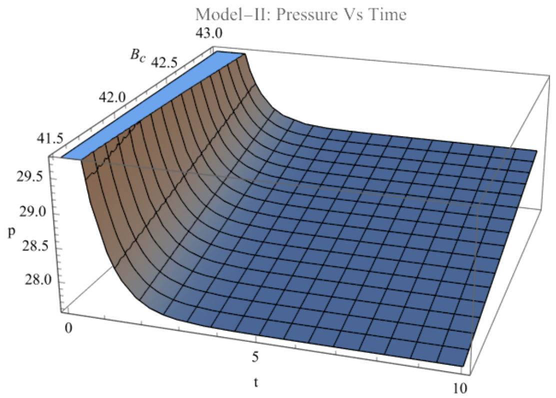

Figure 5. Pressure verses time has been plotted by considering the values $D = 0.2, n = -0.9, c_2 = -1, Q_4 = 5$

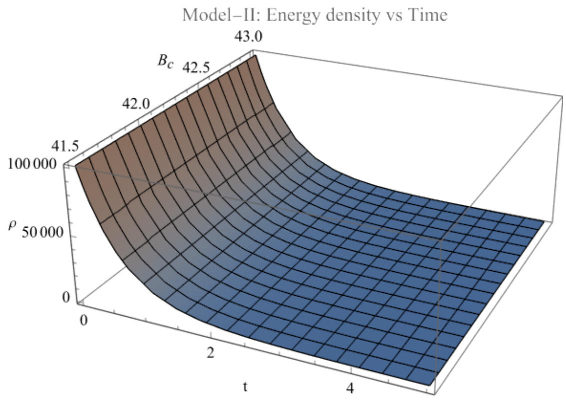

Figure 6. Energy density verses time has been plotted by considering the values $D = 0.2, n = -0.9, c_2 = -1, Q_5 = 10\times 10^4$

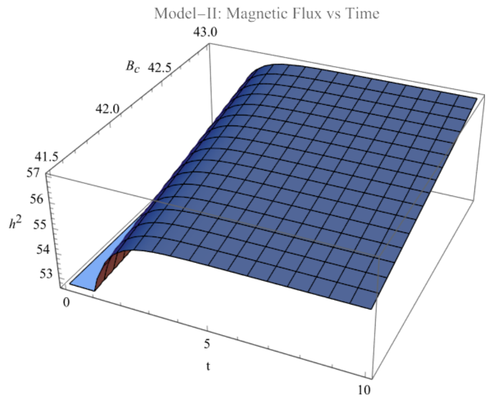

Figure 7. Magnetic flux verses time has been plotted by considering the values $D = 0.2, n = -0.9, c_2 = -1, Q_6 = -10$

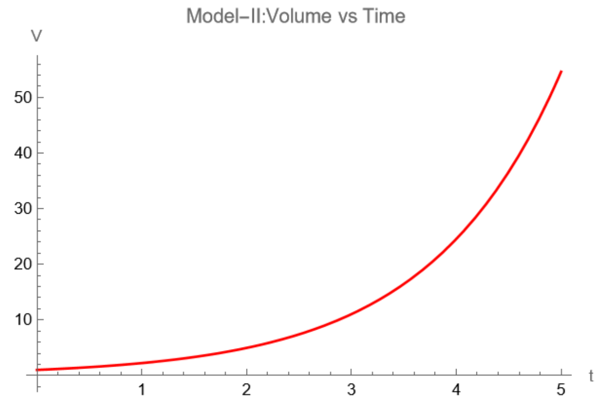

Figure 8. Volume verses time has been plotted by considering the values $D = 0.2,\ c_2 = -1,$

8. CONCLUSION

In this paper, we have investigated a five-dimensional model by considering five-dimensional Bianchi Type-I spacetime with strange quark matter in the framework of Saez and Ballester theory of gravity. By using special law of variation of Hubble's parameter, we have solved the field equations to obtain the values of metric potentials A, B and C. Here we have discussed two models, for $n \neq 0$ and for $n = 0$, by considering the equation of state for strange quark matter and obtained the physical parameters pressure p, density ρ, magnetic flux h^2, mean Hubble's parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2, spatial volume V and discussed their physical behavior in details. We have observed that for model-I ($n \neq 0$) as $t \to \infty$ the scale factors and volume of the universe became infinitely large, whereas the scalar field φ, mean anisotropy parameter, shear scalar tends to 0. For large value of time t, the model approaches to isotropy, the pressure, density, mean Hubble's parameter becomes constant. Thus, the constructed model resembles with the accelerating expansion of the universe.

In model-II ($n = 0$), we have obtained the deceleration parameter $q = -1$ which leads to $dH/dt = 0$. This gives the maximum value of the mean Hubble's parameter, which shows the fastest rate of accelerating expansion of the universe. Thus, the graphical results of the obtained models are in good agreement with the observational data.

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П'ЯТИВИМІРНА КОСМОЛОГІЧНА МОДЕЛЬ Б'ЯНЧІ ТИПУ I З ДИВНОЮ КВАРКОВОЮ МАТЕРІЄЮ В РАМКАХ ТЕОРІЇ ГРАВІТАЦІЇ САЕЗА-БАЛЛЕСТЕРА

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У цій статті ми побудували п'ятивимірну космологічну модель Б'янкі типу I з дивною кварковою матерією в контексті теорії гравітації Саес-Баллестера. Ми обговорили п'ятивимірну космологічну модель за допомогою спеціального правила варіації для параметра Хаббла у формі $H = Da^{-1}$ та рівнянні стану дивної кваркової матерії. Обговорювалися дві різні моделі для $n \neq 0$ і $n = 0$. Крім того, прискорене розширення Всесвіту обговорювалося з використанням різних фізичних параметрів разом із їх графічним зображенням.

Ключові слова: *Bianchi Type-I; дивна кваркова матерія; теорія Саеза-Баллестера*