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ANISOTROPIC BARROW HOLOGRAPHIC DARK ENERGY MODELS IN SCALAR-TENSOR THEORY OF GRAVITATION

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In this research, we have derived the solution of the field equations of the scalar-tensor theory of gravitation, proposed by Saez and Ballester(Phys. Lett. A113, 467:1986) within the frame-work of Bianchi type-III Universe. We have analyzed the interacting and non-interacting anisotropic Barrow Holographic Dark Energy (BHDE) models by assuming the time dependent deceleration parameter ($q(t)$). Further, we have discussed the several cosmological parameters such as energy densities of pressureless dark matter and BHDE, skewness, deceleration, equation of state parameters, ω_{BH}-ω'_{BH} plane and stability of the both interacting and non-interacting models. Also, we have observed that in our non-interacting and interacting models deceleration and equation of state parameters support the recent observational data.

Keywords: *Bianchi type-III Universe; Cosmology; Saez-Ballester theory*

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1. INTRODUCTION

Precision Cosmology [1] measurements have definitively shown that our Universe is experiencing an accelerated phase expansion [2, 3, 4, 5, 6, 7, 8, 9]. However, the fuel of this mechanism is not yet known, leaving room for disparate explanations. Tentative descriptions can be basically grouped into two classes: on one side, Extended Gravity Theories [10] aim at solving the puzzle by modifying the geometric part of Einstein–Hilbert action in General Relativity. On the other side, one can introduce new degrees of freedom in the matter sector, giving rise to dynamical Dark Energy models. In this context, a largely followed approach is the so called Holographic Dark Energy (HDE) model [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] which is based on the use of the holographic principle at cosmological scales.

The holographic DE model (HDE) suggests, this model is originated from holographic principle and its energy density can be expressed by $\rho_{de} = \frac{3C^2M_p^2}{L^2}$, here C^2 is a numerical constant, M_p^2 is the reduced Planck mass and L denotes the size of the current Universe such as the Hubble scale [33, 34]. In addition, the holographic DE has some problems and cannot explain the time line of a flat FRW Universe [35, 36]. One of the proposed solutions for the HDE problems is the consideration of various entropies. One of the considered entropy is Tsallis entropy which has been used in many papers [37, 38, 39, 40]. In recent years, various entropy formalism have been used to discuss the gravitational and cosmological setups. Also, some new holographic DE models are constructed such as Tsallis HDE [41], Renyi HDE (RHDE) [42] and Sharma-Mittal HDE [43]. Kaniadakis [44, 45, 46], Barrow [47, 48, 49, 50, 51, 52, 53, 54, 55] entropies, which arise from the effort to introduce non-extensive, relativistic and quantum gravity corrections in the classical Boltzmann–Gibbs statistics, respectively. While predicting a richer phenomenology comparing to the standard Cosmology, generalized HDE models suffer from the absence of an underlying Lagrangian. This somehow questions their relevance in improving our knowledge of Universe at fundamental level. Hence, with this motivation, in this research, we consider the HDE with Barrow entropy formalism i.e., Barrow HDE (BHDE).

Saridakis [56], constructed the BHDE, by using the usual HP, however applying the Barrow entropy instead of the BH entropy. Also, for the limiting case as $\Delta = 0$ the BHDE possesses standard HDE, although The BHDE, in general, is a new scenario with cosmological behavior and richer structure. While standard HDE is given by the inequality $\rho_{BH} \leq SL^{-4}$, here L denotes horizon length, and under the imposition [57] then $\rho_{BH} = C(\frac{1}{L})^{2-\Delta}$, here C is a parameter. If we take into consideration the IR cut off L as the Hubble horizon (i.e., $L = H^{-1}$), then the energy density of BHDE is obtained as

$$\rho_{BH} = CH^{2-\Delta} \tag{1}$$

Saridakis [58], using Barrow entropy presented a modified cosmological scenario besides the Bekenstein-Hawking one. For the evolution of the effective DE density parameter, the analytical expression was obtained

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and shown the DM to DE era of the Universe. Using the Barrow entropy on the horizon in place of the standard Bekenstein-Hawking one, the potency of the generalized second law of thermodynamics has also been examined [59]. Mamon et al. [60] studied interacting BHDE model and also the validity of the generalized second law by assuming dynamical apparent horizon as the thermodynamic boundary. Anagnostopoulos et.al. [61] have studied observational constraints on BHDE. Pradhan et al. [62] have analysed FRW cosmological models with BHDE in the back-ground of BD theory of gravitation.

Srivastava and Sharma [63] have studied BHDE with Hubble horizon as IR cut-off. Adhikary et al. [64] constructed a BHDE in the case of non-flat Universe in particular, considering closed and open spatial geometry and observed that the scenario can describe the thermal history of the Universe, with the sequence of matter and dark energy epochs. Sarkar and Chattopadhyay [65] have analysed BHDE reconstruct $f(R)$ gravity as the form of back-ground evolution and point out the equation of state can have a transition from quintessence to phantom with the possibility of little Rip singularity. Xu and Lu [66] have investigated the non-interacting HDE with the Hubble radius as IR cut-off cannot explain the current accelerated expansion of Universe in the BD theory. Aditya et al. [67] have discussed anisotropic new HDE model in the frame-work of SB scalar tensor theory of gravitation. Sadri [68] has studied observational constraints on interacting THDE model. Zadeh et al. [69] have investigated the cosmic evolution of THDE in Bianchi type-I model filled with DM and THDE interacting with each other throughout a sign-changeable interacting with different IR cut offs. Chandra et al. [70] have discussed THDE in Bianchi type-I by using hybrid expansion law with K-essence. Sobhanbabu and Santhi [71] have investigated Kantowski–Sachs THDE model with sign-changeable interaction with the back-ground of scalar tensor theory of graviatation. Priyanka et al. [72] have discussed generalized BHDE with Granda–Oliver (GO) cut-off. Aditya et al. [73] have studied observational constraint on interacting THDE in logarithmic BD theory. Ghaffari et al. [74] have investigated interacting and non-interacting THDE models by considering the Hubble horizon as the IR cutoff within BD scalar theory. Jawad et al. [75] have studied cosmological implications of THDE in BD scalar theory. Santhi and Sobhanbabu [76] have analyzed anisotropic interacting and non-interacting THDE models in the frame-work of SB theory of gravitation.

Abdulla et al.[77] have investigated Dynamics of an Interacting BHDE Model and its Thermodynamic Implications. Recently, Ghaffari et.al. [78] have analysed BHDE in the frame-work of BD cosmology. Koussor et al. [79] have studied anisotropic BHDE model in symmetric teleparallel gravity. Very recently, Sobhanbabu et al. [80] have investigated Kantowski–Sachs interacting and non-interacting BHDE models in SB theory of gravitation.

Hence, motivated with the above discussion and observations, in the current work, we have studied anisotropic BHDE models in the back-ground of SB theory. The plan of the work as follows: In Section-II, we have derived field equations of SB theory and its cosmological solution with the help of Bianchi type-III Universe in the presence of two minimally interacting fields: DM and BHDE components. In Section-III, we have constructed non-interacting and interacting BHDE models along with their physical discussions. Finally, we summarize our results in conclusion Section-IV.

2. METRIC AND COSMOLOGICAL SOLUTION OF SB FIELD EQUATIONS

In the current work, we consider the anisotropic Bianchi type-III in the form

$$ds^2 = dt^2 - X^2(t)dx^2 - Y^2(t)e^{-2x}dy^2 - Z^2(t)dz^2, \tag{2}$$

where $X(t)$, $Y(t)$ and $Z(t)$ are the metric potentials, as functions of cosmic time (t). The following are the some physical parameters which are useful to find the solution of the SB field equations for the Universe BT-III. The average scale factor and volume are defined as

$$a(t) = (XYZ)^{\frac{1}{3}}, \quad V(t) = a(t)^3 = XYZ \tag{3}$$

The average Hubble parameter $H(t)$ is defined as

$$H(t) = \frac{\dot{a}}{a} = \frac{1}{3}\Big(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big) \tag{4}$$

The deceleration parameter q is given by

$$q(t) = -\frac{a\ddot{a}}{\dot{a}^2} \tag{5}$$

The SB field equations for matter and BHDE distribution are (with $8\pi G = C = 1$) [83] given by

$$G_{\mu\nu} - w\phi^n\Big(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda}\Big) \;=\; -(T_{\mu\nu} + \bar{T}_{\mu\nu}), \tag{6}$$

and the scalar field ϕ satisfies the following equation

$$2\phi^n\phi^{,\mu}_{,\mu} + n\phi^{n-1}\phi_{,\lambda}\phi^{,\lambda} \;=\; 0, \tag{7}$$

where $G_{\mu\nu}$ represents the Einstein tensor and $T_{\mu\nu}$ & $\bar{T}_{\mu\nu}$ are energy momentum tensors for pressure-less dark matter and BHDE respectively. For physical interpretation, the energy momentum tensors for matter and BHDE can be written as

$$T_{\mu\nu} = diag[1, 0, 0, 0]\rho_M, \tag{8}$$

and

$$\bar{T}_{\mu\nu} = diag[1, -\omega_{BH}, -(\omega_{BH} + \alpha_{BH}), -(\omega_{BH} + \alpha_{BH})]\rho_{BH}, \tag{9}$$

where ρ_{BH}, ρ_M are energy densities of BHDE and matter and p_{BH} is the pressure of BHDE. $\omega_{BH} = \frac{p_{BH}}{\rho_{BH}}$ is an equation of state (EoS) parameter and the skewness parameters α_{BH} are the deviations from y and z axes. So, the field equations for the discussed metric can be written as

$$\frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{Y}\dot{Z}}{YZ} - \frac{w}{2}\phi^n\dot{\phi}^2 \;=\; -\omega_{BH}\ \rho_{BH}, \tag{10}$$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Z}}{Z} + \frac{\dot{X}\dot{Z}}{XZ} - \frac{w}{2}\phi^n\dot{\phi}^2 \;=\; -(\omega_{BH} + \alpha_{BH})\ \rho_{BH}, \tag{11}$$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\dot{X}\dot{Y}}{XY} - \frac{1}{X^2} - \frac{w}{2}\phi^n\dot{\phi}^2 \;=\; -(\omega_{BH} + \alpha_{BH})\ \rho_{BH}, \tag{12}$$

$$\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{X}\dot{Z}}{XZ} - \frac{1}{X^2} + \frac{w}{2}\phi^n\dot{\phi}^2 \;=\; \rho_M + \rho_{BH}, \tag{13}$$

$$\frac{\dot{X}}{Y} - \frac{\dot{Z}}{Z} = 0, \tag{14}$$

$$\ddot{\phi} + \Big(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\dot{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0, \tag{15}$$

and the continuity equation of the matter and BHDE as

$$\dot{\rho_M} + \Big(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\rho_M + \dot{\rho_{BH}} + \Big(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\Big(1 + \omega_{BH}\Big)\rho_{BH} + \Big(\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\alpha_{BH}\rho_{BH} = 0. \tag{16}$$

On integration, Eq.(14) yields $Y = c_1 Z$, where $c_1$is an integration constant. It can be taken as unity, without loss of generality, so that we have

$$Y = Z. \tag{17}$$

In view of Eq.(17), the field equations (10)-(15) reduce to

$$\frac{\ddot{X}}{X} + \frac{\ddot{Z}}{Z} + \frac{\dot{X}\dot{Z}}{XZ} - \frac{w}{2}\phi^n\dot{\phi}^2 \;=\; -\omega_{BH}\ \rho_{BH}, \tag{18}$$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Z}}{Z} + \frac{\dot{X}\dot{Z}}{XZ} - \frac{w}{2}\phi^n\dot{\phi}^2 \;=\; -(\omega_{BH} + \alpha_{BH})\ \rho_{BH}, \tag{19}$$

$$2\frac{\ddot{X}}{X} + \frac{\dot{X}^2}{X^2} - \frac{1}{X^2} - \frac{w}{2}\phi^n\dot{\phi}^2 \;=\; -(\omega_{BH} + \alpha_{BH})\ \rho_{BH}, \tag{20}$$

$$\frac{\dot{X}^2}{X^2} + 2\frac{\dot{X}\dot{Z}}{XZ} - \frac{1}{X^2} + \frac{w}{2}\phi^n\dot{\phi}^2 \;=\; \rho_M + \rho_{BH}, \tag{21}$$

$$\ddot{\phi} + \Big(\frac{2\dot{X}}{X} + \frac{\dot{Y}}{Y}\Big)\dot{\phi} + \frac{n\dot{\phi}^2}{2\phi} = 0, \tag{22}$$

and

$$\dot{\rho_M} + \Big(2\frac{\dot{X}}{X} + \frac{\dot{Z}}{Z}\Big)\Big((1 + \omega_{BH}) + \rho_{BH}\Big)\rho_M + \dot{\rho_{BH}} + \Big(\frac{\dot{X}}{X} + \frac{\dot{Z}}{Z}\Big)\alpha_{BH}\rho_{BH} = 0. \tag{23}$$

The above SB field equations (18)-(22) constitute a system of five non-linear equations with seven unknowns: X, Z, ϕ, ω_{BH}, ρ_{BH}, ρ_M, and α_{BH}. In order to get a deterministic solution, we take the following plausible physical conditions: Here, we consider the fact that expansion scalar is directly proportional to shear scalar which leads [81] to a relation between the metric potentials:

$$X = Z^m, \tag{24}$$

$m \neq 1$ is a positive constant. In this paper, we assume a well-motivated ansatz considered by Abdusattat et al. [82] which puts a constraint on the function form of the deceleration parameter $q(t)$ as

$$q = -\frac{\beta}{t^2} + \gamma - 1, \tag{25}$$

here $\beta > 0$ and $\gamma > 1$.

Hence, from the Eqs. (5), (24), and (25), we find the metric potentials as

$$X = Y = \left[t^2 + \frac{\beta}{\gamma}\right]^{\frac{3m}{2\gamma(2m+1)}}, \quad Z = \left[t^2 + \frac{\beta}{\gamma}\right]^{\frac{3}{2\gamma(2m+1)}}. \tag{26}$$

Now through a proper choice of coordinates and constants the metric (2) with the help of Eq.(26) can be written as

$$ds^2 = dt^2 - \left[t^2 + \frac{\beta}{\gamma}\right]^{\frac{3m}{\beta(2m+2)}} dx^2 - \left[t^2 + \frac{\beta}{\gamma}\right]^{\frac{m}{\gamma(2m+2)}} e^{2x} dy^2 - \left[t^2 + \frac{\beta}{\gamma}\right]^{\frac{3}{\gamma(2m+2)}} e^{-2x} dz^2. \tag{27}$$

Thus, Eq. (27) describes BT-III BHDE model in SB scalar tensor theory of gravitation.

The average scale parameter and volume of the model respectivly, given by

$$a = \left[t^2 + \frac{\beta}{\gamma}\right]^{\frac{1}{2\gamma}} \ \ \& \ \ V = \left[t^2 + \frac{\beta}{\gamma}\right]^{\frac{3}{2\gamma}} \tag{28}$$

The Hubble parameter(H) of the model can be obtained as

$$H = \frac{t}{\gamma(t^2 + \frac{\beta}{\gamma})} \tag{29}$$

The energy density of the BHDE model is given by

$$\rho_{BH} = CH^{4-2\Delta}, \tag{30}$$

where C is a paremeter.

Now with help of Eqs.(29) and (30), the energy density of BHDE is obtained as

$$\rho_{BH} = C\left[\frac{t}{\gamma(t^2 + \frac{\beta}{\gamma})}\right]^{2-\Delta} \tag{31}$$

Using Eqs. (18), and (20) we get the skewness parameters as

$$\alpha_{BH} = \left[\frac{9mH^2}{(2m+1)^2} + \left(\frac{t}{\gamma H}\right)^{-\frac{3m}{2\gamma(2m+1)}} + 3\frac{\Big(m+1)\Big)\dot{H}}{(2m+1)} - 9\frac{m^2H^2}{\Big(2m+1)\Big)^2}\right]\left[CH^{2-\Delta}\right] \tag{32}$$

where $H = \frac{t}{\gamma(t^2 + \frac{\beta}{\gamma})}$

Now using Eqs. (22) & (26), we have the scalar field ϕ is

$$\phi^{\frac{n+2}{2}} = \frac{n+2}{2}\int \phi_0\left(t^2 + \frac{\beta}{\gamma}\right)^{\frac{-3}{2\gamma}} dt + c_2, \tag{33}$$

where ϕ_0 and c_2 are integration constants.

Using Eqs. (21), (31), and (33), we get

$$\rho_M = 9\frac{m(m+2)H^2}{(2m+1)^2} - \left(\frac{\gamma H}{t}\right)^{\frac{3m}{\gamma(2m+1)}} + \frac{w}{2}\phi_0^2\left(\frac{\gamma H}{t}\right)^{\frac{3}{\gamma}} - CH^{2-\Delta} \tag{34}$$

The behavior of skewness parameter (α_{BH}) versus cosmic time (t) is plotted in Fig. 1 for the various values of γ. It is observed that the skewness parameter is positive throughout the evolution and it is initially bouncing behavior later decreases as Universe evolves. Figs. 2 & 3 describe the behavior of energy density of dark matter and BHDE against cosmic time (t) for the different values of γ. It is observed that ρ_M is positive throughout the evolution. From Fig. 3, it is observed that at initial epoch ρ_{BH} increases, reaches a maximum value and decreases at late times.

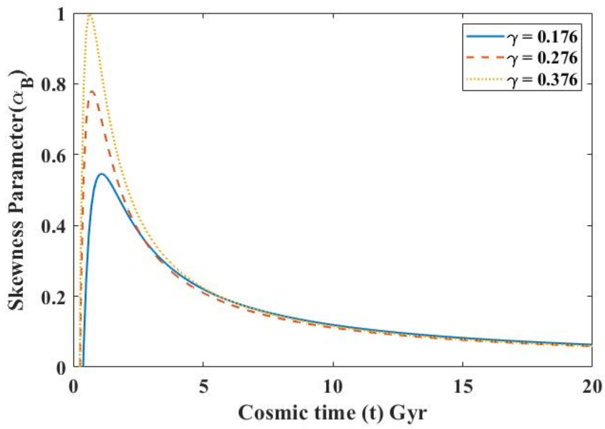

Figure 1. Plot of skewness parameter (α_B) versus cosmic time (t) for $n = 0.198$, $\xi_0 = 0.03$, $\xi_1 = 0.02$, $\xi_2 = 0.01$, $\alpha = 0.2904$, and $k = 0.593, 0.596$, and 0.599.

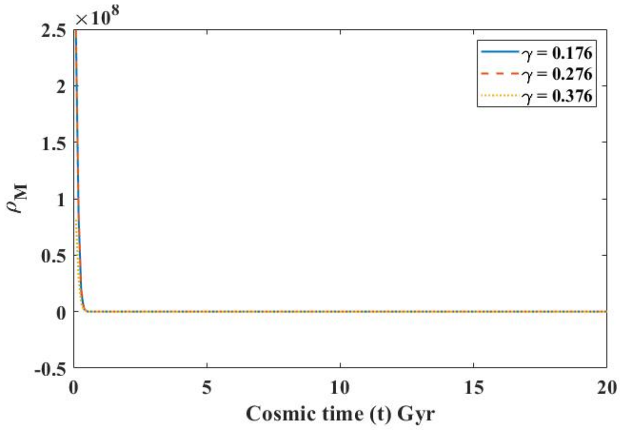

Figure 2. Plot of energy density (ρ_M) of matter versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$.

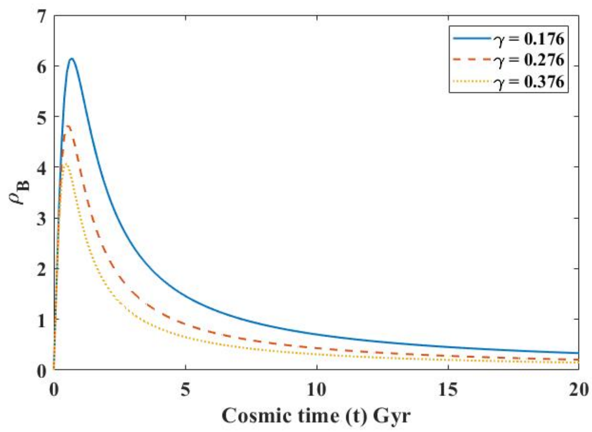

Figure 3. Plot of energy density (ρ_{BH}) of BHDE versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$.

Non-interacting model

Here, we consider the non-interacting dark matter and BHDE. Hence, both of these conserve separately, so that we have from Eq. (16),

$$\dot{\rho_M} + \Big(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\rho_M = 0, \tag{35}$$

$$\dot{\rho}_{BH} + \Big(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\Big(1 + \omega_{BH}\Big)\rho_{BH} + \alpha_B\Big(\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\rho_{BH} = 0 \tag{36}$$

Using Eqs. (26), (31) and (33), we get the EoS parameter (ω_{BH}) of BHDE model

$$\omega_{BH} = -1 - \left\{\left[\frac{m+1}{2m+1}\left[\frac{9mH^2}{(2m+1)^2} + \Big(\frac{t}{\gamma H}\Big)^{-\frac{3m}{2\gamma(2m+1)}} + 3\frac{\Big(m+1)\Big)\dot{H}}{(2m+1)} - 9\frac{m^2H^2}{\Big(2m+1)\Big)^2}\right]\left[CH^{2-\Delta}\right] - \frac{(2-\Delta)\dot{H}}{3H^2}\right]\right\}, \tag{37}$$

where $\dot{H} = \frac{\beta - \gamma t^2}{\gamma\left(\beta + \gamma t^2\right)^2}$

Fig. 4 represents the behavior of equation of state (EoS) parameter in terms of cosmic time (t) for the non-interacting model in different values of γ. It can be seen that EoS parameter completely varies in aggressive phantom region and finally tends to -1.

Taking the derivative of Eq. (42) with respect to lna, we get

$$\omega'_{BH} = -\frac{\alpha\dot{}_{BH}}{(2m+1)H} - \frac{(2-\Delta)(H\ddot{H} - 2\dot{H}^2)}{3H^2}\Big\}, \tag{38}$$

where $\ddot{H} = \frac{2t\left(\gamma t^2 - 3\beta\right)}{\left(\beta + \gamma t^2\right)^3}$ and

$$\dot{\alpha}_{BH} = \left\{\left[\frac{18mH\dot{H}}{(2m+1)^2} - \frac{3m\big(H - t\dot{H}\big)}{2\gamma(2m+1)H^2}\Big(\beta\Big)^{\frac{3m}{2\gamma(2m+1)}}\Big(\frac{t}{H}\Big)^{-\frac{3m}{2\gamma(2m+1)}-1} + \frac{3m\ddot{H}}{2m+1} - \frac{18m^2H\dot{H}}{\big(2m+1\big)^2}\right] \times \left[C\Big[\frac{t}{\gamma(t^2 + \frac{\beta}{\gamma})}\Big]^{\Delta-2}\right] - \left[\frac{9mH^2}{(2m+1)^2} + \Big(\frac{t}{\gamma H}\Big)^{\frac{-3m}{2\gamma H(2m+1)}} + \frac{3\dot{H}(m+1)}{(2m+1)} - \frac{9m^2H^2}{\big(2m+1\big)^2}\right] \times \left[(2-\Delta)H^{-1}\dot{H}\right]\right\}, \tag{39}$$

Fig. 5, represents the $\omega_{BH} - \omega'_{BH}$ plane for the non-interacting BHDE model for the different values of γ. It

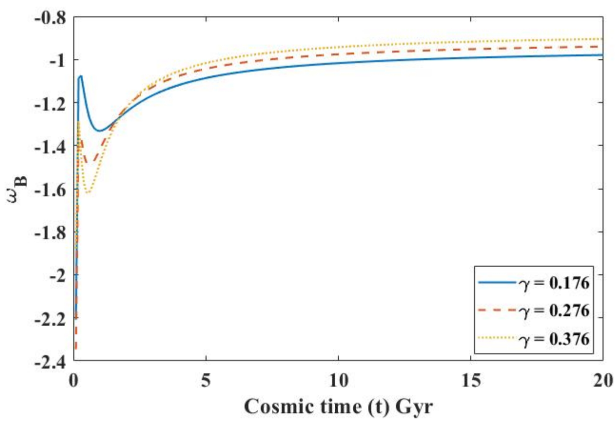

Figure 4. Plot of EoS parameter (ω_{BH}) versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$.

is observed that the $\omega_{BH} - \omega'_{Bh}$ plane corresponds to thawing region. The plot of squared speed of sound (v_s^2) versus cosmic time (t) is shown in Fig. 6. we can observe that squared speed of sound ($v_s^2 < 0$) represents our non-interacting BHDE model is unstable.

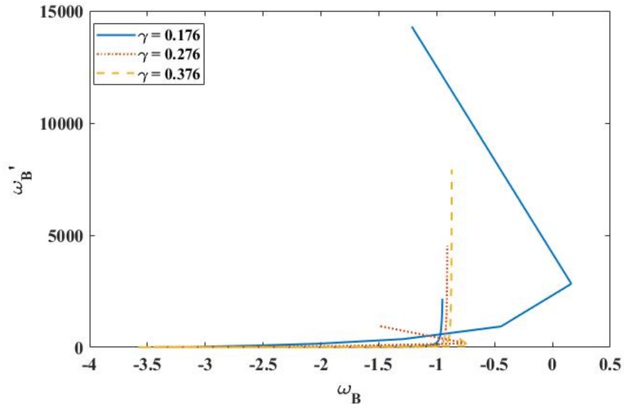

Figure 5. Plot of ω_{BH} versus $\omega_{BH}^{'}$ for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$.

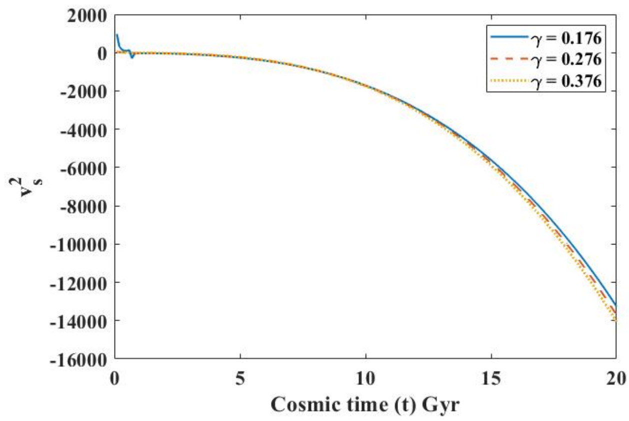

Figure 6. Plot of squared speed of sound (v_s^2) versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$.

INTERACTING MODEL

In this case, we consider that both dark matter and BHDE are interacting with each other. Hence, we can write the energy conservation equation for matter and BHDE as

$$\dot{\rho_M} + \Big(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\rho_M = Q, \tag{40}$$

$$\dot{\rho}_{BH} + \Big(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\Big(1 + \omega_{BH}\Big)\rho_{BH} + \alpha_B\Big(\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\rho_{BH} = -Q \tag{41}$$

where the quantity Q denotes interaction between DE components. From the Eqs. (40) and (41), we can say that the total energy is conserved. Since there is no natural information from fundamental physics on the interaction term Q, one can only study it to a phenomenological level. Various forms of interaction term extensively considered in literature include $Q = 3d^2H\rho_M$, $Q = 3d^2H\rho_{BH}$ and $Q = 3d^2H(\rho_M + \rho_{BH})$. Where, d is a coupling constant and positive c means that DE decays into DM, while negative d means DM decays into DE. Here we consider $Q = 3d^2H\rho_{BH}$ as the interaction term with the coupling parameter d^2.

From Eqs. (24), (29), and (44) we find the EoS parameter ω_{BH} as

$$\left.\begin{aligned}\omega_{BH} = -1 - d^2 - \Bigg[\frac{m+1}{2m+1}\Bigg[\frac{9mH^2}{(2m+1)^2} + \Big(\frac{t}{\gamma H}\Big)^{-\frac{3m}{2\gamma(2m+1)}} + 3\frac{\Big(m+1\Big)\dot{H}}{(2m+1)} - 9\frac{m^2H^2}{\Big(2m+1\Big)^2}\Bigg]\Bigg[CH^{2-\Delta}\Bigg] \\ - \frac{(2-\Delta)\dot{H}}{3H^2}\Bigg]\end{aligned}\right\}, \tag{42}$$

Taking the derivative of Eq. (40) with respect to lna, we get

$$\omega_{BH}^{'} = -\frac{\alpha \dot{B}H}{(2m+1)H} - \frac{(2-\Delta)(H\ddot{H} - 2\dot{H}^2)}{3H^2}\Big\}, \tag{43}$$

where $\ddot{H} = \frac{2t\left(\gamma t^2 - 3\beta\right)}{\left(\beta + \gamma t^2\right)^3}$ and

$$\begin{aligned}\dot{\alpha}_{BH} = &\left[\frac{18mH\dot{H}}{(2m+1)^2} - \frac{3m\left(H - t\dot{H}\right)}{2\gamma(2m+1)H^2}\left(\beta\right)^{\frac{3m}{2\gamma(2m+1)}}\left(\frac{t}{H}\right)^{-\frac{3m}{2\gamma(2m+1)}-1} + \frac{3m\ddot{H}}{2m+1} - \frac{18m^2H\dot{H}}{(2m+1)^2}\right] \\ &\times\left[C\left[\frac{t}{\gamma(t^2 + \frac{\beta}{\gamma})}\right]^{\Delta-2}\right] - \left[\frac{9mH^2}{(2m+1)^2} + \left(\frac{t}{\gamma H}\right)^{\frac{-3m}{2\gamma H(2m+1)}} + \frac{3\dot{H}(m+1)}{(2m+1)} - \frac{9m^2H^2}{(2m+1)^2}\right] \\ &\times\left[(2-\Delta)H^{-1}\dot{H}\right]\end{aligned}\Bigg\}, \tag{44}$$

The plot of EoS parameter (ω_{BH}) against cosmic time (t) for various values of γ and d^2 are depicted in the

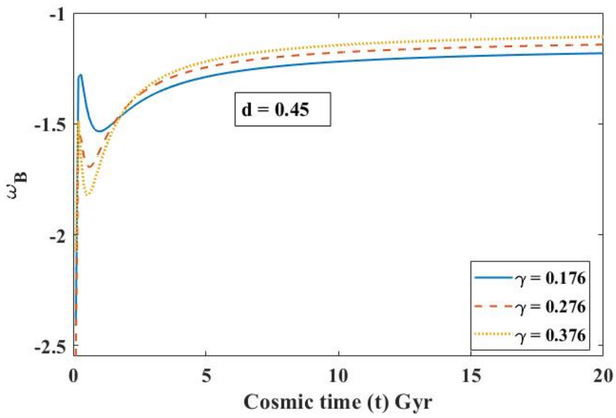

Figure 7. Plot of EoS parameter of BHDE versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$ $d = 0.45$.

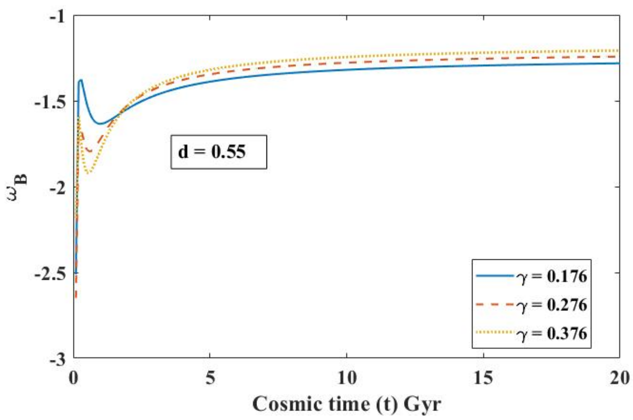

Figure 8. Plot of EoS parameter of BHDE versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$ $d = 0.55$.

Fig. 7, 8 & 9 for the interacting model. It can be observed that the EoS parameter completely varies in aggressive phantom region for all values of coupling parameter d^2 and γ. The $\omega_{BH} - \omega_{BH}^{'}$ plane is used to represents the dynamical property of dark models, where $\omega_{BH}^{'}$ is the evolutionary form of ω_{BH}, here prime indicates derivative with respect to lna. In Figs. 10, 11, & 12, we plot the behavior of $\omega_{BH} - \omega_{BH}^{'}$ plane for three different values

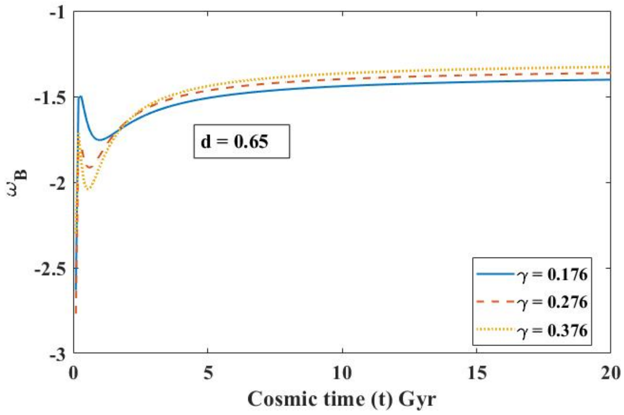

Figure 9. Plot of Plot of EoS parameter of BHDE versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$ $d = 0.65$.

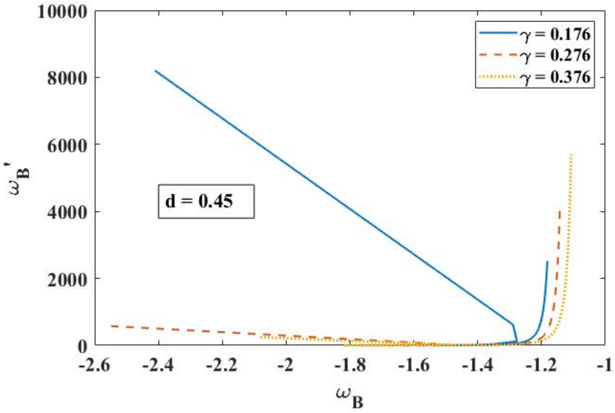

Figure 10. Plot of ω_{BH} versus $\omega^{'}_{BH}$ for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$ $d = 0.45$.

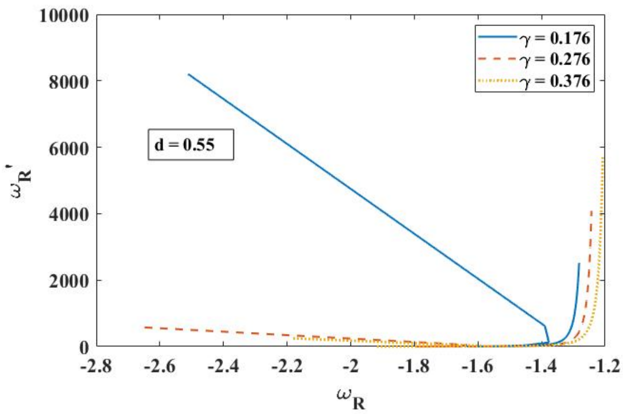

Figure 11. Plot of ω_{BH} versus $\omega^{'}_{BH}$ for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$ $d = 0.45$.

of d^2 and γ. It can be seen that the $\omega_{BH} - \omega^{'}_{BH}$ plane, for interacting BHDE model corresponds to the thawing region ($\omega^{'}_{BH} > 0$ and $\omega_{BH} < 0$) for all the three values of coupling parameter d^2 and γ. Figs. 13, 14, & 15 elaborates the plot of squared speed of sound (v_s^2) versus cosmic time t. The trajectories represents the negative behavior throughout evolution of the Universe which represents the interacting model is unstable for different values of d^2 and γ.

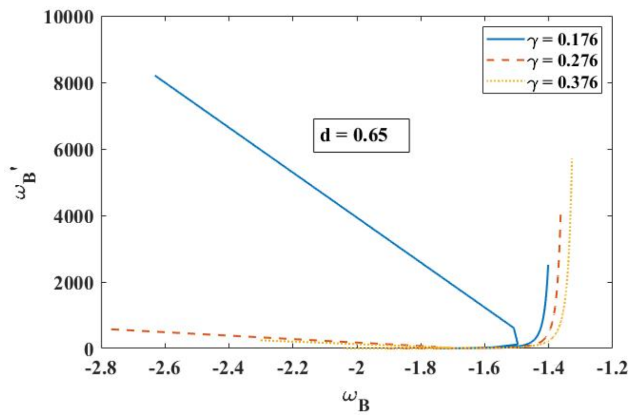

Figure 12. Plot of ω_{BH} versus $\omega_{BH}^{'}$ for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$ $d = 0.65$.

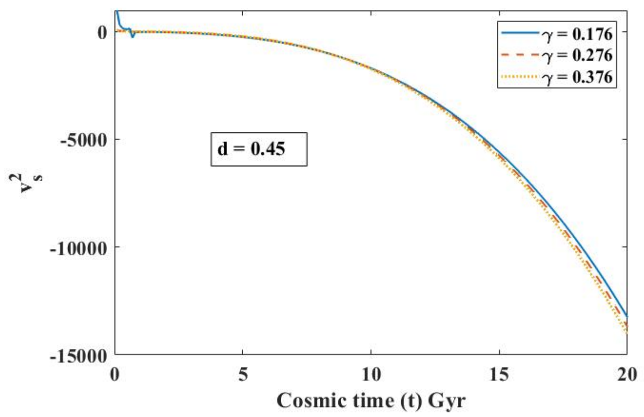

Figure 13. Plot of squared speed of sound v_s^2 versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$ $d = 0.45$.

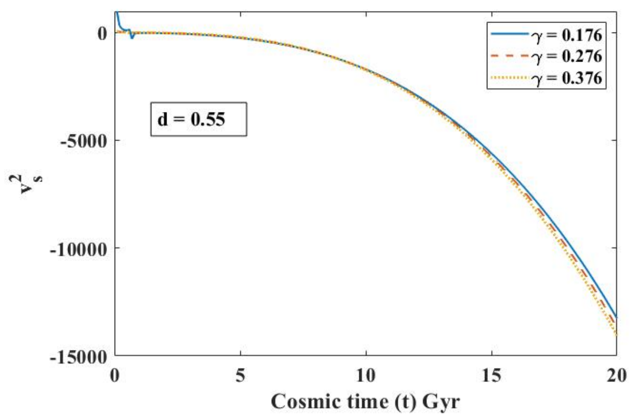

Figure 14. Plot of squared speed of sound v_s^2 versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$, and $d = 0.55$.

The nature of expansion of the model can be explained using the cosmological parameter called as deceleration parameter (DP). The DP for our both models (non-interacting and interacting) is same and given by

$$q = -\frac{\alpha}{t^2} + \beta - 1 \tag{45}$$

Fig. 16 depicts the behavior of DP versus cosmic time t for the different values of γ. We can observed that

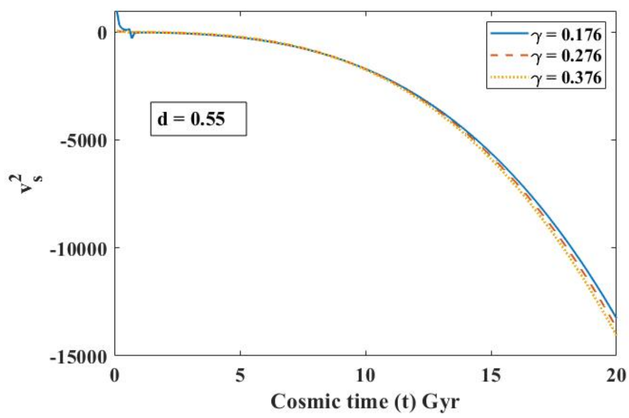

Figure 15. Plot of squared speed of sound v_s^2 versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, $w = 1000$ $d = 0.65$.

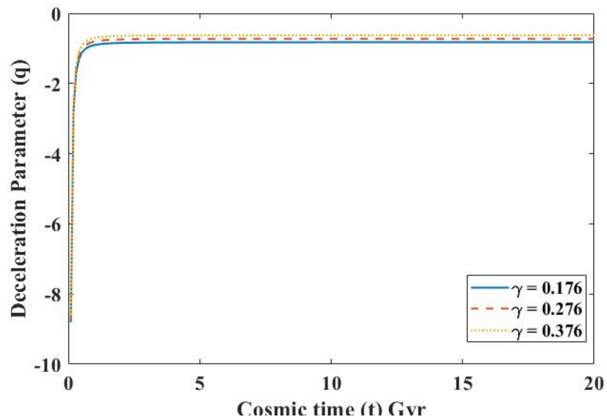

Figure 16. Plot of deceleration parameter (q) versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$, and $w = 1000$.

our models (Interacting and non-interacting) exhibits negative behavior throughout the evolution and finally it tends to -1, which represents our models accelerating behavior. Also, we can seen that the models exhibit accelerated expansion at initial epoch and finally approaches to exponential expansion of the Universe.

In recent years there are many number of DE models have proposed to explain the accelerated expansion of the Universe. The two new parameters formulated by Sahni et al. [84] named as statefinder pair (r, s) by using the deceleration and Hubble parameters defined as follows:

$$r = \frac{\dddot{a}}{aH^3} = \frac{2\gamma(1-4\gamma)t + (1+2\gamma)(t^2 + \frac{\beta}{\gamma})\gamma t}{t^2} \tag{46}$$

$$s = \frac{r-1}{3(q-\frac{1}{2})} = \frac{4\gamma t(1-4\gamma) + 2\gamma(1+2\gamma)(t^2 + \frac{\beta}{\gamma})t - t^2}{3\big((2\gamma-3)t^2 - 2\beta\big)} \tag{47}$$

In Fig.17, we have plotted the trajectories of $r-s$ plane for the three values of γ. It is observe that $r-s$ plane for the three values $\gamma = 0.176$, 0.276 and 0.376 meets the Λ model. We also, observe that the $r-s$ plane belongs to the Chaplygin gas model ($s < 0$ and $r > 1$) for $\gamma = 0.176$ and 0.276. For $\gamma = 0.376$ the $r-s$ plane corresponding to the dark energy models such as phantom ($s > 0$) and quintessence ($r < 1$).

The Om diagnostic parameter tool has been proposed by Sahni et al. [85] as a complementary to the statefinder parameter, which helps to distinguish the present matter density contrast Om in different models more effectively. This is also a geometrical diagnostic that explicitly depends on redshift (z) and the Hubble parameter (H). It is defined as follows:

$$Om(x) = \frac{h(x)^2 - 1}{x^3 - 1}, \tag{48}$$

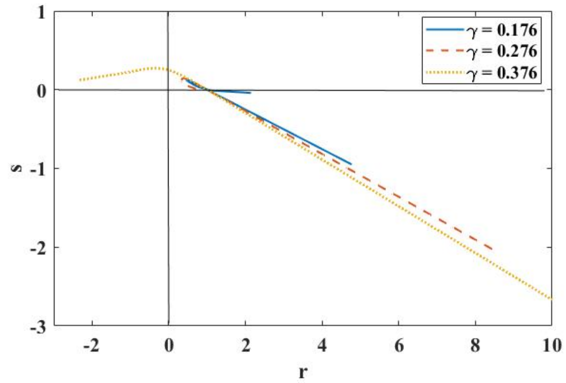

Figure 17. Plot of r versus s for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$.

where $h(x) = \frac{H(x)}{H_0}$, $x = (1 + z)$ and H_0 is the present value of the Hubble parameter.

$$Om(x) = \frac{x^{4\gamma}\left(x^{-2\gamma} - \frac{\beta}{\gamma}\right)}{\gamma^2\left(x^3 - 1\right)} \tag{49}$$

Fig. 18, we have plotted the evolution of Om diagnostic parameter versus cosmic time (t). It can be observed

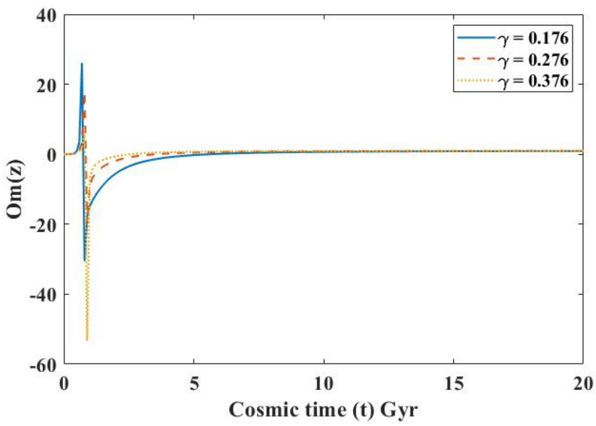

Figure 18. Plot of $Om(z)$ versus cosmic time (t) for $m = 0.925$, $\beta = 0.798$, $\gamma = 0.376$, $\gamma = 0.476$, $\gamma = 0.576$.

that the slope of Om diagnostic parameter is negativ, which represents the quintessence behavior of the Universe. This behavior is consistent with recent observational data.

3. CONCLUSIONS

In this paper, we have studied the accelerated expansion by assuming the BHDE in BT-III Universe within the frame-work of SB scalar–tensor theory of gravity. Using the relation between the metric potentials and the variable DP $q = -\frac{\beta}{t^2} + \gamma - 1$, we have obtained the solution of SB field equations with this solution, we have studied various cosmological parameters to analyze the viability of the non-interacting and non-interacting models and our conclusions are the following:

- The behavior of the skewness parameter is positive throughout the evolution and it is initially bouncing behavior later decreases as Universe evolves.The energy density of DM is observed that ρ_M is positive throughout the evolution. The energy density of BHDE is observed that at initial epoch ρ_{BH} increases, reaches a maximum value and decreases at late times.

- The trajectory of EoS parameter completely varies in aggressive phantom region and finally tends to -1. The $\omega_{BH} - \omega'_{BH}$ plane observed that the $\omega_{BH} - \omega'_{Bh}$ plane corresponds to thawing region. The squared speed of sound ($v_s^2 < 0$) represents unstable for the non-interacting cosmological models for various values of γ.

- The plot of EoS parameter can be observed that the EoS parameter completely varies in aggressive phantom region for all values of coupling parameter d^2 and γ for interacting model. The $\omega_{BH} - \omega'_{BH}$ plane is used to represents the dynamical property of dark models, The behavior of $\omega_{BH} - \omega'_{BH}$ plane, for interacting BHDE model corresponds to the thawing region ($\omega'_{BH} > 0$ and $\omega_{BH} < 0$) for all the three values of coupling parameter d^2 and γ.

- The trajectories of v_s^2 represents the negative behavior throughout evolution of the Universe which represents the interacting model is unstable for different values of d^2 and γ. Also, it is worthwhile to mention here that the present values of EoS parameter of our modelS are in agreement with the modern Plank observational data given by Aghanim et al. [86]. It gives the constraints on EoS parameter of BHDE as follows:
$$\omega_{BH} = -1.56^{+0.60}_{-0.48}(Planck + TT + lowE)$$
$$\omega_{BH} = -1.58^{+0.52}_{-0.41}(Planck + TT, EE + lowE)$$
$$\omega_{BH} = -1.57^{+0.50}_{-0.40}(Planck + TT, TE, EE + lowE + lensing)$$
The EoS parameter ω_{BH} of our model lie within the above observational limits which shows the consistency of our results with the above cosmological observational data.

- The DP for the both Interacting and non-interacting models exhibits negative behavior throughout the evolution and finally it tends to -1, which represents our models accelerating behavior of the Universe. The deceleration parameter q of our model is consistent with the observational data [87] given as
$$q = -0.6401 \pm 0.187(BAO + Masers + TDSL + Panthelon + H_0)$$
$$q = -0.930 \pm 0.218(BAO + Masers + TDSL + Panthelon + H_z).$$

- The trajectories of $r - s$ plane for the different values of γ meets the Λ model. We also, observe that the $r - s$ plane belongs to the Chaplygin gas model ($s < 0$ and $r > 1$) for $\gamma = 0.176$ and 0.276. For $\gamma = 0.376$ the $r - s$ plane corresponding to the dark energy models such as phantom ($s > 0$) and quintessence ($r < 1$).

- The Om diagnostic parameter is negative, which represents the quintessence behavior of the Universe for the different values of γ. This behavior is consistent with recent observational data.

Data Availability Statement: This manuscript has no associated data.

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АНІЗОТРОПНІ ГОЛОГРАФІЧНІ МОДЕЛІ ТЕМНОЇ ЕНЕРГІЇ БАРРОУ В СКАЛЯРНО-ТЕНЗОРНОЇ ТЕОРІЇ ГРАВІТАЦІЇ

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У цьому дослідженні ми отримали розв'язок польових рівнянь скалярно-тензорної теорії гравітації, запропонованої Саезом і Баллестером (Phys. Lett. A113, 467:1986) у рамках типу Б'янкі III Всесвіт. Ми проаналізували взаємодіючі та невзаємодіючі анізотропні моделі голографічної темної енергії Барроу (BHDE), припустивши залежний від часу параметр уповільнення ($q(t)$). Крім того, ми обговорили кілька космологічних параметрів, таких як щільність енергії темної матерії без тиску та BHDE, асиметрія, уповільнення, рівняння параметрів стану, ω_{BH}-ω'_{BH} площина та стабільність як взаємодіючі, так і невзаємодіючі моделі. Крім того, ми помітили, що в наших невзаємодіючих і взаємодіючих моделях уповільнення та рівняння параметрів стану підтверджують останні дані спостережень.
Ключові слова: *Всесвіт типу Бьянкі-III; космологія; теорія Саеза-Баллестера*