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# MHD FLOW PAST A STRETCHING POROUS SURFACE UNDER THE ACTION OF INTERNAL HEAT SOURCE, MASS TRANSFER, VISCOUS AND JOULES DISSIPATION

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The paper investigates two-dimensional, steady, nonlinear laminar boundary layer heat and mass transfer MHD flow past a stretching porous surface embedded in a porous medium under the action of internal heat generation with the consideration of viscous and joules heat dissipation in the presence of a transverse magnetic field. The two-dimensional governing equations are solved by using MATLAB built in bvp4c solver technique for different values of physical parameters. The numerical values of various flow parameters such as velocity, temperature, concentration are calculated numerically and analysed graphically for various values of the non-dimensional physical parameters of the problem followed by conclusions. The study concludes opposite behaviour of transverse and longitudinal velocity under the action of suction velocity in addition to the effects of heat source on fluid velocities, temperature and concentration.
**Keywords**: *MHD-flow; Porous-surface; Internal heat-source; Mass-transfer; Viscous and Joules Dissipation*
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## INTRODUCTION

Since last few years the two-dimensional boundary layer flow, heat and mass transfer over a porous stretching surface under various geometrical situations with heat generation has been a great attention because of its practical applications in the fields of science-technology and industry; for instance, in petroleum industry, polymer technology, power generation, crude oil processing, aerodynamic heating and many others. In the field of nuclear technology MHD convection flow is employed to study the magnetic behaviour of plasmas in fusion reactors, liquid metal cooling of nuclear reactors, electro-magnetic casting etc. It may be pointed out that many metallurgical processes involve the colling of continuous strips by drawing them through a quiescent fluid where in the process of drawing these strips are many times strips are stretched. On the other hand, by playing a role like an energy source the viscous dissipation changes the temperature distributions that in turn lead to affect heat transfer rates as well. The merit of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Moreover, in MHD flows, the joules dissipation acts as a volumetric heat source. Due to abundant applications the heat transfer over a porous surface is a much practical interest of research today. The heat generated from the combustion process is used to convert fluid into high-pressure steam that can drive turbines connected to a generator converting thermal energy into electrical energy. To be more specific instances, heat treated materials travelling between a feed roll and wind-up roll or materials manufactured by extrusion, glass-fiber and paper production, colling of metallic sheets or electronic chips, crystal growing are few to be added. In these cases, the final product of desired characteristics depends on the rate of colling in the process of stretching. MHD flow of variable physical parameters has been investigated to a large extent by many researchers because of its numerous applications in the field of science, technology, industry, in case of extraction of geo-thermal energy, and in many such situations.

In 1977, T.C. Chaim [1] have discussed about magnetohydrodynamic heat transfer over a non-isothermal stretching sheet. In 1979, A. Chakrabarti et al. [2] studied about the hydromagnetic flow and heat transfer over a stretching sheet. About heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation is discussed by K. Vajravelu et al. [3]. The problem of viscous dissipation, joule heating and heat source/sink on non-Darcy MHD natural convection flow over an isoflux permeable sphere in a porous medium is numerically analyzed by K.A. Yih [4]. B. Ganga et al. [5] investigated about the non-linear hydrodynamic flow and heat transfer due to a stretching porous surface with prescribed heat flux and viscous dissipation effects. Viscous dissipation effects on MHD natural convection flow over a sphere in the presence of heat generation are discussed by Md. M. Alam et al. [6]. Saxena and Dubey [7] studied about unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion. Chen [8] discussed about Combined heat and mass transfer in MHD free convection from a vertical surface with ohmic heating and viscous dissipation. Effects of viscous and joules dissipation on MHD flow, heat and mass transfer past a stretching porous surface embedded in a porous medium is analyzed by Devi et al. [9]. Abel et al. [10] discussed about viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and ohmic dissipations. Sajid et al. [11] studied about non-similar analytic solution for MHD flow and heat transfer in a third-order fluid over a stretching sheet. Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity: a numerical reinvestigation was investigated by Pantokratoras [12]. Sonth et al. [13]

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discussed about heat and mass transfer in a visco-elastic fluid over an accelerating surface with heat source/sink and viscous dissipation. Eldahab et al. [14] discussed about viscous dissipation and joule heating effects on MHD free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ios-slips currents. Flow and heat transfer due to a stretching porous surface in presence of transverse magnetic field was discussed by Tak et al. [15].

In recent times, Goswami et al. [16] discussed about the Unsteady MHD free convection flow between two heated vertical parallel plates in the presence of a uniform magnetic field. Khan et al. [17] studied on magnetohydrodynamics Prandtl fluid flow in the presence of stratification and heat generation. Induced magnetic field effect on MHD free convection flow in nonconducting and conducting vertical microchannel was discussed by Goud et al. [18]. Waqas et al. [19] studied about thermo-solutal robin conditions significance in thermally radiative nanofluid under stratification and magneto hydrodynamics. Effect of chemical reaction and joule heating on MHD generalized Couette flow between two parallel vertical porous plates with induced magnetic field and Newtonian heating/cooling was discussed by J. Ming' ang' a [20]. Influence of MHD mixed convection flow for Maxwell nanofluid through a vertical cone with porous material in the existence of variable heat conductivity and diffusion was studied by Kodi et al. [21]. Kodi et al. [22] studied about radiation absorption on MHD free conduction flow through porous medium over an unbounded vertical plate with heat source.

Motivating with the above works, we have tried to investigate the effect of viscous and joules dissipation on a fully developed MHD flow where heat and mass transfer past a stretching porous surface embedded in a porous medium under the influence of heat generation due to an internal heat source in presence of transverse magnetic field. The two-dimensional governing equations are solved by using MATLAB built in bvp4c solver technique for different values of physical quantities influencing the physics. The numerical values of various flow parameters such as velocity, temperature, concentration are calculated numerically and analysed graphically for various values of the non-dimensional physical parameters of the problem. The study concludes opposite behaviour of transverse and longitudinal velocity under the action of suction velocity in addition to the effects of heat source on fluid velocities, temperature and concentration.

## MATHEMATICAL ANALYSIS

We have considered two-dimensional, steady, nonlinear MHD laminar boundary layer flow with heat and mass transfer of a incompressible, viscous and electrically conducting fluid over a porous surface embedded in a porous medium under the action of internal heat source in the presence of a transverse magnetic field applied parallel to y-axis of strength $B_0$ . The situation is similar to the case of a polymer sheet emerging out of a slit and subsequently being stretched at x = 0, y = 0 that often happens in the process of extrusion of polymer, rayon etc. The speed of the flow at a point on the sheet is assumed to be proportional to some positive power of its distance (x) from the slit while the boundary layer approximations are applicable. It is considered that the flow has viscous and Joule heat dissipation whereas the induced magnetic field, the electrical field due to polarization of charges and the external electrical field are negligible. Considering these conditions, the governing boundary layer equations of equation of continuity, momentum, energy, diffusion with viscous and joules dissipation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}-\frac{\upsilon}{K_p}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{C_p}\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\sigma B_0^2}{\rho C_p}\right)u^2+\frac{S^*(T-T_\infty)}{\rho C_p} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions are

$$\left.\begin{array}{l} u=ax^m,\quad v=v_w(x),\ T=T_w(x)=T_\infty+T_0x^n \\ C=C_w(x)=C_\infty+C_0x^n \quad \text{at } y=0 \\ u=0,\ T=T_\infty,\ C=C_\infty \quad at\ y\to\infty \end{array}\right\} \tag{5}$$

Here, u and v are the velocity components along x and y respectively. $K_p$ is the permeability of the medium, σ is the electrical conductivity of the fluid, ρ is the density of the fluid, $B_0$ is the applied magnetic field, T is the temperature of the fluid, $S^*$ is the coefficient of heat source, K is thermal conductivity, $T_w$ is wall temperature, $T_\infty$ is the temperature far away from the surface, $C_p$ is specific heat at constant pressure, C is the species concentration of the fluid, $C_w$ is species concentration of the fluid near the wall, $C_\infty$ is species concentration of the fluid away from the wall, D is the diffusivity coefficient; $a, T_0, C_0$ are dimensional constants, m is index of power-law velocity and n is index of power-law variation of wall temperature which is constant.

Following [15], we introduce the stream function

$$\left.\begin{aligned} &\Psi(x,y) = [\tfrac{2\upsilon x U(x)}{1+m}]^{\frac{1}{2}} F(\eta),\\ &\text{where, } \eta = [\tfrac{(1+m)U(x)}{2\upsilon x}]^{\frac{1}{2}}\, y\,,\\ &\upsilon_w(x) = -\lambda\sqrt{\tfrac{\upsilon a(m+1)}{2}}\, x^{\frac{(m-1)}{2}},\ \text{where } \lambda > 0 \text{ for suction at the stretching plate.}\\ &n = 2m, \end{aligned}\right\} \quad (6)$$

the velocity components are

$$u = \frac{\partial\Psi}{\partial y}, \quad v = -\frac{\partial\Psi}{\partial x}. \quad (7)$$

The equation of continuity is satisfied by the above conditions and introduce the non-dimensional form of temperature and the concentration is given bellow as

$$\theta = \frac{T-T_\infty}{T_w-T_\infty}, h = \frac{C-C_\infty}{C_w-C_\infty}. \quad (8)$$

For the feasibility of the analytic solutions of the problem, we take the value $\beta = 1$, i.e. m=1 that makes velocity of the stretching plate as ax, a being a positive constant; this means the plate stretches with a velocity varying linearly with distance. Using equations 6,7 & 8, equations 2, 3 & 4 are reducing to a nonlinear system of equations given as

$$F''' + FF'' - F'^2 - (M^2 + R_1^{-1})F' = 0 \quad (9)$$

$$\theta'' + PrF\theta' - 2PrF'\theta = -EcPr[F''^2 + M^2F'^2] + SPrReB \quad (10)$$

$$h'' + ScFh' - 2SchF' = 0 \quad (11)$$

The corresponding Boundary conditions are now

$$\left.\begin{aligned} &F(0) = \lambda,\ F'(0) = 1,\ F'(\infty) = 0,\\ &\theta(0) = 1,\ \theta(\infty) = 0\,,\\ &h(0) = 1,\ h(\infty) = 0\,. \end{aligned}\right\} \quad (12)$$

where, $V_0$ is a scale of suction velocity, B is a dimensionless velocity ratio.

## METHOD OF SOLUTION

The couple of non-linear differential equations (9), (10) & (11) with respect to the boundary conditions (12) are solved by using MATLAB inbuilt function 'bvp4c' for different values of physical parameters. In this method we convert the non-linear differential equations into a set of first order differential equations for which we have considered the followings.

$$F = y(1), F' = y(1)' = y(2), F'' = y(2)' = y(3),$$

$$\theta = y(4), \theta' = y(5),$$

$$h = y(6), h' = y(7).$$

The transformed first order differential equations are

$$y(3)' = -Y(1) * y(3) + y(2) * y(2) + (M^2 + R_1^{-1}) * y(2)$$

$$y(5)' = -Pr * y(1) * y(5) * +2 * Pr * y(2) * y(4) - Ec * Pr * [y(3)^2 + M^2 * (y(2))^2] + S * Pr * Re * B,$$

$$y(7)' = -Sc * y(1) * y(7) + 2 * Sc * y(6) * y(2)\,,$$

The boundary conditions are reduced to

$$y0(1) - \lambda; y0(2) - 1; y0(4) - 1; y0(6) - 1; y1(2) - 0; y1(4) - 0; y1(6) - 0.$$

## RESULTS AND DISCUSSIONS

The numerical calculations have been carried out for different values of heat source parameter (S), magnetic parameter ($M^2$), suction velocity parameter (λ), Eckert number (Ec), Prandtl number (Pr) and Schmidth number (Sc). We assume $B = 1$; so that the problem has a physical point of view.

In Fig. 1&2, the dimensionless transverse and longitudinal velocity profiles for different values of magnetic parameter with constant values of heat source parameter, suction parameter, Eckert number, Prandtl number, Schmidth number and permeability parameter are shown. It is clear from Fig.1 that the transverse velocity rises sharply near the plate for a smaller distance from the plate ( η ≅ 1.5) thereafter, becomes uniform away from the plate for all values of magnetic field. The trend is quite opposite in case of longitudinal velocity Fig.2 where it decreases sharply initially then becomes almost uniform or unchanged away from the plate. However, both transverse and longitudinal velocity decreases for increases of magnetic field parameter $M^2$, this is because of the fact that the magnetic field exerts a restraining force on the fluid flow.

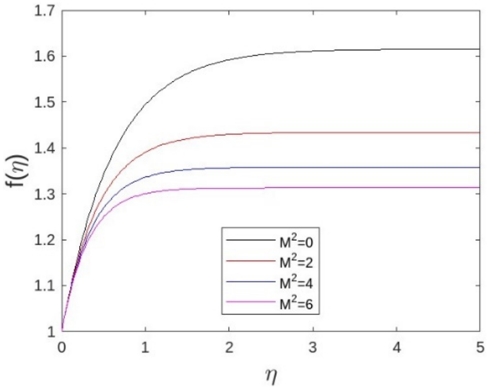


**Figure 1.** Non-dimensional transverse velocity Profiles for different values of $M^2$, at $\lambda = 2, R_1 = 100, Pr = 0.71, Ec = 0.2, S = 2.0$

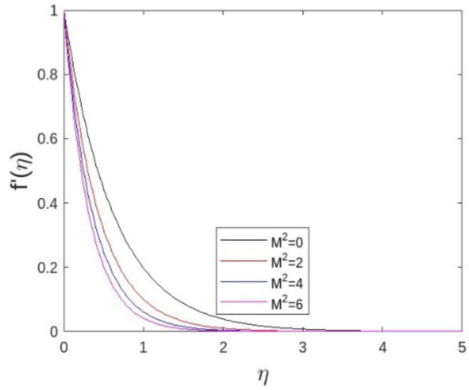


**Figure 2**. Non-dimensional longitudinal velocity Profiles for different values of $M^2$, at $\lambda = 2, R_1 = 100, Pr = 0.71, Ec = 0.2, S = 2.0$

The effect of suction parameter over the non-dimensional transverse and longitudinal velocity profiles are shown in the Figs. 3&4 for different values of suction velocity parameter (λ). It is seen that near the plate (η ≅ 0.0 to 1.0) the transverse velocity rises sharply, thereafter, almost uniform away from the plate (η ≥ 1.0). Longitudinal velocity has exponential decrease near the plate (η ≅ 0.0 to 2.0) and thereafter gradually becomes ineffective. These nature of variation of both transverse and longitudinal velocity are same for all the values of suction velocity parameter (λ). For higher value of λ. Magnitude of transverse velocity is higher while opposite for longitudinal velocity.

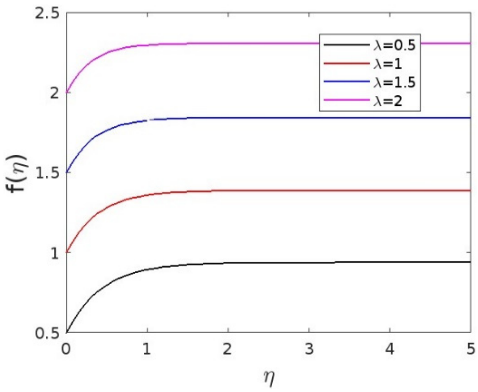


**Figure 3**. Non-dimensional transverse velocity profile for different values of λ, at $M^2 = 3, R_1 = 100, Pr = 0.71, Ec = 0.2, S = 2$ .

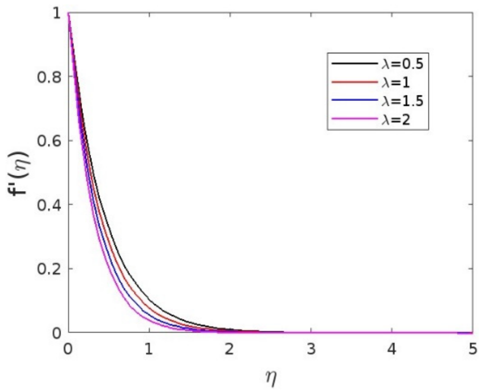


**Figure 4.** Non-dimensional longitudinal velocity profile for different values of λ, for $M^2 = 3.0, R_1 = 100, Pr = 0.71, Ec = 0.2, S = 2.0$

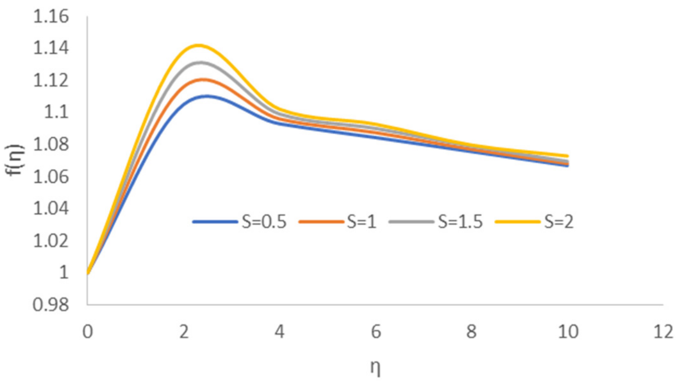


**Figure 5**. Non-dimensional transverse velocity profile for different values of S, for $M^2 = 3, R_1 = 100, Pr = 0.71, Ec = 0.2, \lambda = 1, Re = 4.$

In Fig.5, transverse velocity profile is shown for various values of heat source parameter S ≅ 0.5 to 2.0. Transverse velocity increases sharply within the range (η ≅ 0.0 to 2.0), thereafter, decreases gradually for η ≥ 2.3. The magnitude of transverse velocity is higher for higher values of heat source parameter (S).

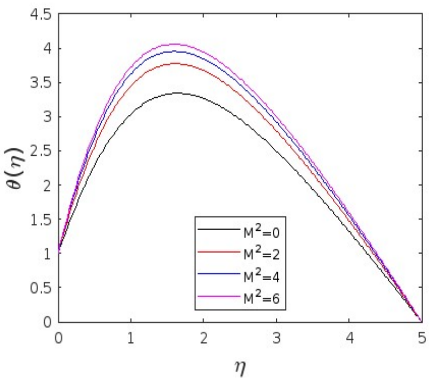


**Figure 6**, Temperature profile for different values of $M^2$, at $\lambda = 1, R_1 = 100, Pr = 0.71, Ec = 0.2, S = 2$,Re=4.0

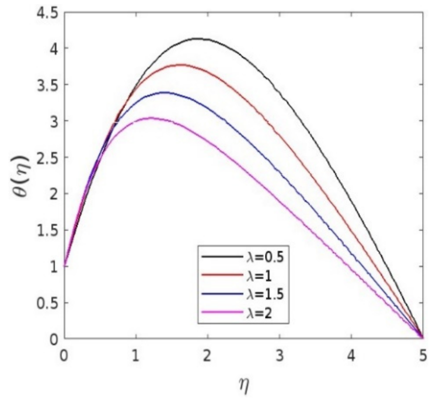


**Figure 7**, Temperature profile for different λ, for $M^2 = 2, R_1 = 100, Ec = 0.2, \lambda = 1, Re = 4.0$

The effects of magnetic field parameter and suction velocity parameter over the temperature are shown in the Fig. 6 &7 respectively. In fig.6, temperature sharply to a certain point ( η ≅ 1.5), thereafter, decreases gradually away from the plate. This nature of variation is same for all the values of magnetic field parameter M. temperature has the peak value at (η ≅ 1.5). In fig.7, the rise of suction velocity (λ) has very less impact on temperature near the plate (η ≅ 1.0) within which it increases sharply, thereafter, deceases gradually away from the plate η ≥ 1.0. An increase in suction velocity parameter λ causes decrease in temperature distribution.

In Fig. 8, the effect of Pr on temperature profile is shown; temperature experienced some increases subject to rise of Pr. An increase in Ec small enhancement in the profile of temperature as observed in Fig. 9, this may be due to the frictional heating the heat energy is stored in the liquid. The effect of Ec is distinct near the plate η ≤ 2.0.

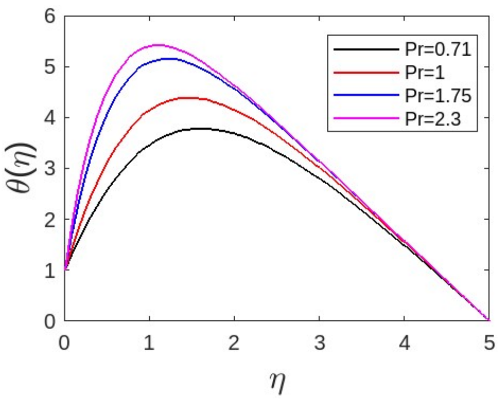


**Figure 8**, Temperature profile for different values of Pr, at $M^2 = 2, R_1 = 100, Ec = 0.2, \lambda = 1, Re = 4.0$

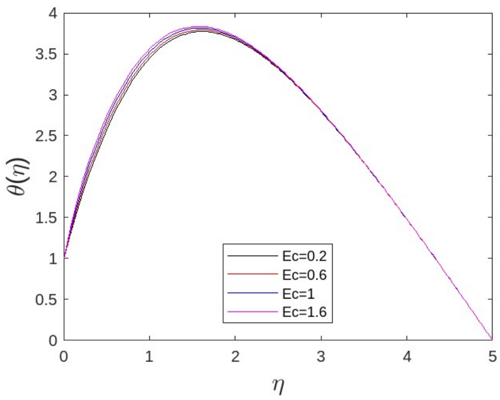


**Figure 9**, Temperature profile for different Ec, at $M^2 = 2, \quad R_1 = 100, \lambda = 1, Re = 4, Pr = 0.71.$

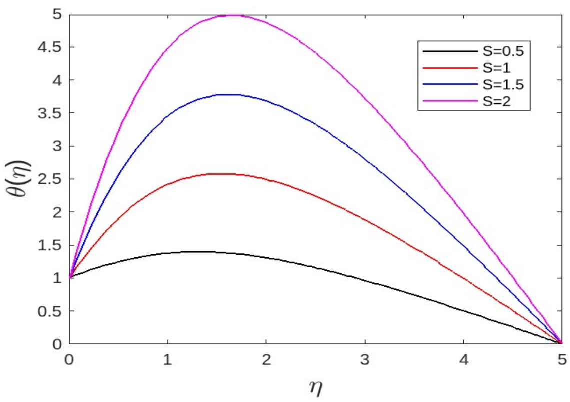


**Figure 10**. Temperature profile for different values of S, for $M^2 = 2, R_1 = 100, Pr = 0.71, Ec = 0.2, \lambda = 1, Re = 4.$

In Fig. 10, The temperature profile is shown for various values of heat source parameter S, it is seen that the temperature has clear increase as heat source parameter enhances. In Fig. 11-14, Fluid concentration h (η) is shown for values of physical parameters M, λ Sc & S. Fluid concentration h (η) decreases exponentially from the plate away from it. Higher the magnetic field, h (η) is higher. This is in contrary to the variation of h (η) with suction velocity λ. Fluid concentration decreases with the increases of λ, Fig. 12. When the Schmidt number Sc is increased, h (η) decreases, Fig. 13. Distribution of fluid concentration for different values of heat source parameter S is shown in fig.14; although in the plot h (η) is seen to be unchanged with the variation of S, but when we observe their numerical values, then it may be concluded the h (η) is higher for higher values of S, e.g. at η=4; for S=1.0 , h (η=4) ≅ 0.91693 while at S=2, h (η=4) ≅ 0.91776; thus fluid concentration increases with the rise of heat source parameter.

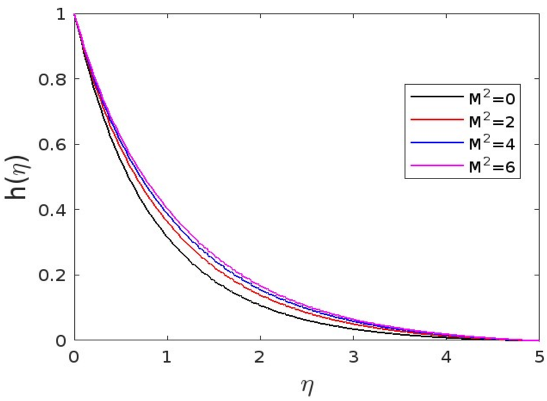


**Figure 11**, Dimensionless concentration distribution for different $M^2$ at $R_1 = 100, Sc = 0.62, \lambda = 1$.

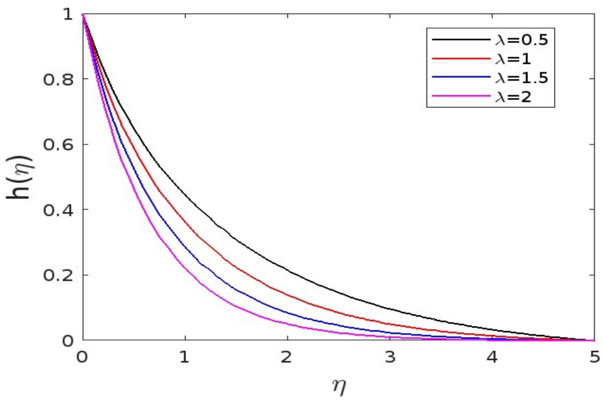


**Figure 12**, Dimensionless concentration distribution for different λ at for $M^2 = 2, R_1 = 100, Sc = 0.62$

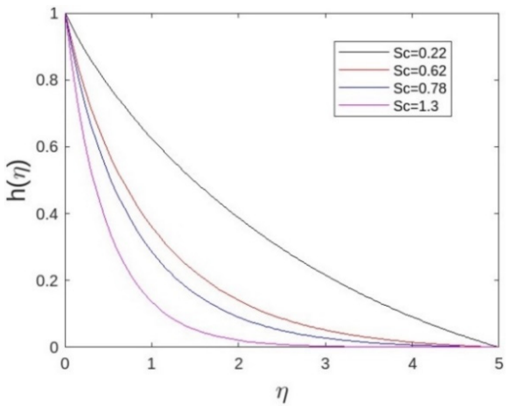


**Figure 13**, Dimensionless concentration distribution for different values of Sc, at $M^2 = 2, R_1 = 100, \lambda = 1.0$

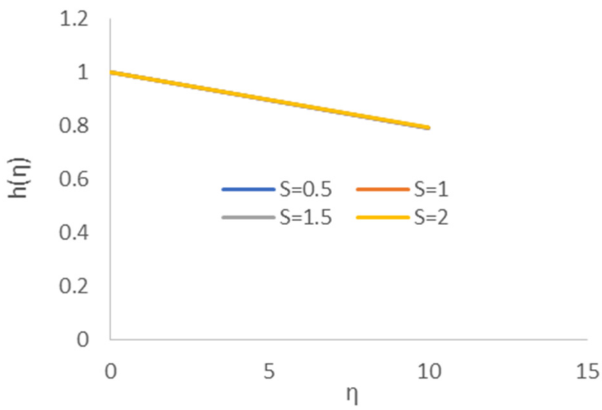


**Figure 14**, Dimensionless concentration distribution for different values of Sc, at $M^2 = 2, R_1 = 100, \lambda = 1, Sc = 0.62$.

## CONCLUSIONS

From the above results and discussions following conclusions can be made in brief.

- With the rise of suction velocity, transverse velocity increases whereas longitudinal velocity, temperature and concentration decreases.
- With the rise of magnetic field both transverse and longitudinal velocity decreases, whereas fluid temperature and fluid concentration decreases.
- The effect of heat source is that it increases that the fluid velocity transverse and longitudinal, temperature and concentration.
- Temperature distribution has increase with the rise of Prandtl number and also Eckert number
- Rise of Schmidth number reduces the fluid concentration

**Appendices**

$Pr = \frac{\mu C_p}{K}$ is Prandtl number

$R_1 = \frac{K_p a}{\upsilon}$ is permeability parameter

$M^2 = \upsilon \frac{2\sigma B_0^2}{\rho a(1+m)}$ is magnetic parameter

$\beta = \frac{2m}{m+1}$ , is stretching parameter

$Re = \frac{V_0 L}{\upsilon}$, is Reynold number

$Sc = \frac{\upsilon}{D}$ is Schmidt number

$E_c = \frac{a^2}{C_p T_0}$ is Eckert number

$\upsilon = \frac{\mu}{\rho}$ is kinematic viscosity

$S = \frac{S^* \upsilon}{\rho C_p V_0^2}$, is the heat source parameter

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## МГД ПОТІК ПОВЗ ПОРИСТОЇ ПОВЕРХНІ, ЩО РОЗТЯГУЄТЬСЯ, ПІД ДІЄЮ ВНУТРІШНЬОГО ДЖЕРЕЛА ТЕПЛА, МАСОПЕРЕНОСУ, В'ЯЗКОЇ ТА ДЖОУЛЕВОЇ ДИСИПАЦІЇ

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У статті досліджено двовимірний, стаціонарний, нелінійний ламінарний МГД-теплообмін прикордонного шару, повз розтягнуту пористу поверхню, вбудовану в пористе середовище, під дією внутрішнього теплоутворення з урахуванням в'язкої та джоулевої тепловиділення за наявності поперечне магнітне поле. Двовимірні керівні рівняння розв'язуються за допомогою MATLAB, вбудованого в розв'язувач bvp4c, для різних значень фізичних параметрів. Числові значення різних параметрів потоку, таких як швидкість, температура, концентрація, обчислюються чисельно та аналізуються графічно для різних значень безрозмірних фізичних параметрів задачі з подальшими висновками. Дослідження робить висновок про протилежну поведінку поперечної та поздовжньої швидкості під дією швидкості всмоктування на додаток до впливу джерела тепла на швидкість рідини, температуру та концентрацію.
**Ключові слова:** *МГД-потік; пориста поверхня; внутрішнє джерело тепла; масообмін; в'язка і джоулева дисипація*