

## MODELLING AND SIMULATING THE HEAT TRANSFERENCE IN CASSON EMHD FLUID MOTION EXACERBATED BY A FLAT PLATE WITH RADIANT HEAT AND OHMIC HEATING

 Bamdeb Dey<sup>a</sup>,  Dovine Dukru<sup>a\*</sup>,  Tusar Kanti Das<sup>b</sup>,  Jintu Mani Nath<sup>c</sup>

<sup>a</sup>Department of Mathematics, Assam Don Bosco University, Guwahati-781017, Assam, India

<sup>b</sup>Department of Mathematics, Dudhnoi College, Dudhnoi-783124, Assam, India

<sup>c</sup>Department of Mathematics, Mangaldai College, Mangaldai-784125, Assam, India

\*Corresponding author email: [dovinedukru1@gmail.com](mailto:dovinedukru1@gmail.com)

Received March 17, 2024; revised April 18, 2024; accepted April 30, 2024

The current study presents the results of a numerical investigation of thermal radiation's consequences, ohmic heating, and electromagnetic hydrodynamic drag on the Casson fluid flow across a flat surface. By incorporating suitable similarity parameters, the equations that regulate the system are converted into non-linear ordinary differential equations. The MATLAB Bvp4c algorithm is used for computing nonlinear ODEs numerically. To optimize the industrial and ecological processing, it is crucial to study the flow of Casson fluids (including drilling muds, fossilised coatings, different sedimentation, and specific lubricating petroleum products, polyethylene dissolves, and a range of colloids) in the presence of heat transmission. Graphics and tables have been employed to present computational findings for various spans of the tangible variables that dictate the velocity and temperature distributions. The fluid rate decreases when the magnetic and Casson parameters rise, whereas fluid velocity increases as the local electric parameters grow. This exemplifies the intricate relationship between electromagnetic radiation and fluid mechanics. Growing Eckert number, thermal radiation, specific heat, and Biot number boost temperature profiles, whereas growing Casson parameter and local electric parameters diminish them, showing diverse impacts on heat transmission phenomena. Additionally, this inquiry pertains to the coefficient of skin friction and Nusselt values were covered. New experimental studies will benefit from this theoretical work, nevertheless.

**Keywords:** Heat transfer; Casson fluid; EMHD; Thermal Radiation; Ohmic heating

**PACS:** 44.05.+e, 44.20.+b, 44.40.+a, 47.50.+d, 47.50.Cd

### INTRODUCTION

Research on fluids is based on how they physically behave. Non-Newtonian fluids, a fluid type with many real-world applications, are examined here. Non-Newtonian fluids have grown in significance in recent years owing to their widespread use in many different industries, including engineering, aerodynamics and applications, fabrication, coating, chemical processing, and many more. In the aforementioned fluids, shifts in stress and fluidity have a self-contained connection. The intricacy of unconventional fluids underlying physical properties means that no single model can adequately capture all of their attributes; materials exhibiting this trait include mud, plasma, coatings, and polymeric solutions. A good illustration of an unconventional fluid with viscoelastic, resembling a solid is the Casson fluid. Casson flow has several real-world uses in industries like nanotechnologies, food production, mineral extraction, environmental research, and extractive activities. Accordingly, M. Shuaib et al. [1] evaluate the thermal and thermophysical characteristics of Casson fluid motion as it is induced by a stochastic porous deformable surface. They noticed that as the overall pattern of the thermal source characteristic flourished, the energy conversion rate of the Casson fluid also boosted, and the overall mass movement rate of the medium climbed in tandem. In their study of MHD of Casson fluid circulation via a permeable medium, A.B. Vishalakshi et al. [2] discovered that the main physical utilization is to raise the cooling down process and the Prandtl coefficient sustains the fluid's temperature consistently. A Caputo-Fabrizio applicability to the Casson fluid over an unstable boundary layer has been illustrated by S. Abbas et al. [3]. Heat radiation's impact on the time-varying Casson fluid motion as a function of an enormously propelled slanted surface, energy, and solutal radiative boundary circumstances, and other factors was studied by Endalew and Sarkar [4]. Computing the Casson flow rate across an elevated irregular surface across a Darcy-Forchheimer opaque media under the consequences of viscous dispersion, MHD, radiation, reactants, and Joule combustion was done quantitatively by S. Jaffrullah et al. [5]. S. J. Reddy et al. [6] investigated how evaporation on electrically executing, dense, impermeable, and hybrid Casson and tiny fluids that resemble an adiabatic porous exponentially expanded surface. As the coefficients of the Casson fluid constraints, magnetic attribute, and suction component increase, authors observed that the flow trajectories decline.

The EMHD micropump operates through the use of the Lorenz effect, which arises from interplay between an external field of electric current and magnetic fields. The EMHD micropump offers several benefits over competing models, including an intuitive production process, perpetually flowing power, and the ability to pump in both directions. As M. Buren et al. [7] revealed, it has several potential applications, including fluid thumping, fluidic system flow surveillance, and fluid swirling and blending. Study findings into EMHD micropump applications, such as pivoting EMHD turbines, EMHD flow in perforated surfaces, etc., have long been popular due to the technology's potential to reduce industry-wide fossil fuel use and associated costs. In a recent publication, K. Tian et al. [8] utilized a modified time-fractional Maxwell paradigm to offer predictive and numerical algorithms for EMHD flows. In their study,

A. Ali et al. [9] examined the implications of energy and differential thermal flux on EMHD nanofluid circulation across an extending surface. E. A. Algehyne et al. [10] conducted a simulation focusing on the chemically susceptible EMHD circulation of a tiny fluid via a unidirectional Riga surface, specifically examining the effects of frequency escapes and convective boundary situations. This study by I. Qamar et al. [11] investigated how activation energy was affected of Arrhenius and varying thermal efficiency on EMHD fluid motion through a porous material over an exponentially radiated constricting surface. The inertia within the Carreau composite nanofluid's EMHD Darcy-Forchheimer escape flows via a stretchy layer in an opaque material exhibiting temperature-variant attributes was examined by Mkhathshwa and Khumalo [12]. Their discovery revealed that temperature-variant heat transfer and energy radiation interact together to improve the thermal energy transmission capabilities of the Carreau composite nanofluids. Other researchers have made newer investigations that consider the impact of EMHD available in their works [13–15]. Many industries and areas of technical fields rely on mechanics of fluids which includes heat transmission as one of its core components. The enormous consumption of energy incurred by fluid thermal exchange operations has recently brought them a lot of attention in fields including electronics, aircraft, chemicals, freezing, and advanced engineering. This is closely reminiscent of the study of B. Dey et al. [16], who simulated the heat transfer in viscous fluids over a flat surface at various temperatures and found that fluid dispersion, in addition to changed physical and thermal consequences, significantly impacts various fluid attributes, including heat. The impact of materials on the movement and conduction of heat through a horizontally stretched sheet of microbes and nanoparticles has been investigated by R. Khan et al. [17]. The transmission of heat and circulation modelling for erratic motion in symmetrical pipes with vortex generators were studied by K.S. Rambhad et al. [18]. Using Newton's principle of heating and steady heat transition, P. Jayalakshmi et al. [19] explored the heat transmission characterization of Sisko fluid motion across an elastic sheet in a conductive domain. Hu H. P. [20] addressed fluid motion and heat transmission in a narrow channel exhibiting arranged microgrooves. References [21–23] also provide some recent and similar studies on heat transmission processes.

At the same time, the Ohmic heating repercussions, also known as Joule heating, occur when electromagnetic energy is converted into heat energy because of an electric field's existence and to the electrostatic resistance of the medium. Among the many industrial and manufacturing uses of joule heating are fluorescent lighting, cooking appliances with electric radiators, thermistors, electric fuse panels, and food preservation, among many others. In their discussion of physical processes involving micropolar fluid flows via an elongated surface, B. S. Goud et al. [24] use the RKF-45 and firing techniques to examine the interplay among ohmic radiation and the effects on MHD. M. Hasan et al. [25] examined the results of ohmic heating on unconventional flow rates accompanied by thermal transfer in an asymmetrical permeable conduit. The ohmic dissipative circulation of fluid in an opaque medium along a stretching sheet involving radiant heat underwent investigation by Samuel and Fayemi [26]. The researchers looked at the effects of varying fluidity and chemical reactions. B.J. Gireesha et al. [27] inquired about the ohmic heating's effect on Casson fluid's coupled convection flow in multi-homogeneous gradients (MHD) while taking the cross-diffusion phenomenon into account.

Thermal radiation on the environment has grown significantly important due to its extensive use in technical fields, particularly in the manufacture of parts and machinery, space exploration, power plants, etc. Idowu and Sani [28] examined the effects of heat absorption on the flow's dynamics among stable and undulating surfaces. Their specific focus was on the third-grade circulation of fluid across stagnant surfaces. A study by B. Dey et al. [29] attempted to investigate how radiation affects the laminar surface layer circulation of a mechanically executing, unstable, viscid, intangible liquid moving over an ascending semi-infinite platform that penetrates a permeable channel. M. Prameela et al. [30] looked at how radiant heat worked and the Schmidt coefficient on the MHD motion of a medium around an annulus using the Rosseland model. In their study, M. A. Kumar et al. [31] assessed the implication of thermal energy on laminar adiabatic fluid flow across a spontaneously initiated upright surface in the context of MHD heating. According to what Choudhury and Dey [32] said when an outside magnetic field exists and time-varying extraction acts in an aspect normal to the movement, an inexhaustible conducting visco-elastic fluid flows unsteadily via an opaque medium through a semi-infinite lateral translucent surface.

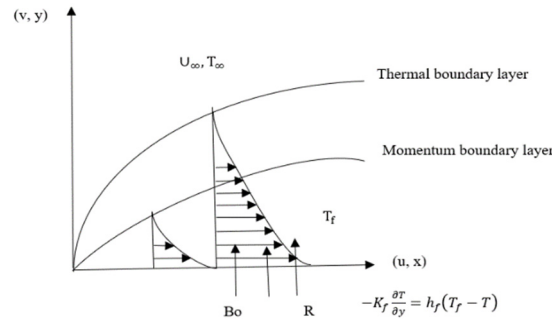
In light of the aforementioned literature, several investigations on Casson fluid flow are being carried out under different conditions. Nevertheless, there has been scant research on how EMHD affects Casson fluid flow. Hydrothermal warehouses, energy reimbursement, healthcare engineering, thermonuclear plant development, and many more applications are possible with non-Newtonian fluids. So far, no research has demonstrated a scenario where the EMHD Casson fluid is subjected to an amalgamation of thermal radiation, conservative boundary conditions, the ohmic heating impact, and suction through a flat surface. The results of this investigation have several possible applications, such as processes in biological engineering (such as distributing drugs and bloodstream flow), chemical extraction (such as viscoplastic fluids), geophysical mechanics (such as flows of magma and sediment), alimentary processing (including microfluidics), sustainability engineering (such as effluent procedure), and many more fields.

The following unanswered questions are the basis for this examination:

- i) How are the magnetic and local electric field factors influencing the flow of a fluid?
- ii) What effects do ohmic heating parameters and thermal radiation have on heat transfer?
- iii) To what extent do these flow characteristics affect skin friction and the Nusselt number?
- iv) In the context of a convective surface boundary, how does thermal radiation affect the boundary layer?

**MATHEMATICAL FORMULATION:**

The current model integrates the EMHD effect alongside the ohmic heating over a steady state Casson non-Newtonian fluid flow. The prescribed flow is configured by flat plate with varying temperature across boundary. In addition to it non-linear radiation impact is imparted in the thermal boundary layer equation. Furthermore, the convective boundary condition is presumed at the vicinity of the plate. For a flat plate, the key elements of the flow rate are ‘u’ in a lateral to the separation layer’s outermost edge and ‘v’ in the other axis as demonstrated in Figure 1.



**Figure 1.** Flow interpretation

The boundary layer mathematical expressions for flow and temperature, based on the assumptions, can be expressed in the following dimensional form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_f \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_f}{\rho} (B_0^2 u - E_0 B_0), \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_b \left[ \frac{\partial^2 T}{\partial y^2} - \frac{1}{k_f} \frac{\partial q_r}{\partial y} \right] + \frac{\mu_f}{\rho C_p} \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_f}{\rho C_p} (u B_0 - E_0)^2. \tag{3}$$

With the assumption concerning the x-path thermal heat flow being modest, the number in  $q_r$  Equation (3) shows the y-path thermal heat flow. The Rosseland diffusion approach may be used to simplify the thermal heat transfer  $q_r$  about an optically thick fluid, as stated in [35] as:

$$q_r = \frac{4\sigma_f \partial T^4}{3k^* \partial y}. \tag{4}$$

Boundary circumstances refer to:

$$\text{At } y = 0; u = 0, v = 0, -K_f \frac{\partial T}{\partial y} = h_f (T_f - T).$$

$$\text{At } y \rightarrow \infty; u \rightarrow U_\infty, T \rightarrow T_\infty. \tag{5}$$

Similarity transformation:

$$\eta = y \sqrt{\frac{U_\infty}{\nu_f x}}, u = U_\infty f'(\eta), v = \frac{1}{2} \sqrt{\frac{U_\infty \nu_f}{x}} (\eta f'(\eta) - f), \theta = \frac{T - T_\infty}{T_f - T_\infty} \tag{6}$$

$$T_f - T_\infty = Ax^n, P_r = \frac{\nu_f}{\alpha_b} = \frac{\mu C_p}{k}, R = \frac{4\sigma_f (T_f - T_\infty)^3}{k^* K}, C_t = \frac{T_\infty}{T_f - T_\infty}$$

From (2)

$$\left( 1 + \frac{1}{\beta} \right) f'''' + \frac{1}{2} f f'' - M f' + M E_1 = 0. \tag{7}$$

Where  $M = \frac{\sigma_f x B_0^2}{\rho U_\infty}$ ,  $E_1 = \frac{E_0}{B_0 U_\infty}$  =local electric parameter

From (3)

$$\left[ 1 + \frac{4}{3} R [C_t + \theta]^3 \right] \theta'' + 4R [C_t + \theta]^2 \theta'^2 + \frac{1}{2} P_r f \theta' + P_r E_c \left( 1 + \frac{1}{\beta} \right) f''^2 + M P_r E_c (f' - E_1)^2 = 0. \tag{8}$$

$$P_r = \frac{\nu_f}{\alpha_b} = \frac{\mu_f}{\rho \alpha_b}, E_c = \frac{U_\infty^2}{(T_f - T_\infty) C_p}$$

Given the aforementioned relevant boundary restrictions:

$$f(0) = 0, f'(0) = 1; \theta'(0) = -Bi(1 - \theta(0)), \tag{9}$$

$$f'(\eta) \rightarrow 0; \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty.$$

Skin friction and Nusselt number expression are as follows:

$$\text{Skin friction} = \left(1 + \frac{1}{\beta}\right) f''(0)$$

$$\text{Nusselt Number} = \left(1 + \frac{4}{3}R\right) \theta'(0)$$

### METHODOLOGY

This section describes how to solve the non-dimensional regressive momentum and temperature equations numerically. To numerically compute equations 7 and 8, a set of approaches based on the Lobatto 3A formula is utilized, specifically the `bvp4c` algorithm in MATLAB [33-34]. For complex, multivariate formulas, the `bvp4c` solver reliably produces correct solutions because of its mastery of enhanced accuracy threshold valued issues. It improves computing performance and reduces cognitive stress with its adaptable grid refining feature. In this method, initial predictions that are in line with the limiting constraints are necessary for the solution estimation. Afterwards, a different strategy called finite variance is used to adjust and enhance the original approximations via recurrent procedures, using the specified beginning circumstances. Simplifying the issue into a framework of initial ordinary differential equations (ODEs) is necessary for carrying out this process. When it comes to computationally addressing such non-dimensional stochastic conventional differential equations, `bvp4c` is the way to go because its degree of convergence rate is  $10^{-6}$  times faster than other approaches.

### RESULTS AND DISCUSSION

In this part, we simulate the motion of a flat plate subjected to Casson fluid and look at how different physical variables, which do not have any dimensions, affect the model. This study looked at how ohmic heating, electromagnetic radiation, and heat transmission factors affect fluid movement. Comprehensive tables and vivid illustrations are part of the content-based evaluation technique. Unless otherwise specified, the values of  $M=1.5$ ,  $R=1$ ,  $E_1=0.1$ ,  $Ec=0.05$ ,  $Pr=7$ ,  $Ct=0.2$ ,  $\beta=0.8$ , and  $Bi=0.2$  are retrieved. The visual representation shows the changing behaviour of velocities  $f'(\eta)$  and the temperature  $\theta(\eta)$  in the erratic mathematical problem with configurable parameters. Figures 2-4 illustrate the velocity trajectories of the fluid concerning its attributes. The velocity gradients for the  $M$  ramifications are shown in Figure 2.

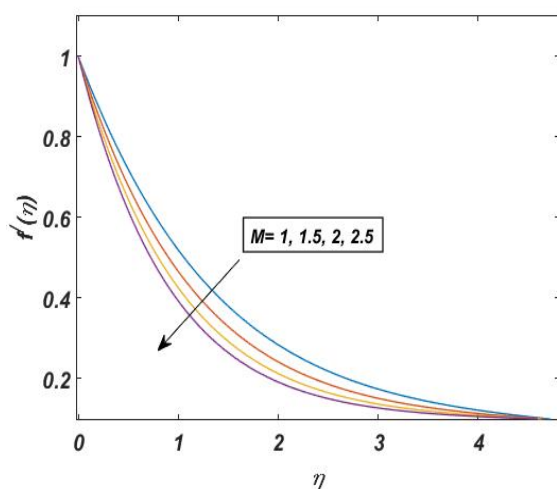


Figure 2. Effects of  $M$  on the velocity contour

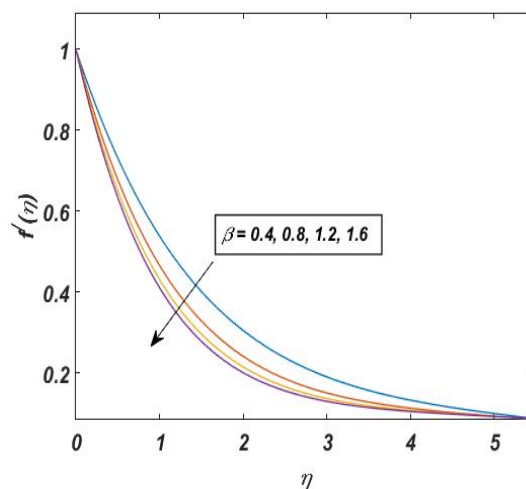


Figure 3. Effects of  $\beta$  on the velocity contour

The graph shows that when the magnetic coefficient  $M$  increases, the fluid's rate decreases. The Lorentz impact, which impedes the stream and is executed by the resisting effect, reduces the optimum trajectory and, consequently, the overall uniformity of the notable frontier section. The effect of attraction on a fluid is amplified as its magnetic field increases. Advantages of the Lorentz force phenomenon include pump-less mechanisms, accurate flow monitoring, and control of fluid flows in conductive fluids when subjected to electromagnetic radiation. The rheological behaviour of unconventional fluids, specifically those having stress gradients, like specific pigments, tissues, and sediments, can be described using Casson's paradigm. The fluid's resistance from flowing increases as the Casson parameter  $\beta$  is increased, necessitating more substantial shear forces to get an equivalent mobility. Figure 3 makes this clear that the fluid velocity drops as  $\beta$  rises. The hypothesis underlying this is that when  $\beta$  increases, the viscosity of the fluid thickens, causing it to be harder for the fragments of the medium to pass among themselves.

How the local magnetic characteristics influences  $E_1$  on the velocity deviation is examined in Figure 4. According to this graph, how the distribution of velocity and the frontier layer thicknesses have both grown as the magnitude of  $E_1$  increases indicating that a higher force is exerted regardless of the fluid's transition as a result of the tugging effectiveness. Flow behaviour and thermal stratification are frequently affected by electromagnetic radiation. This change is capable of enhancing the conveyance of heat, which raises the thermal contour because heat is transferred against its origin more effectively, as seen in Figure 5.

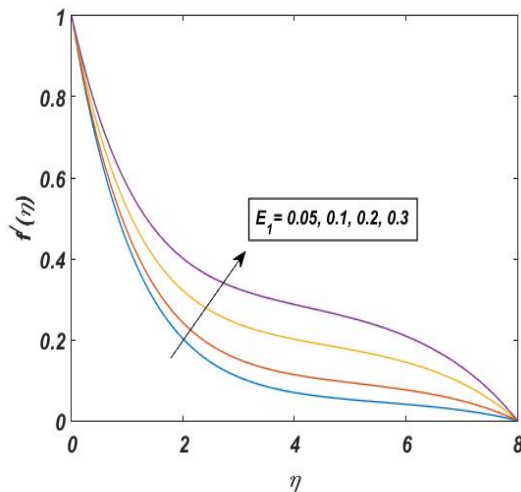


Figure 4. Effects of  $E_1$  on velocity contour

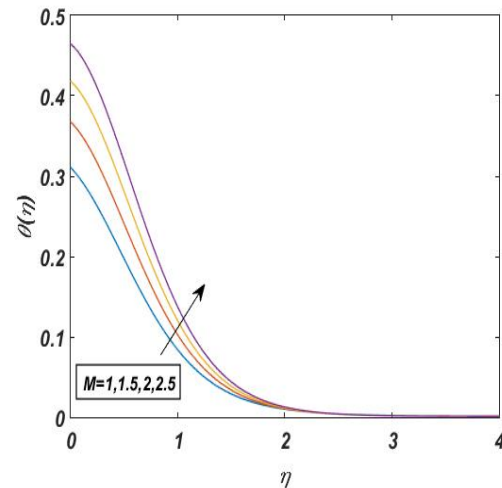


Figure 5. Effects of  $M$  on thermal contour

The relationship between transformation of enthalpy and kinetic energy becomes more significant as the Eckert number rises. An enhancement in the Eckert number might result in an upsurge in the temperature gradient as illustrated in Figure 6, since heat transfer through convection is accelerated. This is especially true when examining variations in temperature in circulation hypotheses. It is crucial to comprehend the effects of deviations in the Eckert value in uses in aviation, such as designing of propellers for aircraft and radiation shielding for a spaceship. Scientists and engineers can use this information to create cooling techniques that can handle the extreme conditions that are produced subsequent execution and after landing into the global atmosphere effectively. Casson's model includes specifications for fluidity and yield stress. Increasing Casson's attributes influences the pace of flow of the medium to rise. Figure 7 shows that energy transmission as heat increases due to an elevated viscosity, which in turn causes an enhanced temperature gradient.

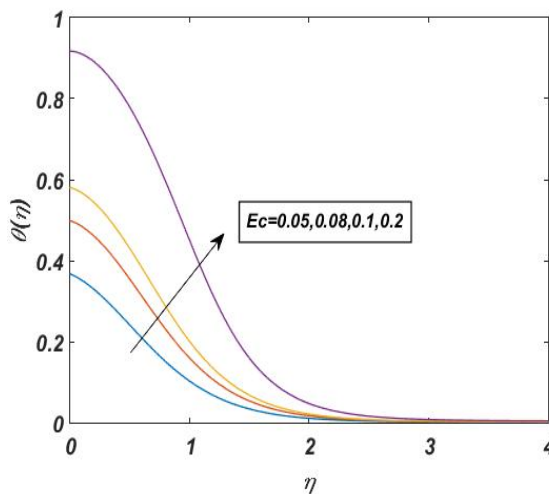


Figure 6. Effects of  $Ec$  on thermal contour

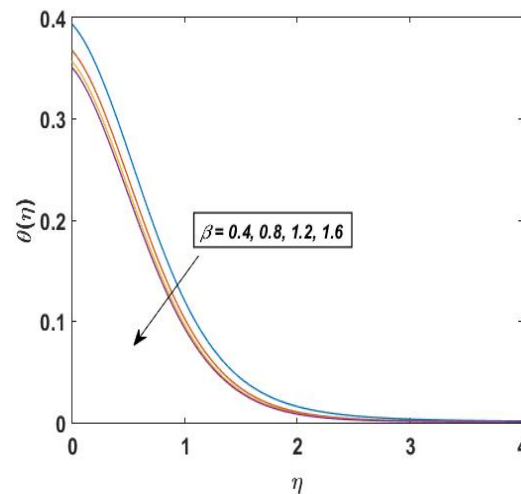


Figure 7. Effects of  $\beta$  on thermal contour

Maximizing efficacy, guaranteeing superior quality, and optimizing workflows all hinge on comprehending the impact of thermal variation on Casson's properties and the fluid's behaviour. In many fields, scientists and mathematicians deal with non-Newtonian fluids; hence, this knowledge aids in decision-making. Figure 8 shows how improved heat transmission and absorption mechanisms can cause temperature distributions in practical applications to rise when thermally radiated parameters like efficiency or absorbance are increased. In thermal solar panels, for example, substances with an elevated absorption rate can soak up brighter light, which in turn increases the thermal output. For manufacturing processes such as furnaces or combustion engines, elevated efficiency allows for better transmission of energy from heated surfaces to the ambient atmosphere, resulting in elevated system heat. Enhanced thermal emission characteristics, including the temperature distribution of a fluid's movement, can be altered by a shift in the local electric parameters,  $E_1$ . The existence of electromagnetic radiation changes the trajectory of heat across the medium, inducing known as electrothermal phenomena. The emission of energy via joule combustion improves as  $E_1$  increases because it places higher forces on energized particles inside the fluid. As apparent in Figure 9, the total thermal distribution of the fluid's motion lowers as a result of this additional energy dissipating as heat. An increasing thermal pattern in a fluid's flow rate is typically associated with an improvement in its specific heat factors,  $C_t$ . An increase in  $C_t$  indicates that the medium has a greater capacity for preserving thermal energy per mass. An improved temperature contour is the outcome of a greater ability to store heat since more of the energy is dispersed within the medium. Figure 10 shows the uniformity of the fluid's

temperature distribution circulation increases when the specific heat component rises because the fluid can hold and transport more heat. The impact is most noticeable in processes wherein heat transport is crucial, including in power plants, industrial operations, and natural occurrences. In general, the temperature persona grows as the Biot number (Bi) rises in a fluid's motion. The Biot number shows how much of an influence dispersion or transmission has on a material's thermal conductivity relative to how much heat transfer there is at the surface. If the Bi is higher, then convection is more important than conduction. Figure 11 shows that a higher thermal distribution amid the fluid's stream occurs when Bi increases because convection appears more prevalent and the fluid carries additional energy beyond its outermost layer. Hence, because convective radiation is amplified, larger Biot numbers cause fluid circulation mechanisms to have greater thermal gradients.

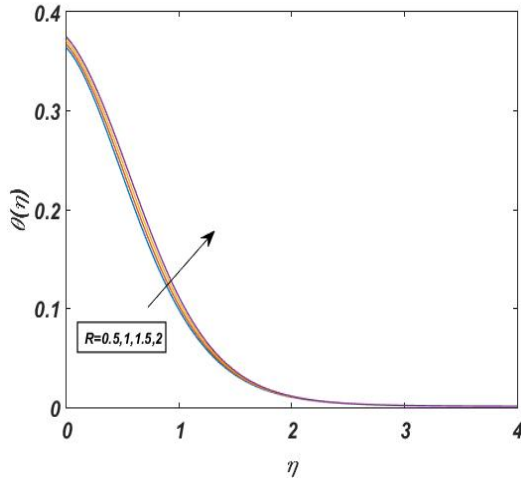


Figure 8. Effects of R on thermal contour

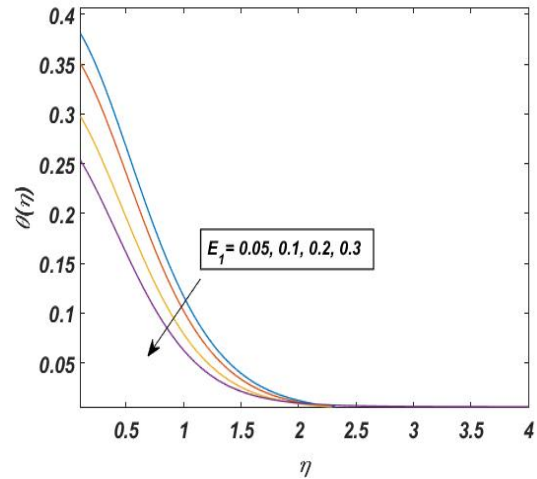


Figure 9. Effects of  $E_1$  on thermal contour

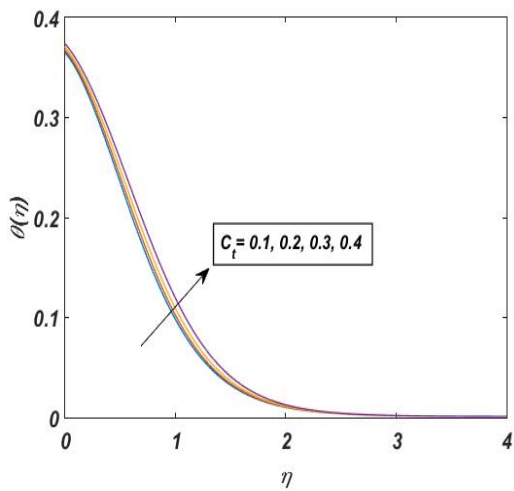


Figure 10. Effects of  $C_t$  on thermal contour

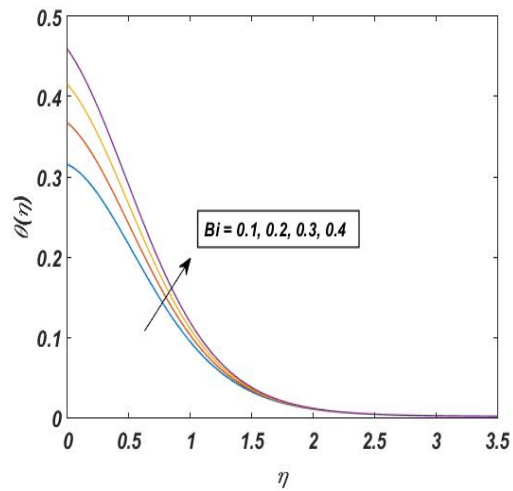


Figure 11. Effects of Bi on thermal contour

The values of skin friction and the Nusselt number are displayed in Tables 1 for different flow characteristics. The concept of skin friction is fundamental to comprehending how fluids behave as they approach substantial materials; it has consequences for many technical and physiological processes, including drag, heat transmission, radiating energy, and flow consistency. For fluid-flow technologies to function well in a variety of contexts, skin friction must be effectively managed. To slow down a moving fluid, Lorentz forces are created when a significant magnetic field induces energy in it. Furthermore, electromagnetism modifies the topology of the surface layer, which leads to less viscous drag and improved flow. Consequently, lessens skin friction by increasing the magnetic specifications, which alter the fluid mechanics via MHD effects. However, when the Casson parameter rises, the resultant stress rises as well, which in turn raises the resistance to flow and shear tension at the fluid-solid interaction, all of which contribute to greater skin friction.

Concerning convective heat transmission, the Nusselt number is fundamental to fluid mechanics. A fluid motion technique's radiative-to-thermal conductivity ratio may be measured using this method. Optimising temperatures in radiators and thermal exchangers are only two examples of the numerous engineering uses that benefit from a comprehension of the Nusselt number. As heat radiation increases, the Nusselt number drops because convection becomes less efficient. Because of the impairment to thermal conduction caused by more substantial layer thickness (shown by higher Casson parameters) and more reluctant thermal dispersion (indicated by higher Biot numbers), the Nusselt number drops. Furthermore, the Nusselt number, which quantifies the strength of heat conduction transport, increases as the

magnetic factors do as a result of magnetoconvection. A higher Eckert number indicates an improved kinetic energy to thermal transference ratio, which improves the efficacy of laminar heat transmission. By fortifying magnetohydrodynamic streams' radiative thermal transfer processes, both elements add to higher Nusselt numbers.

**Table 1.** Skin friction and Nusselt number

Flow factor		Skin friction	Nusselt number
R	0.5	-1.7512	-0.2121
	1	-1.7512	-0.2953
	1.5	-1.7512	-0.3775
	2	-1.7512	-0.4587
M	1	-1.4715	-0.3217
	1.5	-1.7512	-0.2953
	2	-1.9933	-0.2716
	2.5	-2.2095	-0.2500
Ec	0.05	-1.7512	-0.2953
	0.08	-1.7512	-0.2340
	0.1	-1.7512	-0.1957
	0.2	-1.7512	-0.0385
$\beta$	0.4	-2.1871	-0.2831
	0.8	-1.7512	-0.2959
	1.2	-1.5802	-0.3003
	1.6	-1.4875	-0.3031
Bi	0.1	-1.7512	-0.1597
	0.2	-1.7512	-0.2953
	0.3	-1.7512	-0.4095
	0.4	-1.7512	-0.5050

## CONCLUSION

Ohmic dissipation is considered in the context of an unconventional Casson fluid flowing steadily across a flat surface. Additionally, an algorithmic strategy for the effect of radiant heat on the creation of thermal frontier layers on a surface with a convective flow limit is covered. Significant tangible constraints are imposed on the indeterminate form of the inconsistent PDEs used in the assessment. Applying the bvp4c method from MATLAB's estimate strategy, numerical solutions to similarity formulations are achieved. In this study, contour graphs and tables are used to investigate the impact that M, beta,  $E_1$ , R,  $C_1$ , Ec, Bi, and Pr have on the velocity and temperature. According to the findings, the outcomes are as follows:

- When the magnetic coefficient M's efficacy causes a decrease in its flow rate, the fluid's motion is transformed into thermal energy, leading to an increase in the surrounding temperature. Increased amounts of elongation indicate a stronger inhibitor for motion, which in turn causes flows to stall dramatically as viscosity grows.
- Raising the Casson parameter makes the media more viscous, which makes it harder for fragments to slip about in it. Particles are less mobile and there is more resistance to flow when the Casson factor is high. This indicates that the fluid's non-Newtonian behaviour is more noticeable, which affects its flow properties and real-world uses.
- For substantial amounts of the Casson parameters, it has been observed that the extent of the boundary layer, which determines momentum, promptly decreases.
- Casson flow velocities are raised when local electric parameters are raised, suggesting possibilities for improved control and manipulation in areas like as microfluidics, medication delivery, and industrial processes. If we can better understand this link, we may optimize processes in the related areas of technology and medicine to make them more efficient and effective.
- The temperature distribution becomes more pronounced as the temperature difference ( $C_1$ ) grows.
- Heat transfer mechanisms are amplified when the Biot number and thermal emission variable are increased, leading to elevated temperature gradients. This finding is fundamental for improving industrial thermal operations, making sure energy is used efficiently, and boosting efficiency in many technical areas including harvesting solar energy and appliance condensation.

## FUTURE SCOPE

Various numerical approaches have also been successful in resolving this issue. Other complicated geometrical structures may be amenable to this method's generalization. This scenario may be supplemented with an assortment of non-Newtonian models. Several crucial physical properties may be used to change the flow fluid's behaviour. So, a lot of potential studies is lying around doing nothing.

### Nomenclature

$u$ and $v$ are velocity component along and perpendicular to the plane respectively	$B_0$ is strength of magnetic field
$g$ is gravitational acceleration	$K$ is thermal efficiency
$\beta$ Casson parameter	$C_p$ is the specific heat at persistent pressure
$q_r$ is radiative heat flux	$K_T$ is thermal diffusion ration
$\sigma_f$ is Stefan Boltzmann constant,	$\eta$ is dimensionless co-ordinate,
$k^*$ is mean absorption	$\mu$ is dynamic viscosity
$T$ is temperature of the fluid close to the plate	$Pr$ is Prandtl number
$T_f$ is base plate temperature	$\theta$ is non-dimensional temperature
$T_\infty$ is far-field temperature	$\Psi$ is stream function
$\rho$ is fluid density	$M$ is magnetic aspect
$h_f$ heat transfer coefficient	$C_t$ Temperature difference parameter
$\nu_f$ is kinematic viscosity	$R$ is thermal radiation parameter
$E_1$ is local electric parameter	$U_\infty$ is local electric parameter
$E_0$ is electric field	

### ORCID

-  Bamdeb Dey, <https://orcid.org/0000-0002-9002-676X>; 
  Tusar Kanti Das, <https://orcid.org/0000-0001-8105-4366>  
 Jintu Mani Nath, <https://orcid.org/0000-0002-9596-2936>

### REFERENCES

- [1] M. Shuaib, M. Anas, H.U. Rehman, A. Khan, I. Khan, and S.M. Eldin, "Volumetric thermo-convective casson fluid flow over a nonlinear inclined extended surface," *Scientific Reports*, **13**(1), 6324 (2023). <https://doi.org/10.1038/s41598-023-33259-z>
- [2] A.B. Vishalakshi, U.S. Mahabaleswar, M.H. Ahmadi, and M. Sharifpur, "An MHD Casson fluid flow past a porous stretching sheet with threshold non-Fourier heat flux model," *Alexandria Engineering Journal*, **69**, 727-737 (2023). <https://doi.org/10.1016/j.aej.2023.01.037>
- [3] S. Abbas, Z.U. Nisa, M. Nazar, M. Amjad, H. Ali, & A.Z. Jan, "Application of heat and mass transfer to convective flow of casson fluids in a microchannel with Caputo–Fabrizio derivative approach," *Arabian Journal for Science and Engineering*, **49**, 1275–1286 (2024). <https://doi.org/10.1007/s13369-023-08351-1>
- [4] M.F. Endalew, and S. Sarkar, "Modeling and Analysis of Unsteady Casson Fluid Flow due to an Exponentially Accelerating Plate with Thermal and Solutal Convective Boundary Conditions," *Journal of Applied Mathematics*, **2023**, 3065357 (2023). <https://doi.org/10.1155/2023/3065357>
- [5] S. Jaffrullah, W. Sridhar, and G.R. Ganesh, "MHD Radiative Casson Fluid Flow through Forchheimer Permeable Medium with Joule Heating Influence," *CFD Letters*, **15**(8), 179-199 (2023). <https://doi.org/10.37934/cfdl.15.8.179199>
- [6] S.J. Reddy, P. Valsamy, and D.S. Reddy, "Numerical Solutions of Casson-Nano Fluid Flow Past an Isothermal Permeable Stretching Sheet: MHD, Thermal Radiation and Transpiration Effects," *Journal of Nanofluids*, **12**(6), 1503-1511 (2023). <https://doi.org/10.1166/jon.2023.2034>
- [7] M. Buren, Y. Jian, L. Chang, Q. Liu, and G. Zhao, "AC magnetohydrodynamic slip flow in microchannel with sinusoidal roughness," *Microsystem Technologies*, **23**, 3347-3359 (2017). <https://doi.org/10.1007/s00542-016-3125-7>
- [8] K. Tian, S. An, G. Zhao, and Z. Ding, "Two-Dimensional Electromagnetohydrodynamic (EMHD) Flows of Fractional Viscoelastic Fluids with Electrokinetic Effects," *Nanomaterials*, **12**(19), 3335 (2022). <https://doi.org/10.3390/nano12193335>
- [9] A. Ali, H.S. Khan, S. Saleem, and M. Hussan, "EMHD nanofluid flow with radiation and variable heat flux effects along a slandering stretching sheet," *Nanomaterials*, **12**(21), 3872 (2022). <https://doi.org/10.3390/nano12213872>
- [10] E.A. Algehyne, A.F. Alharbi, A. Saeed, A. Dawar, P. Kumam, and A.M. Galal, "Numerical analysis of the chemically reactive EMHD flow of a nanofluid past a bi-directional Riga plate influenced by velocity slips and convective boundary conditions," *Scientific Reports*, **12**(1), 15849 (2022). <https://doi.org/10.1038/s41598-022-20256-x>
- [11] I. Qamar, M.A. Farooq, M. Irfan, and A. Mushtaq, "Insight into the dynamics of electro-magneto-hydrodynamic fluid flow past a sheet using the Galerkin finite element method: Effects of variable magnetic and electric fields," *Frontiers in Physics*, **10**, 1002462 (2022). <https://doi.org/10.3389/fphy.2022.1002462>
- [12] M.P. Mkhathshwa, and M. Khumalo, "Irreversibility scrutinization on EMHD Darcy–Forchheimer slip flow of Carreau hybrid nanofluid through a stretchable surface in porous medium with temperature-variant properties," *Heat Transfer*, **52**(1), 395-429 (2023). <https://doi.org/10.1002/htj.22700>
- [13] M. Irfan, M.A. Farooq, and T. Iqra, "A new computational technique design for EMHD nanofluid flow over a variable thickness surface with variable liquid characteristics," *Frontiers in Physics*, **8**, 66 (2020). <https://doi.org/10.3389/fphy.2020.00066>
- [14] S.M. Atif, M. Abbas, U. Rashid, and H. Emadifar, "Stagnation point flow of EMHD micropolar nanofluid with mixed convection and slip boundary," *Complexity*, **2021**, 3754922 (2021). <https://doi.org/10.1155/2021/3754922>
- [15] S. Khatun, M.M. Islam, M.T. Mollah, S. Poddar, and M.M. Alam, "EMHD radiating fluid flow along a vertical Riga plate with suction in a rotating system," *SN Applied Sciences*, **3**, 452 (2021). <https://doi.org/10.1007/s42452-021-04444-4>
- [16] B. Dey, J.M. Nath, T.K. Das, and D. Kalita, "Simulation of transmission of heat on viscous fluid flow with varying temperatures over a flat plate," *JP Journal of Heat and Mass Transfer*, **30**, 1-18 (2022). <http://dx.doi.org/10.17654/0973576322052>
- [17] R. Khan, A. Ahmad, M. Afraz, and Y. Khan, "Flow and heat transfer analysis of polymeric fluid in the presence of nanoparticles and microorganisms," *Journal of Central South University*, **30**(4), 1246-1261 (2023). <https://doi.org/10.1007/s11771-023-5300-1>
- [18] K.S. Rambhad, V.P. Kalbande, M.A. Kumbhalkar, V.W. Khond, and R.A. Jibhakate, "Heat transfer and fluid flow analysis for turbulent flow in circular pipe with vortex generator," *SN Applied Sciences*, **3**(7), 709 (2021). <https://doi.org/10.1007/s42452-021-04664-8>



- [19] P. Jayalakshmi, M. Obulesu, C.K. Ganteda, M.C. Raju, S.V. Varma, and G. Lorenzini, "Heat Transfer Analysis of Sisko Fluid Flow over a Stretching Sheet in a Conducting Field with Newtonian Heating and Constant Heat Flux," *Energies*, **16**(7), 3183 (2023). <https://doi.org/10.3390/en16073183>
- [20] H.P. Hu, "Theoretical Study of Convection Heat Transfer and Fluid Dynamics in Microchannels with Arrayed Microgrooves," *Mathematical Problems in Engineering*, **2021**, 3601509 (2021). <https://doi.org/10.1155/2021/3601509>
- [21] B. Dey, and R. Choudhury, "Slip Effects on Heat and Mass Transfer in MHD Visco-Elastic Fluid Flow Through a Porous Channel," in: *Emerging Technologies in Data Mining and Information Security: Proceedings of IEMIS 2018*, Vol.1 (Springer, Singapore, 2018), pp. 553-564.
- [22] R. Choudhury, B. Dey, and B. Das, "Hydromagnetic oscillatory slip flow of a visco-elastic fluid through a porous channel," *Chemical Engineering*, **71**, 961-966 (2018). <https://doi.org/10.3303/CET1871161>
- [23] A.H. Mirza, B. Dey, and R. Choudhury, "The detrimental effect of thermal exposure and thermophoresis on MHD flow with combined mass and heat transmission employing permeability," *International Journal of Applied Mechanics and Engineering*, **29**(1), 90-104 (2024). <https://doi.org/10.59441/ijame/181556>
- [24] B.S. Goud, and M.M. Nandeppanavar, "Ohmic heating and chemical reaction effect on MHD flow of micropolar fluid past a stretching surface," *Partial Differential Equations in Applied Mathematics*, **4**, 100104 (2021). <https://doi.org/10.1016/j.padiff.2021.100104>
- [25] M.M. Hasan, M.A. Samad, and M.M. Hossain, "Effects of Hall Current and Ohmic Heating on Non-Newtonian Fluid Flow in a Channel due to Peristaltic Wave," *Applied Mathematics*, **11**(04), 292 (2020). <https://doi.org/10.4236/am.2020.114022>
- [26] D.J. Samuel, and I.A. Fayemi, "Impacts of variable viscosity and chemical reaction on Ohmic dissipative fluid flow in a porous medium over a stretching sheet with thermal radiation," *Heat Transfer*, **52**(7), 5022-5040 (2023). <https://doi.org/10.1002/htj.22915>
- [27] B.J. Gireesha, K.G. Kumar, M.R. Krishnamurthy, S. Manjunatha, and N.G. Rudraswamy, "Impact of ohmic heating on MHD mixed convection flow of Casson fluid by considering cross diffusion effect," *Nonlinear Engineering*, **8**(1), 380-388 (2019). <https://doi.org/10.1515/nleng-2017-0144>
- [28] A.S. Idowu, and U. Sani, "Thermal radiation and chemical reaction effects on unsteady magnetohydrodynamic third grade fluid flow between stationary and oscillating plates," *International Journal of Applied Mechanics and Engineering*, **24**(2), 269-293 (2019). <https://doi.org/10.2478/ijame-2019-0018>
- [29] B. Dey, B. Kalita, and R. Choudhury, "Radiation and chemical reaction effects on unsteady viscoelastic fluid flow through porous medium," *Frontiers in Heat and Mass Transfer (FHMT)*, **18**, 1-8 (2022). <https://doi.org/10.5098/hmt.18.32>
- [30] M. Prameela, D.V. Lakshmi, and J.R. Gurejala, "Influence of thermal radiation on mhd fluid flow over a sphere," *Biointerface Res. Appl. Chem.* **5**(12), 6978-6990 (2021). <https://doi.org/10.33263/BRIAC125.69786990>
- [31] M.A. Kumar, Y.D. Reddy, V.S. Rao, and B.S. Goud, "Thermal radiation impact on MHD heat transfer natural convective nano fluid flow over an impulsively started vertical plate," *Case studies in thermal engineering*, **24**, 100826 (2021). <https://doi.org/10.1016/j.csite.2020.100826>
- [32] R. Choudhury, and B. Dey, "Unsteady thermal radiation effects on MHD convective slip flow of visco-elastic fluid past a porous plate embedded in porous medium," *International Journal of Applied Mathematics and Statistics*, **57**(2), 215-226 (2018).
- [33] A. Paul, J.M. Nath, and T.K. Das, "Thermally stratified Cu-Al<sub>2</sub>O<sub>3</sub>/water hybrid nanofluid flow with the impact of an inclined magnetic field, viscous dissipation and heat source/sink across a vertically stretching cylinder," *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, **104**(2), e202300084 (2023). <https://doi.org/10.1002/zamm.202300084>
- [34] A. Paul, J.M. Nath, and T.K. Das, "An investigation of the MHD Cu-Al<sub>2</sub>O<sub>3</sub>/H<sub>2</sub>O hybrid- nanofluid in a porous medium across a vertically stretching cylinder incorporating thermal stratification impact," *Journal of Thermal Engineering*, **9**(3), 799-810 (2023). <https://doi.org/10.18186/thermal.1300847>
- [35] B.K. Jha, B.Y. Isah, and I.J. Uwanta, "Combined effect of suction/injection on MHD free-convection flow in a vertical channel with thermal radiation," *Ain Shams Engineering Journal*, **9**(4), 1069-1088 (2018). <https://doi.org/10.1016/j.asej.2016.06.001>

## МОДЕЛЮВАННЯ ТА ІМІТАЦІЯ ВПЛИВУ ТЕПЛООБМІНУ НА ЕМНД ПОТІК CASSON РІДИНИ, ПОСИЛЕНИЙ ПЛОСКОЮ ПЛАСТИНОЮ З ПРОМЕНЕВИМ ТА ОМІЧНИМ НАГРІВОМ

Бамдеб Дей<sup>а</sup>, Довін Дукру<sup>а</sup>, Тусар Канті Дас<sup>б</sup>, Джінту Мані Нат<sup>с</sup>

<sup>а</sup>Факультет математики, Університет Дона Боско в Ассамі, Гувахаті-781017, Ассам, Індія

<sup>б</sup>Департамент математики, Дудхой коледж, Дудхой, Ассам, Індія

<sup>с</sup>Департамент математики, коледж Мангалдай, Мангалдай-784125, Ассам, Індія

Поточне дослідження представляє результати чисельного дослідження впливу теплового випромінювання, омічного нагріву та електромагнітного гідродинамічного опору на потік рідини Кассона через плоску поверхню. Використовуючи відповідні параметри подібності, рівняння, які регулюють систему, перетворюються на нелінійні звичайні диференціальні рівняння. Алгоритм MATLAB Ввр4с використовується для чисельного обчислення нелінійних ODE. Для оптимізації промислової та екологічної обробки вкрай важливо вивчити потік рідини Кассона (включаючи бурові розчини, скам'янілі покриття, різні відкладення та специфічні мастильні нафтопродукти, поліетилен, що розчиняється, і ряд колоїдів) за наявності теплопередачі. Графіки та таблиці були використані для представлення обчислювальних результатів для різних діапазонів відчутних змінних, які визначають розподіл швидкості та температури. Швидкість рідини зменшується, коли магнітні параметри та параметри Кассона зростають, тоді як швидкість рідини збільшується зі зростанням локальних електричних параметрів. Це є прикладом складного зв'язку між електромагнітним випромінюванням і механікою рідини. Зростає число Еккерта, теплове випромінювання, питома теплоємність і число Біо підвищують профілі температури, тоді як зростання параметра Кассона та локальних електричних параметрів зменшує їх, демонструючи різні впливи на явища теплопередачі. Крім того, цей запит стосується коефіцієнта шкірного тертя та значень Нуссельта. Проте нові експериментальні дослідження виграють від цієї теоретичної роботи.

**Ключові слова:** теплообмін; рідина Кассона; EMHD; теплове випромінювання; омічний нагрів