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NUMERICAL APPROACH TO BURGERS' EQUATION IN DUSTY PLASMAS WITH DUST CHARGE VARIATION

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In this paper, the Crank-Nicholson method is applied to solve the one-dimensional nonlinear Burgers' equation in warm, dusty plasmas with dust charge variation. After obtaining numerical results, a thorough analysis is conducted and compared against analytical solutions. On the basis of the comparison, it is evident that the numerical results obtained from the analysis are in good agreement with the analytical solution. The error between the analytical and numerical solutions of the Burgers' equation is calculated by two error norms, namely L_2 and L_∞. A Von-Neumann stability analysis is performed on the present method, and it is found to be unconditionally stable according to the Von-Neumann analysis.

Keywords: *Warm Dusty plasmas; Burgers' equation; Crank-Nicolson method; von Neumann stability analysis*

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1. INTRODUCTION

Many real-life problems are represented by nonlinear partial differential equations, including plasma physics, acoustics, fluid mechanics, etc. One notable model equation is the nonlinear Burgers' equation, initially introduced by Bateman [1] and later recognized as a mathematical model for turbulence by Johannes Martinus Burgers [2]. The Burgers' equation is a partial differential equation that is used as a simplified version of the Navier-Stokes equation [3]. The Burgers' equation is a combination of convection and diffusion terms and has the same nonlinear and dissipative terms as the Navier-Stokes equation. This equation is primarily used for studying turbulence and shock wave theory in the context of nonlinear and dissipative phenomena. Burgers' equation in dusty plasmas that describes the nonlinear phenomenon of the shock structure formation on the acoustic wave originating from dust charge fluctuation dynamics.

The study of dusty plasmas, characterized by the presence of charged microparticles suspended in a plasma medium, has garnered significant attention in both experimental and theoretical research in recent years because dusty plasma plays an important role in studying the different types collective process in space environment,namely lower and upper mesosphere, radiofrequency, plasma discharge, planetary rings, plasma crystals, commentary tail, asteroid zones, planetary magnetosphere, interplanetary spaces, interstellar medium, earth's environment etc. [4, 5]. A dusty plasma is characterised by intense interactions between the dust particles and the nearby plasma species, which have a significant influence on plasma behaviour. Charged dust particles influence not only the equilibrium and stability of the plasma system but also exhibit fascinating dynamical properties, such as dust acoustic waves, dust ion-acoustic waves, and dust cyclotron waves. These waves have the potential to have a significant impact on the overall dynamics of a plasma due to the collective behaviour of dust particles. The Burgers' equation, which includes the effects of both convection and diffusion, is one of the fundamental equations used to describe the dynamics of dusty plasmas [6]. Researchers have been greatly interested in this equation since it was first presented because of its many practical applications, including gas dynamics, shock theory, traffic flows, viscous flow, and turbulence. Over the past few decades, numerous numerical methods have been developed and applied for solving Burgers' equation [7, 8, 9, 10, 11, 12, 13, 14]. These methods include finite element methods, finite difference methods, least-squares finite element methods, and spectral methods.

The Burgers' equation was studied by Wei and Gu in 2002, and they employed the Conjugate Filter Approach as a method for solving the equation [15]. Additionally, N.A. Mohamed [16] introduced new fully implicit schemes for solving the unsteady one-dimensional and two-dimensional equations. Singh and Gupta [17] have developed a new fourth order modified cubic B-spline (mCB) based upon collocation technique (mCBCT4) to determine approximate solution of Burgers' equation. Yusuf et al. [18] applied finite element collocation method with strang splitting to finding exact solutions of Burgers' type equation. Xu et al. [19] proposed a novel numerical scheme to solve Burgers' equation. Inan and Bahadir [20], developed implicit and fully implicit

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exponential finite difference methods for numerical solution of the one-dimensional Burgers' equation. Inan and Bahadir [21] solved Burgers' equation numerically using a Crank-Nicolson exponential finite difference method. Mittal and Jain [22] have implemented modified cubic B-splines collocation method to solve nonlinear Burgers' equation.Wani and Thakar [23] developed a modified Crank-Nicolson type method for numerical solution of Burgers' equation. Mohamed [24] provided a new numerical scheme based on the finite difference method for solving the nonlinear one-dimensional Burgers' equation.

Yaghoobi and Najafi [25] constructed implicit non-standard finite difference scheme for solving the nonlinear Burgers' equation. An efficient numerical solution based on Milne method was presented in [26]. Shallal et al. [27] solved Burgers' Equation by a cubic Hermite finite element method. A numerical technique is formulated for solving the coupled viscous Burgers' equation (CVBE) by employing cubic B-spline and the Hermite formula [28]. Hussain [29] introduces a hybrid radial basis function (HRBF) approach for the numerical solution of the quasi-linear viscous Burgers' equation.

In this research article, the Crank-Nicholson method is applied to solve the Burgers' equation in warm dusty plasmas, taking dust charge fluctuations into account. The Crank-Nicholson method, a finite difference-based scheme, provides a robust and accurate numerical approach by employing an implicit midpoint rule, which combines the advantages of explicit and implicit schemes. This approach surpasses other numerical techniques in terms of precision.The behavior of plasma can be significantly impacted by the fluctuating charge levels of dust particles. Therefore, the inclusion of dust charge variation is crucial. The numerical method described in this paper aims to provide a comprehensive analysis of the influence of varying dust charge on the behavior of nonlinear waves and shock structures in dusty plasmas.

The manuscript is organized as follows: In Section 2, we introduce the governing equations for dusty plasmas with variable dust charge, and it provides a detailed discussion on the derivation of Burgers' equation within the context of dusty plasmas. In Section 3, we provide an overview of the Crank-Nicolson method used for the solution of the equation. Section 4 presents the stability analysis of the technique. In Section 5, we present the results and discussions, wherein we analyze the numerical solutions obtained and thoroughly discuss their implications.

2. BASIC EQUATIONS AND DERIVATION OF BURGERS' EQUATION

The fundamental equations governing the behavior of dust-charged grains in a fluid description consist of the equations of continuity and momentum, which can be expressed as follows[30]:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}\left(n_d \nu_d\right) = 0 \tag{1}$$

$$\frac{\partial \nu_d}{\partial t} + \nu \frac{\partial \nu_d}{\partial x} + \frac{\varsigma_d}{n_d}\frac{\partial p_d}{\partial x} = z_d \frac{\partial \psi}{\partial x} + \zeta \frac{\partial^2 v_d}{\partial x^2} \tag{2}$$

$$\frac{\partial p_d}{\partial t} + \upsilon \frac{\partial p_d}{\partial x} + 3 p_d \frac{\partial v_d}{\partial x} = 0 \tag{3}$$

The Poisson's equation is given as

$$\frac{\partial^2 \psi}{\partial x^2} = z_d \lambda n_d + (1-\lambda)\, n_e - n_i \tag{4}$$

The distribution of electron and ion density can be characterized using a Boltzmann distribution, that is.

$$n_e = n_{e_0} exp\left(\psi\right) \tag{5}$$

$$n_i = n_{i_0} exp\left(-\beta\psi\right) \tag{6}$$

In the given context, where n_d, n_e, n_i, v_d, p_d, ψ, x, and t represent the dust particle number density, electron number density, ion number density, dust fluid velocity, dust fluid pressure, electrostatic potential, spatial variable, and time, respectively, and they are normalized by n_{d_0} (unperturbed dust particle number density), n_{e_0} (unperturbed electron particle number density), and n_{i_0} (unperturbed ion particle number density); $\lambda = \frac{n_{d_0}}{n_{i_0}}$, $\beta = \frac{T_e}{T_i}$, and $\varsigma_d = \frac{T_d}{T_e}$, where T_d, T_e, and T_i are the temperatures for dust, electron, and ion. λ_d is the fluid velocity normalized to the dust acoustic speed $C_d = \left(\frac{z_d n_{d_0} e\lambda + 3\varsigma_d K_B T_e q}{m_d q}\right)^{\frac{1}{2}}$ with $q = (1-\lambda)\, n_{e_0} + \beta n_{i_0}$, and K_B, m_d, and z_d being the Boltzmann constant, dust acoustic mass, and charged number of dust particles. p_d is the pressure normalized to $n_{d_0} K_B T_d$; ψ is the electrostatic wave potential normalized by $\left(\frac{K_B T_i}{e}\right)$, with e being the electron charge; the spatial variable is normalized to the dust Debye length $\vartheta_d = \left(\frac{3\sigma_d K_B T_e m_d}{4\pi n_{d_0}\left(z_d^2 + qe\right)}\right)^{\frac{1}{2}}$, and the time variable is normalized to the dust period $\varpi_{pd}^{-1} = \left(\frac{m_d}{4\pi n_{d_0} z_d^2 e^2}\right)^{\frac{1}{2}}$. The coefficient of viscosity ζ is a normalized quantity given by $\varpi_{pm}\vartheta_m^2 m_d n_{d_0}$.

The plasma system maintains overall charge neutrality through the following relationship:

$$z_d \lambda n_{d_0} + (1-\lambda)\, n_{e_0} = n_{i_0} \tag{7}$$

The following stretched coordinates are taken into consideration in order to obtain the Burgers' equation:

$$\chi = \rho\,(x - \vartheta t)\,; \zeta = \rho^2 t \tag{8}$$

Here, ϑ represents the phase velocity of the wave along the x direction, normalized by the acoustic velocity, while ρ serves as a dimensionless expansion parameter, quantifying the strength of dispersion.
The equation of Burgers is formulated in [30] as:

$$\frac{\partial \psi^{(1)}}{\partial \zeta} + A\phi^{(1)} \frac{\partial \psi^{(1)}}{\partial \chi} = C \frac{\partial^2 \psi^{(1)}}{\partial \chi^2} \tag{9}$$

where

$$A = \frac{z_d \lambda \left\{\vartheta^2 n_{i_0} - (1-\lambda)\, n_{e_0}\right\} (\vartheta - 3\varsigma_d) - 3\left(\vartheta^2 + \varsigma_d\right) \left\{(1-\lambda)\, n_{e_0} + \beta n_{i_0}\right\}^2}{2\vartheta z_d \lambda \left\{(1-\lambda)\, n_{e_0} + \beta n_{i_0}\right\}} \tag{10}$$

and

$$C = \frac{\zeta}{2\vartheta} \tag{11}$$

Employing the tanh-method [31], the solution for the shock wave is derived as

$$\psi^1\,(\chi, \zeta) = \psi_m \left\{1 - tanh\left(\frac{\Omega}{\Upsilon}\right)\right\} \tag{12}$$

Where $\Omega = \chi - Mt$,$\psi_m = \frac{M}{A}$ and $\Upsilon = \frac{2B}{M}$ The variables ψ_m and Υ represent the amplitude and width of the shock waves, respectively, while M denotes the Mach number. The profile of the shock wave is influenced by the nonlinearity coefficient A and dissipation coefficient C, both of which are functions of plasma parameters.

3. CRANK-NICOLSON METHOD

Section 2 introduces the derivation of the Burgers equation within the context of dusty plasmas, considering variations in dust charge, and presents the solution for shock waves. Here, we proceed to apply the Crank-Nicholson method to solve the derived Burgers equation.The Crank-Nicolson method, proposed by Crank and Nicolson [24], is a numerical scheme and is a combination of the forward Euler method and the backward Euler method, which provides improved accuracy and stability. We simplify the equation 9 by introducing the transformations $\psi^1\,(\chi, \zeta) = u\,(x, t) \cong u_{ix,jt} \cong u_{i,j}$ The equation 9 can be expressed as

$$\frac{\partial u}{\partial t} + Au\frac{\partial u}{\partial x} = C\frac{\partial^2 u}{\partial x^2} \tag{13}$$

Let us consider the discretization of the Burgers' equation by using the Crank-Nicholson method:

$$\frac{\partial u}{\partial x} = \frac{u_{i+1,j+1} - u_{i-1,j+1}}{2h} + \frac{u_{i+1,j} - u_{i-1,j}}{2h} \tag{14}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{2h^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{2h^2} \tag{15}$$

$$\frac{\partial u}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{k} \tag{16}$$

Now substituting eqs (26)-(28) into eq (25), we obtain

$$\begin{aligned} &\frac{u_{i,j+1} - u_{i,j}}{k} + Au_{i,j}\Big(\frac{u_{i+1,j+1} - u_{i-1,j+1}}{2h} + \frac{u_{i+1,j} - u_{i-1,j}}{2h}\Big) \\ &= C\Big(\frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{2h^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{2h^2}\Big) \end{aligned} \tag{17}$$

Let $a = \frac{Ck}{2h^2}$,$r = \frac{Ak}{2h}$,the equation (29) will become

$$\begin{aligned} (1+2a)\, u_{i,j+1} - a\,(u_{i-1,j+1} + u_{i+1,j-1}) &= (1-2a)\, u_{i,j} + a\,(u_{i-1,j} + u_{i+1,j}) \\ &\quad - r\left[u_{i,j}\,(u_{i+1,j+1} - u_{i-1,j+1}) + u_{i,j}\,(u_{i+1,j} - u_{i-1,j})\right] \end{aligned} \tag{18}$$

4. STABILITY ANALYSIS OF THE CRANK-NICHOLSON METHOD

The stability analysis was developed by the mid-twentieth century Hungarian mathematician and father of the electronic computer John von Neumann. The Von Neumann stability theory in which the growth factor of a Fourier mode is defined as

$$u_{i,j} = \xi^j e^{Ikhi} = \xi^j e^{I\theta i} \tag{19}$$

Where $I = \sqrt{-1}$,ξ^j is the amplitude at time level k is the wave number and $h = \Delta x$. To investigate the stability of the numerical scheme, the Burgers' equation has been linearized by ignoring the nonlinear term and then obtained the differential equation by applying the Crank-Nicholson method to the linearized Burgers' equation. The linearized Burgers' equation is given as below:

$$\frac{\partial u}{\partial t} = C\frac{\partial^2 u}{\partial x^2} \tag{20}$$

Applying the Crank-Nicolson method to equation 20, we get

$$(1+2a)\, u_{i,j+1} - a\left(u_{i-1,j+1} + u_{i+1,j+1}\right) = (1-2a)\, u_{i,j} + a\left(u_{i-1,j} + u_{i+1,j}\right) \tag{21}$$

Substitute 19 in 21, we get

$$(1+2a)\,\xi^j e^{I\theta i}\xi - a\left(\xi^j e^{I\theta i}\xi e^{-I\theta} + \xi^j e^{I\theta i}\xi e^{I\theta}\right) = (1-2a)\,\xi^j e^{I\theta i} + a\left(\xi^j e^{I\theta i}e^{-I\theta} + \xi^j e^{I\theta i}e^{I\theta}\right) \tag{22}$$

$$(1+2a)\,\xi - a\left(\xi e^{-I\theta} + \xi e^{I\theta}\right) = (1-2a) + a\left(e^{-I\theta} + e^{I\theta}\right) \tag{23}$$

$$(1+2a)\,\xi - a\xi cos\theta = (1-2a) + acos\theta \tag{24}$$

$$\xi = \frac{1-2a+acos\theta}{1+2a-acos\theta} \tag{25}$$

Where the quantity ξ in equation 19 is called the amplification factor. Since $0 \le cos\theta \le 1$ When $cos\theta = 0$,$\xi = \frac{1-2a}{1+2a} \le 1$ When $cos\theta = 1$,$\xi = \frac{1-2a+a}{1+2a-a} = \frac{1-a}{1+a} \le 1$ Hence, $\xi \le 1$ is always satisfied for any value of a where 1 is the upper limit for ξ.For stability, we must have $|\xi| \le 1$,which means $-1 \le \xi \le 1$. Now, we consider the lower limit for ξ.

$$-1 \le \xi \tag{26}$$

$$-1 \le \frac{1-2a+acos\theta}{1+2a-acos\theta} \tag{27}$$

$$-1\,(1+2a-acos\theta) \le 1-2a+acos\theta \tag{28}$$

$$acos\theta - 1 - 2a \le 1 - 2a + acos\theta \tag{29}$$

$$-1-2a \le 1-2a \tag{30}$$

$$2a - 2a \le 1+1 \tag{31}$$

$0 \le 2$ which is always true.

It implies that the lower limit for ξ is satisfied for any value of a. Thus the Crank-Nicholson method is unconditionally stable according to the linear analysis.

5. RESULTS AND DISCUSSION

The analytical solution 12 can be rewritten as ,

$$u(x,t) = \frac{M}{A}\left\{1 - tanh\frac{M}{2C}\,(x - Mt)\right\} \tag{32}$$

To proceed the numerical solution of Burgers' equation,we consider the initial condition as

$$u(x,0) = \frac{M}{A}\left\{1 - tanh\frac{Mx}{2C}\right\} \tag{33}$$

and the boundary conditions

$$u(0,t) = \frac{M}{A}\left\{1 + tanh\frac{M^2 t}{2C}\right\} \tag{34}$$

$$u(1,t) = \frac{M}{A}\left\{1 - tanh\frac{M}{2C}\,(1 - Mt)\right\} \tag{35}$$

Due to the dependence of the nonlinear coefficient A and dissipation coefficient C on different plasma parameters, we have considered a range of values for A and C, corresponding to the various plasma parameters. The validity of the present technique is evaluated using the absolute error which is defined by

$$\left|u_i^{Analytical} - u_i^{Numerical}\right| \tag{36}$$

Also,L_2 and L_∞ error norms, defined by

$$L_2 = \sqrt{h\sum_{j=1}^{N}\left|u_j^{analytical} - u_j^{numerical}\right|^2} \tag{37}$$

$$L_\infty = max\left|u_j^{analytical} - u_j^{numerical}\right| \tag{38}$$

are presented graphically for various values of nonlinear coefficient and dissipation coefficient for chosen space and time steps to check the accuracy and effectiveness of the method.

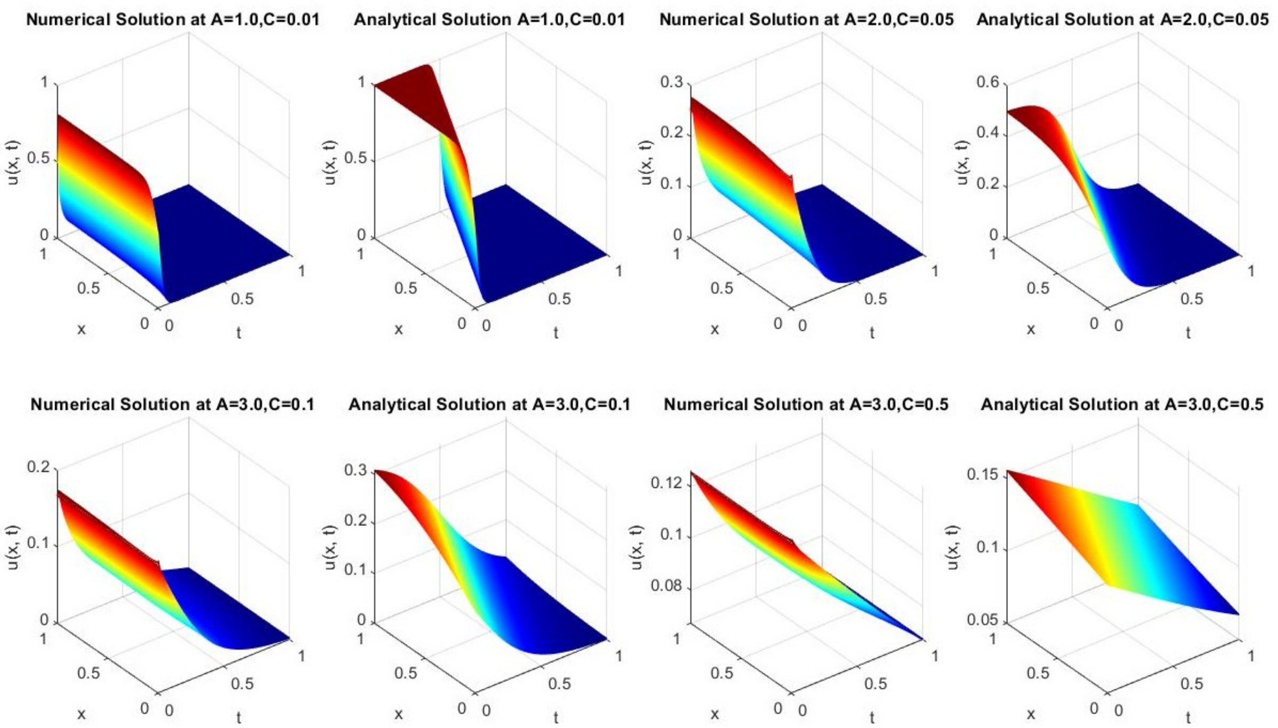

Figure 1. Comparison of Analytical and numerical solution of Burgers' equation at various values of A and C.

The comparison between the analytical and numerical solutions of the Burgers' equation in dusty plasma using the Crank-Nicholson method has been presented in Figure 1. It has been observed that the numerical solution obtained through the Crank-Nicholson method demonstrates good agreement with the analytical solution. The figure clearly illustrates that the presence of shock wave structures is observed when the dissipation coefficient is reduced to a smaller value. As C becomes larger, the diffusive behavior becomes more prominent and suppress the formation of shocks and maintain a more diffusive behavior. The wavefronts become smoother and propagate slower as C increases.When the nonlinear coefficient (A) is increased, the advection term becomes dominant, leading to the formation of steep gradients and shock waves in the solution.

Table 1. Absolute error between the numerical and analytical values at A = 1.0, C = 0.01

x	t	Numerical value	Analytical value	Absolute error
0	0	0.5000	1.0000	0.5000
0.1	0.1	0.071419	1.0000	0.9286
0.2	0.2	0.022251	1.0000	0.9777
0.3	0.3	0.0059356	0.99995	0.9940
0.4	0.4	0.0011129	0.99331	0.9922
0.5	0.5	0.0001371	0.5000	0.4999
0.6	0.6	1.0961e-05	0.0066929	0.0067
0.7	0.7	5.7353e-07	4.5398e-05	4.4824e-05
0.8	0.8	1.9973e-08	3.059e-07	2.8593e-07
0.9	0.9	4.7392e-10	2.0612e-09	1.5872e-09
1.0	1.0	0.0	1.3888e-11	1.3888e-11
L_2				0.442952
L_∞				0.996807

Table 2. Absolute error between the numerical and analytical values at A = 2.0, C = 0.05

x	t	Numerical value	Analytical value	Absolute error
0	0	0.25	0.49665	0.2467
0.1	0.1	0.12951	0.49101	0.3615
0.2	0.2	0.075079	0.47629	0.4012
0.3	0.3	0.047618	0.4404	0.3928
0.4	0.4	0.030846	0.36553	0.3347
0.5	0.5	0.019766	0.25	0.2302
0.6	0.6	0.012283	0.13447	0.1222
0.7	0.7	0.0072688	0.059601	0.0523
0.8	0.8	0.0039561	0.023713	0.0198
0.9	0.9	0.0017256	0.0089931	0.0073
1.0	1.0	2.2699e-05	0.0033464	0.0033
L_2				0.155768
L_∞				0.403353

Table 3. Absolute error between the numerical and analytical values at A = 3.0, C = 0.1

x	t	Numerical value	Analytical value	Absolute error
0	0	0.16667	0.30805	0.1414
0.1	0.1	0.11332	0.2936	0.1803
0.2	0.2	0.079198	0.27252	0.1933
0.3	0.3	0.0585	0.24369	0.1852
0.4	0.4	0.044279	0.20749	0.1632
0.5	0.5	0.033691	0.16667	0.1330
0.6	0.6	0.025343	0.12585	0.1005
0.7	0.7	0.018454	0.089647	0.0712
0.8	0.8	0.012529	0.060809	0.0483
0.9	0.9	0.0072105	0.039734	0.0325
1.0	1.0	0.002231	0.025286	0.0231
L_2				0.077955
L_∞				0.193327

Table 4. Absolute error between the numerical and analytical values at A = 4.0, C = 0.5

x	t	Numerical value	Analytical value	Absolute error
0	0	0.125	0.15561	0.0306
0.1	0.1	0.11556	0.14967	0.0341
0.2	0.2	0.1064	0.14361	0.0372
0.3	0.3	0.098681	0.13746	0.0388
0.4	0.4	0.09209	0.13124	0.0392
0.5	0.5	0.0864	0.125	0.0386
0.6	0.6	0.081442	0.11876	0.0373
0.7	0.7	0.077087	0.11254	0.0355
0.8	0.8	0.073233	0.10639	0.0332
0.9	0.9	0.069802	0.10033	0.0305
1.0	1.0	0.067235	0.094385	0.0271
L_2				0.021960
L_∞				0.039163

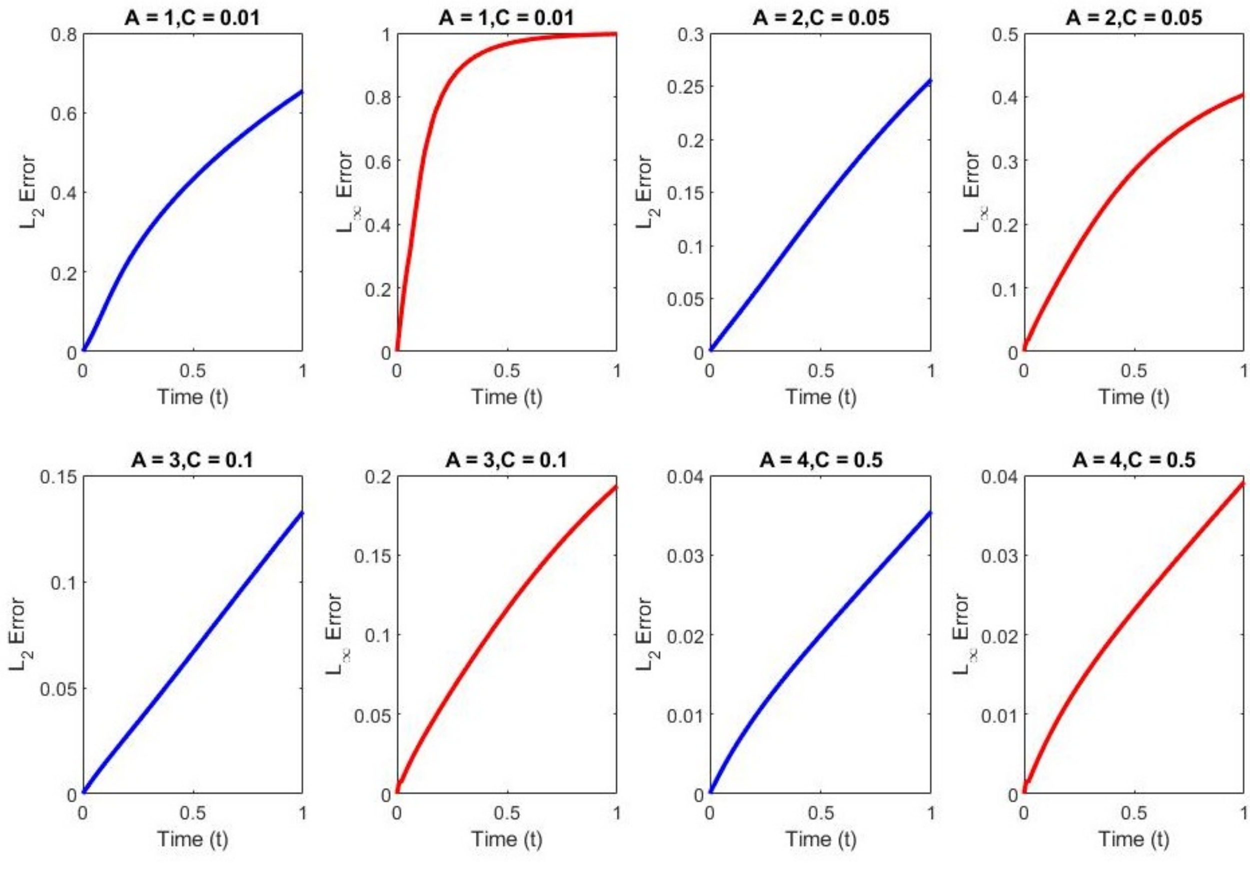

Figure 2. L_2 and L_∞ error norms at various values of A and C.

It has been observed from the Table 1-4 and Figure 2 that the value of L_2 and L_∞ decrease as the value nonlinear coefficient A and the dissipation coefficient C increases. As it is seen from the Table 1-4, the error norms L_2 and L_∞ are sufficiently small and satisfactorily acceptable. A decreasing trend in the L_2 and L_∞ error norm as the mesh size or time step is refined indicates improved accuracy and convergence of the numerical scheme. A lower L_2 and L_∞ error norm indicates better accuracy and convergence of the numerical scheme.

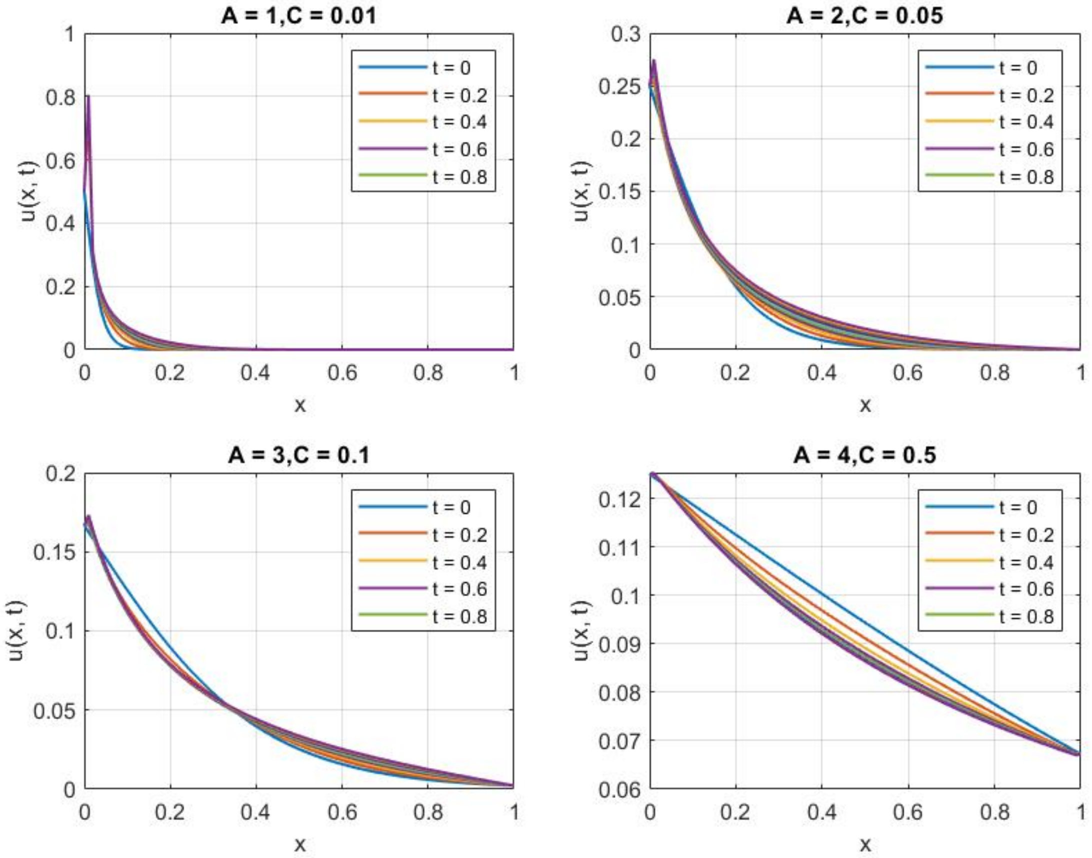

Figure 3. Numerical solutions at different times for (a) $A = 1, C = 0.01$,(b)$A = 2, C = 0.05$,(c) $A = 3, C = 0.1$ and (d)$A = 4, C = 0.5$.

The graphs of the numerical solution at different times for increasing values of the nonlinear coefficient will show more pronounced changes in the solution profile, with sharper transitions and larger gradients.The graphs of the numerical solution at different times for increasing values of the dissipation coefficient will exhibit smoother profiles with reduced oscillations and less pronounced sharp transitions.

6. CONCLUSION

In this study, one dimensional Burgers' equation is numerically solved using the Crank-Nicholson method and the behavior of shock wave profiles are investigated in warm dusty plasmas considering dust charge variation. The graphs of the numerical results are plotted to compare with the analytical results and it is clear from the comparison that the graphs of numerical results are close with the results obtained by analytically and better than numerical solutions obtained by some other methods in literature. The propagation of the shock waves for various values of nonlinear coefficient and dissipation coefficient have been observed and it is found that the wave front become more sharper as the dissipation coefficient decreases. The absolute error is computed for checking the accuracy and efficiency of the present technique. From the study, it has been noted that the accuracy and efficiency of the technique depends on the value of dissipation coefficient and the result will get better when the dissipation coefficient takes smaller value.

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ЧИСЕЛЬНИЙ ПІДХІД ДО РІВНЯННЯ БЮРГЕРСА В ЗАПИЛЕНІЙ ПЛАЗМІ ЗІ ЗМІНОЮ ЗАРЯДУ ПИЛУ

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У цій статті застосовано метод Кренка-Ніколсона для вирішення одновимірного нелінійного рівняння Бюргерса в теплій запорошеній плазмі зі зміною заряду пилу. Проведено аналіз отриманих чисельних результатів та порівняння з аналітичними результатами. На основі порівняння очевидно, що числові результати, отримані в результаті аналізу, добре узгоджуються з аналітичним рішенням. Похибка між аналітичним і чисельним розв'язками рівняння Бюргерса обчислюється за двома нормами похибки, а саме L_2 і L_∞. Аналіз стабільності виконується за методом фон-Неймана, і він виявляється безумовно стабільним згідно з аналізом.

Ключові слова: *гаряча пилова плазма; рівняння Бюргерса; метод Кранка-Нікольсона; аналіз стійкості фон Неймана*