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EXISTENCE OF SMALL AMPLITUDE KDV AND MKDV SOLITONS IN A MAGNETIZED DUSTY PLASMA WITH $q-$NONEXTENSIVE DISTRIBUTED ELECTRONS

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The existence and propagating characteristics of small amplitude dust-ion-acoustic (DIA) Korteweg-de Vries (KdV) and modified KdV solitons in a three component magnetized plasma composed of positive inertial ions with pressure variation, noninertial electrons and negative charged immobile dust grains are theoretically and numerically investigated when the electrons obey a $q-$nonextensive velocity distribution. Utilizing the reductive perturbation method, to derive KdV and modified KdV equations and obtain the DIA soliton solutions along with the corresponding small amplitude potentials. This study shows that there are compressive and/or rarefactive solitons and no soliton at all, due to the parametric dependency on the first-order nonlinear coefficient through the number density of positive ions and negative dust grains and the electron nonextensivity. The coexistence of compressive and rarefactive solitons appears by raising the measure of nonlinearity coefficient to the second-order using the modified KdV equation. The properties such as speed, amplitude, width etc. of the propagating soliton are numerically discussed.

Keywords: *Dust ion acoustic wave; Magnetized plasma; $q-$nonextensive distribution; Reductive perturbation method; KdV equation*

PACS: 52.27.Lw; 52.25.Xz; 52.35.Fp; 52.35.Qz; 52.35.Sb

1. INTRODUCTION

The propagation of nonlinear electrostatic waves in many physical situation is an fascinating and recently growing field of research in plasma dynamics; however its exploration had started in the long past. During last few decades, the discovery of dust charged grains in the plasma medium have opened a great deal of interest in the minds of modern plasma workers, because its presence is drastically change in the characteristics of waves in plasma. The role and influence of dust particles are beautifully explained by several experts in space and astrophysical plasmas such as in Saturn's rings, Earth's ionosphere and magnetosphere, in Planetary rings and magnetosphere, in Cometary tails, interstellar medium as well as in laboratory plasmas (not mentioned here). The formation and existence of nonlinear electrostatic waves in a magnetized plasma consisting of charged dust particles has been extensively investigated theoretically [1–4] and experimentally [5–8]. DIA waves are weak-frequency waves which involve in the movement of massive ions and form compression and rarefaction region just like in travelling sound waves; in which, the inertia is attributed to the number density of ions, whereas the thermal pressure of electrons is assumed to establish the restoring force which is responsible for initiating the plasma waves, and the negatively charged dust grains are expected to remain stationary in this realism. The understanding of the nonlinear propagation of DIA solitons in an unmagnetized plasma having three components namely, positive ions, electrons, and negatively charged dust grains is now theoretically [9–12] and experimentally [13, 14] well-established. The presence of an external magnetic field in a plasma is not only affects the existence and direction of DIA modes, but also introduces new approach of propagation and inherent oscillations. Many studies have published into how the magnetic field influences the dynamic characteristics of DIA waves, considering both linear and nonlinear properties. For instance, Ghosh et al. [15] conducted theoretical investigations into DIA wave propagation in a magnetized dusty plasma with charge fluctuations using the reductive perturbation method. Anowar and Mamun [16] derived a KdV equation to describe the oblique propagation of solitary waves in an adiabatic magnetized dusty plasma. While the oblique propagation of large amplitude DIA solitary waves in a magnetized dusty plasma are studied by Saha and Chatterjee [17]. Besides, it has been established by many researchers in various relevant scenarios that the presence of a magnetic field significantly alters the inherent characteristics of DIA waves as they propagate through plasmas. [18–22]. Recently, Abdus et al. [23] employed the reductive perturbation approach to theoretically investigate the influence of higher-order nonlinear and dispersive effects on the fundamental characteristics of DIA solitary waves in a magnetized dusty plasma.

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Nonlinear electrostatic waves in plasmas can be treated mathematically in a variety of ways, but two main approaches are commonly employed. The most popular method involves employing the Sagdeev pseudo-potential method to study the arbitrary amplitude of waves in plasmas. On the other hand, the evolution equation for the small amplitude electrostatic waves may be extracted using a reductive perturbation method. The velocity distribution function of plasma particle plays a crucial role in influencing the nonlinear behavior of plasma waves. In many instances, the Maxwellian velocity distribution function is the standard choice for describing electron's behavior. However, in recent years, there has been a notable increase in interest regarding the study of particle distribution in plasma using the Boltzmann Gibbs Shannon entropy. This concept was originally introduced by Renyi [24] and has garnered significant attention. The generalization of Boltzmann-Gibbs entropy in the non-equilibrium states with the q-nonextensive entropy suggested by Tsallis [25]. By citing this approach many researchers have paid more attention to employ the nonextensive distribution for the number density of particles in plasma [26–32]. The q-nonwxtensive distribution function shows distinct behaviors based on the values of q, which determines the quantity of the nonextensivity of the system being studied. For $q < 1$, the distribution function indicates the plasma with higher number of superthermal particles compared to that of Maxwellian case (superextensivity), whereas for $q > 1$, the distribution function shows the plasma with large number of low-speed particles compared to that of Maxwellian case (subextensivity). Moreover, if $q = 1$, the distribution fuctiuon is then reduced to common Maxwell-Boltzmann velocity distribution [33].

The main objective of the paper is to the investigate the propagating behavior of nonlinear DIA solitary waves in a three component magnetized plasma consisting of inertial ions, nonextensive electrons and charged dust grains. For this work, the reductive perturbation method is used to investigate the nonlinear DIA waves and we emphasize the DIA solitary waves of small amplitude. The paper is organized as follows: in Section-1, we have given the usual introduction; in Section-2, we give the basic set governing equations for describing the plasma model; in Section-3 and Section-4, we derivation of the KdV and modified KdV equation respectively, where the result and discussion are made and Finally we summarize our work in Section-5. At the end, the references are included.

2. THE BASIC EQUATIONS FOR PLASMA SYSTEM

We consider a magnetized collisonless plasma system consisting of positively charged inertial ions, negatively charged dust grains and noninertial electrons which obey $q-$nonextensive distributions. Therefore, at equilibrium $z_i n_{i0} = n_{e0} + z_d n_{d0}$, where n_{i0}, n_{e0} and n_{d0} are the particle number densities of ion, electron and dust respectively at equilibrium, while z_i and z_d are the ion and dust charge numbers. An uniform external magnetic field is assumed along $z-$direction in the plasma, *i.e.* $\vec{B} = B_0\hat{z}$. The charges carried by the dust grains are considered to remain constant, and their effects on the dynamics of DIA waves is ignored. As the plasma possesses a finite ion temperature, we keep the ion pressure gradient term in our considerations. The dynamics of nonlinear waves structures in such a plasma system are governed by the following unnormalized fluid equations

$$\frac{\partial N}{\partial T} + \overrightarrow{\nabla}'.(N\overrightarrow{V}) = 0, \tag{1}$$

$$\frac{\partial \overrightarrow{V}}{\partial T} + (\overrightarrow{V}.\overrightarrow{\nabla}')\overrightarrow{V} = -\frac{z_i e}{m}\overrightarrow{\nabla}'\Phi + \frac{z_i e}{m}(\overrightarrow{V} \times \overrightarrow{B}) - \frac{1}{mN}\overrightarrow{\nabla}' P, \tag{2}$$

$$\frac{\partial P}{\partial T} + (\overrightarrow{V}.\overrightarrow{\nabla}')P + \gamma P(\overrightarrow{\nabla}'.\overrightarrow{V}) = 0, \tag{3}$$

$$\nabla'^2\Phi = 4\pi e[N_e + z_d n_{d0} - z_i N]. \tag{4}$$

where N, $\overrightarrow{V}$, m, P, e, N_e and Φ are respectively the ion number density, ion fluid velocity, mass of an ion, ion fluid pressure, electronic charge, electron number density and electrostatic potential. And $\gamma = C_{ip}/C_{iv}$ is the adiabatic index, where C_{ip} (C_{iv}) is the specific heat of ion at constant pressure (volume). We have taken adiabatic index, $\gamma = 3$. To normalize the set of equations (1)-(4), we consider the dimensionless variables as follows:

$$t = \frac{T}{\omega_{pi}^{-1}}, \quad n = \frac{N}{n_{i0}}, \quad n_e = \frac{N_e}{n_{e0}}, \quad \vec{v} = \frac{\overrightarrow{V}}{c_i}, \quad \phi = \frac{e\Phi}{k_b T_e}, \quad \overrightarrow{\nabla} = \frac{\overrightarrow{\nabla}'}{\lambda_D^{-1}}, \quad p = \frac{P}{p_{i0}}.$$

with the characteristic ion plasma frequency ω_{pi}, the electron Debye length λ_D and equilibrium ion pressure p_{i0} are given by

$$\omega_{pi} = \sqrt{\frac{4\pi n_{i0} e^2 z_i^2}{m}}; \quad \lambda_D = \sqrt{\frac{k_b T_e}{4\pi n_{i0} e^2 z_i}} \quad \text{and} \quad p_{i0} = n_{i0} k_b T_i.$$

Thus, the ion acoustic speed $c_i = \omega_{pi}\lambda_D = \sqrt{z_i k_b T_e/m}$. Where T_e (T_i) and k_b are the characteristic electron (ion) temperature and the Boltzmann constant respectively. As the electron velocity distribution is assumed to be $q-$nonextensive, so the normalized expression for the number density of electron is given by [34–36]

$$n_e = [1 + (q-1)\phi]^{\frac{(q+1)}{2(q-1)}}. \tag{5}$$

Where the parameter q stands for the strength of electrons nonextensivity and it is a real number greater than -1. The normalized form of the set of equations (1)-(4) can be written in the component form as

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} + \frac{\partial (nv)}{\partial y} + \frac{\partial (nw)}{\partial z} = 0, \tag{6}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial \phi}{\partial x} + \Omega v - \frac{\sigma}{n}\frac{\partial p}{\partial x}, \tag{7}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial \phi}{\partial y} - \Omega u - \frac{\sigma}{n}\frac{\partial p}{\partial y}, \tag{8}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial \phi}{\partial z} - \frac{\sigma}{n}\frac{\partial p}{\partial z}, \tag{9}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} + 3p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0, \tag{10}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 1 - n + a_1\phi + a_2\phi^2 + a_3\phi^3 + \ldots, \tag{11}$$

with the coefficients $a_1,\ a_2,\ a_3,\ \ldots$ appear in the last equation are expressed by

$$a_1 = \frac{(1-\mu)(1+q)}{2}, \quad a_2 = \frac{a_1(3-q)}{4}, \quad a_3 = \frac{a_2(5-3q)}{6} \tag{12}$$

In the above equations $\overrightarrow{V} = (u, v, w)$, where $u,\ v$, and w are the ion fluid velocities along $x,\ y$ and z axes. We have defined $\Omega = \frac{\omega_{ci}}{\omega_{pi}}$, in which $\omega_{ci} = \frac{ez_iB_0}{m}$ is ion gyrofrequency, $\sigma = \frac{T_i}{z_iT_e}$ is ion-to-electron temperature ratio and $\mu = \frac{z_dn_{d0}}{z_in_{i0}} < 1$ is dust-to-ion number density ratio.

3. THE KDV EQUATION AND SMALL AMPLITUDE WAVES

3.1. Derivation Of KdV Equation:

To investigate the dynamics of propagating DIA waves of small amplitude, the reductive perturbation method is applied to the equations (6)-(11) to derive nonlinear KdV equation for the present plasma model. For this, the independent space variables (x, y, z, t) are stretched to (ξ, τ) by

$$\xi = \epsilon^{1/2}(l_xx + l_yy + l_zz - U_pt), \quad \tau = \epsilon^{3/2}t. \tag{13}$$

Where ϵ $(0 < \epsilon \ll 1)$ is a dimensionless expansion parameter, is represents the level of the perturbation, U_p is the phase velocity of the waves, and $l_x,\ l_y$ and l_z are the direction cosines of the wave vector $\vec{k}$ along the $x,\ y$ and z axes respectively so that $l_x^2 + l_y^2 + l_z^2 = 1$. We now write all the dependent variables in the power series of ϵ about their equilibrium state as

$$\left.\begin{aligned}
n &= 1 + \epsilon n_1 + \epsilon^2 n_2 + \epsilon^3 n_3 + \ldots,\\
p &= 1 + \epsilon p_1 + \epsilon^2 p_2 + \epsilon^3 p_3 + \ldots,\\
u &= \epsilon^{3/2} u_1 + \epsilon^2 u_2 + \epsilon^{5/2} u_3 + \ldots,\\
v &= \epsilon^{3/2} v_1 + \epsilon^2 v_2 + \epsilon^{5/2} v_3 + \ldots,\\
w &= \epsilon w_1 + \epsilon^2 w_2 + \epsilon^3 w_3 + \ldots,\\
\phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^3 \phi_3 + \ldots,
\end{aligned}\right\} \tag{14}$$

As a result of drift $\vec{E} \times \vec{B}$ in a magnetized plasma causes u_1 and v_1 to be smaller [37]. Now, on substituting the transformations (13) and the expansions (14) into the equations (6)-(11) and then equating the lowest order terms of ϵ, we get the first order perturbed quantities as

$$-U_p\frac{\partial n_1}{\partial \xi} + l_z\frac{\partial w_1}{\partial \xi} = 0, \tag{15}$$

$$l_x\frac{\partial \phi_1}{\partial \xi} + \sigma l_x\frac{\partial p_1}{\partial \xi} - \Omega v_1 = 0, \tag{16}$$

$$l_y \frac{\partial \phi_1}{\partial \xi} + \sigma l_y \frac{\partial p_1}{\partial \xi} + \Omega u_1 = 0, \tag{17}$$

$$-U_p \frac{\partial w_1}{\partial \xi} + l_z \frac{\partial \phi_1}{\partial \xi} + \sigma l_z \frac{\partial p_1}{\partial \xi} = 0, \tag{18}$$

$$-U_p \frac{\partial p_1}{\partial \xi} + 3 l_z \frac{\partial w_1}{\partial \xi} = 0, \tag{19}$$

$$n_1 - a_1 \phi_1 = 0. \tag{20}$$

Integrating (15),(18) and (19) by using the boundary conditions: $n_1 = p_1 = w_1 = 0$ and $\phi_1 = 0$ as $|\xi| \longrightarrow \infty$, and then we express above first-order quantities as a function of ϕ_1, namely

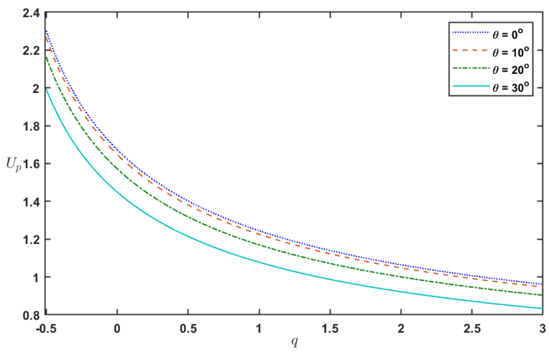
$$\left.\begin{aligned} n_1 &= \frac{l_z}{U_p} w_1 = a_1 \phi_1, \\ u_1 &= -\frac{l_y}{\Omega}(1 + 3\sigma a_1)\frac{\partial \phi_1}{\partial \xi}, \\ v_1 &= \frac{l_x}{\Omega}(1 + 3\sigma a_1)\frac{\partial \phi_1}{\partial \xi}, \\ w_1 &= -\frac{l_z}{U_p}(1 + 3\sigma a_1)\phi_1, \\ p_1 &= 3n_1 = \frac{3l_z}{U_p} w_1 = 3a_1\phi_1. \end{aligned}\right\} \tag{21}$$

Moreover, the expression for phase velocity can obtained as

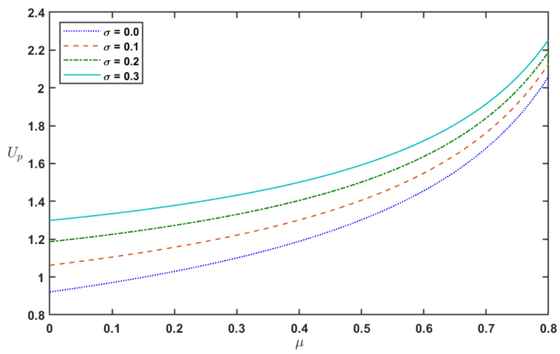
$$U_p = \ l_z \sqrt{\frac{1}{a_1} + 3\sigma}. \tag{22}$$

Form expression (22), the phase velocity U_p of DIA waves depends on ions and electrons temperature by σ, the dust and ion number density by μ, nonextensive parameter q and the direction cosine $l_z = \cos\theta$, where θ is the obliqueness angle between $\vec{B}$ and the wave vector $\vec{k}$.

In Figure[1a], we showed the variation of phase velocity U_p versus nonextensive parameter q with varying obliqueness angle θ, while the variation of U_p versus dust-to-ion number density ratio μ with varying ion-to-electron temperature ratio σ as depicted in Figure[1b]. Where we found that the phase velocity of propagating DIA waves drops (Figure[1a]) with the increasing of q and also with the increasing of θ. That means the phase velocity is advances for the parallel propagating and more superthermal electrons than the obliquely propagating and large low-speed electrons. On the other hand, the phase velocity grows (Figure[1b]) with the increasing of μ as well as σ. That is the the phase velocity is higher for the increase in ion temperature and more populated negative dust particles than the increase in electron temperature and less populated negative dust particles.



(a)



(b)

Figure 1. The variation of phase velocity U_p **(a)** versus nonextensive parameter q with varying obliqueness angle θ, with fixed ion-to-electron temperature $\sigma = 0.1$ and dust-to-ion number density $\mu = 0.2$; and **(b)** versus μ with varying σ and fixed $\theta = 15^o,\ q = 1.2$.

Now, for the second-order perturbed quantities, we equate the coefficients of next higher order terms in ϵ from the equations (6)-(11), we obtain the following equations

$$-U_p \frac{\partial n_2}{\partial \xi} + \frac{\partial n_1}{\partial \tau} + l_x \frac{\partial u_2}{\partial \xi} + l_y \frac{\partial v_2}{\partial \xi} + l_z \frac{\partial w_2}{\partial \xi} + l_z \frac{\partial (n_1 w_1)}{\partial \xi} = 0, \tag{23}$$

$$-U_p\frac{\partial u_1}{\partial \xi}-\Omega v_2=0 \implies v_2=-\frac{U_p}{\Omega}\frac{\partial u_1}{\partial \xi}, \tag{24}$$

$$-U_p\frac{\partial v_1}{\partial \xi}+\Omega u_2=0 \implies u_2=\frac{U_p}{\Omega}\frac{\partial v_1}{\partial \xi}, \tag{25}$$

$$-U_p\frac{\partial w_2}{\partial \xi}-U_pn_1\frac{\partial w_1}{\partial \xi}+\frac{\partial w_1}{\partial \tau}+w_1l_z\frac{\partial w_1}{\partial \xi}+l_z\frac{\partial \phi_2}{\partial \xi}+n_1l_z\frac{\partial \phi_1}{\partial \xi}+\sigma l_z\frac{\partial p_2}{\partial \xi}=0, \tag{26}$$

$$-U_p\frac{\partial p_2}{\partial \xi}+\frac{\partial p_1}{\partial \tau}+l_zw_1\frac{\partial p_1}{\partial \xi}+3l_x\frac{\partial u_2}{\partial \xi}+3l_y\frac{\partial v_2}{\partial \xi}+3l_z\frac{\partial w_2}{\partial \xi}+l_zp_1\frac{\partial w_1}{\partial \xi}=0, \tag{27}$$

$$-\frac{\partial^2 \phi_1}{\partial \xi^2}-n_2+a_1\phi_2+a_2\phi_1^2=0 \implies n_2=a_1\phi_2+a_2\phi_1^2-\frac{\partial^2 \phi_1}{\partial \xi^2}. \tag{28}$$

Eliminating w_2 and p_2 from (23), (26) and (27) and then putting the values of v_2, u_2 and n_2 from (24), (25) and (28) and using the values of first-order quantities from (21), the KdV equation is obtained as

$$\frac{\partial \varphi}{\partial \tau}+\mathcal{A}\varphi\frac{\partial \varphi}{\partial \xi}+\mathcal{B}\frac{\partial^3 \varphi}{\partial \xi^3}=0. \tag{29}$$

with $\phi_1=\varphi$ and the the nonlinear coefficient $\mathcal{A}$ and dispersion coefficient $\mathcal{B}$ are given by

$$\mathcal{A}=\frac{1}{2a_1U_p}\left\{4a_1^2U_p^2-a_1l_z^2-2l_z^2\left(\frac{a_2}{a_1}\right)\right\}, \tag{30}$$

$$\mathcal{B}=\frac{1}{2a_1U_p}\left\{\frac{l_z^2}{a_1}+(1-l_z^2)\frac{a_1U_p^4}{l_z^2\Omega^2}\right\}. \tag{31}$$

The nonlinear term $\mathcal{A}$ (causing wave steepening) and the dispersion term $\mathcal{B}$ (causing wave broadening) are crucial factors not only for the structure of the propagation of DIA solitary waves but also for specifying the soliton's characteristics. So it is important to analysed on the the parametric dependence of these two terms

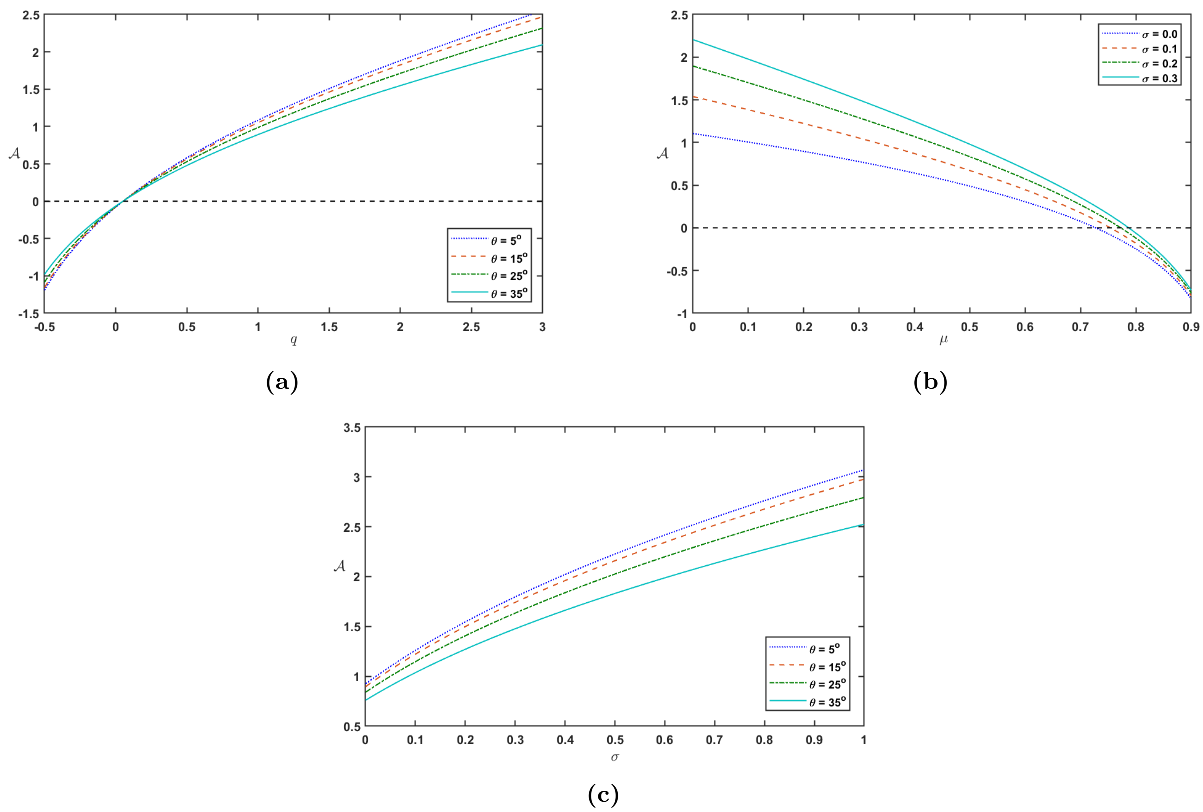


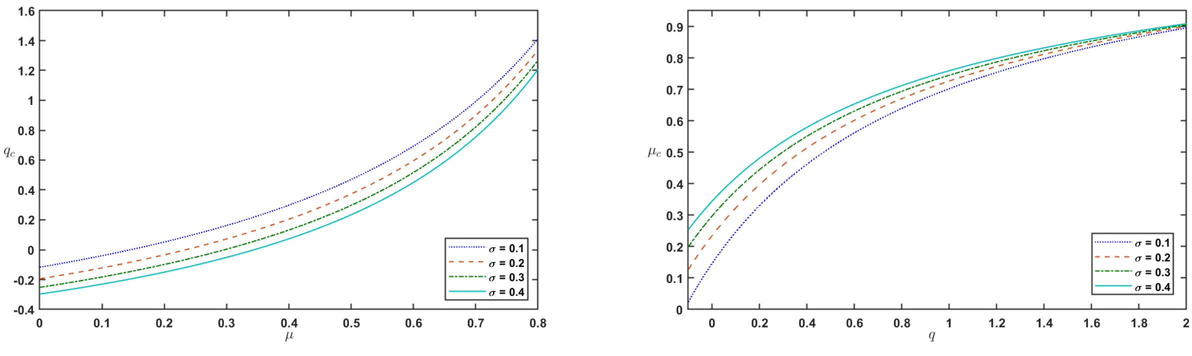
Figure 2. The variation of nonlinear term $\mathcal{A}$ **(a)** versus nonextensive parameter q with varying obliqueness angle θ, with fixed ion-to-electron temperature $\sigma=0.1$ and dust-to-ion number density $\mu=0.2$; **(b)** versus μ with varying σ and fixed $\theta=15^o$, $q=1.2$; and **(c)** versus σ with varying θ and fixed $\mu=0.2$, $q=1.2$.

in our considered plasma system. From expressions (30) and (31), both $\mathcal{A}$ and $\mathcal{B}$ are the functions obliqueness angle θ, nonextensive parameter q, ion-to-electron temperature ratio σ and dust-to-ion number density ratio μ. Besides, the dispersion term $\mathcal{B}$ is also a function of external magnetic field strength B_0 through Ω but the nonlinearity is unaffected at all. Interestingly, we can see that both $\mathcal{A}$ and $\mathcal{B}$ become zero for limiting $\theta \rightarrow 90^o$, in this case the propagating DIA solitons does not exist, that is the waves are electrostatic is abolished for the larger values of θ, and they should instead be electromagnetic in nature [38]. Again, the influences of the external magnetic field disappears for $\theta = 0^o$, in this case the terms $\mathcal{A}$ and $\mathcal{B}$ become to the condition for unmagnetized plasma system. Thus, we have consider small value for obliqueness angle θ $(0 < \theta < 55^o)$ in this investigation. It is seen from the expression(31) that the dispersion $\mathcal{B}$ acquires only positive values in varying different physical parameters under consideration and it increases with θ, σ and μ and decreases with q and Ω (figures not shown here). In order to inspect the parametric effects on the nonlinear term $\mathcal{A}$, we have plotted the variation of $\mathcal{A}$ versus nonextensive parameter q with varying obliqueness angle θ; $\mathcal{A}$ versus dust-to-ion number density ratio μ with varying ion-to-electron temperature ratio σ and $\mathcal{A}$ versus σ with varying θ in Figure[2]. Where, we find that the nonlinearity increases with the increase of q and also increase of σ, while it decreases with the increase of μ and also increase of θ. Form first two panels of Figure[2], we have found that the nonlinear term $\mathcal{A}$ changes its sign from positive to negative or vice versa and it become zero for a critical composite value of nonextensive parameter (say q_c) for a fixed value of μ, or a critical composite value of dust-to-ion number density ratio (say μ_c), for a fixed value of q. That means the KdV soliton can changed its types from compressive to rarefactive or vice versa in the considered plasma system. Now, by solving the equation $A(q, \sigma, \mu, l_z) = 0$ for μ and q separately, both q_c and μ_c are found as

$$q_c = \frac{-\left[3(1-\mu)\{4\sigma(1-\mu)+1\}+1\right]}{12\sigma(1-\mu)^2} \pm \frac{\sqrt{\left[3(1-\mu)\{4\sigma(1-\mu)+1\}+1\right]^2 - 24\sigma(1-\mu)^2\left[3(1-\mu)\{2\sigma(1-\mu)+1\}-3\right]}}{12\sigma(1-\mu)^2}, \quad (32)$$

$$\mu_c = \frac{3\{4\sigma(1+q)+1\}}{12\sigma(1+q)} \pm \frac{\sqrt{9\{4\sigma(1+q)+1\}^2 - 24\sigma\{6\sigma(1+q)^2+4q\}}}{12\sigma(1+q)}, \quad (33)$$

Since these critical values are identifies the polarity of DIA solitary waves, so it is important to analysed them. From expressions (32) and (33), we seen that q_c (μ_c) is a explicit function of σ and μ (q), and both q_c and μ_c are undefined when $\sigma = 0$. For $-1 < q < q_c$ (or $1 > \mu > \mu_c$) with fixed μ (or q), $\mathcal{A} < 0$; And for $q > q_c$ (or $0 \leq \mu < \mu_c$) with fixed μ (or q), $\mathcal{A} > 0$. The variation of q_c versus μ and μ_c versus q with different values of σ are shown in Figure[3], and we found that the value of q_c (μ_c) increases with μ (q). We have also predicted the value of q_c is reduced while μ_c is raised as the increase ion-to-electron temperature ratio σ in our considered plasma system.



(a) **(b)**

Figure 3. The variation of critical value **(a)** q_c versus μ and **(b)** μ_c versus q with varying $\sigma > 0$

3.2. Solitary Wave Solution

To obtain the stationary wave solutions of the KdV equation (29), we introduce a new transformation variable $\chi = \xi - \nu\tau$, where ν is the travelling wave velocity in the linear $\chi-$ space. Then, the KdV equation (29) becomes the ordinary differential equation,

$$-\nu\frac{d\varphi}{d\chi} + \mathcal{A}\varphi\frac{d\varphi}{d\chi} + \mathcal{B}\frac{d^3\varphi}{d\chi^3} = 0 \quad (34)$$

This equation is known the reduced KdV equation and its well-known solution (*i.e.*, solitary wave solution) is given by

$$\varphi = \varphi_m \operatorname{sech}^2\left(\frac{\chi}{\delta}\right) \tag{35}$$

where $\varphi_m = \dfrac{3\nu}{\mathcal{A}}$ and $\delta = \sqrt{\dfrac{4\mathcal{B}}{\nu}}$, represents the peak amplitude and the width of the pulse of solitary waves respectively and here both ν and $\mathcal{B}$ are positive values.

Now, integrating twice equation(32) over χ, and using the boundary conditions: $\varphi = \frac{d\varphi}{d\chi} = \frac{d^2\varphi}{d\chi^2} = 0$ as $|\chi| \longrightarrow \infty$, we have

$$\frac{1}{2}\left(\frac{d\varphi}{d\chi}\right)^2 + P(\varphi) = E_c \tag{36}$$

This is the form of law of conservation of energy, in which E_c is the integration constant and is acted as the entire energy of the system. The term $\frac{1}{2}(d\varphi/d\chi)^2$ is presumed as kinetic energy while $P(\psi)$ indicates the potential energy that is defined as

$$P(\varphi) = \left[\frac{\mathcal{A}}{6\mathcal{B}}\right]\varphi^3 - \left[\frac{\nu}{2\mathcal{B}}\right]\varphi^2 \tag{37}$$

The properties of this potential as **(i)** $P(\varphi) = P'(\varphi) = 0$ and $P''(\varphi) < 0$, for $\varphi = 0$; **(ii)** $P(\varphi) = 0,\ P'(\varphi) \neq 0$, for $\varphi = \varphi_m$ and $P(\varphi) < 0$ in between 0 and φ_m. That is the potential $P(\varphi)$ has double roots, one root is $\psi = 0$, at which $P(\psi)$ reaches its highest value value also and other root is $\varphi = \varphi_m$. Thus, we can also analysed the dynamical characteristics of DIA solitary waves in the considered magnetized plasma system through the amplitude potential $P(\varphi)$ for different core plasma parameters under consideration.

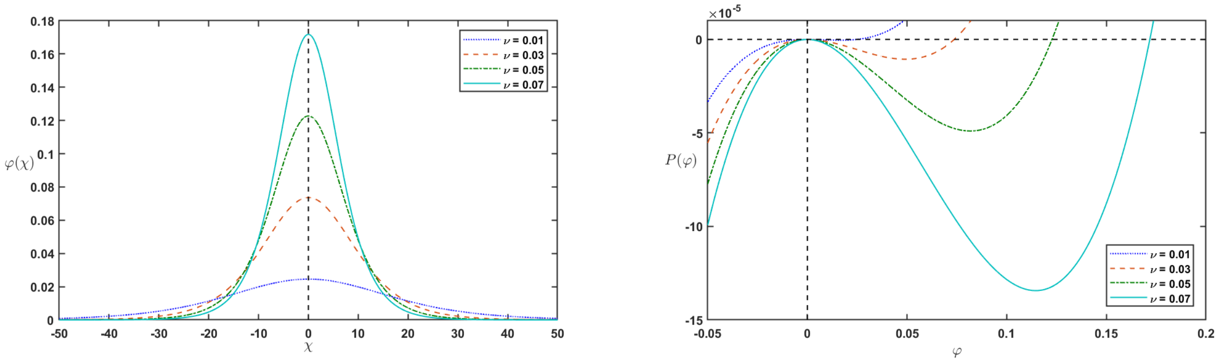
3.3. Numerical Discussions for Parametric effects

In order to discuss the parametric effects on the dynamical characteristics of DIA solitons for small amplitude limit by plotting both the solitary wave profile $\varphi(\chi)$ given in equation (35) against the linear parameter χ and the amplitude potential $P(\varphi)$ given in equation (37) against the electrostatic potential φ are as depicted in Figures[4-9]. It is important to notice one thing that, where the curve of the potential $V(\varphi)$ crosses the $\varphi-$axis from below at some point of φ, from that point we predicted the soliton's amplitude φ_m.

In Figures[4a-4b], we showed the variation of $\varphi(\chi)$ versus χ and the variation of $P(\varphi)$ versus φ with different values of travelling wave velocity ν for fixed other parameters, where we observed only one type of solitons *i.e.,* compressive solitons propagates and the amplitude of the pulse of compressive DIA soliton increase while width decrease with the increases in ν. In Figures[5a-5b], we showed the variation of $\varphi(\chi)$ versus χ and also $P(\varphi)$ versus φ with different values of obliqueness angle θ and fixed other parameters, where we found that the propagating DIA soliton is compressive and both the amplitude and width of the compressive solitary pulse to increase with obliqueness angle θ. For the wave propagates along the external magnetic field (i.e., $\theta = 0^o$), the values of the amplitude and width gets smaller and as θ increases, both the amplitude and width increases. That means, we predicts that the energy of the propagating DIA soliton is directly influenced by the obliqueness propagating angle. Likewise, the variation of $\varphi(\chi)$ versus χ and also $P(\varphi)$ versus φ with different values of ion-to-electron temperature ratio σ and fixed other parameters are shown in Figures[6a-6b], where we observed the compressive DIA soliton and its amplitude of the solitary pulse is seen to decrease, while width to increase as the value of σ gets higher. That is, in the considered plasma system by increasing (decreasing) the temperature of ion (electron) species with keeping the electron (ion) temperature fixed, will typically change the geometrical structure of the propagating DIA soliton.

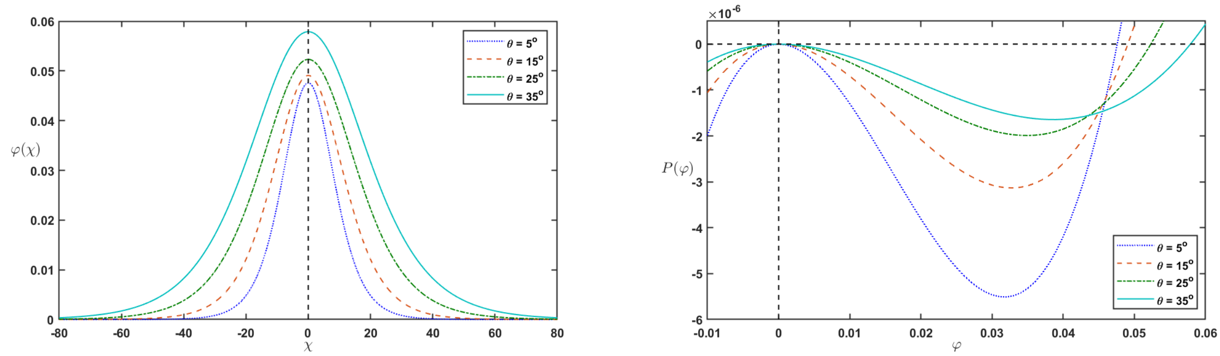
In Figures[7a-7b], we showed the variation of $\varphi(\chi)$ versus χ and also the variation of $P(\varphi)$ versus φ with different values of parameters q and fixed other parameters: $\theta = 15^o,\ \sigma = 0.1,\ \mu = 0.2,\ \Omega = 0.3$ and $\nu = 0.02$, where we found that the soliton type transformed from rarefactive (negative potential) to compressive (positive potential), which is obvious from our results that the sign of nonlinearity $\mathcal{A}$ changes from negative to positive for varying q. It is observed that both amplitude and width of the rarefactive DIA solitary pulse to increase as the value of q increases in between -1 and q_c, whereas both amplitude and width of the compressive DIA solitary pulse to decrease as the value of $q > q_c$. For the chosen parametric values, we obtain $q_c = 0.0516$. Thus, it is predicting that the electron nonextensivity makes a noticeable impact on the dynamics of DIA soliton in the present plasma system. An analogous result is obtained due to the variation of dust-to-ion number density ratio μ with fixed other parameters: $\theta = 15^o,\ \sigma = 0.1,\ q = 1.2,\ \Omega = 0.3$ and $\nu = 0.02$, as shown in the Figures[8a-8b]. That is, the propagating DIA solitons can transits from compressive to rarefactive with varying μ. Both the amplitude and width of the compressive DIA solitary pulse are seen to increases for the increase of μ in between 0 and μ_c; while the amplitude and width of the rarefactive DIA solitary pulse are seen to decreases for the increase of $\mu > \mu_c$, in which $\mu_c = 0.7539$. That is, increasing the population of negatively charged dust grains with fixing the ion number density in our considered plasma system can leads to the transformation of soliton type from compressive to rarefactive. Lastly, in Figures[9a-9b], we showed the variation of $\varphi(\chi)$ versus

χ and also the variation of $P(\varphi)$ versus φ with different values of external magnetic field strength B_0 by Ω and fixed other parameters, where we have found that the propagating DIA soliton is compressive and the width of the solitary pulse is to reduces, while the amplitude is seen to remain constant as the value of Ω get increased. Hence, the amplitude of DIA solitons is unaffected by the external magnetic field B_0, but their width is significantly affected.



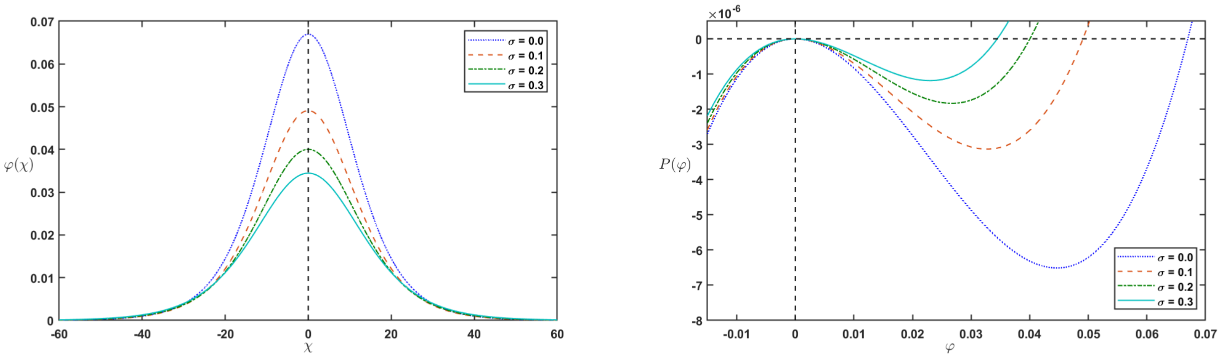
(a) **(b)**

Figure 4. The variation of **(a)** solitary wave profile $\varphi(\chi)$ versus χ, and **(b)** small amplitude potential $P(\varphi)$ versus φ with varying ν. where $\theta = 15^o$, $\sigma = 0.1$, $q = 1.2$, $\mu = 0.2$ and $\Omega = 0.3$.



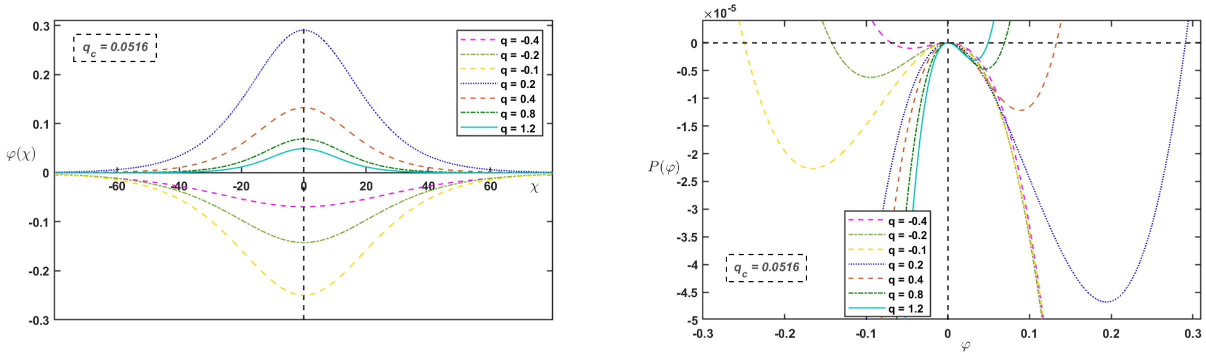
(a) **(b)**

Figure 5. The variation of **(a)** solitary wave profile $\varphi(\chi)$ versus χ, and **(b)** small amplitude potential $P(\varphi)$ versus φ with varying θ. where $\sigma = 0.1$, $q = 1.2$, $\mu = 0.2$, $\Omega = 0.3$ and $\nu = 0.02$.



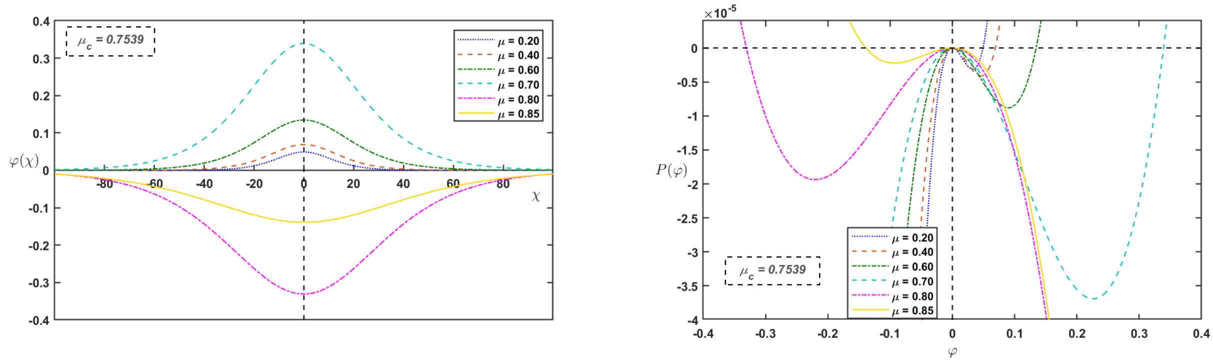
(a) **(b)**

Figure 6. The variation of **(a)** solitary wave profile $\varphi(\chi)$ versus χ, and **(b)** small amplitude potential $P(\varphi)$ versus φ with varying σ. where $\theta = 15^o$, $q = 1.2$, $\mu = 0.2$, $\Omega = 0.3$ and $\nu = 0.02$.



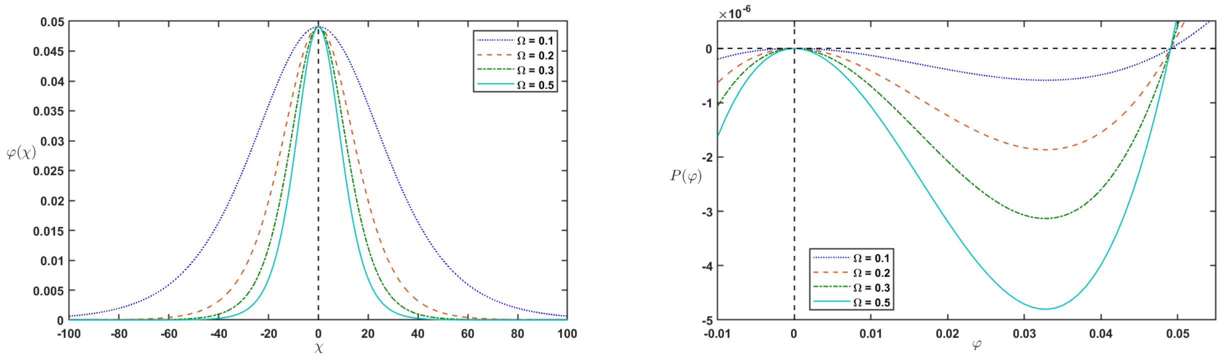
(a) **(b)**

Figure 7. The variation of **(a)** solitary wave profile $\varphi(\chi)$ versus χ, and **(b)** small amplitude potential $P(\varphi)$ versus φ with varying q. where $\theta = 15^o,\ \sigma = 0.1,\ \mu = 0.2,\ \Omega = 0.3$ and $\nu = 0.02$.



(a) **(b)**

Figure 8. The variation of **(a)** solitary wave profile $\varphi(\chi)$ versus χ, and **(b)** small amplitude potential $P(\varphi)$ versus φ with varying μ. where $\theta = 15^o,\ \sigma = 0.1,\ q = 1.2,\ \Omega = 0.3$ and $\nu = 0.02$.



(a) **(b)**

Figure 9. The variation of **(a)** solitary wave profile $\varphi(\chi)$ versus χ, and **(b)** small amplitude potential $P(\varphi)$ versus φ with varying Ω. where $\theta = 15^o,\ \sigma = 0.1,\ q = 1.2,\ \mu = 0.2$ and $\nu = 0.02$.

From the observation mentioned above, it has been evidently noted that the propagating DIA soliton represented by KdV equation (29) is shifts from a positive potential to a negative potential as a result of changes in electron nonextensivity q and dust and ion number density via μ in the considered plasma system. The amplitude of the soliton becomes infinite nearly at either $q = q_c$ or $\mu = \mu_c$, for which $\mathcal{A} = 0$. In this context, the KdV equation (29) fails to described the model. In order to explore dynamics of DIA solitary waves in this critical scenario, we must considered the evolution equation having second higher order nonlinearity as modified KdV equation and will be described in the next following section.

4. THE MKDV EQUATION AND SMALL AMPLITUDE WAVES

4.1. Derivation Of mKdV Equation

To study the solitary waves at the critical number density region μ_c, we derive the modified kdV (mKdV) equation, for small but finite amplitude DIA solitary waves. For this, we again use the reductive perturbation method and introduce a modified stretching of independent variables as

$$\xi = \epsilon(l_x x + l_y y + l_z z - U_p t), \quad \tau = \epsilon^3 t \tag{38}$$

For this approach, we use the following dependent variables in the power series of ϵ as

$$\left.\begin{aligned} n &= 1 + \epsilon n_1 + \epsilon^2 n_2 + \epsilon^3 n_3 + \dots \\ p &= 1 + \epsilon p_1 + \epsilon^2 p_2 + \epsilon^3 p_3 + \dots \\ u &= \epsilon^2 u_1 + \epsilon^3 u_2 + \epsilon^4 u_3 + \dots \\ v &= \epsilon^2 v_1 + \epsilon^3 v_2 + \epsilon^4 v_3 + \dots \\ w &= \epsilon w_1 + \epsilon^2 w_2 + \epsilon^3 w_3 + \dots \\ \phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^3 \phi_3 + \dots \end{aligned}\right\} \tag{39}$$

Now, using (38) and (39) into the equations (6)-(9) and equating the coefficients of smallest order of ϵ (i.e., ϵ^2 from (6)-(8) and ϵ from (9)), we obtain the first order terms which are same as (21)-(22) given in Subsection[3.1]. For second order terms of ϵ, we equate the coefficients of ϵ^3 from (6)-(8) and ϵ^2 from (9) and we obtain

$$-U_p\frac{\partial n_2}{\partial\xi} + l_x\frac{\partial u_1}{\partial\xi} + l_y\frac{\partial v_1}{\partial\xi} + l_z\frac{\partial w_2}{\partial\xi} + l_z n_1\frac{\partial w_1}{\partial\xi} + l_z w_1\frac{\partial n_1}{\partial\xi} = 0 \tag{40}$$

$$-U_p\frac{\partial u_1}{\partial\xi} + l_x\frac{\partial\phi_2}{\partial\xi} + l_x n_1\frac{\partial\phi_1}{\partial\xi} + \sigma l_x\frac{\partial p_2}{\partial\xi} - \Omega(v_2 + n_1 v_1) = 0 \tag{41}$$

$$-U_p\frac{\partial v_1}{\partial\xi} + l_y\frac{\partial\phi_2}{\partial\xi} + l_y n_1\frac{\partial\phi_1}{\partial\xi} + \sigma l_y\frac{\partial p_2}{\partial\xi} + \Omega(u_2 + n_1 u_1) = 0 \tag{42}$$

$$-U_p\frac{\partial w_2}{\partial\xi} - U_p n_1\frac{\partial w_1}{\partial\xi} + l_z w_1\frac{\partial w_1}{\partial\xi} + l_z\frac{\partial\phi_2}{\partial\xi} + l_z n_1\frac{\partial\phi_1}{\partial\xi} + \sigma l_z\frac{\partial p_2}{\partial\xi} = 0 \tag{43}$$

$$-U_p\frac{\partial p_2}{\partial\xi} + 3l_x\frac{\partial u_1}{\partial\xi} + 3l_y\frac{\partial v_1}{\partial\xi} + 3l_z\frac{\partial w_2}{\partial\xi} + 3l_z p_1\frac{\partial w_1}{\partial\xi} + l_z w_1\frac{\partial p_1}{\partial\xi} = 0 \tag{44}$$

$$n_2 - a_2\phi_2 - a_1\phi_1^2 = 0 \tag{45}$$

Integrating (40),(43) and (44) by using the boundary conditions $n_1 = n_2 = p_1 = p_2 = w_1 = w_2 = u_1 = v_1 = 0$ as $|\xi| \longrightarrow \infty$ and then using the first-order quantities from (21), we write the second-order quantities in terms of ϕ's, we get

$$\left.\begin{aligned} n_2 &= a_2\phi_2 + a_1\phi_1^2 \\ u_2 &= \frac{U_p l_x}{\Omega^2}(1+3\sigma a_1)\frac{\partial^2\phi_1}{\partial\xi^2} - \frac{l_y}{\Omega}\left\{(1+3\sigma a_1)\frac{\partial\phi_2}{\partial\xi} + 3\sigma(a_1+2a_2)\phi_1\frac{\partial\phi_1}{\partial\xi}\right\} \\ v_2 &= \frac{U_p l_y}{\Omega^2}(1+3\sigma a_1)\frac{\partial^2\phi_1}{\partial\xi^2} + \frac{l_x}{\Omega}\left\{(1+3\sigma a_1)\frac{\partial\phi_2}{\partial\xi} + 3\sigma(a_1+2a_2)\phi_1\frac{\partial\phi_1}{\partial\xi}\right\} \\ w_2 &= \frac{l_z}{U_p}\left\{(1+3\sigma a_1)\phi_2 + \left(\frac{a_1}{2} + 3\sigma(a_2+a_1^2)\right)\phi_1^2\right\} \\ p_2 &= 3n_2 + 3n_1^2 = 3a_1\phi_2 + 3(a_2+a_1^2)\phi_1^2 \end{aligned}\right\} \tag{46}$$

Similarly, for third order terms of ϵ, we equate the coefficients ϵ^4 from (6) and (8) and ϵ^3 from (9), we obtain

$$-U_p\frac{\partial n_3}{\partial\xi} + \frac{\partial n_1}{\partial\tau} + l_x\frac{\partial(u_2+n_1u_1)}{\partial\xi} + l_y\frac{\partial(v_2+n_1v_1)}{\partial\xi} + l_z\frac{\partial(n_1w_2+n_2w_1)}{\partial\xi} + l_z\frac{\partial w_3}{\partial\xi} = 0 \tag{47}$$

$$-U_p\frac{\partial u_2}{\partial\xi}-U_pn_1\frac{\partial u_1}{\partial\xi}+l_x\frac{\partial\phi_3}{\partial\xi}+l_xn_1\frac{\partial\phi_2}{\partial\xi}+l_xn_2\frac{\partial\phi_1}{\partial\xi}+\sigma l_x\frac{\partial p_3}{\partial\xi}+l_zw_1\frac{\partial u_1}{\partial\xi}-\Omega(v_3+n_1v_2+n_2v_1)=0 \quad (48)$$

$$-U_p\frac{\partial v_2}{\partial\xi}-U_pn_1\frac{\partial v_1}{\partial\xi}+l_y\frac{\partial\phi_3}{\partial\xi}+l_yn_1\frac{\partial\phi_2}{\partial\xi}+l_yn_2\frac{\partial\phi_1}{\partial\xi}+\sigma l_y\frac{\partial p_3}{\partial\xi}+l_zw_1\frac{\partial v_1}{\partial\xi}+\Omega(u_3+n_1u_2+n_2u_1)=0 \quad (49)$$

$$-U_p\frac{\partial w_3}{\partial\xi}-U_pn_1\frac{\partial w_2}{\partial\xi}-U_pn_2\frac{\partial w_1}{\partial\xi}+l_z\frac{\partial\phi_3}{\partial\xi}+l_zn_1\frac{\partial\phi_2}{\partial\xi}+l_zn_2\frac{\partial\phi_1}{\partial\xi}+\sigma\frac{\partial p_3}{\partial\xi}+\frac{\partial w_1}{\partial\tau}+l_xu_1\frac{\partial w_1}{\partial\xi}$$
$$+l_yv_1\frac{\partial w_1}{\partial\xi}+l_zw_1n_1\frac{\partial w_1}{\partial\xi}+l_z\frac{\partial(w_1w_2)}{\partial\xi}=0 \quad (50)$$

$$-U_p\frac{\partial p_3}{\partial\xi}+l_xu_1\frac{\partial p_1}{\partial\xi}+l_yv_1\frac{\partial p_1}{\partial\xi}+l_zw_1\frac{\partial p_2}{\partial\xi}++3l_x\frac{\partial u_2}{\partial\xi}+3l_y\frac{\partial v_2}{\partial\xi}+3l_z\frac{\partial w_3}{\partial\xi}+\frac{\partial p_1}{\partial\tau}+3l_xp_1\frac{\partial u_1}{\partial\xi}$$
$$+3l_yp_1\frac{\partial v_1}{\partial\xi}+3l_zp_1\frac{\partial w_2}{\partial\xi}+l_zw_2\frac{\partial p_1}{\partial\xi}+3l_zp_2\frac{\partial w_1}{\partial\xi}=0 \quad (51)$$

$$-\frac{\partial^2\phi_1}{\partial\xi^2}+a_1\phi_3+2a_2\phi_1\phi_2+a_3\phi_1^3-n_3=0 \quad (52)$$

Now, eliminating p_3, w_3 and n_3 from equations (47), (49)-(52) and substituting the values of first and second order terms given in (21) and (46), and using the expression (22), we found a nonlinear equation of the form,

$$\frac{\partial\phi_1}{\partial\tau}+\mathcal{A}'\phi_1^2\frac{\partial\phi_1}{\partial\xi}+B'\frac{\partial^3\phi_1}{\partial\xi^3}+C'\frac{\partial(\phi_1\phi_2)}{\partial\xi}=0 \quad (53)$$

where the coefficient $\mathcal{A}'$ is given by

$$\mathcal{A}'=\frac{1}{2a_1U_p}\left[12a_1a_2U_p^2-13l_z^2(a_1^2+a_2)+14a_1^3U_p^2+\frac{2a_1l_z^4}{a_1U_p^2}(a_1^2+2a_2)-3l_z^2\left(\frac{a_3}{a_1}\right)\right] \quad (54)$$

and the coefficients B' and C' are exactly same as that of $\mathcal{B}$ and $\mathcal{A}$ respectively given in (30) and (31) in the Subsection-3.1. But at the critical regime, *i.e.*, at $\mu=\mu_c$ or $q=q_c$, $\mathcal{A}=C'=0$. Thus, if we consider $\mathcal{B}=B'$, and $\phi_1=\psi$, equation (53) becomes the standard mKdV equation as

$$\frac{\partial\psi}{\partial\tau}+\mathcal{A}'\psi^2\frac{\partial\psi}{\partial\xi}+\mathcal{B}\frac{\partial^3\psi}{\partial\xi^3}=0 \quad (55)$$

with the second order nonlinear coefficient $\mathcal{A}'$ and the dispersion coefficient $\mathcal{B}$. From the definition of the expression (31), we have $\mathcal{B}>0$ for all plasma parametric values. Therefore, here we analyse the the parametric dependence of the term $\mathcal{A}'$ in our considered plasma system. For this, we showed the variation of $\mathcal{A}'$ versus μ for varying σ at $q=q_c$ and also versus q for varying σ at $\mu=\mu_c$ in the Figures[10a-10b], where we observed that when $q=q_c$, the second order nonlinearity increases for an increase of μ and also an increase of σ; moreover when $\mu=\mu_c$, the second order nonlinearity increases for an increase of q in between -1 and 1.7 and after $q\approx 1.7$ it gets decreases. Thus, in both the cases the value of $\mathcal{A}'$ is seen to be positive.

4.2. Solitary Wave Solution

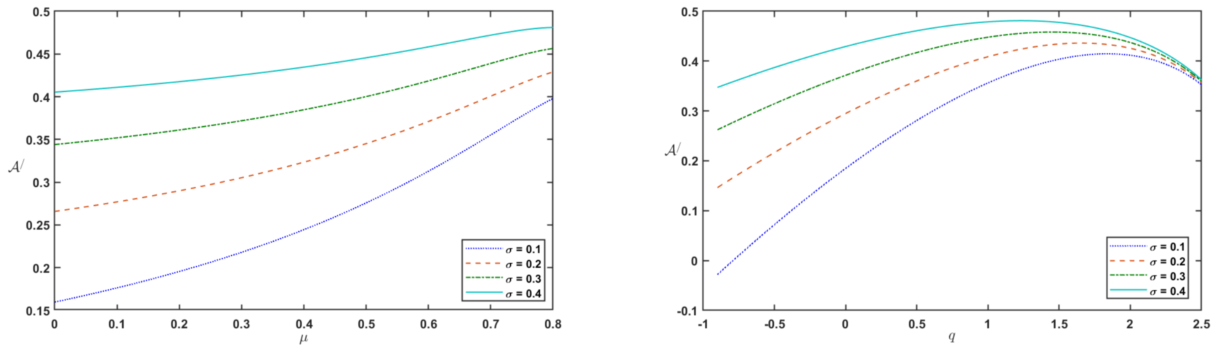
To obtain the stationary wave solution of (55), we use the same transformation given in Subsection-3.2. So, mKdV equation(55) is transformed into the reduced mKdV equation as

$$-\nu\frac{d\psi}{d\chi}+\mathcal{A}'\psi^2\frac{d\psi}{d\chi}+\mathcal{B}\frac{d^3\psi}{d\chi^3}=0 \quad (56)$$

and we obtain two stationary solitary wave solution as

$$\psi=\pm\,\psi_m\,\mathrm{sech}\left(\frac{\chi}{\Delta}\right) \quad (57)$$

where $\psi_m=\sqrt{6\nu/\mathcal{A}'}$ and $\Delta=\sqrt{\mathcal{B}/\nu}$ are respectively the amplitude and width of solitary waves represented by the mKdV equation(55) and ν is the travelling wave velocity in the linear $\chi-$space. And the positive and negative indicators are respectively associated to the compressive and rarefactive DIA soliton.



(a) **(b)**

Figure 10. The variation of second order nonlinearity $\mathcal{A}'$ **(a)** versus dust-to-ion number density μ when $q = q_c$ and **(b)** versus nonextensive parameter q when $\mu = \mu_c$, with varying σ and $\theta = 15^o$.

Integrating equation(56) twice with the boundary conditions: $\psi = \frac{d\psi}{d\chi} = \frac{d^2\psi}{d\chi^2} = 0$ as $|\chi| \longrightarrow \infty$ and proceeding the same way as given in Subsection-3.2, we obtain the small amplitude potential energy equation as

$$P_m(\psi) = \left[\frac{\mathcal{A}'}{12\mathcal{B}}\right]\psi^4 - \left[\frac{\nu}{2\mathcal{B}}\right]\psi^2 \tag{58}$$

This potential has also the same characteristics that of the potential $P(\varphi)$ given in (37), *i.e.*, at $\psi = 0$, both its value and its first derivative vanish, while the second derivative is negative. This indicates that $P_m(\psi)$ has a maximum value and a root at origin. Also, $\psi = \psi_m$ is the other root of the potential $P_m(\psi)$, which is the amplitude of the mKdV solitons. In the region of the vanishing first order nonlinear term $\mathcal{A}$, two types of solitons compressive and rarefactive appear to coexist in the expressions (57) and (58).

4.3. Numerical Discussions for Parametric effects

We have analysed the parametric effects (mainly the effects of obliqueness angle θ, the ion-to-electron temperatures via σ, the electron nonextensive parameter q, dust-to-ion number density via μ and external magnetic field strength B_0 via Ω, at when q_c or μ_c) on the geometrical behaviour of DIA soliton represented by the mKdV equation (55) in the considered magnetized plasma system by plotting both the modified solitary wave profile $\psi(\chi)$ given in equation (57) against the linear parameter χ and the corresponding small amplitude potential $P_m(\psi)$ given in equation (58) against the electrostatic potential ψ.

In the Figure[11], we showed the variation of $\psi(\chi)$ versus χ and also $P_m(\psi)$ versus ψ with varying σ and μ in separate panels when $q = q_c$. And also we depicted the variation of $\psi(\chi)$ versus χ and also $P_m(\psi)$ versus ψ with varying σ and q separately in different panels in the Figure[12] when $\mu = \mu_c$. Where we find that in both the situations, the amplitude decreases while width increases of the pulse of compressive and rarefactive (DIA) modified solitons (as shown in Figures [11a-11b] & [12a-12b]) for an increase in ion temperature. The similar result is to visible with the variation of μ (or q) at fixed $q = q_c$ (or $\mu = \mu_c$), that is the amplitude reduces whereas the width raises of both the pulse of compressive and rarefactive (DIA) modified solitons with an increasing values of μ (or $q < 1.7$) as seen in Figures[11c-11d] (Figures[12c-12d]). However, in the case of the variation of electron nonextensivity q at the critical scenario μ_c, it is worth to noticed that the propagating modified DIA solitons show opposite characteristics for $q \geq 1.7$ in the considered plasma system, for which both the amplitude and width of the pulse compressive and rarefactive DIA solitons are seen to enhanced in this particular case (Figure is not included here). By this numerical examination, we can predict that the solitary waves in the critical region $q = q_c$ are to form more taller and wider than in the critical region $\mu = \mu_c$ for any other plasma parametric values.

5. RESULTS & CONCLUSIONS

In this manuscript, we have theoretically investigated the existence and propagation characteristics of DIA solitary waves in a magnetized plasma in presence of inertial ions, noninertial electrons which obey $q-$nonextensive velocity distribution and negative dust grains. The ion pressure as a variable is taken into consideration and the Possion's equation is taken to making the plasma system self-consistent. The nonlinear KdV and modified KdV equations are derives by adopting reductive perturbation method that describes the existence of the small amplitude DIA waves in the considered system. The solution of these two equations and the corresponding small amplitude Sagdeev type virtual potential is obtained to analyse the characteristics

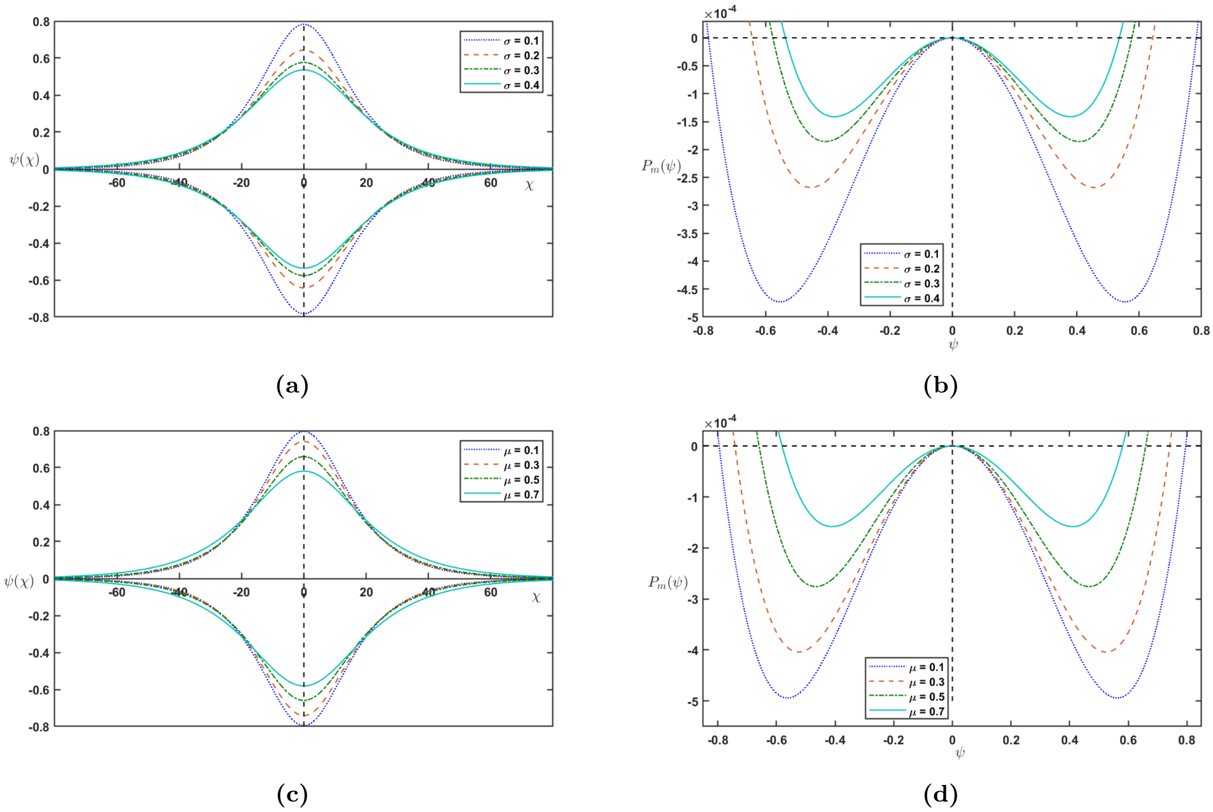


Figure 11. The variation of modified solitary wave profile $\psi(\chi)$ versus χ and small amplitude potential $P_m(\psi)$ versus ψ **(a)-(b)** with varying σ and $\theta = 15^o$, $\mu = 0.2$, $\Omega = 0.3$ and **(c)-(d)** with varying μ and $\theta = 15^o$, $\sigma = 0.1$, $\Omega = 0.3$. In all the panels, $q = q_c$ and $\nu = 0.02$.

of the DIA solitons in such a plasma system. The effects of different plasma parameters such as obliqueness angle (θ), electron nonextensivity (q), dust-to-ion number density ratio (μ), ion temperature (via ion-to-electron temperature ratio σ), external magnetic field (via Ω) etc. on the dynamical characteristics of propagating DIA solitary waves are studied. The results that have been noticed in our theoretical investigation can be succinctly summarized as follows.

1. The basic nature of the propagating DIA solitons that is amplitude, width and speed, are virtually affected by the core plasma parameters viz θ, σ, μ, q and Ω.
2. The phase velocity (U_p) of the waves advances for the parallel propagating than for the obliquely propagating along the magnetic field. While the phase velocity is lower in plasma having large low-speed electrons than the superthermal electrons.
3. The phase velocity is faster in a dusty plasma owning hot ions than in a dusty plasma with cold ion. Besides, the phase velocity achieves higher (lower) values in a plasma having more (less) number of negative dusts then in a plasma having less (more) number of positive ions.
4. The dispersion coefficient $\mathcal{B}$ is a positive quantity, while the nonlinear coefficient $\mathcal{A}$ can be a positive and a negative quantity, depending on the plasma parametric values. Therefore, in our considered plasma system the existence of compressive and/or rarefactive DIA solitary structures possible.
5. The change in the soliton types from compressive to rarefactive or vice-versa is predicting mainly through the deviation of electron nonextensivity by q and also the dust and ion number density by μ. At an appropriate value of nonextensive parameter q (i.e., q_c) with fixed other parameters or dust-to-ion number density ratio μ (i.e., μ_c) with fixed other parameters, the coefficient $\mathcal{A} = 0$, consequently the amplitude of the pulse of solitary structure become infinite. That is, it can be say that there does not exist any soliton for this condition.
6. Both the width and amplitude of pulse of soliton is found to increase with the obliqueness propagation angle $\theta \leq 55^o$, But the width decreases for $\theta \geq 55^o$ and the amplitude (width) of the DIA soliton is seen to be infinity (zero) as $\theta \longrightarrow 90^o$, which implying the possibility for the of DIA solitary waves propagation for $0 \leq \theta \leq 55^o$.
7. The increasing of ion temperature (by σ) in the plasma, lead to increase the amplitude and decrease the width of the pulses of the propagating DIA soliton.

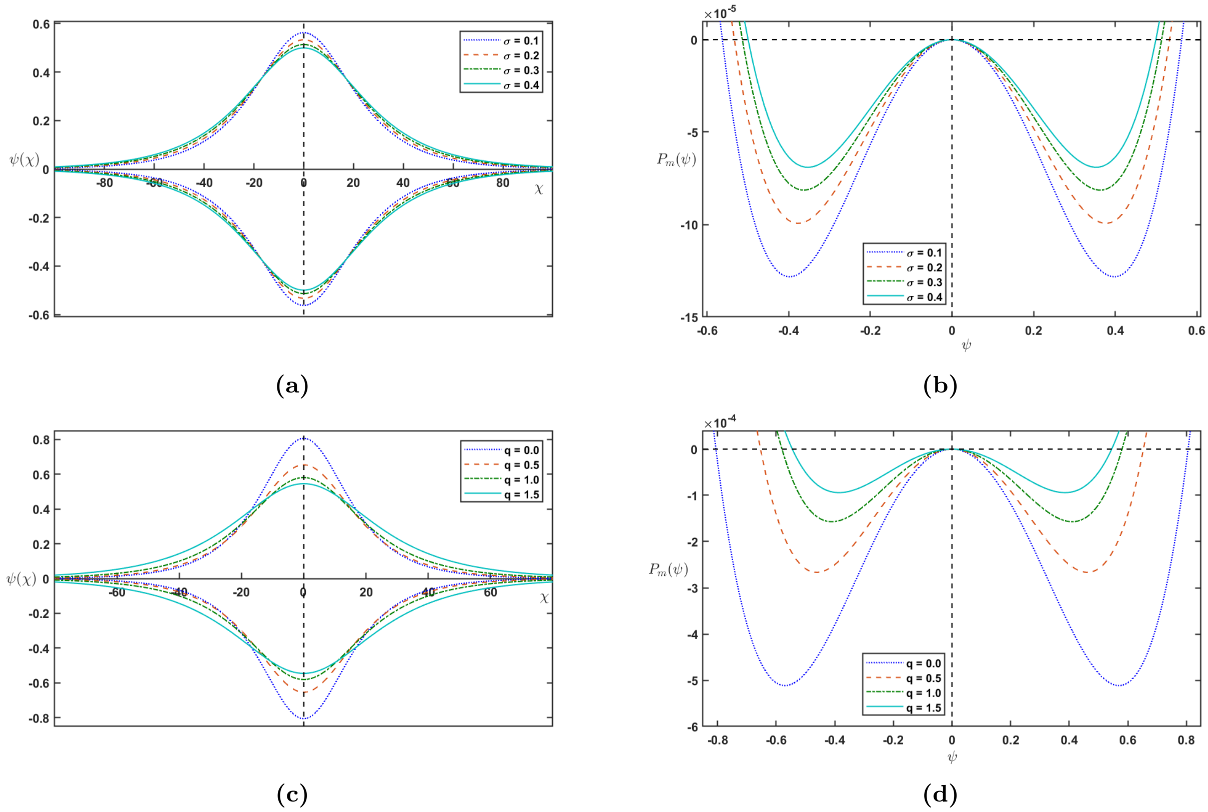


Figure 12. The variation of modified solitary wave profile $\psi(\chi)$ versus χ and small amplitude potential $P_m(\psi)$ versus ψ **(a)-(b)** with varying σ and $\theta = 15^o$, $q = 1.2$, $\Omega = 0.3$ and **(e)-(f)** with varying q and $\theta = 15^o$, $\sigma = 0.1$, $\Omega = 0.3$. In all the panels, $\mu = \mu_c$ and $\nu = 0.02$.

8. The strength of external magnetic field has a significant impact on the width of the propagating DIA soliton and width reduces with increasing the strength of the magnetic field, but it does not have any effect on the amplitude of the soliton.
9. Both the amplitude and width of pulse of propagating compressive (rarefactive) soliton is found to decrease (increase) with the increases of electron nonextensivity. Compressive soliton is obtained after the point q_c and rarefactive soliton is obtained before that point. However, the same but opposite characteristics is found with increase (decrease) of the number density of dust (ion) in the plasma.
10. At the critical q_c or μ_c, a second order nonlinearity $\mathcal{A}'$ which is a positive quantity, is obtained via mKdV equation. And it is predicted that the coexistence of compressive and rarefactive solitons are feasible in the considered plasma system.
11. The amplitude of both compressive and rarefactive modified soliton decreases, while width increases with ion temperature and also with dust-to-ion number density, μ (electron nonextensivity q) at fixed $q = q_c$ ($\mu = \mu_c$). However, the amplitude is seen to higher in the region q_c compared to the region μ_c.

Finally, we draw the conclusion that our present theoretical findings should be useful for better understanding the dynamical nature of small but finite amplitude DIA solitons in both astrophysical and space contexts as well as in future laboratory investigations in which the considered plasma model are existed.

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ІСНУВАННЯ СОЛІТОНІВ KdV ТА mKdV МАЛОЇ АМПЛІТУДИ В НАМАГНІЧЕНІЙ ЗАПИЛЕНІЙ ПЛАЗМІ З q−НЕЕКСТЕНСИВНИМИ РОЗПОДІЛЕНИМИ ЕЛЕКТРОНАМИ

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Існування та характеристики поширення пилово-іонно-акустичних (DIA) солітонів Кортевега-де Фріза (KdV) і модифікованих KdV солітонів малої амплітуди в трикомпонентній намагніченій плазмі, що складається з позитивних інерційних іонів зі зміною тиску, неінерційних електронів і негативно заряджених нерухомих частинок пилу теоретично та чисельно досліджено, коли електрони підкоряються q−неекстенсивному розподілу швидкостей. Використовуючи метод редуктивних збурень, отримати KdV і модифіковані рівняння KdV і отримати солітонні рішення DIA разом із відповідними потенціалами малої амплітуди. Це дослідження показує, що існують стискаючі та/або розріджені солітони та відсутні солітони взагалі через параметричну залежність від нелінійного коефіцієнта першого порядку через щільність позитивних іонів і негативних частинок пилу та неектенсивність електронів. Співіснування стискаючих і розріджених солітонів з'являється шляхом підвищення міри коефіцієнта нелінійності до другого порядку за допомогою модифікованого рівняння KdV. Чисельно обговорюються такі властивості, як швидкість, амплітуда, ширина тощо.

Ключові слова: *пилова іонна акустична хвиля; намагнічена плазма;* q*−неекстенсивний розподіл; редуктивний метод збурень; рівняння KdV*