

EFFECT OF RAREFACTIONS AND CONVECTIVE HEAT CHANGE ON FREE CONVECTIVE UNSTEADY MHD FLOW IN A SLIP-FLOW REGIME PAST A VERTICAL WALL WITH CONVECTIVE SURFACE BOUNDARY CONDITION

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Received March 7, 2024; revised July 10, 2024; accepted July 20, 2024

The study investigates the unsteady free convective two-dimensional MHD flow past a vertical porous plate with convective surface boundary condition in porous medium in slip flow regime under the action of variable suction velocity. Analytical solutions are obtained for the system using perturbation technique that converts non-linear coupled governing partial differential equations into non-dimensional form of ordinary differential equations. Effects of variable suction velocity, rarefactions parameter and heat change parameter are analysed and discussed graphically for various values of effective physical parameter such as Grasshof number, Magnetic field parameter, Prandtl number, Permeability parameter on fluid velocity and temperature, skin friction, and heat transfer.

Keywords: *Magneto hydrodynamic flow; Permeability; Rarefaction; Convective heat change; Variable suction-velocity*

PACS: 47.35.Tv, 47.65.-d ; 47.56.+v

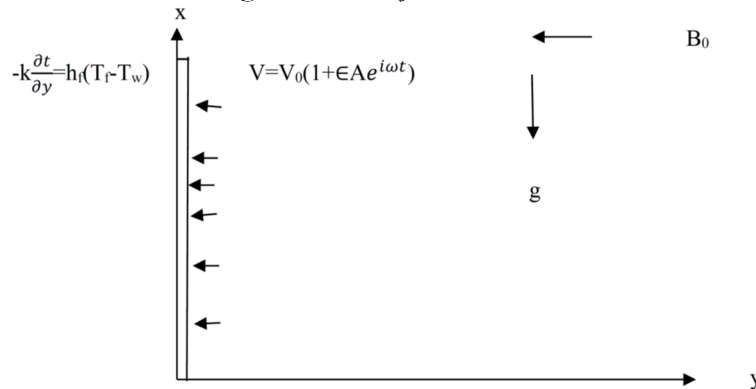
INTRODUCTION

Magneto hydrodynamic (MHD) flow under various physical situations has been of great interest in the recent years because of numerous applications in the field of science and technology, and industry; e.g. in production of Rayon and Nylon, Purification of crude oil, MHD generator and pumps, functioning of a nuclear reactor, etc. Such flows that are buoyancy induced, density varied driven by temperature gradient in presence magnetic field occurs in many situations in our everyday life as well; in the atmospheric flow (movement of air around solar system), extracting geothermal energy, exploring oil from ground etc. Over the years, many researchers have studied unsteady free MHD convective flow past a flat solid plate with oscillating temperature or with a constant temperature. Martynenko et al. [1], Das et al. [2], Hossain et al. [3], Sahoo et al. [4] have studied on such unsteady MHD free convective flow with suction parameter and with heat source. MHD flow over a moving permeable vertical surface using perturbation technique was studied by Abdelkhalek [5]. Flow over a solid wall with no-slip boundary condition where fluid particles acquire no velocity but have finite tangential velocity commonly called a slip-flow regime. Such flow over a solid surface or wall plays importance role in many practical applications in science-technology, industry, atmospheric phenomenon etc. Sharma and Chaudhary [6], Khaled and Vafai [7], Mehmood and Ali [8], Chaudhary and Jha [9], Hayat et al. [10] have studied heat transfer problems which have effect of slip flow regime on Unsteady MHD flow. Makinde [11], Gangadhar et al. [12], Olanjrewaju et al. [13] and Garg et al. [14] have studied such type of fluid flow under the condition convective heating of surfaces. Garg et al. [15] have studied free convective unsteady MHD flow past a vertical plate with convective surface boundary condition in slip flow regime. N. Kalita et al. [16] studies on a unsteady flow which passed over an vertically plate in presence of some chemical reaction. S. Sarma et al. [17] investigate on different frequency of waves on unsteady MHD flow over different temperature in porous medium. A. Selvaraj et al. [19] studies on MHD parabolic flow which passed over isothermal plate with rotation effect in mass, heat diffusion. Mv Krishna [20] investigate on Unsteady MHD rotating flow in various temperature in Jeffrays fluid. P.K. Pattnaik et al. [20] studies on the effect of chemical reaction in MHD flow over a stretching sheet. R.S. Nath et al. studies on various stratification effect in Magnetohydrodynamics Nanofluid which passed over an vertical plate in porous medium in the presence of radiation, heat source.

In this paper, two dimensional unsteady free convective viscous MHD flow past a vertical plate in porous medium in slip-flow regime under the action of variable suction velocity has been studied. A uniform magnetic field acts perpendicular to the surface and a suction velocity varies with time. Analytical equations are obtained for the flow system using perturbation technique. The objective this study is to analysis the rarefaction effect and convective heat change within the medium of the of the free convective flow system in presence of magnetic field past a vertical wall with surface boundary condition in slip flow regime. The flow patterns are analysed and discussed with the help of a graphs for different values of Magnetic field, Suction velocity, medium permeability, Grasshof number, Prandtl number, etc.

FORMULATION OF PROBLEMS

Consider an unsteady free two-dimensional flow which passes through porous medium with convective surface boundary condition in slip flow regime. The variable suction velocity $V=V_0(1+\epsilon Ae^{i\omega t})$ and magnetic field are applied perpendicular to the wall. We take x-axis along the wall and y-axis normal to the wall. As shown in Figure below.



Since the plate is considered to be infinite in x-direction, therefore its flow of variable are function of y and t only. After neglecting viscous and magnetic dissipation along with Boussinesq approximations, the governing equations can be written as

$$\frac{\partial u'}{\partial t} - V_0(1 + \epsilon Ae^{i\omega t'}) \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) - \frac{\sigma}{\rho} B_0^2 u' \frac{y}{K} u', \tag{1}$$

$$\frac{\partial T}{\partial t'} - nV_0(1 + \epsilon Ae^{i\omega t'}) \frac{\partial T'}{\partial y'} = \alpha \frac{\partial^2 T}{\partial y'^2}, \tag{2}$$

where u and v are the x and y components of the velocities respectively, A is the variable suction velocity, T is the temperature, ω is the frequency, ν is the kinematic viscosity of the fluid, ρ is the fluid density, $\alpha (= \frac{k}{\rho c_p})$ is the thermal diffusivity of the fluid, β is the thermal expansion coefficient, k is the thermal conductivity, g is the acceleration due to gravity, σ is the electrical conductivity, B_0 is the magnetic field in y direction.

The boundary condition are:

$$u' = L \left(\frac{\partial u'}{\partial y'} \right), -k \left(\frac{\partial T}{\partial y'} \right) = h_f (T_f - T_w) \text{ at } y' = 0, \tag{3}$$

$$u' \rightarrow 0, T \rightarrow T_\infty \text{ as } y' \rightarrow \infty, \tag{4}$$

where $T_w = T(0, t)$ and L is some body dimension defined by characteristics dimension flow field.

We now introduce the non-dimensional variables into equations 1-4 as follows

$$y = \frac{y' V_0}{\nu}, t = \frac{t' V_0^2}{4\nu}, \omega = \frac{4\nu\omega'}{V_0^2}, u = \frac{u'}{V_0}, \theta = \frac{T - T_\infty}{T_f - T_\infty}, k = \frac{\nu^2}{K V_0^2}, \tag{5}$$

Now, equation 1 and 2 reduces to

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \epsilon Ae^{i\omega t}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - Mu - \frac{1}{K} u, \tag{6}$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} - (1 + \epsilon Ae^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2}, \tag{7}$$

The boundary condition (3) and (4) in the dimensional form below

$$u = H \frac{\partial u}{\partial y}, \frac{\partial \theta}{\partial y} = -H_f(1 - \theta) \text{ at } y = 0$$

$$u = 0, \theta = 0 \text{ as } y \rightarrow \infty, \tag{8}$$

where $Gr = \frac{g\beta\nu}{V_0^3}(T_f - T_\infty)$ is the Grashof number, $M = \frac{\sigma B_0^2}{\rho V_0^2}$ is the magnetic field parameter, $H = \frac{LV_0}{\nu}$ is the Rarefaction parameter, $H_f = \frac{\nu h_f}{kV_0}$ is the convective Heat change parameter and $Pr = \frac{\mu c_p}{k}$ is the prandtl number

SOLUTION OF THE PROBLEM

In order to solve the boundary value problem, it is first reduced to ordinary differential equation from partial differential equation using perturbation technique. Assuming small amplitude $\epsilon \ll 1$, the velocity u and temperature θ near the plate can be represented as

$$u(y,t) = u_0(y) + \epsilon e^{i\omega t} u_1(y) + \epsilon^2 e^{2i\omega t} u_2(y), \quad (9)$$

$$\theta(y,t) = \theta_0(y) + \epsilon e^{i\omega t} \theta_1(y) + \epsilon^2 e^{2i\omega t} \theta_2(y). \quad (10)$$

Substituting the value of equation (9) and (10) into equation (6) and (7) and comparing the coefficient of identical powers of ϵ we obtain

$$u_0''(y) + G_r \theta_0(y) + u_0'(y) - \xi u_0(y) = 0, \quad (11)$$

$$u_1''(y) + u_1'(y) + Au_1'(y) - \frac{1}{4} i\omega u_1(y) + G_r \theta_1(y) - \xi u_1(y) = 0, \quad (12)$$

$$u_2''(y) + u_2'(y) + Au_2'(y) - \frac{1}{2} i\omega u_2(y) + G_r \theta_2(y) - \xi u_2(y) = 0, \quad (13)$$

$$\theta_0''(y) + P_r \theta_0'(y) = 0, \quad (14)$$

$$\theta_1''(y) - P_r \frac{i\omega}{4} \theta_1(y) + P_r \theta_1'(y) + AP_r \theta_0'(y) = 0, \quad (15)$$

$$\theta_2''(y) - \frac{i\omega}{2} P_r \theta_2(y) + P_r \theta_2'(y) + AP_r \theta_1'(y) = 0. \quad (16)$$

The corresponding boundary condition (8) reduces to

$$u_0 = H \frac{\partial u_0}{\partial y}, u_1 = H \frac{\partial u_1}{\partial y}, u_2 = H \frac{\partial u_2}{\partial y} \frac{\partial \theta_0}{\partial y} - H_f(1 - \theta_0), \frac{\partial \theta_1}{\partial y} = H_f \theta_1 \text{ at } y=0$$

$$\frac{\partial \theta_2}{\partial y} = H_f \theta_2 \text{ at } y=0, \quad (17)$$

$$u_0 = 0, u_1 = 0, u_2 = 0, \theta_0 = 0, \theta_1 = 0, \theta_2 = 0 \text{ as } y \rightarrow \infty, \quad (18)$$

Solving the set of equation (11) – (16) under boundary condition (18), we get

$$u_0(y) = A_7 e^{-m_1 y} - A_8 e^{-Pr y}, \quad (19)$$

$$u_1(y) = A_{10} e^{-m_2 y} - A_{11} e^{-Pr y} - A_{12} e^{-m_1 y} + A_9 e^{-m_3 y}, \quad (20)$$

$$u_2(y) = A_{13} e^{-m_4 y} + A_{14} e^{-m_3 y} - A_{15} e^{-Pr y} + A_{16} e^{-m_2 y} - A_{17} e^{-m_1 y}, \quad (21)$$

$$\theta_0(y) = A_2 e^{-Pr y}, \quad (22)$$

$$\theta_1(y) = -A_3 e^{-m_2 y} + A_4 e^{-Pr y}, \quad (23)$$

$$\theta_2(y) = A_5 e^{-m_3 y} + A_6 e^{-Pr y} + A_{18} e^{-m_2 y}, \quad (24)$$

The final equation for temperature and velocity are as follows:

$$\theta(y) = A_2 e^{-Pr y} - A_3 e^{-m_2 y} + A_4 e^{-Pr y} + A_5 e^{-m_3 y} + A_6 e^{-Pr y} + A_{18} e^{-m_2 y}, \quad (25)$$

$$u(y) = A_7 e^{-m_1 y} - A_8 e^{-Pr y} + A_{10} e^{-m_2 y} - A_{11} e^{-Pr y} - A_{12} e^{-m_1 y} + A_9 e^{-m_3 y} + A_{13} e^{-m_4 y} +$$

$$+ A_{14} e^{-m_3 y} - A_{15} e^{-Pr y} + A_{16} e^{-m_2 y} - A_{17} e^{-m_1 y} \quad (26)$$

Skin-Friction

The equation of Skin Friction is

$$\tau_x = \left(\frac{\partial u}{\partial y} \right)_{y=0} = -m_1 A_7 + Pr A_8 + \epsilon (-A_{10} m_2 + A_{11} Pr + A_{12} m_1 - A_9 m_3) e^{-\omega t} -$$

$$-\epsilon^2 (A_{13} m_4 + A_{14} m_3 - A_{15} Pr + A_{16} m_2 - A_{17} m_1). \quad (27)$$

Coefficient of Heat Transfer

The coefficient of Heat transfer is Nusselt number which is dimensionless

$$Nu = \left(\frac{\partial \theta}{\partial y} \right)_{y=0} = -A_2 Pr + A_3 m_2 - A_4 Pr - A_5 m_3 - A_6 Pr. \quad (28)$$

RESULTS AND DISCUSSION

The purpose the study is to investigate effects of variable suction velocity A , rarefactions parameter H and convective heat change parameter H_f on fluid velocity and temperature, skin friction, and heat transfer represented by Nusselt number at the plate. Momentum and energy equations of the flow system are solved by perturbation technique. Expressions for fluid temperature and velocity, skin-friction and Nusselt Number on the wall are shown in equation 1.25 - 1.28 respectively. While calculating the numerical values of physical quantities of our interest, the values of non-

dimensional quantities are considered as $Gr=5.0$, $\omega=5.0$, $Pr=0.71$, $K=1.0$, $H=0.4$, $H_f=1.0$, $M=1.5$. The effects of A , H & H_f on transient velocity and temperature profiles, Skin friction and Nusselt number over the wall are shown in Figures (i) - (xv) as given below.

Fig(i) shows velocity profile ($y \geq 0.0$) for various values of suction parameter A ($\cong 0.0$ to 5.0) at $M=1.5$ & 2.5 . The plots show that fluid velocity decreases gradually away from the wall and then rises slowly. Near the wall ($y \cong 0.0$ to 1.0) fluid velocity higher for higher values of A while away ($y \cong 0.1$ to 7.0) velocity is lower for higher values of A . Further away from the wall ($y \geq 7.0$), fluid velocity is less variable with rise of A . Nature of this variations are same for variation of magnetic parameter $M \cong 1.5$ & 2.5 , however, the critical point from where the nature of variation of fluid velocity rises slowly is decreased with the increase of M (e.g. for $M=1.5$ at $y \cong 1.0$ whereas, for $M \cong 2.5$ at $y \cong 0.6$). Rise of value of M causes decrease of u at a point $y \geq 0.0$.

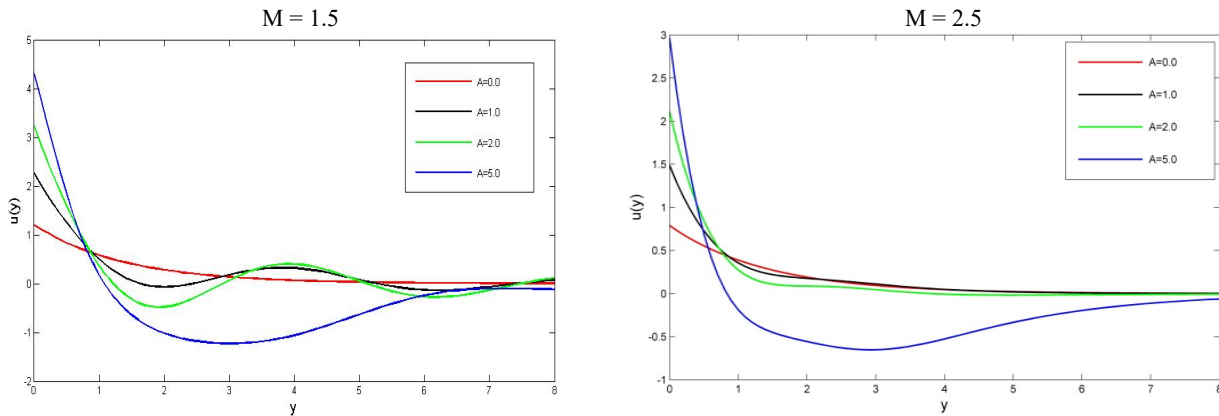


Figure (i). Effect of suction parameter A on velocity profiles

Fig(ii) shows velocity profile ($y \geq 0.0$) for various values of heat change parameter H_f ($\cong 0.1$ to 2.0) at $M=1.5$ & 2.5 . The plots show that fluid velocity decreases gradually from the wall to a certain extent and then rises slowly. Near the wall ($y \cong 0.0$ to 1.5) fluid velocity higher for higher values of H_f while away ($y \cong 0.1$ to 8.0) velocity is less for higher values of H_f . Further away from the wall ($y \geq 8.0$), variation of fluid velocity is less variable with rise of H_f . The nature of this variation is same for M ($\cong 1.5$ & 2.5). Rise of value of M decreases the values of u at a point $y \geq 0.0$.

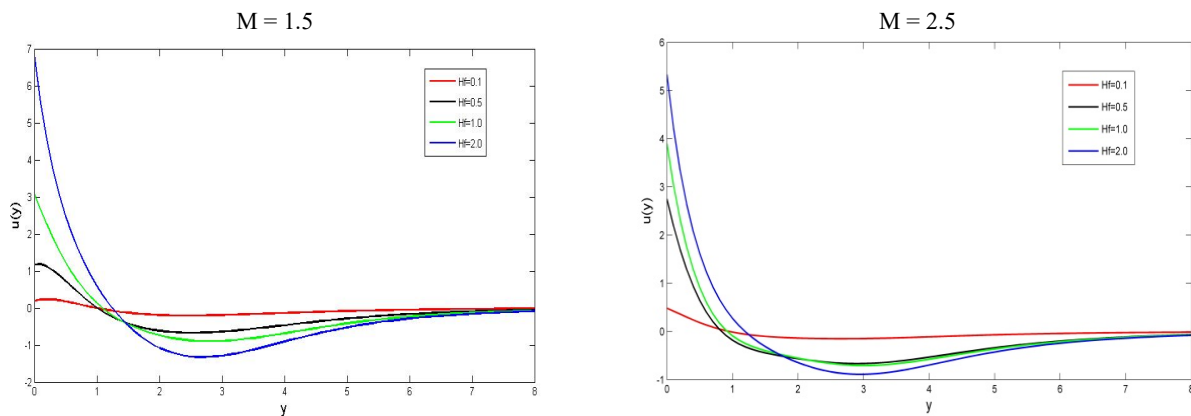


Figure (ii). Effect of H_f on velocity profiles

Fig(iii) shows effect of Rarefaction parameter (H) on velocity field at $y \geq 7.0$ at $M=1.5$ & 2.5 . For all values of H , fluid velocity near the wall decreases, then gradually increases away from it. Near the wall, at a point $y \leq 6.0$ fluid velocity decreases with the rise of H ($\cong 0.0$ to 5.0); away from it gradually becomes less significant; nature of these variation are same for magnetic parameter M ($\cong 1.5$ & 2.5). Fig(iv) shows velocity profile ($y \geq 0.0$) for various values of K ($\cong 1.0$ to 10.0) at $M=1.5$ & 2.5 . For all values of K , near the wall $y \leq 6$ velocity decreases with the rise of K and slowly increases away from it. For higher values of K , magnitude of velocity also more. Nature of variations are same for M ($\cong 1.5$ & 2.5). Fig(v) shows velocity profile ($y \geq 0.0$) for various values of Prandtl number (Pr) for $M=1.5$ & 2.5 . As Pr varies from 0.71 to 10.0 , fluid velocity (u) has significant change near the wall but not that away from it. Near the wall $y \leq 1.0$ fluid velocity increases with the rise of Pr . Higher the value of Pr higher is the fluid velocity (u) near the wall. But away from it velocity is almost independent to the variation to the variation of Pr . This nature of variation is same for $M=1.5$ & 2.5 . Fig(vi) show velocity profile ($y \geq 0.0$) for various values of M ($\cong 0.01$ to 2.0). With the rise of magnetic field M velocity gradually decreases; the rate of decrease is much higher near the wall $y \leq 2.0$. Away from it, the effect of magnetic field gradually

becomes less significant. Fig(vii) show velocity profile ($y \geq 0.0$) for various values of $Gr(=1.0$ to $15.0)$. Near the wally ≤ 3.5 , fluid velocity largely depends upon changing values of Gr . Higher the value of Gr , fluid velocity is higher. Away from the wall $y \geq 3.5$ (approx.) velocity gradually becomes less dependent to Gr .

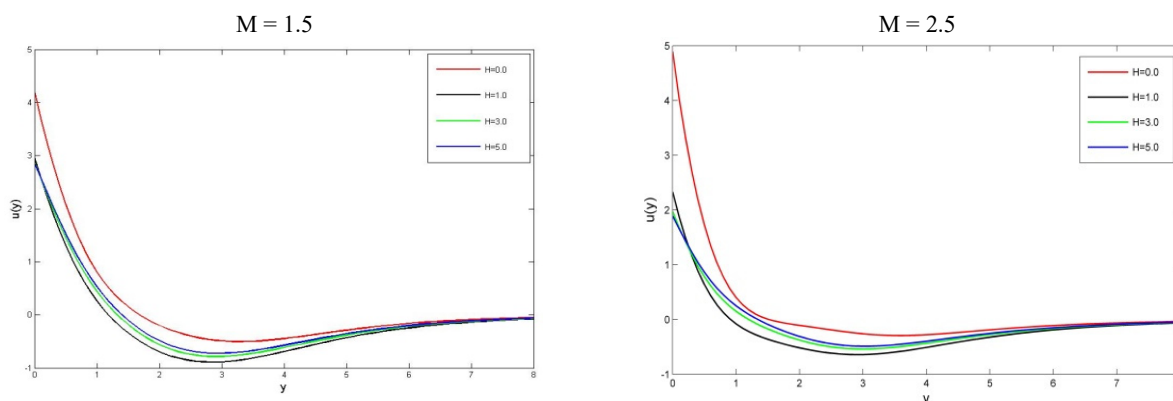


Figure (iii). Effect of H on velocity profiles

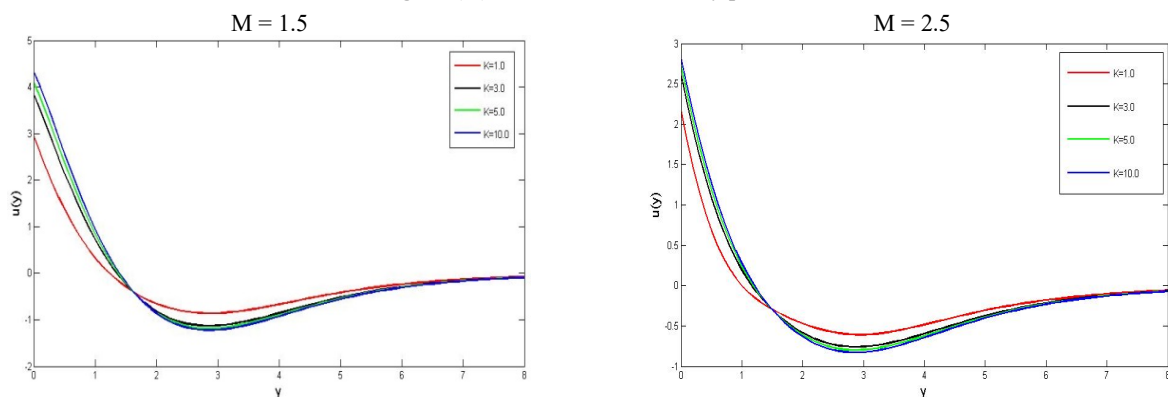


Figure (iv). Effect of K on velocity profile

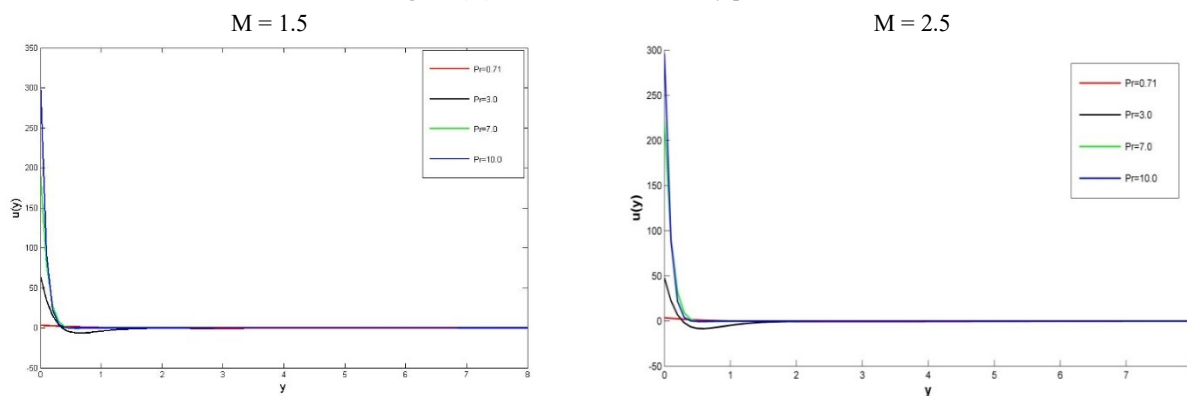


Figure (v). Effect of Pr on velocity profile

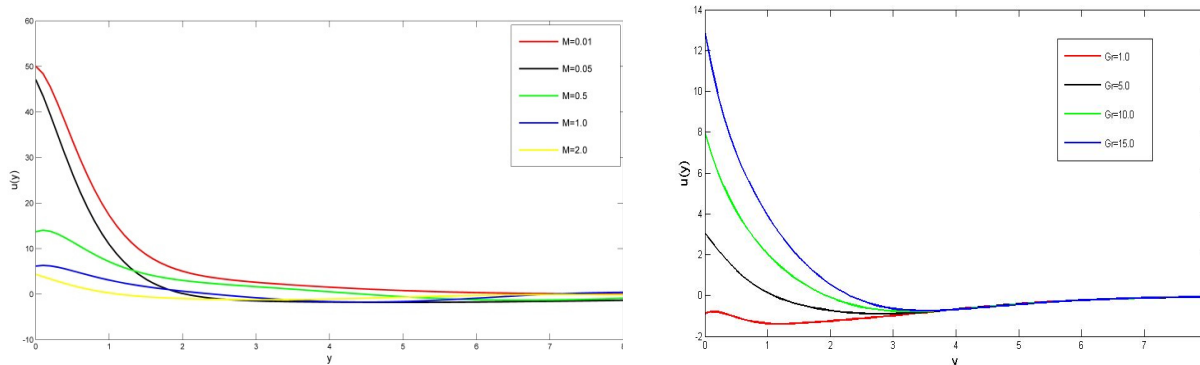


Figure (vi). Effect of M on velocity profile

Figure (vii). Effect of Gr on velocity profile

Fig(viii) show temperature (θ) profile ($y \geq 0.0$) for various values frequency ω (frequency by which suction velocity changes). It is seen that away from the wall, with the rise of ω ($\cong 1.0$ to 2.0), θ decreases gradually. Effect of ω is significantly effective near the wall $y \leq 4.0$ while away from it much less. Fig(ix) show temperature (θ) profile ($y \geq 0.0$) for various values of Heat change parameter H_f ; the plots show much significant effect of H_f on fluid temperature near the wall in compare to away from it. In the neighbourhood of the wall, with the rise of H_f ($\cong 0.1$ to 5.0), temperature(θ) first increases and then decreases sharply toward $y \cong 5.0$; thereafter almost no impact of H_f on temperature (θ). Within the very near to the wall $y \leq 0.5$ temperature rises then decreases sharply towards $y \cong 5.0$ (approx.). Fig(x) show temperature (θ) profile ($y \geq 0.0$) for various values of Prandtl number (Pr). It is seen that with the increase of Pr ($\cong 0.1$ to 0.5), θ decreases gradually away from the wall. The rate of decrease is significant within near the wall $y \leq 3.0$, beyond which it is very less. Fig(xi) show temperature (θ) profile ($y \geq 0.0$) for various values of Suction parameter A. It is seen that rise of A ($\cong 1.0$ to 5.0) fluid temperature θ first increases then decreases slowly away from the wall. At near to the wall $y \leq 0.5$ temperature rises up but then decreases sharply to the away from it. At away from the wall, temperature has less effective with rise of A.

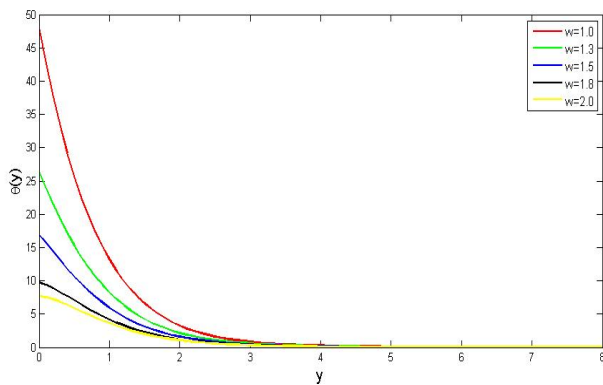


Figure (viii). Effect of ω on temperature profile

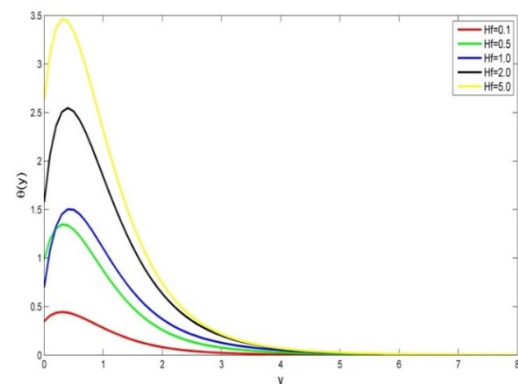


Figure (ix). Effect of H_f on temperature profile

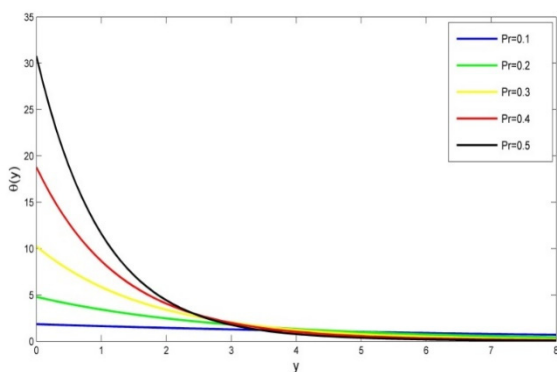


Fig (x): Effect of Pr on temperature profile

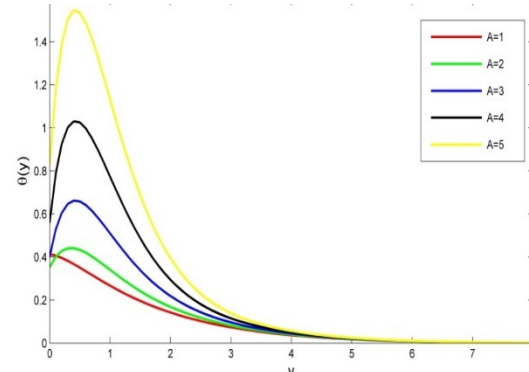


Fig (xi): Effect of A on temperature profile

Fig(xii) shows effect of rarefaction parameter H & Suction parameter A on skin friction τ_w . It is seen that skin friction on the wall gradually decreases with the rise of A for all values of H ($\cong 0.0$ to 1.0). Skin friction on the wall increases with the rise of H. this nature of variation is almost same for $M \cong 1.5$ & 2.5 .

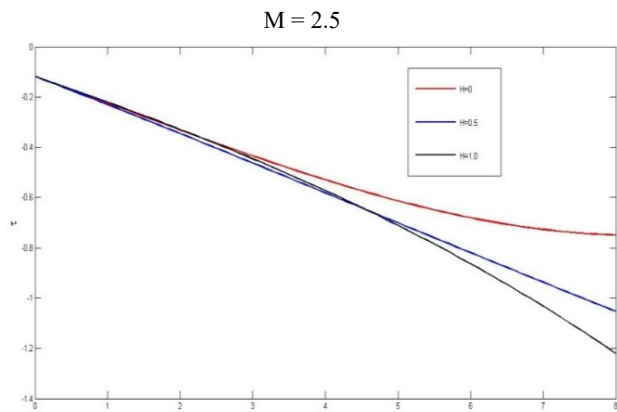
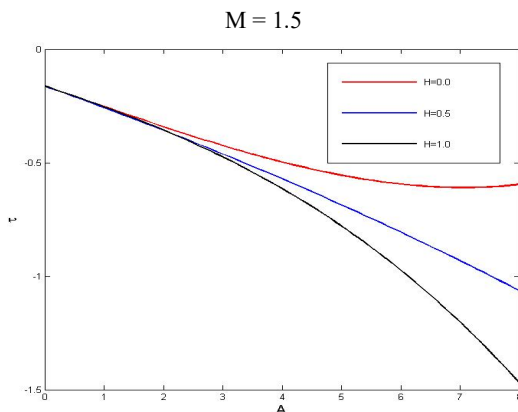


Figure (xii). Effect of H on skin friction

Fig(xiii) shows effect of heat change parameter H_f & Suction parameter A on skin friction on the wall. It is seen that τ gradually decreases with the rise of A for all values of H_f ($\cong 0.2$ to 1.0). skin friction τ decreases from higher values of H_f , nature of this variation is same for $M = 1.5$ & 2.5 . Fig. (xiv) show effect the variation of Nusselt number Nu (i.e. the rate of heat transfer) with different values of Heat change parameter H_f & Suction parameter A . Nu gradually increases with the increase of A . Further, Nu is higher for higher values of H_f . Fig. (xv) show effect the variation of Nusselt number Nu with different values of Prandtl number Pr and & Suction parameter A . Nu gradually increases with the increase of A ; Nu is higher for higher values of Pr (0.2 to 5.0).

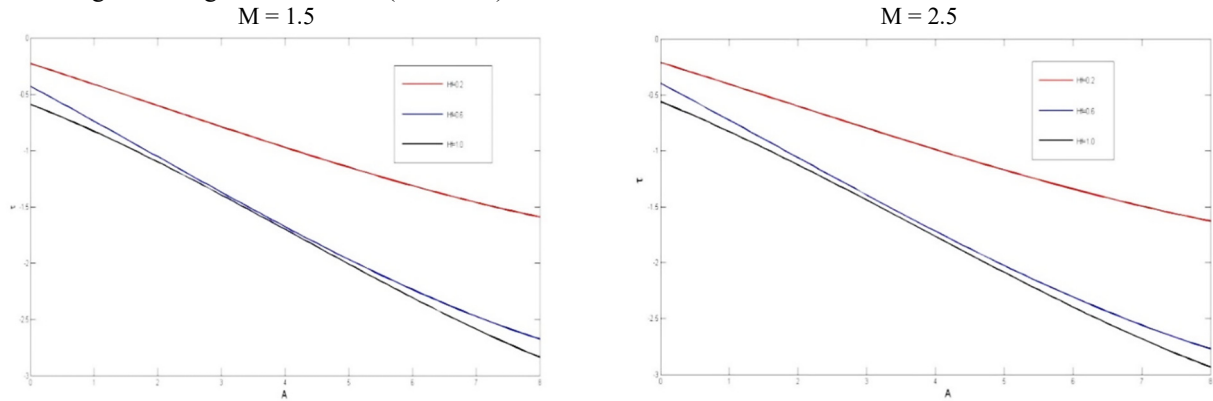


Figure (xiii). Effect of H_f on skin friction

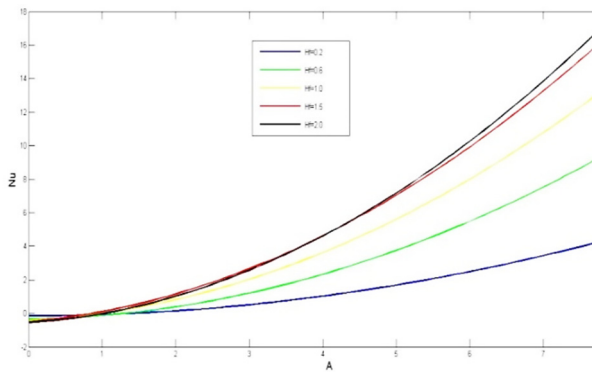


Figure (xiv). Effect of H_f on Nusselt number

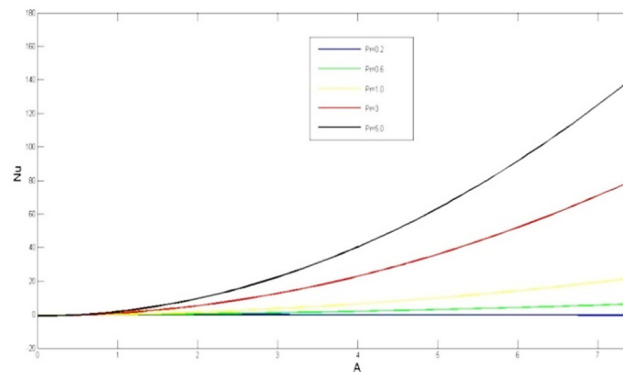


Figure (xv). Effect of Pr on Nusselt number

CONCLUSIONS

- (i) The effect of slip-flow regime is such that fluid velocity is maximum at the wall and away from it gradually decreases. Effects of suction parameter, heat change parameter, Rarefaction parameter and medium permeability are distinctively visible at the wall while away from it less dependent. At the wall, and near it fluid velocity increases directly with the increase of suction parameter, heat change parameter and permeability parameter; whereas decreases with the increase of effects of rarefaction parameter. Change of Prandtl number has direct impact on velocity within very near the wall only. Nature of these variations is same for rise of magnetic field.
- (ii) Within the slip flow regime Grashoff number has direct effect while magnetic field has opposite effect.
- (iii) Unlike the fluid velocity, fluid temperature, within the slip-flow regime initially increases and then decreases exponentially away from the wall. With the rise of heat change parameter and suction parameter, temperature rises whereas it decreases with the rise of frequency.
- (iv) Skin friction on the wall decreases with the rise of suction parameter whereas it decreases for higher values of heat change parameter and rarefaction parameter. This nature variation is same for increase of magnetic field.
- (v) Nusselt number that represents the rate of heat transfer within the fluid, increases with rise of suction parameter; it increases with the rise of heat change parameter, rarefaction parameter and Prandtl number.

APPENDICES

$$A_2 = \frac{H_f}{Pr + H_f}, A_3 = \frac{(H_f + Pr)A_4}{m_2 - H_f}, A_4 = \frac{i4AA_2Pr}{\omega}, A_5 = \frac{A_6(1 + Pr)}{1 - m_3}, A_6 = \frac{-2iAP_rA_4e^{-Pr\gamma}}{\omega},$$

$$A_7 = \frac{A_6(1 + HP_r)}{1 + Hm_1}, A_8 = \frac{G_rA_2}{Pr^2 - Pr - \xi}, A_9 = \frac{-(1 + Hm_2)A_{10} + (1 + HP_r)A_{11} + (1 + Hm_1)A_{12}}{1 + Hm_3},$$

$$A_{10} = \frac{G_rA_3}{m_2^2 - m_2 - (\xi + \frac{i\omega}{4})}, A_{11} = \frac{G_rA_4 - AA_8}{Pr^2 - Pr - (\xi + \frac{i\omega}{4})}, A_{12} = \frac{AA_7}{m_1^2 - m_1 - (\xi + \frac{i\omega}{4})},$$

$$A_{13} = \frac{-(1+m_3)A_{14}+(1+Pr)A_{15}-(1+m_2)A_{16}+(1+m_1)A_{17}}{1+Hm_4}, A_{14} = \frac{-G_r A_8 + AA_9 m_3}{m_3^2 - m_3 - (\xi + \frac{i\omega}{2})},$$

$$A_{15} = \frac{-G_r A_6 + AA_{11} Pr}{Pr^2 - Pr - (\xi + \frac{i\omega}{2})}, A_{16} = \frac{AA_{10} m_2 e^{-m_2 y}}{m_2^2 - m_2 - (\xi + \frac{i\omega}{2})}, A_{17} = \frac{AA_{12} m_1 e^{-m_1 y}}{m_1^2 - m_1 - (\xi + \frac{i\omega}{2})}, m_1 = \frac{-1 + \sqrt{1 - 4(m + \frac{1}{K})}}{2}$$

$$m_2 = -\frac{Pr}{2} (1 + \sqrt{1 + \frac{i\omega}{Pr}}), m_3 = \frac{-Pr}{2} (1 + \sqrt{1 + \frac{2i\omega}{Pr}}), m_4 = \frac{-1 + \sqrt{1 + 4(m + \frac{1}{K}) + 2i\omega}}{2}$$

ORCID

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ВПЛИВ РОЗРІДЖЕНЬ ТА КОНВЕКТИВНОЇ ТЕПЛОЗМІНИ НА ВІЛЬНИЙ КОНВЕКТИВНИЙ НЕСТАЦІЙНИЙ МГД-ПОТІК У РЕЖИМІ КОВЗАННЯ ПОВЗ ВЕРТИКАЛЬНУ СТІНКУ З ГРАНИЧНОЮ УМОВОЮ КОНВЕКТИВНОЇ ПОВЕРХНІ

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Досліджено нестационарний вільний конвективний двовимірний МГД-течія повз вертикальну пористу пластину з граничною умовою конвективної поверхні в пористому середовищі в режимі ковзання під дією змінної швидкості всмоктування. Аналітичні розв'язки отримані для системи з використанням методу збурень, який перетворює нелінійні зв'язані керівні диференціальні рівняння в часткових похідних у безрозмірну форму звичайних диференціальних рівнянь. Вплив змінної швидкості всмоктування, параметра розрідження та параметра зміни тепла аналізується та обговорюється графічно для різних значень ефективного фізичного параметра, такого як число Грассофа, параметр магнітного поля, число Прандтля, параметр проникності, на швидкість рідини та температуру, тертя шкіри та теплопередачу.

Ключові слова: *магнітогідродинамічний потік; проникність; розрідження; конвективна зміна тепла; змінна швидкість всмоктування*