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BARROW HOLOGRAPHIC DARK ENERGY MODEL IN BIANCHI TYPE-III UNIVERSE WITH QUINTESSENCE

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In this paper, we study a spatially homogeneous and anisotropic Bianchi type-III universe containing cold dark matter and Barrow holographic dark energy within the framework of General Relativity. We assume the cold dark matter and Barrow holographic dark energy to be non-interacting and obtain exact solutions of the Einstein field equations by considering a hybrid expansion law and assuming that the expansion scalar is proportional to the shear scalar. We examine the physical and kinematical properties of the resulting model using parameters such as the Hubble parameter, the anisotropic parameter, the deceleration parameter, the equation of state parameter, the jerk parameter etc. We also examine whether the energy conditions are violated or validated. We find that the Null, Weak, and Dominant energy conditions are fulfilled, while the Strong Energy Condition is violated, which supports the accelerated expansion of the universe. The Statefinder diagnostics have been conducted based on recent cosmological observations. In addition, we reformulated the correspondence between quintessence scalar field and Barrow holographic dark energy model.

Keywords: *Cosmic accerleration; Barrow holographic dark energy; Bianchi type-III; Cold dark matter; Deceleration parameter; Equation of state parameter*

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1. INTRODUCTION

The observational data from Supernovae Type Ia [1, 2] reveal that the universe is currently undergoing a phase of accelerated expansion and various other probes such as fluctuation of Cosmic Microwave Background Radiation (CMBR) [3], Sloan Digital Sky Survey (SDSS) [4], Wilkinson Microwave Anisotropy Probe (WMAP) [5], Large Scale Structure (LSS) [6], Baryonic Acoustic Oscillations [7], Gravitational Lensing [8], Chandra X-ray observatory [9] etc., strongly support the accelerated expansion of the universe at late times as well as the inflationary phase at the early stage of the universe. The observations demonstrate that the universe is dominated by some unusual cosmic fluid, namely the dark energy (DE), which is thought to be the driving force behind the ongoing expansion of the universe. According to standard cosmology, the dark energy exerts a huge negative pressure which makes up about three quaters of the total content of the universe.

In search of this mysterious dark energy various candidates are proposed in the literature including the cosmological constant Λ introuduced by Albert Einstein in his field equations [10] which is the simplest and the most obvious theoretical candidate with the equation of state (EoS) parameter $\omega = -1$. However, there exist notable conceptual challenges that arise, including issues of fine-tuning and the cosmic coincidence problems [11]. So, to overcome these problems, several researchers have developed a varity of dynamically evolving scalar field models such as quintessence [12], k-essence [13], exotic fluid models like Chaplygin gas models [14], tachyon [15] etc.

An alternative solution to the puzzle of dark energy, called holographic dark energy (HDE), is also proposed in the literature which arises from a quantum gravitational principle called the Holographic Principle. The fundamental concept behind the Holographic Principle is that the quantity of possible configurations, known as degrees of freedom, that contribute to the entropy of a physical system is proportional to the surface area that surrounds the system, rather than its volume. G.'t Hooft [16] initially proposed this concept as an explanation for the thermodynamics of black hole physics. Subsequently, Fischler and Susskind [17] expanded upon this principle, applying it to the cosmological context by suggesting that the gravitational entropy flows through the past light-cone of that particular surface. To implement the holographic principle and formulate holographic dark energy, one utilizes the expression for black hole entropy. As a result, various versions of the theory can be derived by employing different entropy expressions. Li [18] investigated the dark energy model by considering the Holographic Principle with the choice of future event horizon as the IR cut-off which gives the desired scenerio of an accelerating universe. Tsallis and Cirto in 2013 developed a holographic dark energy model namely Tsallis holographic dark energy (THDE) model by utilising the Tsallis generalized entropy $S_\delta = \gamma A^\delta$ where A is horizon area, γ is unknown parameter and δ is a non-additive parameter [19]. A new holographic

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dark energy (NHDE) model is propose by considering the Granda and Oliveros cut-off [20] whose energy density is given by the the square of Hubble parameter and its time derivative. An another holographic dark energy model called Rényi holographic dark energy (RHDE) model was developed by using the Rényi entropy [21].

A noval black hole entropy relation has recently been proposed by Barrow [22] which emerges from the inclusion of quantum gravitational effects, potentially introducing complex and fractal characteristics to the area of a black hole, namely $S_B = \left(\frac{A}{A_{pl}}\right)^{1+\frac{\Delta}{2}}$, where A is the standard horizon area, A_{pl} is the Planck area and the exponent Δ with $0 \le \Delta \le 1$ quantifies the quantum-gravitational deformation. $\Delta = 0$ corresponds to the standard Bekenstein-Hawking entropy, i.e., the standard smooth structure while $\Delta = 1$ corresponds to the most intricate and fractal structure. The standard holographic dark energy is given by the inequality $\rho_{\mathrm{HDE}} L^4 \le S$, where L denotes horizon length, and under the imposition $S \propto A \propto L^2$ [23], the Barrow entropy will give the energy density of Barrow holographic dark energy (BHDE) as $\rho_{\mathrm{BHDE}} = 3c^2 M_p^2 L^{\Delta-2}$, where c is a parameter and M_p is the Planck mass. Recently Saridakis [24] constructed the BHDE by applying the Barrow entropy instead of the usual Bekenstein-Hawking entropy. Srivastava and Sharma [25] also investigate the BHDE with Hubble horizon as IR cut-off for flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe. Paul *et al.* [26] investigated Bianchi type-I universe in the presence of BHDE where they found that the new exponent plays an important role to identify the nature of the univese.

Even though the universe on a large scale appears to be homogeneous and isotropic, described well by the Friedmann-Lemaître-Robertson-Walker model, it is worth noting that there is no observational evidence that definitively rules out the existence of an anisotropic universe. Anisotropies might have been present in the early universe, as suggested by measurements of the anisotropy of the Cosmic Microwave Background (CMB). To better understand the dynamics of the evolving universe, spatially homogeneous and anisotropic Bianchi models are often considered, as they represent the simplest models with an anisotropic background and play a significant role in describing the large-scale behavior of the universe. Bianchi type models offer a way to include the influence of anisotropy, effectively connecting the homogeneous and isotropic FLRW models with the inhomogeneous and anisotropic models. Consequently, numerous researchers have explored spatially homogeneous and anisotropic Bianchi cosmological models in various contexts.

The cosmological constant Λ associated with the vacuum energy density provides crucial information about the universe's large-scale behavior. A small value of Λ suggests that the observable universe is large and relatively flat. The inspiration to involve the cosmological constant Λ is that most DE models usually feature only a single component for DE. However, since the cold dark matter consists of more than one component, it may be possible that DE also consist of multiple components. Recently, a new DE model named ΛHDE has been proposed in which the DE contains two DE components namely the cosmological canstant Λ and the holographic dark energy [**?**]. Therefore, in our present study, we consider the dark energy to be composed of two components: the cosmological constant Λ and BHDE.

In this paper, we consider a spatially homogeneous and anisotropic Bianchi type-III universe filled with pressureless cold dark matter and non-interacting Barrow holographic dark energy with IR cutoff as Hubble horizon $L = H^{-1}$. The paper is organised as follows: In section 2, we derive the modified Einstein field equations for the Bianchi type-III metric. In section 3, we obtain exact solutions of the field equations by considering a hybrid expansion law and assuming that the expansion scalar is proportional to the shear scalar. Additionally, various cosmologically significant parameters are taken into account. In section 4, we explore the physical and kinematical properties of the model and also analyze the energy conditions. In Section 5, we discuss the Statefinder diagnostics and analyze its implications. The correspondence between BHDE with quintessence scalar field is explained in Section 6. In section 7, we provide some concluding remarks on our findings.

2. THE METRIC AND FIELD EQUATIONS

We consider the spatially homogeneous and anisotropic Bianchi type-III space-time given by the metric

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2lx} dy^2 + C^2 dz^2 \tag{1}$$

where A, B, C are the functions of the cosmic time t only and l is a positive constant.
The Einstein modified field equations in natural unit are given by

$$R_{ij} - \frac{1}{2} R g_{ij} + g_{ij}\Lambda = (T_{ij} + \bar{T}_{ij}) \tag{2}$$

where R_{ij} is the Ricci tensor, $R = g^{ij} R_{ij}$ is the Ricci scalar, g_{ij} is the metric tensor, Λ is the cosmological constant, T_{ij} and $\bar{T}_{ij}$ are the energy momentum tensors of cold dark matter and Barrow HDE respectively. T_{ij} and $\bar{T}_{ij}$ are defined by

$$T_{ij} = \rho_m u_i u_j \tag{3}$$

and

$$\bar{T}_{ij} = (\rho_{\mathrm{BHDE}} + p_{\mathrm{BHDE}})u_i u_j + g_{ij}p_{\mathrm{BHDE}} \tag{4}$$

where ρ_m is the energy density of cold dark matter, and ρ_{BHDE} and p_{BHDE} are the energy density and pressure of the Barrow HDE respectively. The four velocity u_i satisfy $u_i u^i = -1$. Here, we consider the Barrow HDE density [24] given by

$$\rho_{\mathrm{BHDE}} = DH^{2-\Delta} \tag{5}$$

where D is a constant and H is the Hubble parameter.

In comoving coordinate system, the Einstein field equations (2) with the help of equations (3) and (4) for the metric (1) lead to the following system of field equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = -p_{\mathrm{BHDE}} + \Lambda \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = -p_{\mathrm{BHDE}} + \Lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{l^2}{A^2} = -p_{\mathrm{BHDE}} + \Lambda \tag{8}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{l^2}{A^2} = \rho_m + \rho_{\mathrm{BHDE}} + \Lambda \tag{9}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{10}$$

Here, an over dot (.) denotes differentiation with respect to the cosmic time t.
On integrating, from equation (10), we obtain

$$B = mA \tag{11}$$

where m is a constant of integration.
Looking at the equation (11), the field equations (6)-(10) can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = -p_{\mathrm{BHDE}} + \Lambda \tag{12}$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{l^2}{A^2} = -p_{\mathrm{BHDE}} + \Lambda \tag{13}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{l^2}{A^2} = \rho_m + \rho_{\mathrm{BHDE}} + \Lambda \tag{14}$$

The continuity equation is derived from the energy conservation law $(T^{ij} + \bar{T}^{ij})_{;j} = 0$ as

$$\dot{\rho}_m + \dot{\rho}_{\mathrm{BHDE}} + 3H(\rho_m + \rho_{\mathrm{BHDE}} + p_{\mathrm{BHDE}}) = 0 \tag{15}$$

We assume the cold dark matter and Barrow HDE to be non-interacting, therefore they conserve independently as

$$\dot{\rho}_m + 3H\rho_m = 0 \tag{16}$$

and

$$\dot{\rho}_{\mathrm{BHDE}} + 3H(\rho_{\mathrm{BHDE}} + p_{\mathrm{BHDE}}) = 0 \tag{17}$$

We now present some cosmological parameters which can be used to understand the evolution of the universe. For the given metric, the average scale factor a which parameterize the relative expansion of the universe, the spatial volume V and the mean Hubble parameter H which measures the rate of expansion of the universe are defined by

$$a = (ABC)^{\frac{1}{3}} \tag{18}$$

$$V = a^3 = ABC \tag{19}$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{20}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters along the x, y, z directions respectively. The expansion scalar θ, the shear scalar σ, the average anisotropy parameter A_m are defined by

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{21}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6} \tag{22}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{23}$$

where $\Delta H_i = H_i - H$ $(i = 1, 2, 3)$.
The expansion scalar θ measures the relative rate of expansion (or contraction) of the universe, the shear scalar θ gives the measure of the deformation of the cosmic structure caused by density fluctuation while the anisotropy parameter A_m measures the deviation from isotropy.

3. COSMOLOGICAL SOLUTIONS OF THE FIELD EQUATIONS

We have three independent field equations (12)-(14) with six unknowns A, C, ρ_m, ρ_{BHDE}, p_{BHDE} and Λ together with the equation (5). So, to obtain an exact solution of the system, we need two additional constraints related to these parameters.

As a first constraint, we assume the expansion scalar θ to be proportional to the shear scalar σ which gives the relationship between the metric potentials as

$$C = A^k \tag{24}$$

where $k \neq 1$ is a positive constant.

Secondly, following the Akarsu *et al.* [28], we consider the average scale factor a to obey the hybrid expansion law given by

$$a(t) = a_0 t^\alpha e^{\beta(t-1)} \tag{25}$$

where α and β are non-negative constants. The relation (25) gives the power law for $\beta = 0$ and exponential law for $\alpha = 0$.
Then, from equations (11), (18) and (24), we get the directional scale factors as

$$A = m^{\frac{-1}{k+2}}\{a_0 t^\alpha e^{\beta(t-1)}\}^{\frac{3}{k+2}} \tag{26}$$

$$B = m^{\frac{k+1}{k+2}}\{a_0 t^\alpha e^{\beta(t-1)}\}^{\frac{3}{k+2}} \tag{27}$$

$$C = m^{\frac{-k}{k+2}}\{a_0 t^\alpha e^{\beta(t-1)}\}^{\frac{3k}{k+2}} \tag{28}$$

Therefore, the metric (1) becomes

$$ds^2 = -dt^2 + \{a_0 t^\alpha e^{\beta(t-1)}\}^{\frac{6}{k+2}}\left(m^{\frac{-2}{k+2}}dx^2 + m^{\frac{2(k+1)}{k+2}}e^{-2lx}dy^2\right) + m^{\frac{-2k}{k+2}}\{a_0 t^\alpha e^{\beta(t-1)}\}^{\frac{6k}{k+2}}dz^2 \tag{29}$$

4. PHYSICAL AND KINEMATICAL PROPERTIES OF THE MODEL

For our model (29), the directional Hubble parameters H_1, H_2, H_3 and mean Hubble parameter H are obtained as

$$H_1 = H_2 = \frac{3}{k+2}\left(\beta + \frac{\alpha}{t}\right), \quad H_3 = \frac{3k}{k+2}\left(\beta + \frac{\alpha}{t}\right) \tag{30}$$

$$\therefore \quad H = \left(\beta + \frac{\alpha}{t}\right) \tag{31}$$

The spatial volume V, the expansion scalar θ, the shear scalar σ and the average anisotropy parameter A_m of the model are obtained as

$$V = \{a_0 t^\alpha e^{\beta(t-1)}\}^3 \tag{32}$$

$$\theta = 3H = 3\left(\beta + \frac{\alpha}{t}\right) \tag{33}$$

$$\sigma^2 = 3\frac{(k-1)^2}{(k+2)^2}\left(\beta + \frac{\alpha}{t}\right)^2 \tag{34}$$

$$A_m = 2\frac{(k-1)^2}{(k+2)^2} \tag{35}$$

From equations (31)-(35), we find that the spatial volume V is zero at the beginning of the universe. Therefore, our model starts with a Big Bang singularity. We plot the graph of scale factor a in figure 1. The Hubble parameter H, the expansion scalar θ, and the shear scalar σ are all diverse at $t = 0$ and decrease with an increase in cosmic time t up to a fixed limit. Figure 2 depicts the evolution of the Hubble parameter H over cosmic time t. The anisotropy parameter A_m and the shear scalar σ become nonzero constant for $k \neq 1$, so the model will remain anisotropic throughout the evolution of the universe. Both the parameters are equal to zero for $k = 1$. Therefore, the universe can become isotropic only when $k = 1$.

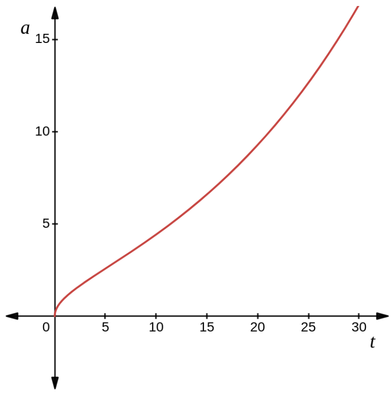

Figure 1. Evolution of the scale factor a versus cosmic time t for $\alpha = 0.49$ and $\beta = 0.04$

The deceleration parameter (q) is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2} \tag{36}$$

Using equation (25), the deceleration parameter q for our model is obtained as

$$q = -1 + \frac{\alpha}{(\beta t + \alpha)^2} \tag{37}$$

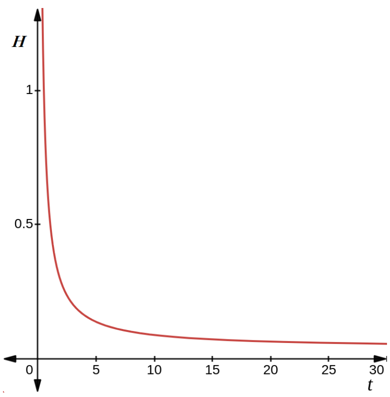

Figure 2. Evolution of the Hubble parameter H versus cosmic time t for $\alpha = 0.49$ and $\beta = 0.04$

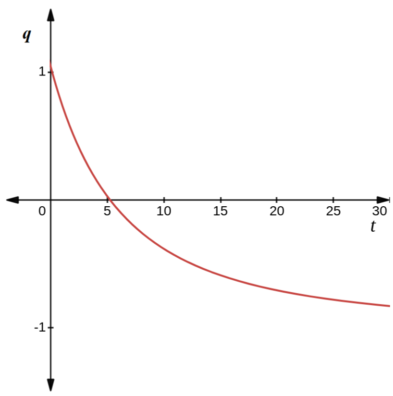

Figure 3. Evolution of the deceleration parameter q versus cosmic time t for $\alpha = 0.49$ and $\beta = 0.04$

The deceleration parameter q is a measure of whether the expansion of the universe is accelerating, decelerating or uniform. The expansion is accelerating for $q < 0$, decelerating for $q > 0$ and uniform for $q =$

0. Observations indicate that the universe is currently expanding at an accelerating rate, with a deceleration parameter in the range of $-1 \le q < 0$.

From equation (37), it is clear that the universe evolves with variable deceleration parameter when both α and β are non zero. Figure 3 demonstrates the evolution of deceleration parameter q versus cosmic time t and shows that the deceleration parameter q is positive in the early universe but transition to negative at time $t = \frac{\sqrt{\alpha}-\alpha}{\beta}$ and $t = -\frac{(\sqrt{\alpha}+\alpha)}{\beta}$. Here, we do not consider the case $t = -\frac{(\sqrt{\alpha}+\alpha)}{\beta}$ as we consider our universe to begin at time $t = 0$ (in billion years). Hence, cosmic acceleration took place at the time $t = \frac{\sqrt{\alpha}-\alpha}{\beta}$, which implies that the parameter α falls within the range of $0 < \alpha < 1$. This transition marks the switch from early decelerating phase of the universe to its current accelerating phase. In this paper, we choose $\alpha = 0.49$ and $\beta = 0.04$ as these smaller values of α and β result in the universe's acceleration occurring around approximately 5.5 billion years, aligning with observational data. A relatively larger value of β would indicate acceleration happening much earlier in the evolution of the universe.

Using equations (31) and (25) in equation (16), to obtain the matter energy density ρ_m as

$$\rho_m = M\{a_0 t^{\alpha} e^{\beta(t-1)}\}^{-3} \tag{38}$$

where M is a constant.

Using equation (31) in equation (5), we obtain the Barrow HDE density ρ_{BHDE} as

$$\rho_{\text{BHDE}} = D\left(\beta + \frac{\alpha}{t}\right)^{2-\Delta} \tag{39}$$

From equations (38) and (39), we notice that both the energy densities are decreasing functions of cosmic time t. Figure 4 depicts the evolution of matter energy density ρ_m which demonstrates that it is high at early stage of the universe and tends to zero at late times. The BHDE density given by the equation (39) behaves like the standard HDE for $\Delta = 0$. The behavior of BHDE will differ from the standard one, depending on the Δ parameter, resulting to a distinct cosmological scenario. To understand the evolution of BHDE, we plot it against the cosmic time t in figure 5 for different values of Δ. From figure 5, we can see that for $\Delta = 0$, 0.5 and 1.5, the BHDE density decrease with an increase of cosmic time t and tend to a constant value at late times. Whereas, for $\Delta = 2$, the BHDE density is constant throughout the evolution of the universe. In this scenario, the BHDE behaves like a cosmological constant and the model is reffered as ΛCDM model. The physical implications of the decrease in energy densities suggest that the volume of the universe is increasing.

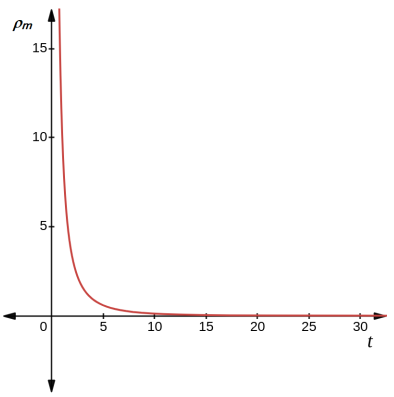

Figure 4. Evolution of matter energy density ρ_m versus cosmic time t for $M = 10$, $\alpha = 0.49$, $\beta = 0.04$ and $a_0 = 1$

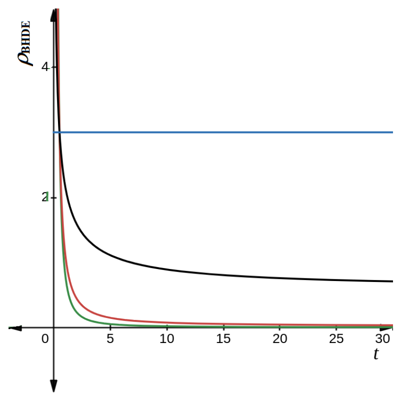

Figure 5. Evolution of BHDE density ρ_{BHDE} versus cosmic time t for $D = 3$, $\alpha = 0.49$, $\beta = 0.04$ and $\Delta = 0$ (green line), $\Delta = 0.5$ (red line), $\Delta = 1.5$ (black line), $\Delta = 2$ (blue line)

Using equations (39) and (31) in equation (17), we get pressure p_{BHDE} of the Barrow HDE as

$$p_{\text{BHDE}} = -D\left(\beta + \frac{\alpha}{t}\right)^{2-\Delta} + \frac{D\alpha(2-\Delta)}{3t^2}\left(\beta + \frac{\alpha}{t}\right)^{-\Delta} \tag{40}$$

Figure 6 displays the Barrow holographic dark energy pressure p_{BHDE} with respect to cosmic time t, as given by the equation (40). In case of $\Delta = 0$ and 0.5, the pressure is positive in the initial stage and decreases as cosmic time t increases and eventually become negative at late times. For $\Delta = 1.5$, the pressure p_{BHDE} is extremely negative at the beginning and rises gradually as cosmic time t increases until it reaches a certain constant value. However for $\Delta = 2$, the pressure is constantly negative in the entire evolution of the universe. This indicates that the universe is undergoing accelerated expansion for all the values of Δ, as the pressure remains negative throughout the evolution.

Using equations (26), (28) and (40) in equation (12), we obtain the expression for cosmological constant Λ as

$$\Lambda = \frac{9(k^2+k+1)}{(k+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2 - \frac{3\alpha(k+1)}{t^2(k+2)} - D\left(\beta+\frac{\alpha}{t}\right)^{2-\Delta} + \frac{D\alpha(2-\Delta)}{3t^2}\left(\beta+\frac{\alpha}{t}\right)^{-\Delta} \tag{41}$$

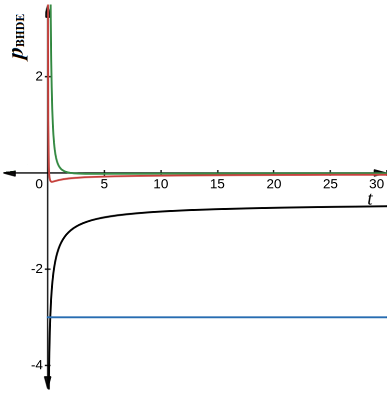

Figure 6. Evolution of BHDE pressure p_{BHDE} versus cosmic time t with $\alpha = 0.49$, $\beta = 0.04$, $D = 3$ and $\Delta = 0$ (green line), $\Delta = 0.5$ (red line), $\Delta = 1.5$ (black line), $\Delta = 2$ (blue line)

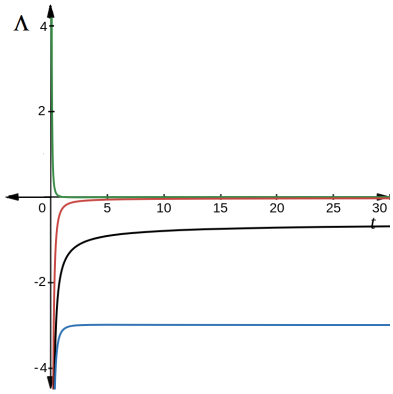

Figure 7. Evolution of cosmological constant Λ versus cosmic time t with $\alpha = 0.49$, $\beta = 0.04$, $D = 3$, $k = 0.1$ and $\Delta = 0$ (green line), $\Delta = 0.5$ (red line), $\Delta = 1.5$ (black line), $\Delta = 2$ (blue line)

Figure 7 illustrates the evolution of Λ with respect to cosmic time t for different value of Δ. From figure 7, we observe that Λ is positive but decreasing for $\Delta = 0$ and negative but increasing for $\Delta = 0.5$, 1.5 and 2 with cosmic time t. When Δ is set to 0 and 0.5, the value of Λ approaches zero at the later stages of the universe. However, for $\Delta = 1.5$ and 2, Λ remains at a constant negative value. Despite the negative value of Λ for $\Delta = 0.5$, 1.5, and 2, it increases with cosmic time t. In both scenarios, the negative pressure at late times signifies the accelerated expansion of the universe. Biswa and Mazumdar [29] and Anjan *et al.* [30] have investigated cosmological models that feature a negative Λ. Currently, the determination of Λ is not only complicated, but it is also uncertain.

Using equations (39) and (40), the equation of state parameter (EoS parameter) ω_{BHDE} of Barrow HDE is obtained as

$$\omega_{\mathrm{BHDE}} = \frac{p_{\mathrm{BHDE}}}{\rho_{\mathrm{BHDE}}} = -1 + \frac{\alpha(2-\Delta)}{3t^2\left(\beta+\frac{\alpha}{t}\right)^2} \tag{42}$$

Equation (42) implies that the EoS parameter ω_{BHDE} in our model is a strictly decreasing function of cosmic time t for $\Delta < 2$. Figure 8 depicts the variation of EoS parameter ω_{BHDE} with cosmic time t for different values of Δ. The figure displays that, for $\Delta = 0$, 0.5 and 1.5, the EoS parameter ω_{BHDE} in our model varies in the quintessence region ($-1 < \omega_{\mathrm{BHDE}} < -\frac{1}{3}$) after a particular point in time during its evolution. Additionally, we can note that the EoS parameter ω_{BHDE} converges to -1 at late times and eventually approach the ΛCDM ($\omega_{\mathrm{BHDE}} = -1$) model in future. When $\Delta = 2$, the EoS parameter ω_{BHDE} remains constant at a value of -1 throughout its evolution. As previously mentioned, in this scenario, the Barrow holographic dark energy behaves like the cosmological constant. Considering all the cases of our model, we can conclude that the model will have a more accelerating effect at higher values of cosmic time t.

The density parameters are read as

$$\Omega_m = \frac{\rho_m}{3H^2},\ \Omega_{\mathrm{BHDE}} = \frac{\rho_{\mathrm{BHDE}}}{3H^2},\ \Omega_\sigma = \frac{\sigma^2}{3H^2},\ \Omega_\Lambda = \frac{\Lambda}{3H^2},\ \Omega_l = \frac{l^2}{3H^2A^2}$$

where Ω_m, $\Omega_{\rm BHDE}$, Ω_Λ, Ω_σ and Ω_l represent the dimensionless density parameters for matter, Barrow HDE, vacuum energy, shear anisotropy and l parameter respectively.

From equation (14), the total energy density parameter (Ω) is obtained as

$$\Omega \equiv \Omega_m + \Omega_{\rm BHDE} + \Omega_\sigma + \Omega_\Lambda + \Omega_l = 1 \tag{43}$$

Therefore, equation (43) imply

$$\Omega = \frac{M}{3\left(\beta+\frac{\alpha}{t}\right)^2\{a_0 t^\alpha e^{\beta(t-1)}\}^3} + \frac{3(k^2+k+1)}{(k+2)^2} - \frac{\alpha(k+1)}{t^2(k+2)\left(\beta+\frac{\alpha}{t}\right)^2} + \frac{D\alpha(2-\Delta)}{9t^2\left(\beta+\frac{\alpha}{t}\right)^{2+\Delta}} + \frac{(k-1)^2}{(k+2)^2} + \frac{m^{\frac{2}{k+2}}\times l^2}{3\left(\beta+\frac{\alpha}{t}\right)^2\{a_0 t^\alpha e^{\beta(t-1)}\}^{\frac{6}{k+2}}} = 1 \tag{44}$$

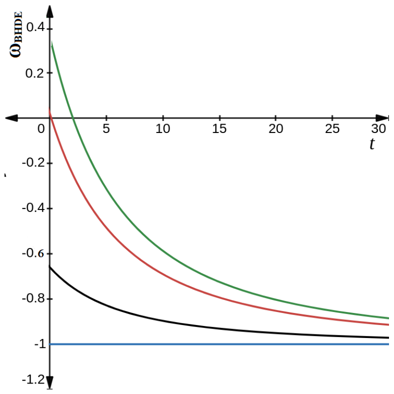

Figure 8. Evolution of EoS parameter $\omega_{\rm BHDE}$ versus cosmic time t with $\alpha = 0.49$, $\beta = 0.04$ and $\Delta = 0$ (green line), $\Delta = 0.5$ (red line), $\Delta = 1.5$ (black line), $\Delta = 2$ (blue line)

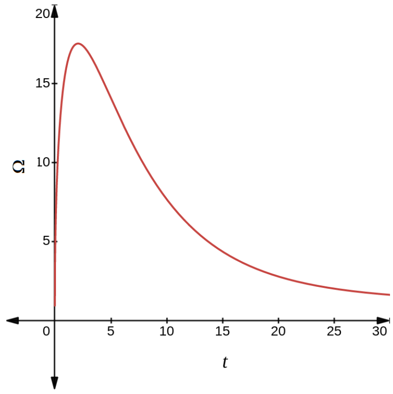

Figure 9. Evolution of overall density parameter Ω versus cosmic time t for $M = 10$, $\alpha = 0.49$, $\beta = 0.04$, $k = 0.1$, $\Delta = 0.5$, $l = 1$, $D = 3$ and $a_0 = 1$

The graph depicted in Figure 9 demonstrates how the total energy density Ω changes over cosmic time t. The total energy density parameter $\Omega > 1$, $\Omega = 1$ and $\Omega < 1$ represent the open, flat and closed universe respectively. As the figure indicates, the total density parameter Ω experiences a rapid increase during the early stages of the universe's evolution, reaching a fixed point before beginning to decrease as time goes on. Eventually, the total density parameter Ω approaches 1 in later times, indicating that the universe becomes flat at late times.

4.1. Energy conditions:

Energy conditions are a set of theoretical inequalities that serve as linear combinations of energy density and pressure within the framework of general relativity. These conditions establish constraints on how matter and energy are distributed in spacetime. Energy conditions help to understand the behavior of gravity and derives many theorems within the classical general relativity such as the collapse of stars [31], the area increase theorem related to black holes [31], the positive mass theorem [32] etc. There are several energy conditions, among which the linear energy conditions are the null energy condition (NEC), the weak energy condition (WEC), the strong energy condition (SEC) and the dominant energy condition (DEC).

The standard energy conditions for T_{ij}, where T_{ij} is the energy-momentum tensor of perfect fluid are defined as:

• Null Energy Condition (NEC): For any null vector k^i, $T_{ij}k^ik^j \geq 0$. This implies the relation

$$\rho + p \geq 0$$

The NEC ensures that the pressure cannot be so negative that it dominates over the energy density of the universe. This means that as the energy density of the universe decreases, the pressure should become less negative with the universe's expansion. Conversely, when the NEC is violated, it signifies that the pressure becomes more negative compared to the energy density, leading to a more accelerating effect in the universe.

• Weak Energy Condition (WEC): For any timelike vector v^i, $T_{ij}v^iv^j \geq 0$ which leads to

$$\rho \geq 0 \quad \text{and} \quad \rho + p \geq 0$$

The WEC includes the NEC and adds the requirement that the energy density of the universe remains positive throughout its evolution. The violation of the WEC implies that either the energy density becomes negative, or the pressure overwhelms the energy density of the universe.

• Strong Energy Condition (SEC): For any timelike vector v^i, $T_{ij}v^iv^j + \frac{1}{2}T \geq 0$, where T is the trace of T_{ij}. This leads to the relation

$$\rho + 3p \geq 0 \quad \text{and} \quad \rho + p \geq 0$$

The SEC implies a positive deceleration parameter, which leads to a decelerating universe. However, if the SEC is violated, it signifies that the universe is undergoing an accelerated expansion phase.

• Dominant Energy Condition (DEC): For any timelike vector v^i, $T_{ij}v^iv^k \geq 0$ and $J_iJ^i \leq 0$ where $J^i = -T^i_jv^j$. This results in the following relation

$$\rho \geq 0 \quad \text{and} \quad \rho + p \geq 0 \quad \text{and} \quad \rho - p \geq 0$$

The DEC is the WEC with an extra condition that the pressure should not exceed the energy density of the universe. The DEC can be interpreted as stating that the energy density is always non-negative and should have dominance over pressure of the universe.

For our model,

$$\rho + p = \frac{M}{\{a_0 t^{\alpha} e^{\beta(t-1)}\}^3} + \frac{D\alpha(2-\Delta)}{3t^2}\left(\beta + \frac{\alpha}{t}\right)^{-\Delta}$$

$$\rho - p = \frac{M}{\{a_0 t^{\alpha} e^{\beta(t-1)}\}^3} + 2D\left(\beta + \frac{\alpha}{t}\right)^{2-\Delta} - \frac{D\alpha(2-\Delta)}{3t^2}\left(\beta + \frac{\alpha}{t}\right)^{-\Delta}$$

$$\rho + 3p = \frac{M}{\{a_0 t^{\alpha} e^{\beta(t-1)}\}^3} - 2D\left(\beta + \frac{\alpha}{t}\right)^{2-\Delta} + \frac{D\alpha(2-\Delta)}{t^2}\left(\beta + \frac{\alpha}{t}\right)^{-\Delta}$$

The above mentioned energy conditions are generally satisfied by the normal matter. However, the SEC is known to be violated in an accelerated expansion phase of the universe. Figure 10 shows a plot of the left side of the energy conditions, which indicates that in our model, initially, the NEC, WEC, SEC and DEC are all satisfied, but at late times, the SEC gets violated. The violation of the SEC results in the acceleration of the universe.

4.2. Jerk parameter (j):

The cosmic jerk parameter j is a measure of the acceleration of the universe, describing how quickly the rate of expansion is changing over cosmic time t. It is a dimensionless quantity defined as the third derivative of the scale factor a with respect to cosmic time t and provides an important information about the universe's expansion. A positive value of jerk parameter j signifies that the universe undergoes a transition from deceleration to acceleration in its expansion. For the popular ΛCDM model the value of jerk parameter j is equal to one.

Mathematically, the jerk parameter j is defined as

$$j = \frac{1}{aH^3}\frac{d^3a}{dt^3} \tag{45}$$

For our model, the jerk parameter j is obtained as

$$j = \frac{\beta^3 + \frac{3\alpha\beta^2}{t} + \frac{(3\alpha^2-3\alpha)\beta}{t^2} + \frac{\alpha^3-3\alpha^2+2\alpha}{t^3}}{\left(\beta + \frac{\alpha}{t}\right)^3} \tag{46}$$

The graph illustrating the jerk parameter j is displayed in figure 11. It is clear from the figure that the jerk parameter j remains positive throughout the universe's evolution, signifying that the rate of expansion of the universe is increasing. Additionally, in figure 11, it can be observed that the jerk parameter j converges to 1 at late times, indicating that the model behaves similar to the ΛCDM model.

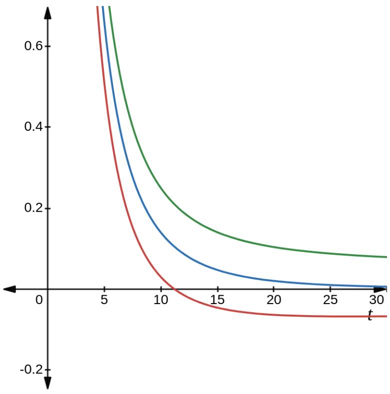

Figure 10. Graph of energy conditions versus cosmic time t for $M = 10$, $a_0 = 1$, $\alpha = 0.49$, $\beta = 0.04$, $D = 3$, $\Delta = 0.5$, $\rho + p$ (blue line), $\rho - p$ (green line) and $\rho + 3p$ (red line)

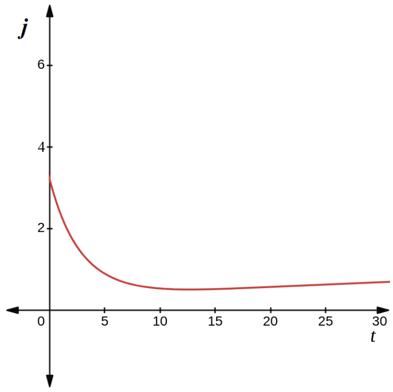

Figure 11. Evolution of jerk parameter j versus cosmic time t for $\alpha = 0.49$ and $\beta = 0.04$

4.3. Snap parameter (s):

The snap parameter s is a dimensionless quantity defined as the fourth order derivative of the scale factor a with respect to cosmic time t. It describes the rate of change of the acceleration of the universe's expansion and helps to understand the dynamics of the universe.

Mathematically, the snap parameter s is defined as

$$s = \frac{1}{aH^4}\frac{d^4a}{dt^4} \tag{47}$$

For our model, the snap parameter s is obtained as

$$s = \frac{\beta^4 + \frac{4\alpha\beta^3}{t} + \frac{(6\alpha^2-6\alpha)\beta^2}{t^2} + \frac{(4\alpha^3-12\alpha^2+8\alpha)\beta}{t^3} + \frac{\alpha^4-6\alpha^3+11\alpha^2-6\alpha}{t^4}}{\left(\beta + \frac{\alpha}{t}\right)^4} \tag{48}$$

Figure 12 depicts the variation of the snap parameter s with respect to cosmic time t. The snap parameter s shows the increasing behavior. At $t \to 0$, it is negative and increases over cosmic time t. Eventually, the snap parameter s converges to 1 at late times. This suggests that the universe is experiencing a phase of accelerated expansion.

4.4. Lerk parameter (l):

The lerk parameter l is also a dimensionless quantity defined as the fifth order derivative of the scale factor a with respect to the cosmic time t.

Mathematically, the lerk parameter l is defined as

$$l = \frac{1}{aH^5}\frac{d^5a}{dt^5} \tag{49}$$

For our model, the lerk parameter l is obtained as

$$l = \frac{\beta^5 + \frac{5\alpha\beta^4}{t} + \frac{(10\alpha^2-10\alpha)\beta^3}{t^2} + \frac{(10\alpha^3-30\alpha^2+20\alpha)\beta^2}{t^3} + \frac{(5\alpha^4-30\alpha^3+55\alpha^2-30\alpha)\beta}{t^4} + \frac{\alpha^5-10\alpha^4+35\alpha^3-50\alpha^2+24\alpha}{t^5}}{\left(\beta + \frac{\alpha}{t}\right)^5} \tag{50}$$

Figure 13 illustrates the variation of the lerk parameter l with respect to cosmic time t. The figure displays that the lerk parameter l has a high value at $t \to 0$ and gradually converges to 1 at late times.

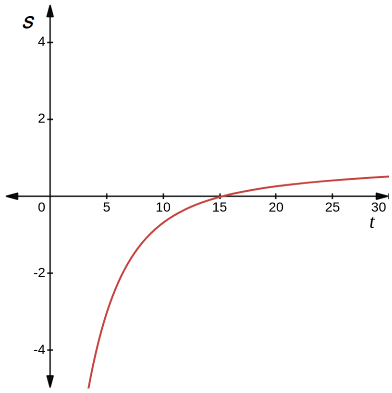

Figure 12. Evolution of snap parameter s versus cosmic time t for $\alpha = 0.49$ and $\beta = 0.04$

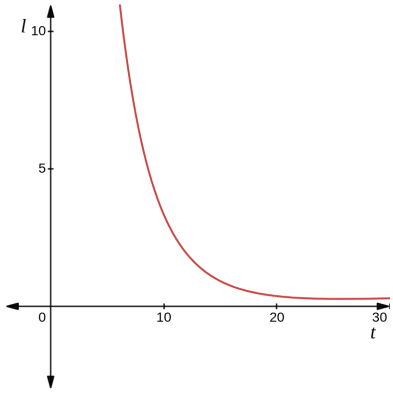

Figure 13. Evolution of lerk parameter l versus cosmic time t for $\alpha = 0.49$ and $\beta = 0.04$

4.5. Coincidence parameter ($\bar{r}$):

The parameter $\bar{r}$, known as the coincidence parameter, is defined as the ratio between two energy densities, i.e. $\bar{r} = \frac{\rho_{\text{BHDE}}}{\rho_m}$, of the universe. Current observations demand that the value of the coincidence parameter should either remain constant or change very slowly in response to the universe's expansion. But the most simplest and widely accepted dark energy model, the ΛCDM model, is inconsistent with this observations. So, numarous alternative models are considered to avoid this coincidence problem. For our model, the coincidence paramter $\bar{r}$ is obtained as

$$\bar{r} = \frac{\{a_0 t^{\alpha} e^{\beta(t-1)}\}^3 D(\beta + \frac{\alpha}{t})^{2-\Delta}}{M} \tag{51}$$

The variation of coincidence parameter $\bar{r}$ versus cosmic time t is shown in figure 14. From figure, we observe that the value of $\bar{r}$ increases rapidly at late times. So, the coincidence problem is not resolved in our model. Since, here we have considered the BHDE and dark matter to be non interacting, so, it will be interesting to see the behavior of coincidence parameter in case of interacting model. Thus, a specific form of interaction between BHDE and dark matter may make the ratio of their densities to remain relatively constant as the universe evolves.

5. STATEFINDER DIAGNOSTIC

To distinguish between various cosmological scenarios related to dark energy, it is essential to have a reliable and precise method to evaluate DE models. For this purpose Sahni *et al.* [33] introduced a diagnostic using the parameter pair $\{r, s\}$, commonly known as the Statefinder diagostic. The pair is effective in distinguishing among various dark energy models, such as the cosmological constant, quintessence, the Chaplygin gas, braneworld models, and models involving interacting dark energy. The dimensionless Statefinder diagnostic is constructed by considering the scale factor a of the universe and its higher order derivative with respect to cosmic time t only. The parameter r represents the hierarchy of geometrical cosmological parameters, succeeding the Hubble parameter H and the deceleration parameter q. On the other hand, s is a parameter derived as a linear combination of q and r in a way that makes it independent of the density associated with dark energy.

The Statefinder diagnostic pair $\{r, s\}$ is defined as

$$r = \frac{1}{aH^3}\frac{d^3a}{dt^3} \quad \text{and} \quad s = \frac{r-1}{3(q-\frac{1}{2})}, \quad \text{where} \quad q \neq \frac{1}{2}$$

For our model,

$$r = \frac{\beta^3 + \frac{3\alpha\beta^2}{t} + \frac{(3\alpha^2-3\alpha)\beta}{t^2} + \frac{\alpha^3-3\alpha^2+2\alpha}{t^3}}{\left(\beta + \frac{\alpha}{t}\right)^3} \tag{52}$$

$$s = \frac{-6\alpha\beta t - 6\alpha^2 + 4\alpha}{3(\beta t + \alpha)\{2\alpha - 3(\beta t + \alpha)^2\}} \tag{53}$$

The $r-s$ plane for these cosmological parameters for ΛCDM and standard CDM (SCDM) are $(1,0)$ and $(1,1)$ respectively. The quintessence DE epochs are given by the region $(r<1, s>0)$ whereas trajectories for Chaplygin gas lie in the range $(r>1, s<0)$. In our model, equations (52) and (53) indicate that the Statefinder diagnostic pair depend on the cosmic time t. As t approaches infinity, the diagnostic pair yields $r=1$ and $s=0$. The figure 15 also confirms that our model coincide with the ΛCDM model during later stages.

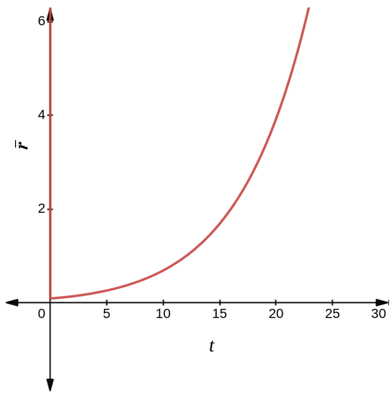

Figure 14. Evolution of coincidence parameter $\bar{r}$ versus cosmic time t for $\alpha=0.49$, $\beta=0.04$, $M=10$, $a_0=1$, $D=3$ and $\Delta=0.5$

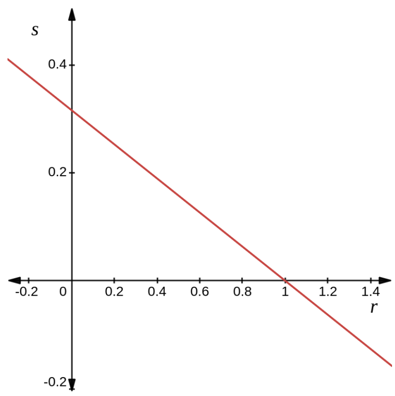

Figure 15. Evolution of the Statefinder parameters r versus s for $\alpha=0.49$ and $\beta=0.04$

6. CORRESPONDENCE BETWEEN THE BHDE MODEL AND QUINTESSENCE SCALAR FIELD MODEL

Quintessence is considered as one of the candidate of dark energy because it has the potential to explain the cosmic coincidence problem and can fit better with observational data compared to the cosmological constant. In order to get the accelerated expansion of the universe the EoS parameter for quintessence must be less than $-\frac{1}{3}$.

In this section, our focus is on investigating the correspondence between BHDE and the quintessence scalar field model. To establish the correspondence, we perform a comparison between the BHDE density and the energy density of the scalar field model. Additionally, we analyze the EoS parameter of BHDE in comparison to the EoS parameter of the scalar field.

The energy density and pressure of the quintessence scalar field model are given by

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) \tag{54}$$

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \tag{55}$$

where ϕ is the scalar field and $V(\phi)$ is the potential of scalar field ϕ.

The EoS parameter for the scalar field is given by

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \tag{56}$$

Equations (54) and (55) gives

$$\dot{\phi}^2 = \rho_\phi + p_\phi \tag{57}$$

and

$$V(\phi) = \frac{\rho_\phi - p_\phi}{2} \tag{58}$$

Using equations (42) and (56), we get

$$-1+\frac{\alpha(2-\Delta)}{3t^2\left(\beta+\frac{\alpha}{t}\right)^2}=\frac{\dot{\phi}^2-2V(\phi)}{\dot{\phi}^2+2V(\phi)} \tag{59}$$

By taking $\rho_\phi = \rho_{\text{BHDE}}$ and $p_\phi = p_{\text{BHDE}}$, we can calculate the kinetic energy $\dot{\phi}^2$ and the scalar potential $V(\phi)$ as

$$\dot{\phi}^2=\frac{D\alpha(2-\Delta)}{3t^2}\left(\beta+\frac{\alpha}{t}\right)^{-\Delta} \tag{60}$$

$$V(\phi)=D\left(\beta+\frac{\alpha}{t}\right)^{2-\Delta}-\frac{D\alpha(2-\Delta)}{6t^2}\left(\beta+\frac{\alpha}{t}\right)^{-\Delta} \tag{61}$$

Thus, we have established the potential $V(\phi)$ and scalar field ϕ for the quintessence scalar field model that corresponds to the BHDE model. The kinetic energy $\dot{\phi}^2$ is plotted in the figure 16. The figure illustrates that the kinetic energy $\dot{\phi}^2$ decreases over cosmic time t and eventually diminishes at late times. Figure 17 depicts the scalar field potential $V(\phi)$ for the quintessence model. It can be seen that the potential $V(\phi)$ is also decreasing function of cosmic time t and tends toward a constant value at late times.

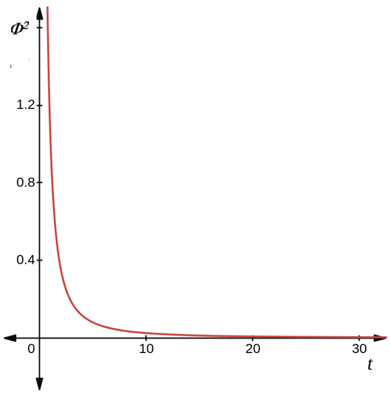

Figure 16. Evolution of kinetic energy $\dot{\phi}^2$ versus cosmic time t for $\alpha = 0.49$, $\beta = 0.04$, $D = 3$ and $\Delta = 0.5$

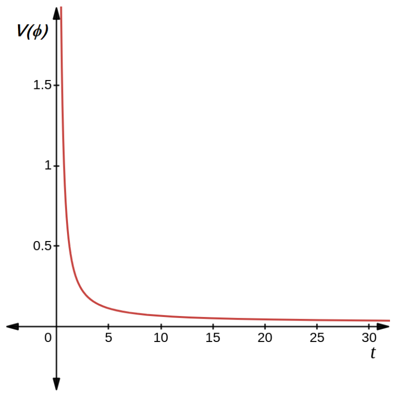

Figure 17. Evolution of the $V(\phi)$ versus t for $\alpha = 0.49$, $\beta = 0.04$, $D = 3$ and $\Delta = 0.5$

7. CONCLUSION

In this paper, we investigate a spatially homogeneous and anisotropic Bianchi type-III universe filled with pressureless cold dark matter and non-interacting Barrow holographic dark energy in the framework of Einstein's General theory of Relativity. We obtain the exact solution of the Einstein field equations by choosing a hybrid expansion law and by assuming the expansion scalar to be proportional to the shear scalar. We investigate various cosmological parameters to discuss the physical and kinematical properties of the model and find that:

- At $t = 0$, the volume V of the universe vanish i.e. the universe starts with zero volume and tends to infinity at $t \to \infty$. So, the universe begins with a Big Bang singularity.
- The Hubble parameter H and the expansion scalar θ are decreasing functions of cosmic time t and $\frac{dH}{dt}$ and $\frac{d\theta}{dt}$ tend to 0 at $t \to \infty$.
- The shear scalar σ^2 diverges at $t = 0$ and converges to a constant value at $t \to \infty$.
- The anisotropy parameter is constant and not equal to zero unless $k = 1$ which implies that the universe remain anisotropic throughout the evolution. But, for $k = 1$ the anisotropy parameter, $A_m = 0$ and the universe become isotropic throughout the evolution.
- The matter energy density ρ_m and Barrow holographic dark energy density ρ_{BHDE} are decreasing functions of cosmic time t. The decrease of energy densities with an increase of cosmic time t leads to the volume expansion of the universe.
- The pressure of Barrow holographic dark energy p_{BHDE} is negative at late times which indicates the accelerated expansion of the universe.
- The EoS parameter ω_{BHDE} of Barrow holographic dark energy varies in quintessence region and tends to ΛCDM model at late times.

• The deceleration parameter q transists from its positive value in the early universe to -1 at late times. This transition shows the early decelerated phase to current accelerated phase of the universe.
• The total energy density parameter Ω converges to 1 as cosmic time $t \to \infty$. This suggests that the universe is approaching towards a flat universe at late times.
• The Null energy condition, the Weak energy condition and the Dominant energy condition are all satisfied but the Strong energy condition is violated at late times which supports the accelerated expansion of the universe.
• The cosmic jerk parameter j, the snap parameter s and the lerk parameter l all converges to 1 at late times. The convergence of j to at late times demonstrates that our model matches the ΛCDM model.
• The coincidence parameter $\bar{r}$ is not constant in the univers's evolution. So, the coincidence probelm is not resolved in our model.
• The Statefinder parameters pass through the point $(1, 0)$ which corresponds to the ΛCDM model.
• The correspondence between Barrow holographic dark energy model and quintessence scalar field model describe the accelerated phase of the expanding universe.

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ГОЛОГРАФІЧНА МОДЕЛЬ ТЕМНОЇ ЕНЕРГІЇ БАРРОУ У ВСЕСВІТІ Б'ЯНКІ ТИПУ III З КВІНТЕСЕНЦІЄЮ

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У цій статті ми досліджуємо просторово однорідний і анізотропний Всесвіт Б'янкі типу III, що містить холодну темну матерію та голографічну темну енергію Барроу в рамках загальної теорії відносності. Ми припускаємо, що холодна темна матерія та голографічна темна енергія Барроу не взаємодіють, і отримуємо точні розв'язки рівнянь поля Ейнштейна, розглядаючи гібридний закон розширення та припускаючи, що скаляр розширення пропорційний скаляру зсуву. Ми перевіряємо фізичні та кінематичні властивості отриманої моделі, використовуючи такі параметри, як параметр Хаббла, параметр анізотропії, параметр уповільнення, параметр рівняння стану, параметр ривка тощо. Ми також перевіряємо, чи порушуються енергетичні умови або підтверджуються. Ми виявили, що умови нульової, слабкої та домінантної енергії виконуються, тоді як умова сильної енергії порушена, що підтримує прискорене розширення Всесвіту. Діагностика Statefinder була проведена на основі останніх космологічних спостережень. Крім того, ми переформулювали відповідність між скалярним полем квінтесенції та голографічною моделлю темної енергії Барроу.

Ключові слова: *космічне прискорення; голографічна темна енергія Барроу; Біанкі тип-III; холодна темна матерія; параметр уповільнення; рівняння параметра стану*