EAST EUROPEAN JOURNAL OF PHYSICS. **1.** 504-511 (2024)
DOI:10.26565/2312-4334-2024-1-56 ISSN 2312-4334

# CHARACTERISTICS OF NONLINEAR DUST ACOUSTIC WAVES *(DAWS)* PROPAGATING IN AN INHOMOGENEOUS COLLISIONLESS MAGNETIZED DUSTY PLASMA

**Hirak Jyoti Dehingia*, Paramananda Deka#**
*Department of Mathematics, Dibrugarh University, Assam, India*
*Corresponding Author e-mail: *hirakjyotidehingia11@gmail.com, #pndeka@dibru.ac.in*
Received January 25, 2024; revised February 11, 2024; in final form February 24, 2024 accepted February 26, 2024

In this paper, we have presented our investigation on the characteristic of nonlinear dust acoustic waves *(DAWs)* propagating in an inhomogeneous collisionless magnetized dusty plasma *(MDP).* In this problem, we have considered a collisionless plasma consisting of nonthermal ions, non-extensive electrons and negatively charged dust grains. Using the *reductive perturbation theory (RPT)* we have derived the modified *Zakharov-Kuznetsov (m-ZK)* equation. The solution of *m-ZK* equation indicates the nonlinear characteristics of the *DASWs* in plasma. Our investigation also predicts how the amplitudes of nonlinear *DASWs* are significantly modified due to the influence of magnetic field, non-extensive electrons and inhomogeneity parameters in plasma. The results obtained in this investigation may be useful for understanding the propagation characteristics and modification of structures of nonlinear waves in both laboratory and astrophysical plasmas.
**Keywords:** *Dusty plasma; RPT;* $m-ZK$ *equation; Inhomogeneous plasma; non-extensive electrons*
**PACS:** 52.27.Lw, 05.45.-a, 95.30.Qd

## I. INTRODUCTION

Solitary waves or solitons are nonlinear wave packets which maintain their shapes during their propagation at a particular speed. Solitons occur due to the mutual cancellation of the nonlinear effects and dispersive effects in the medium. Washimi and Taniuti [1] derived the *Korteweg de-Vries (KdV)* equation to describe the ion-acoustic solitons *(IAS).* Nishikawa and Kaw [2] discussed the propagation of ion-acoustic solitons in an inhomogeneous plasma. Theoretically, Kuehl [3] discussed the propagation and reflection of ion-acoustic solitons in an inhomogeneous plasma. He has observed the variations in the soliton amplitudes of incident and reflected solitons. Nejoh [4] investigated the effects of ion temperature on the characteristics of soliton propagation in a relativistic plasma. A few plasma physics researchers studied the properties and characteristics of solitary waves propagating in the presence of various physical situations such as negative ions [5,6] and dust grains [7] in the plasma and solitary wave excitation in nonequilibrium plasmas [30-33]. Kakad *et al.* [8] provided an experimental study on the validity of fluid theory and chain formation of nonlinear wave propagation in a collisional magnetized plasma. Later, using *Kundu-nonlinear Schrödinger equation (Kundu-NLS)* Shi *et al.* [9] discussed the dynamics of nonlinear nonlocal solitary wave solutions propagating in an inhomogeneous plasma. Rani and Yadav [10] studied the characteristics of electron acoustic-solitary waves *(EASWs)* propagating in a dense magnetized collisional plasma in the presence of degenerate quantum electrons. Recently, Dehingia and Deka [11] have discussed the variations of *IAS* structures propagating in an inhomogeneous plasma in the presence of hot isothermal electrons. They have observed that at a certain point the structure of *IAS* gets deformed due to the presence plasma inhomogeneity during their propagation through the system.

Dusty plasma *(DP)* is a very important research field in plasma physics. *DP* consists of ions, electrons, and charged dust particles. When the dust particles are included in the plasma, the system indicates some complex behaviours in the system. Thus, *DP* is also termed as the multicomponent plasma or complex plasma. These *DPs* are observed in planetary magnetospheres, cometary environments, planetary ring, and nebulas etc. [12]. The study of dusty plasma helps us to understand the astrophysical phenomena, the geophysical theories, and importance of space missions etc. Goertz [13] worked on the fundamental properties of dusty plasma in an astrophysical environment. Many researchers studied the basic properties of dust ion-acoustic waves (DIAWs) [14] and dust-acoustic waves (DAWs) [15] propagating in an inhomogeneous plasma. Shukla and Mamun [16] introduced basic structures, properties and propagation of *DAWs, DIAWs,* dust-cyclotron waves *(DCWs)* and dust lattice waves *(DLWs)* etc. in inhomogeneous plasmas. Using the kinetic theory Baluku and Hellberg [17] provided a brief description on the propagation of *DIAWs* in the presence of $k-$ distributed electrons and negatively charged dust grains in the plasma. Alinejad, and Khorrami [18] studied on the structures of *DAWs* propagation in the presence of trapped ions and polarized Debye sheath in a strongly coupled inhomogeneous plasma. Atteya, Sultana, and Schlickeiser [19] investigated the effect of superthermal electrons, positive ions as well as negative ions in the propagation of *DIAWs* in an inhomogeneous magnetized plasma. Akhtar *et al.* [20] discussed the dynamics of *DAWs* and *DCWs* during their propagation in the magnetized plasma. Rehman, Mahmood, and Hussain [21] studied the behaviour of nonlinear magneto-acoustic waves *(MAWs)* in the presence of warm, collisionless pair-ion (PI) fullerene plasma. In their analysis, they concluded that due to the presence of ion inertial length in the plasma, the effects of wave dispersion are observed in PI plasma. In the study of plasma physics, linear theory is used to study the

***Cite as***: H.J. Dehingia, P. Deka, East Eur. J. Phys. 1, 504 (2024), https://doi.org/10.26565/2312-4334-2024-1-56

small amplitude waves without considering the nonlinearities in the plasma. But in the case of large amplitude waves, nonlinearities cannot be ignored. In the plasma studies, nonlinearities play an important role in the nature, properties and characteristics of the dusty wave phenomena. In experimental and theoretical studies, we observe various nonlinear dusty wave structures such as shock waves, rouge waves, solitons, supersolitons etc. Atteya *et al.* [22] studied the propagation of nonlinear *DAWs* in an inhomogeneous quantum plasma in the presence of magnetic field. They have observed that the low-frequency longitudinal waves have the potential to trap electrons which obtained from the high-intensity magnetic fields during the modulation of plasma species. Using the Pseudopotential method Pakzad and Nobahar [23] discussed the important properties of *DIASWs* propagating in an inhomogeneous unmagnetized plasma in the presence of superthermal electrons, inertial ions, and stationary dust grain particles. They have also analysed the modification of nonlinear wave structures of *DIAS* propagating in an inhomogeneous plasma in the presence of the critical parametric values of superthermal electrons and electron-ion density ratio. Dehingia and Deka [24-27] studied the various properties of *DAWs*, modification in *DASW* structures, effect of dust particles in soliton propagation and propagation of nonlinear waves in the presence of negatively charged dust grains with charge fluctuations in inhomogeneous plasma. There are still many scopes to study on the propagation characteristics of nonlinear wave structures in inhomogeneous plasma depending on the various astrophysical conditions. In this problem, we consider a collisionless magnetized plasma consisting of cold ions, non-extensive electrons and negatively charged dust grains. In this investigation, we have discussed the characteristics of nonlinear *DASWs* propagating in an inhomogeneous collisionless magnetized plasma in the presence of nonthermal ions with non-extensive electrons and negatively charged dust grains.

## II. GOVERNING EQUATIONS

In this article, we have studied the characteristics of nonlinear *DASWs* propagation in the presence of negatively charged mobile dust grains, $q-$ distributed non-extensive electrons of temperatures $T_{e1}$ and $T_{e2}$, and nonthermal ions with finite temperature $T_i$, where $T_i \ll T_{e1} \ll T_{e2}$ in an inhomogeneous magnetized plasma. These temperatures are expressed in the units of energy. The set of governing equations for nonthermal ions, non-extensive electrons and negatively charged dust grains in a dusty plasma system are given by [25-29]

$$\frac{\partial \vec{n}_d}{\partial t} + \vec{\nabla}.(n_d \vec{u}_d) = 0 \tag{1}$$

$$\frac{\partial \vec{u}_d}{\partial t} + (\vec{u}_d.\vec{\nabla})\vec{u}_d + \vec{\nabla}\phi + \alpha(\vec{u}_d \times \hat{z}) = 0 \tag{2}$$

$$\vec{\nabla}^2\phi = n_d + \mu_1[1 + (q_1 - 1)]^{\frac{(q_1+1)}{2(q_1-1)}} + \mu_2[1 + (q_2 - 1)]^{\frac{(q_2+1)}{2(q_2-1)}} - \mu_i[1 + (\beta\phi + \beta\phi^2 + \cdots)e^{-\phi}]^{\frac{(q_2+1)}{2(q_2-1)}} \tag{3}$$

$$n_i = n_{i0}\left[1 + \beta\left(\frac{e\phi}{T_i}\right) + \beta\left(\frac{e\phi}{T_i}\right)^2\right]e^{-\left(\frac{e\phi}{T_i}\right)} \tag{4}$$

$$n_{e1} = n_{e10}\left[1 + (q_1 - 1)\left(\frac{e\phi}{T_i}\right)\right]^{\frac{(q_1+1)}{2(q_1-1)}} \tag{5}$$

$$n_{e2} = n_{e20}\left[1 + (q_2 - 1)\left(\frac{e\phi}{T_i}\right)\right]^{\frac{(q_2+1)}{2(q_2-1)}} \tag{6}$$

Now, for equilibrium condition, the charge neutrality equation is given by $n_{i0} = n_{e10} + n_{e20} + Z_d n_{d0}$ where we consider $n_{e10}$, $n_{e20}$ and $n_{i0}$ are taken as number densities of low temperature and high temperature electrons and ion number density respectively in the plasma. In the above equations, $Z_d$ is dust charge number, $n_d$ is the density of dust grains, and $q_1$ and $q_2$ are the strengths of non-extensivity of two types electrons with temperatures $T_{e1}$ and $T_{e2}$ respectively in the plasma. In this mathematical model, $\vec{u}_d$ reparents the dust fluid velocity of the dust grain particles normalized by $C_d = \sqrt{\frac{Z_d T_i}{m_d}}$, $m_d$ is the mass of dust grains, $\vec{\phi}$ is the electrostatic potential which is normalized by $\frac{T_i}{e}$, time $t$ is also normalized by $\omega_{pd}^{-1} = \sqrt{\frac{m_d}{4\pi n_{d0} Z_d^2 e^2}}$ and the space variable $x$ is normalized by $\lambda_{Dm} = \sqrt{\frac{T_i}{4\pi n_{d0} Z_d e^2}}$. We have also considered the nonthermal parameters are $\beta = \frac{4\gamma}{(1+3\gamma)}$, where $\gamma$ is the parameter which determines the density population of nonthermal ions in the plasma. The values of the parameters $\beta$ and $\gamma$ are taken to be $0 \le \beta \le \frac{4}{3}$ and $\gamma \ge 0$. In the above equations, we have also considered some set of dimensionless quantities such as $\mu_i = \frac{n_{i0}}{Z_d n_{d0}}$, $\mu_1 = \frac{n_{e10}}{Z_d n_{d0}}$, $\mu_2 = \frac{n_{e20}}{Z_d n_{d0}}$, $\alpha = \frac{\omega_{cd}}{\omega_{pd}}$, $\sigma_1 = \frac{T_i}{T_{e1}}$, and $\sigma_2 = \frac{T_i}{T_{e2}}$ etc.

## III. DERIVATION OF MODIFIED ZK-EQUATION

To study the propagation of nonlinear *DASWs* and their characteristics for small amplitude waves, we use the *RPT* to derive the *m-ZK* equation for the plasma system. The solutions of m-ZK equation show the propagation characteristics of the nonlinear *DASWs* in the plasma. To use the *RPT*, we use a set of appropriate stretched coordinates [1, 24-27] for the weakly inhomogeneous plasma is as follows:

$$X = \varepsilon^{\frac{1}{2}}x, Y = \varepsilon^{\frac{1}{2}}y, Z = \varepsilon^{\frac{1}{2}}(z - Vt), \tau = \varepsilon^{\frac{3}{2}}t \tag{7}$$

where the phase velocity $V$ is normalized by $C_d$, and the smallness parameter $\varepsilon$ measures the strength of the dispersion. Here, all the axes $X, Y$, and $Z$ are normalized by Debye length $(\lambda_m)$, and $\tau$ is normalized by ion plasma period $\left(\frac{1}{\omega_{pd}}\right)$ respectively.

To apply the *RPT*, we use expanded dependent variables $u_{dx}, u_{dy}, u_{dz}, n_d$ and $\phi$ along with their perturbed values and in terms of $\varepsilon$ is as follows [26]:

$$\begin{bmatrix} n_d \\ u_{dx} \\ u_{dy} \\ u_{dz} \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_{d1} \\ 0 \\ 0 \\ u_{dz1} \\ \phi_1 \end{bmatrix} + \varepsilon^{\frac{3}{2}} \begin{bmatrix} 0 \\ u_{dx1} \\ u_{dy1} \\ 0 \\ 0 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_{d2} \\ u_{dx2} \\ u_{dy} \\ u_{dz2} \\ \phi_2 \end{bmatrix} + \cdots \tag{8}$$

Now, using Eqs. $(7)$ and $(8)$ into Eqs. $(1) - (6)$, we obtain the 1st order quantities for $z -$ component of momentum and Poisson's equations are as follows:

$$n_{d1} = -\frac{\phi_1}{V^2}, u_{dz1} = -\frac{\phi_1}{V}, \tag{9}$$

$$V = \frac{1}{\sqrt{\mu_i(1-\beta) + \frac{\mu_1\sigma_1}{2}(q_1+1) + \frac{\mu_2\sigma_2}{2}(q_2+1)}}, \tag{10}$$

The above Eq. $(10)$ indicates the phase velocity of the *DASWs* under the influence of non-extensive electrons, nonthermal ions and negatively charged dust grains propagating in an inhomogeneous *MDP*.

Similarly, we obtain 1st order $x$ and $y$ components for the momentum equation are as follows:

$$u_{dx1} = -\frac{1}{\alpha}\frac{\partial\phi_1}{\partial Y}, \tag{11}$$

$$u_{dy1} = \frac{1}{\alpha}\frac{\partial\phi_1}{\partial X}. \tag{12}$$

The above Eqs. $(11)$ and $(12)$ represents the velocities of *DASWs* in $x$ and $y$ components in the plasma. These two equations also satisfy the 2nd order continuity equation for *DASWs* in the plasma. Again, using Eqs. $(7)$ and $(8)$ in Eqs. $(1) - (6)$ and eliminating the 1st order terms of $x$ and $y$ components in momentum and Poisson's equation, we get the 2nd order terms are as follows:

$$u_{dx2} = -\frac{V}{\alpha^2}\frac{\partial^2\phi_1}{\partial Z\partial X}, \tag{13}$$

$$u_{dy2} = -\frac{V}{\alpha^2}\frac{\partial^2\phi_1}{\partial Z\partial Y}, \tag{14}$$

$$\frac{\partial^2\phi_1}{\partial X^2} + \frac{\partial^2\phi_1}{\partial Y^2} + \frac{\partial^2\phi_1}{\partial Z^2} = \mu_1 M + \mu_2 N - \mu_i P + n_{d2}, \tag{15}$$

where,

$$M = (q_2 + 1)\left\{\frac{\sigma_2\phi_2}{2} + \frac{(3-q_2)\sigma_2^2\phi_1^2}{8}\right\}, \tag{16}$$

$$N = (q_2 + 1)\left\{\frac{\sigma_2\phi_2}{2} + \frac{(3-q_2)\sigma_2^2\phi_1^2}{2}\right\}, \tag{17}$$

$$P = (\beta - 1)\phi_2 + \frac{\phi_1^2}{2}. \tag{18}$$

The above Eqs. $(13) - (15)$ represents the $x$ and $y$ components of the dust polarization drift. Now, proceeding the same way we will obtain the higher order terms of continuity equation and $z -$ components from momentum equations. Now, eliminating the higher order terms $n_{d2}$, $u_{dy2}$, and $\phi_2$ finally we obtain the equation is as follows:

$$\frac{\partial\phi_1}{\partial\tau} + EF\phi_1\frac{\partial\phi_1}{\partial Z} + \frac{1}{2}E\frac{\partial}{\partial Z}\left[\frac{\partial^2}{\partial Z^2} + G\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)\right]\phi_1 = 0, \tag{19}$$

where,

$$E = V^3, \tag{20}$$

$$F = \frac{1}{2}\left[\mu_i - \frac{1}{4}\left\{\begin{matrix} \mu_1\sigma_1^2(q_1+1)(3-q_1) \\ +\mu_2\sigma_2^2(q_2+1)(3-q_2) \end{matrix}\right\} - \frac{3}{V^4}\right], \tag{21}$$

$$G = 1 + \frac{1}{\alpha^2}. \tag{22}$$

The above equation (19) represents the $m - ZK$ equation which describes the oblique propagation of *DASWs* in the presence of non-extensive electrons having two distinct temperatures in the *MDP*.

## IV. SOLUTION OF *m-ZK* EQUATION

To study the characteristics of nonlinear *DASWs* propagating in an inhomogeneous plasma, we have considered the plasma with an effect of magnetic field at an angle $\delta$ with Z-axis. Considering the Y-axis fixed, the coordinate axes are assumed to be rotated at an angle $\delta$ which is significantly modified under the influence of magnetic field, non-extensive electrons and inhomogeneity parameters in the plasmas. Thus, we use a set of transformation equations for the independent variables is as follows:

$$\left.\begin{matrix} \rho = X\cos\delta - Z\sin\delta, & \tau = t, \\ \xi = X\sin\delta + Z\cos\delta, & \eta = Y, \end{matrix}\right\}. \tag{23}$$

Using the above transformation equations [1], [26], [27], we rewrite the above $m - ZK$ Eq. (19) is as follows:

$$\frac{\partial\phi_1}{\partial\tau} + \delta_1\phi_1\frac{\partial\phi_1}{\partial\xi} + \delta_2\frac{\partial^3\phi_1}{\partial\xi^3} + \delta_3\phi_1\frac{\partial\phi_1}{\partial\rho} + \delta_4\frac{\partial^3\phi_1}{\partial\rho^3} + \delta_5\frac{\partial^3\phi_1}{\partial\xi^2\partial\rho} + \delta_6\frac{\partial^3\phi_1}{\partial\rho^2\partial\xi} + \delta_7\frac{\partial^3\phi_1}{\partial\xi\partial\eta^2} + \delta_8\frac{\partial^3\phi_1}{\partial\rho\partial\eta^2} = 0, \tag{24}$$

where

$$\left.\begin{aligned} &\delta_1 = EF\cos\delta \\ &\delta_2 = \frac{1}{2}E(\cos^3\delta + G\sin^2\delta\cos\delta) \\ &\delta_3 = -EF\sin\delta \\ &\delta_4 = -\frac{1}{2}E(\sin^3\delta + G\sin\delta\cos^2\delta) \\ &\delta_5 = E\left\{G\left(\sin\delta\cos^2\delta - \frac{1}{2}\sin^3\delta\right) - \frac{3}{2}\sin\delta\cos^2\delta\right\} \\ &\delta_6 = -E\left\{G\left(\sin^2\delta\cos\delta - \frac{1}{2}\cos^3\delta\right) - \frac{3}{2}\sin^2\delta\cos\delta\right\} \\ &\delta_7 = \frac{1}{2}EG\cos\delta \\ &\delta_8 = -\frac{1}{2}EG\sin\delta \end{aligned}\right\}, \tag{25}$$

Now, using the transformation equation [27], the steady state solution for $m - ZK$ equation is given by

$$\phi_1 = \phi_0(H), \tag{26}$$

where $H = \xi - V_0 t$ and $V_0$ represents the constant speed normalized by $C_d$. Using the above transformation equation, the above $m - ZK$ equation can be rewritten in steady state form is as follows:

$$-V_0\frac{d\phi_0}{dH} + \delta_1\phi_0\frac{d\phi_0}{dH} + \delta_2\frac{d^3\phi_0}{dH^3} = 0. \tag{27}$$

Now, using appropriate boundary conditions, i.e., $\phi_1 \to 0$, $\frac{d\phi_1}{dH} \to 0, \frac{d^2\phi_1}{dH^2} \to 0$ as $H \to \pm\infty$. Then solitary wave solution of Eq. (27) is given by

$$\phi_0(H) = \phi_m sech^2(WH), \tag{28}$$

where $\phi_m = \frac{3V_0}{\delta_1}$ and $W = \sqrt{\frac{V_0}{4\delta_1}}$ represents the amplitude and inverse width of the solitary wave solutions respectively. Since $V_0 > 0$, so it is clear from the Eqs. (19), (21) and (24) that based on the sign of $F$, the solitary waves or solitons will only be associated with negative potential ($\phi_m < 0$) of the wave propagation in the plasma.

In the above derivations, we have observed that the Eq. (27) is the one-dimensional steady state form of $m - KdV$ equation. The steady state solution of $m - ZK$ Eq. (27) and standard $m - KdV$ equation both gives the same results in one-dimensional steady state cases. To obtain the localized solitary wave solution, we use appropriate boundary conditions and transformation equations [27] to solve the Eq. (27). In this problem, we have considered the steady state solution of $m - ZK$ Eq. (19) in one-dimensional cases where all $\delta$'s $\to 0$ provided $\delta_1$ and $\delta_2$. Thus, $\delta_1$ and $\delta_2$ will occur in our wave solutions obtained in our above derivations.

## V. RESULTS AND DISCUSSIONS

Using the above results and derivations, we have plotted some figures to describe the propagation and characteristics of nonlinear *DASWs* propagating in an inhomogeneous *MDP* consisting of nonthermal ions, non-extensive electron with different temperatures and negatively charged massive dust grains. In Fig. 1 and Fig. 2, we have plotted the graphs to show the variations of amplitudes of *DASWs* w.r.t. $\sigma_1$ and $\sigma_2$ depending on the various parametric values of non-extensive parameters $q_1$ and $q_2$ respectively. Fig. 3 shows the variations of width $(W)$ of *DASWs* w.r.t. number density of relative electrons $(\mu_1)$ depending on the various parametric values of nonthermal parameter $(\beta)$. Depending on the various values of the magnetic parameter $\alpha$, the variations of width $(W)$ of *DASWs* w.r.t. oblique parameter $\delta$ are also shown in Fig. 4. Finally, Fig. 5 describes the variations of phase velocities of *DASWs* w.r.t. the nonthermal parameter $(\beta)$ propagating in an inhomogeneous magnetized dusty plasma. In this problem, we have also investigated the effect of magnetic field on the propagation of *MDP* under the above considered plasma situations.

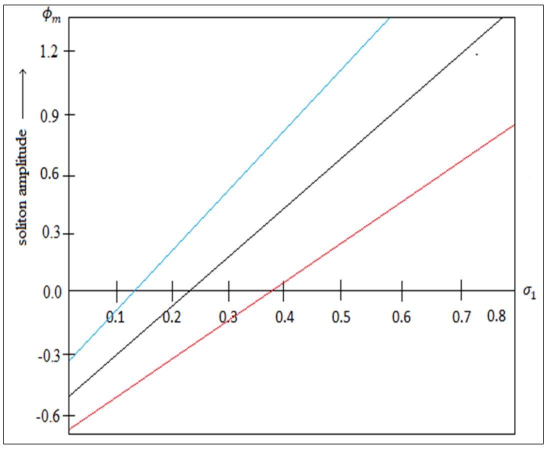


**Figure 1.** Variation of soliton amplitude $(\phi_m)$ of *DASWs* with $\sigma_1$ for various values of $q_1 = 0.3, -0.3, -0.8$ with $\delta = 35^{\circ}$, $q_2 = 1.5, \sigma_2 = 0.05, \mu_1 = 0.3, \mu_2 = 0.2, U = 0.2$ and $\beta = 0.1$

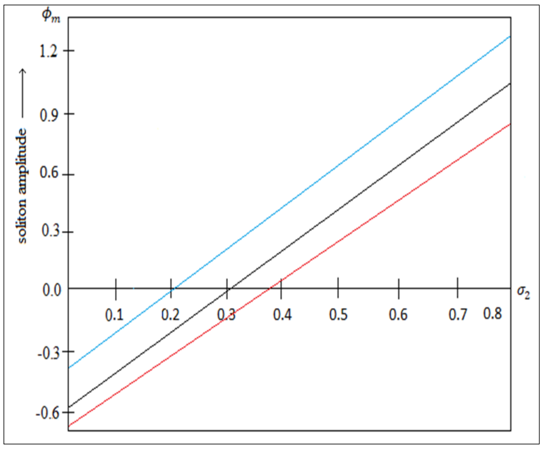


**Figure 2.** Variation of soliton amplitude $(\phi_m)$ of *DASWs* with $\sigma_1$ for various values of $q_2 = 0.4, 0.6, -0.8$ with $q_1 = -0.8, \sigma_1 = 0.3, \sigma_2 = 0.2, U = 0.2, \beta = 0.1$.

To study the effects of magnetic field and propagation characteristics of *DASWs* propagating in an inhomogeneous plasma in the nonthermal ions, non-extensive electron with different temperatures and negatively charged massive dust grains, we have used *RPT* to derive and solve the $m - ZK$ equation for small amplitude waves by using the various dependent variables of perturbed number densities of ions, electrons and dust grains with dust velocities and electrostatic potentials in the plasma. From the above investigations, Fig. 1 indicate that the magnitudes of amplitudes of *DASWs* increases gradually with the increase in the temperature ratios $(\sigma_1)$ and non-extensive parameter $(q_1)$.

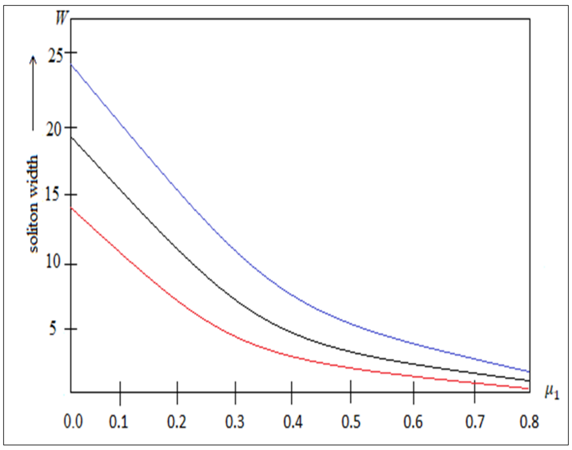


**Figure 3.** Variation of soliton width $(W)$ of *DASWs* with $\mu_1$ for various values of $\beta = 0.2, 0.4, 0.6$ with $\delta = 35^{\circ}$, $q_1 = -0.8, \mu_2 = 1.5, \sigma_1 = 0.3, U = 0.2$ and magnetic field parameter $\alpha = 0.1$.

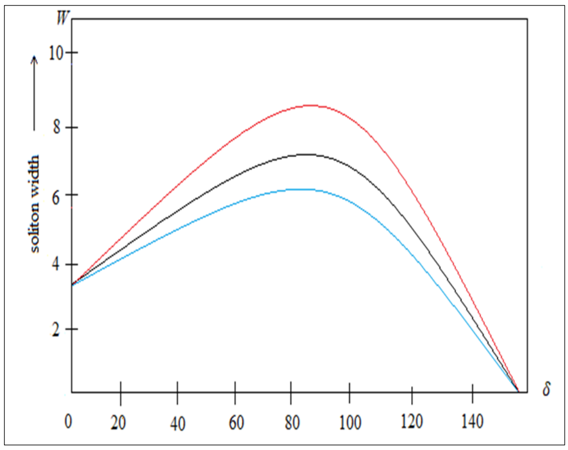


**Figure 4.** Variation of soliton width $(W)$ of *DASWs* with $\delta$ for various values of $\alpha = 0.2, 0.3, 0.4$ with $q_1 = -0.8, \sigma_2 = 0.01, \delta = 35^{\circ}, U = 0.2, \sigma_1 = 0.3, q_2 = 1.5, \mu_1 = 0.3, \mu_2 = 0.2, U = 0.2, \beta = 0.1$.

On the other hand, Fig.2 shows that the magnitudes of widths of *DASWs* decreases gradually with the increase in the temperature ratios $(\sigma_2)$ and non-extensive parameter $(q_2)$. However, in Fig. 3, we have observed that the width of *DASWs* vary adversely with the increasing or decreasing values of number density of relative electrons $(\mu_1)$ depending on the various parametric values of nonthermal parameter $(\beta)$. The above figures also indicate that with the increase in the number density of relative electrons $(\mu_1)$ and the values of nonthermal parameter $(\beta)$ for ions, increases the width of $DASWs$ during their propagation through the plasma. This shows the importance of nonthermal ions which play an important role on stabilizing, forming and the propagation of *DASWs* in an inhomogeneous plasma with various physical situations e.g., nonthermal ions and non-extensive electrons present in the laboratory and astrophysical plasmas such as protostellar disk, circumstellar and interstellar clouds, cometary tails, Saturn's rings, earth's magnetospheres, solar winds and in asteroid zones etc. In Fig. 4, we have seen that the variations in the oblique parameter $\delta$ effects the width $(W)$ of $DASWs$ which increases due to the presence of $\delta$ with lower angles in between $0^{\circ}$and $45^{\circ}$. On the other hand, the width $(W)$ of $DASWs$ which decreases with higher angles of $\delta$ in between $45^{\circ}$ and $90^{\circ}$. Also, when $\delta \to 90^{\circ}$, the amplitude and width of the soliton tends to $\infty$ and $0$ respectively. This concludes that the waves will not be considered as electrostatic anymore. In this investigation, we have considered the magnetic field parameter as $\alpha = \frac{\omega_{cd}}{\omega_{pd}}$, where $\omega_{cd}$ and $\omega_{pd}$ are dust cyclotron frequency and dust plasma frequency respectively. Fig. 4 also shows that while dust cyclotron frequency i.e., $\alpha$ increases, the width $(W)$ of the soliton decreases in the plasma.

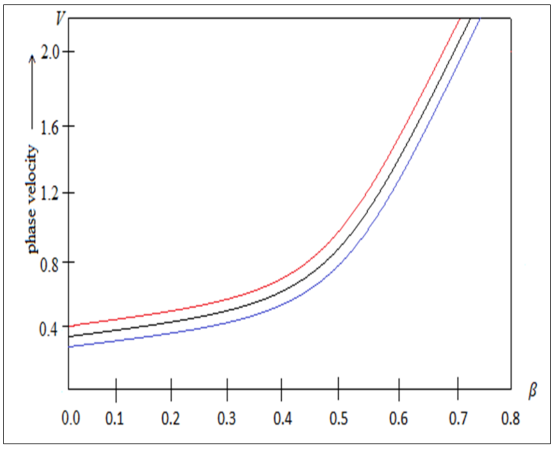


**Figure 5.** Variation of phase velocity $(V)$ of *DASWs* with $\beta$ for various values of $q_1 = q_2 = -0.9, 0.5, 1.5$ with $\delta = 35^{\circ}$, $\mu_1 = 0.3, \mu_2 = 0.2, \sigma_1 = 0.3, \sigma_2 = 0.05$.

In the above derivations, we have also derived the dispersion relation and phase velocity in Eq. $(10)$. However, in Fig. 4, it is also observed with the increase in the values of $\alpha$, the amplitudes of the solitary wave structures become relatively spiky and so that the system become destabilized. On the other hand, the width $(W)$ of *DASWs* decreases with the increase in the value of external magnetic field parameter $\alpha$. In Fig. 5, we have shown the effects of nonthermal ions and non-extensive electrons and the phase velocity of the *DASWs* propagating in an inhomogeneous *MDP*. In Fig. 5, we have also shown the increase of phase velocity of the *DASWs* slowly and steadily but later it increases rapidly with the increasing values of nonthermal parameter $(\beta)$ for ions in the plasma. When the parameters other than $\beta$ are considered to be constant, the phase velocity of the *DASWs* increases rapidly with the increase in critical value of $\beta$. The critical value of nonthermality parameter $(\beta)$ depends on the relatively non-extensive electrons present in the plasma. Also, when the electron non-extensivity increases, the nonthermality of ions increases in the modelled plasma which are shown in Fig. 5. From the above discussions, it is clear that the phase velocity of *DASWs* especially much depend on the nonthermality of ions. However, the phase velocity of *DASWs* less dependent on the non-extensivity of electrons present in the system.

## VI. CONCLUSION

In this paper, we have studied the fundamental properties of *DASWs* propagating obliquely in an *MDP* consisting of nonthermal ions, non-extensive with distinct two temperature electrons and negatively charged dust grains. In this problem, we have also discussed the fundamental characteristics of the *DASWs* propagating in an inhomogeneous plasma under the above considered physical situations. We have observed that with the increase in dust cyclotron frequency, there is an increase in the spikes of amplitudes of *DASWs* structures in the plasma. The above results also indicate that with the increasing values of $\mu_1$ and $\beta$, the width of *DA* solitons increases during their propagation through the inhomogeneous collisionless magnetized plasma. We have also observed that the oblique parameter $\delta$ effects the variations in the width $(W)$ of *DASWs* which increases due to the presence of $\delta$ with lower angles in between $0^{\circ}$and $45^{\circ}$. However, the width $(W)$ of *DA* soliton decreases with the increasing angles of $\delta$ in between $45^{\circ}$ and $90^{\circ}$. In this investigation, the above results also imply that slowly and steadily the phase velocity of the *DA* soliton will increase but later, it will increase rapidly with the increasing values of $\beta$ in the plasma. When the parameters are supposed to be constant except $\beta$, the phase velocity of the *DA* soliton rapidly increases with the rapid increase of critical value of $\beta$. The above results also indicate that with the increase in the magnitude of external magnetic field in *DASWs*, the width of the soliton decreases adversely. The results obtained in this problem may be useful for understanding the propagation of localized electrostatic waves in both laboratory plasmas such as earthquake dynamics, dating of geological and archaeological minerals, dissipative optical lattices, double plasma machines, hot turbulent thermonuclear plasmas, radio frequency discharged plasmas etc. and astrophysical plasmas such as cosmology, galaxy clusters, self-gravitation polytopic systems, solar winds, Auroral regions, noctilucent clouds in Earth's atmosphere. Our results may also be useful for further study of the dynamics of the various nonlinear wave propagation in an inhomogeneous plasma under the various physical astrophysical situations.

**ORCID**
**Hirak Jyoti Dehingia**, https://orcid.org/0000-0002-3119-0842; **Paramananda Deka,** https://orcid.org/0000-0001-9485-9294

**REFERENCES**

[1] H. Washimi, and T. Taniuti, "Propagation of ion-acoustic solitary waves of small amplitude," Phys. Rev. Lett. **17**, 996–998 (1966). https://doi.org/10.1103/PhysRevLett.17.996
[2] N. Nishikawa, and K. Kaw, "Propagation of solitary ion-acoustic waves in inhomogeneous plasmas," Phys. Lett. A, **50**, 455–456 (1975). https://doi.org/10.1016/0375-9601(75)90124-3
[3] H.H. Kuehl, "Reflection of an ion-acoustic soliton by plasma inhomogeneities," Phys. Fluids, **26**(6), 1577–1583 (1983). https://doi.org/10.1063/1.864292
[4] Y. Nejoh, "The effect of the ion temperature on the ion-acoustic solitary waves in a collisionless relativistic plasma," J. Plasma Phys. **37**(3), 487–495 (1987). https://doi.org/10.1017/S0022377800012320
[5] D.K. Singh, and H.K. Malik, "Soliton reflection in a negative ion containing plasma: Effect of magnetic field and ion temperature," Phys. Plasmas, **13**(8), 082104(1–10) (2006). https://doi.org/10.1063/1.2335427
[6] D.K. Singh, and H.K. Malik, "Modified Korteweg–de Vries soliton evolution at critical density of negative ions in an inhomogeneous magnetized cold plasma," Phys. Plasmas, **14**(6), 062113 (2007). https://doi.org/10.1063/1.2743026
[7] D. Xiao, J.X. Ma, Y. Li, Y. Xia, and M.Y. Yu, "Evolution of nonlinear dust-ion-acoustic waves in an inhomogeneous plasma," Phys. Plasmas, **13**(5), 052308 (2006). https://doi.org/10.1063/1.2196247
[8] A. Kakad, Y. Omura, and B. Kakad, "Experimental evidence of ion acoustic soliton chain formation and validation of nonlinear fluid theory," Physics of Plasmas, **20**(6), 062103 (2013). https://doi.org/10.1063/1.4810794
[9] X. Shi, J. Li, and C. Wu, "Dynamics of soliton solutions of the nonlocal Kundu-nonlinear Schrödinger equation," Chaos, **29**, 023120 (2019). https://doi.org/10.1063/1.5080921
[10] N. Rani, and M. Yadav, "Propagation of nonlinear electron acoustic solitons in magnetized dense plasma with quantum effects of degenerate electrons," AIP Conference Proceedings, **2352**, 030008 (2020). https://doi.org/10.1063/5.0052436
[11] H.J. Dehingia, and P.N. Deka, "Structural Variations of Ion-Acoustic Solitons," in: *Nonlinear Dynamics and Applications: Proceedings of the ICNDA 2022* (pp. 97-104). (Springer International Publishing, Cham, 2022).
[12] L. Spitzer Jr., "Review of Publications: Physical Processes in the Interstellar Medium", Journal of the Royal Astronomical Society of Canada, **72**, 349 (1978). https://articles.adsabs.harvard.edu/pdf/1978JRASC..72..349S
[13] C.K. Goertz, "Dusty plasmas in the solar system", Reviews of Geophysics, **27**(2), 271-292 (1989). https://doi.org/10.1029/RG027i002p00271
[14] N.N. Rao, P.K. Shukla, and M.Y. Yu, "Dust-acoustic waves in dusty plasmas", Planetary and space science, **38**(4), 543 (1990). https://doi.org/10.1016/0032-0633(90)90147-I
[15] P.K. Shukla, and V.P. Silin, "Dust ion-acoustic wave", Physica Scripta, **45**(5), 508 (1992). https://doi.org/10.1088/0031-8949/45/5/015
[16] P.K. Shukla, and A.A. Mamun, *Introduction to dusty plasma physics*, (Institute of Physics Publishing Ltd, Bristol, 2015).
[17] T.K. Baluku, and M.A. Hellberg, "Kinetic theory of dust ion acoustic waves in a kappa-distributed plasma", Physics of Plasmas, **22**(8), 083701 (2015). https://doi.org/10.1063/1.4927581
[18] H. Alinejad, and V. Khorrami, "Effects of Polarized Debye Sheath and Trapped Ions on Solitary Structures in a Strongly Coupled Inhomogeneous Dusty Plasma", IEEE Transactions on Plasma Science, **46**(4), 755 (2017). https://doi.org/10.1109/TPS.2017.2749382
[19] A. Atteya, S. Sultana, and R. Schlickeiser, "Dust-ion-acoustic solitary waves in magnetized plasmas with positive and negative ions: The role of electrons superthermality", Chinese journal of physics, **56**(5), 1931 (2018). https://doi.org/10.1016/j.cjph.2018.09.002
[20] N. Akhtar, S.A. El-Tantawy, S. Mahmood, and A.M. Wazwaz, "On the dynamics of dust-acoustic and dust-cyclotron freak waves in a magnetized dusty plasma", Romanian Reports in Physics, **71**, 403 (2019). https://rrp.nipne.ro/2019/AN71403.pdf
[21] H. Ur-Rehman, S. Mahmood, and S. Hussain, "Magneto-acoustic solitons in pair-ion fullerene plasma", Waves in Random and Complex Media, **30**(4), 632-642 (2020). https://doi.org/10.1080/17455030.2018.1549762
[22] A. Atteya, M.A. El-Borie, G.D. Roston, and A.S. El-Helbawy, "Nonlinear dust acoustic waves in an inhomogeneous magnetized quantum dusty plasma", Waves in Random and Complex Media, **33**, 329-344 (2021). https://doi.org/10.1080/17455030.2021.1880030
[23] H.R. Pakzad, and D. Nobahar, "Dust-ion acoustic solitons in superthermal dusty plasmas", New Astronomy, **93**, 101752 (2022). https://doi.org/10.1016/j.newast.2021.101752
[24] H. Dehingia, "Various Aspects of Dust-Acoustic Solitary Waves (DAWs) in Inhomogeneous Plasmas," in: *Plasma Science - Recent Advances, New Perspectives and Applications*, edited by S. Singh, (IntechOpen, 2022). https://doi.org/10.5772/intechopen.109160
[25] H. Dehingia, and P.N. Deka, "Effects of dust particles on dust acoustic solitary waves (DASWs) propagating in inhomogeneous magnetized dusty plasmas (MDPs) with dust charge fluctuations," in: *Int. Conf. Adv. Trans. Phenomena*, **1**(4), 56-58 (2022).
[26] H.J. Dehingia, and P.N. Deka, "Structural Variations of Dust Acoustic Solitary Waves (DASWs) Propagating in an Inhomogeneous Plasma," East European Journal of Physics, (1), 19-27 (2023). https://doi.org/10.26565/2312-4334-2023-1-02

[27] H.J. Dehingia, and P.N. Deka, "Propagation of nonlinear dust-acoustic solitary waves under the effect of non-extensive electrons in inhomogeneous collisional magnetized dusty plasma," J. Korean Phys. Soc. **83**, 337–343 (2023). https://doi.org/10.1007/s40042-023-00854-2
[28] F. Verheest, and M.A. Hellberg, "Nonthermal effects on existence domains for dust-acoustic solitary structures in plasmas with two-temperature ions," Physics of Plasmas, **17**(2), 023701 (2010). https://doi.org/10.1063/1.3299356
[29] N.N. Rao, and P.K. Shukla, "Coupled Langmuir and ion-acoustic waves in two-electron temperature plasmas," Physics of Plasmas, **4**(3), 636-645 (1997). https://doi.org/10.1063/1.872160
[30] V. Maslov, and H. Schamel, "Growing electron holes in drifting plasmas," Physics Letters A, **178**(1-2), 171 (1993). https://doi.org/10.1016/0375-9601(93)90746-M
[31] H. Schamel, and V.I. Maslov, "Adiabatic growth of electron holes in current-carrying plasmas," Physica Scripta, T50, 42 (1994). https://doi.org/10.1088/0031-8949/1994/T50/006
[32] H. Schamel, and V.I. Maslov, "Langmuir Wave Contraction Caused by Electron Holes," Physica Scripta T, **82**, 122 (1999). https://doi.org/10.1238/Physica.Topical.082a00122
[33] V.I. Maslov, "Electron beam excitation of a potential well in a magnetized plasma waveguide," Physics Letters A, **165**(1), 63-68 (1992). https://doi.org/10.1016/0375-9601(92)91055-V

## ХАРАКТЕРИСТИКИ НЕЛІНІЙНИХ ПИЛОВИХ АКУСТИЧНИХ ХВИЛЬ (DAWS), ЩО ПОШИРЮЮТЬСЯ В НЕОДНОРІДНІЙ НАМАГНІЧЕНІЙ ПИЛОВІЙ ПЛАЗМІ БЕЗ ЗІТКНЕНЬ

**Хірак Джйоті Дехінгія, Парамананда Дека**
*Факультет математики, Університет Дібругарх, Ассам, Індія*

У цій статті ми представили наше дослідження характеристик нелінійних пилових акустичних хвиль (DAWS), що поширюються в неоднорідній намагніченій пиловій плазмі (MDP) без зіткнень. У цій задачі ми розглядали плазму без зіткнень, що складається з нетеплових іонів, непротяжних електронів і негативно заряджених частинок пилу. Використовуючи відновну теорію збурень (RPT), ми вивели модифіковане рівняння Захарова-Кузнєцова (m-ZK). Рішення рівняння m-ZK вказує на нелінійні характеристики *DASW* у плазмі. Наше дослідження також передбачає, як амплітуди нелінійних *DASW* значно модифікуються через вплив магнітного поля, непротяжних електронів і параметрів неоднорідності в плазмі. Результати, отримані в цьому дослідженні, можуть бути корисними для розуміння характеристик розповсюдження та модифікації структур нелінійних хвиль як у лабораторній, так і в астрофізичній плазмі.

**Ключові слова:** *запилена плазма; RPT; рівняння m-ZK; неоднорідна плазма; неекстенсивні електрони*