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COSMOLOGICAL DYNAMICS OF ANISOTROPIC KANIADAKIS HOLOGRAPHIC DARK ENERGY MODEL IN BRANS-DICKE GRAVITY

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The present study examines the Kaniadakis holographic dark energy in the context of the Brans-Dicke scalar-tensor theory of gravity (Phys. Rev. 124: 925, 1961). This paper focuses on a background with an anisotropic Kantowski-Sachs space-time that is homogeneous in space. Under these circumstances, the Brans-Dicke scalar field denoted as ϕ is used as a function of the average scale factor $a(t)$. Using a graphical model to analyze the model's physical behaviour is part of the inquiry into the Universe's accelerating expansion. We evaluate the cosmological parameters such as the scalar field, the equation of state parameter and the deceleration parameter. Furthermore, the models' stability is assessed through the application of the squared sound speed (v_s^2). For our models, we derive the widely accepted cosmic planes such as $\omega_{kde} - \omega'_{kde}$ and statefinder (r,s) planes. It is found that the scalar field is a decreasing function of cosmic time and hence the corresponding kinetic energy increases. The deceleration parameter exhibits accelerated expansion of the Universe. It is mentioned here that the equation of state parameter lies in the phantom region and finally attains the ΛCDM model. Also, the $\omega_{kde} - \omega'_{kde}$ plane provides freezing and thawing regions. In addition, the statefinder plane also corresponds to the ΛCDM model. Finally, it is remarked that all the above constraints of the cosmological parameters show consistency with Planck observational data.
Keywords: *Scalar-tensor theory; Scalar field; Holographic dark energy; Kantowski-Sachs model*
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1. INTRODUCTION

Recent observational data on the history of cosmic expansion have enabled the discovery of the universe's accelerating expansion conceivable, as provided in the works of Perlmutter et al. [1] and Riess et al. [2]. As a mysterious and intensely pressured force, dark energy (DE) is thought to be the fundamental reason. Nevertheless, the traits and behaviours of DE remain a mystery. Two main approaches are available to tackle the problem of cosmic acceleration: the first one includes introducing a DE component into the Universe and studying its dynamics. (Caldwell [3]; Padmanabhan [4]; Santhi et al. [5], The alternative, however, involves investigating changes to Einstein's theory of gravitation and viewing them as a flaw in general relativity.

Among dynamical differential equation models, the holographic DE (HDE) model has become an important instrument to study the mystery of DE in recent years. Based on the quantum characteristics of black holes (BHs), which have been thoroughly studied in the literature to analyze the idea of quantum gravity, this study's research was carried out. [6]. The vacuum energy Λ of a system of size L should not be greater than the mass of a BH of the same size, according to the holographic principle, a hypothesis in quantum field theory. Within the context of quantum field theory, this idea is essential to comprehend the genesis of BHs. The study carried out established the formal energy density of HDE by Cohen et al. [7]. The equation provided may be rewritten more academically as follows:

$$\rho_{kde} = 3d^2 m_p^2 L^{-2}. \quad (1)$$

In this case, the Planck mass reduction is represented by m_p, the numerical constant is represented by $3d^2$, and the IR cutoff is indicated as L. The literature has researched several infrared cutoffs in great detail, including the Hubble horizon H^{-1}, the event horizon, the particle horizon, the conformal universe age, the Ricci scalar radius, and the Granda-Oliveros cutoff [8]. Examining the current acceleration of the universe is made possible by the use of HDE models with various IR cutoffs, offering insights into the transition redshift value that signifies the change from early deceleration ($q > 0$) to present acceleration ($q < 0$). This research shows that the transition redshift value aligns with current observational data. Moreover, it could offer a possible resolution to the puzzle of cosmic coincidence, which concerns the puzzling topic of why, in the current state of the universe, the energy densities originating from dark matter and DE display a constant ratio. A respectable degree of agreement between the HDE model and observational data has been demonstrated by numerous investigations [9]. Nojiri and Odintsov [10] presented a methodology in their work that uses

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phantom cosmology and generalized HDE to try and reconcile the early and late epochs of the universe. In research by Ghaffari [11], the HDE model was examined to determine whether the generalized laws of thermodynamics held in the context of the D-dimensional Kaluza-Klein-type FRW world. Various cosmological components of new and updated HDE models have been investigated by Aditya and Reddy [12]. As a generic entropy metric, Kaniadakis statistics have recently been used to examine various gravitational and cosmological implications [13]. The generalized $\mathcal{K}$-entropy (Kaniadakis), a single free parameter entropy of a BH is obtained as [14]

$$S_{\mathcal{K}} = \frac{1}{\mathcal{K}} sinh(\mathcal{K} S_{BH}), \tag{2}$$

where $\mathcal{K}$ is an unknown parameter.

Consequently, a new model of DE known as Kaniadakis Holographic DE (KHDE) [14] is presented utilizing this entropy and holographic DE theory, which exposes considerable properties. Jawad and Sultan [15] have discussed KHDE models in different theories of gravity. As Tsallis and Kaniadakis, Sadeghi et al. [16] have examined the dynamic structures of HDE within the context of the Brans-Dicke theory of gravity.

Many entropy-related formalisms have been applied recently to the development and examination of cosmological models. Several new HDE models have been put out, such as the Renyi HDE (RHDE) model [17], the Tsallis HDE (THDE) [18], and the Sharma-Mittal HDE (SMHDE) [19]. Conversely, in the case of non-interacting cosmic systems, the SMHDE theory exhibits classical stability. The RHDE theory shows better stability when viewed individually and is predicated on the idea that cosmic sectors are not connected. The Tsallis, Renyi, and Sharma-Mittal entropies are investigated in the work by Younas et al. [20] in a flat Friedmann-Robertson-Walker (FRW) universe with Chern-Simons modified gravity. In the THDE, Aditya et al. [21] examined the empirical constraints on the logarithmic Brans-Dicke theory of gravity. The authors Prasanthi and Aditya have conducted a study on the observational restrictions in RHDE [22, 23]. In their study, Sharma and Dubey [23] examined the SMHDE models using several diagnostic methods. In light of the aforementioned research, we have chosen to examine the HDE using a novel entropy formalism known as the SMHDE, with the Hubble horizon serving as the infrared cutoffs in our investigation.

The statistical isotropy of the universe is called into question discovery of large variances in cosmic microwave background radiation at wide angles. Even in the absence of inflation, the universe may have some anisotropic geometry within the framework of cosmological theories. Several researchers have recently become quite interested in investigating different cosmological models with anisotropic backdrops. Within the context of the Brans-Dicke theory of gravity [25], this study attempts to explore the Kantowski-Sachs universe taking into account the effects of pressureless matter and KHDE. The suggested work plan's outline is given below. Both the derivation of field equations and their solutions are covered in Section 2. Section 3 examines the model's physical properties. Section 4 contains the comparison of our work with the observational data. The paper's conclusions and a final summary are presented in the last section.

2. FIELD EQUATIONS AND THE MODEL

There have been several gravitational theories put forth as alternatives to Einstein's general theory of gravity. But the most effective substitute for Einstein's theory is thought to be the scalar-tensor theory created by Brans and Dicke [25]. Assume that the universe is composed of DE with a density of ρ_{kde} and pressure-free matter with an energy density of ρ_m. For the combined scalar and tenor fields, the Brans-Dicke field equations are thus provided in this instance by

$$R_{ij} - \frac{1}{2} R g_{ij} = -\frac{8\pi}{\phi}\left(T_{ij} + \overline{T}_{ij}\right) - \phi^{-1}\left(\phi_{i;j} - g_{ij}\phi_{;\alpha}^{;\alpha}\right) - w\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,\alpha}\phi^{,\alpha}\right), \tag{3}$$

$$\phi_{;\alpha}^{;\alpha} = \frac{8\pi}{(3+2w)}\left(T + \overline{T}\right), \tag{4}$$

and the energy conservation equation is

$$(T_{ij} + \overline{T}_{ij})_{;j} = 0, \tag{5}$$

which is the result of field equations (3) and (4). In this case, R is a Ricci scalar, R_{ij} is a Ricci tensor, and w is a dimensionless coupling constant. T_{ij} and $\overline{T}_{ij}$ are energy-momentum tensors for pressure-less matter and KHDE, which are defined as

$$T_{ij} = \rho_m u_i u_j; \;\; \overline{T}_{ij} = (\rho_{kde} + p_{kde}) u_i u_j - p_{kde} g_{ij}, \tag{6}$$

here p_{kde} and ρ_{kde} are the pressure and energy density of DE respectively and ρ_m is the energy density of matter. The equation of state (ω_{kde}) parameter of DE is defined as $\omega_{kde} = \frac{p_{kde}}{\rho_{kde}}$.

We consider the Kantowski-Sachs space-time in the following form

$$ds^2 = dt^2 - A^2 dr^2 - B^2(d\psi^2 + sin^2\psi d\varphi^2), \tag{7}$$

where A and B are metric potentials and only cosmic time t functions. The Kantowski-Sachs class of metrics describes anisotropic and homogenous yet expanding cosmologies. They also provide models for estimating and comparing the consequences of anisotropies with the FRW class of cosmologies (Thorne [26]). For the Kantowski-Sachs model (7), we define the main parameters:
Hubble's parameter of the model

$$H = \frac{\dot{a}}{a} \tag{8}$$

where

$$a(t) = (AB^2)^{1/3} \tag{9}$$

is the average scale factor. Anisotropic parameter A_h is given by

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{10}$$

where $H_1 = \frac{\dot{A}}{A}, H_2 = H_3 = \frac{\dot{B}}{B}$ are directional Hubble's parameters, which express the expansion rates of the universe in the directions of x, y and z respectively.

Expansion scalar and shear scalar are defined as

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \tag{11}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2, \tag{12}$$

where σ_{ij} is the shear tensor, A_h is the deviation from isotropic expansion and the universe expands isotropically if $A_h = 0$. The deceleration parameter is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{13}$$

If $-1 \le q < 0$, the universe expands at an accelerating rate, decelerating volumetric expansion if $q > 0$. If $q = 0$, the universe expands at a constant rate.

The field equations (3)-(5) for the metric (7) produce the following equations when adopting co-moving coordinates:

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + 2\frac{\dot{B}\dot{\phi}}{B\phi} = -\frac{\omega_{kde}\rho_{kde}}{\phi}, \tag{14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = -\frac{\omega_{kde}\rho_{kde}}{\phi}, \tag{15}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + 3\frac{\dot{\phi}}{\phi}H = \frac{\rho_m + \rho_{kde}}{\phi}, \tag{16}$$

$$\ddot{\phi} + 3\dot{\phi}H = \frac{8\pi}{\phi(3+2w)}(\rho_{kde} - 3p_{kde} + \rho_m), \tag{17}$$

and the conservation equation is given by

$$\dot{\rho}_m + \dot{\rho}_{kde} + 3H(\rho_m + (1 + \omega_{kde})\rho_{kde}) = 0. \tag{18}$$

We assume that there is minimal interaction between the DE, $\overline{T}_{ij} = 0$ and that the pressure-less matter component is minimally interacting, $T_{ij} = 0$, due to the energy conservation equation (5). Consequently, two additive conserved components have been extracted from the energy conservation equation (5): maintenance of the DE's energy-momentum tensor

$$\dot{\rho}_{kde} + 3H(1 + \omega_{kde})\rho_{kde} = 0, \tag{19}$$

and the conservation of the energy-momentum tensor of the pressure-less matter

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{20}$$

here the overhead dot represents ordinary differentiation for cosmic time t.

A, B, ϕ, ω_{kde}, ρ_{kde}, and ρ_m are six unknown variables in the four equations (14)-(17). As a result, some extra constraints are required to solve the above system of equations. We build our computations on the following physically acceptable assumptions:

The shear scalar (σ) is regarded as proportionate to the expansion scalar (θ). As a result, the metric potentials are related to one another. (Collins et al. [27]), i.e.,

$$A = B^k. \tag{21}$$

where $k > 1$ is a constant that accounts for space-time anisotropy (we have taken the integration constant as a unity). The physical foundation for this assumption can be found in observations of the velocity redshift relation for extragalactic sources, which indicate that the Hubble expansion of the universe may achieve isotropy when σ/θ is constant.

In addition, it is common in the literature to employ a power-law relationship between scalar field ϕ and average scale factor $a(t)$ of the form (Johri and Sudharsan [28]; Johri and Desikan [29]) $\phi \propto [a(t)]^n$ where n denotes a power index. Many authors have looked into different aspects of this type of scalar field ϕ. Given the physical significance of the preceding relationship, we employ the following assumption to reduce the mathematical complexity of the system

$$\phi(t) = \phi_0[a(t)]^n, \tag{22}$$

where ϕ_0 is the proportionality constant.
From Eqs. (14), (15), (21) and (22), we obtain the metric potentials as

$$A = \left(\frac{t^2}{k-1} - A_1(k-1)\right)^{\frac{k}{2}}, \tag{23}$$

$$B = \sqrt{\frac{t^2}{k-1} - A_1(k-1)}, \tag{24}$$

where A_1 is integrating constant and $n(k+2) + 3k = 0$. Now, the scalar field ϕ calculated as

$$\phi(t) = \phi_0\left(\frac{t^2}{k-1} - A_1(k-1)\right)^{\frac{1}{6\,n(k+2)}}. \tag{25}$$

Now the metric (7) can be rewritten as

$$ds^2 = dt^2 - \left(\frac{t^2}{k-1} - A_1(k-1)\right)^k dr^2 - \left(\frac{t^2}{k-1} - A_1(k-1)\right)(d\psi^2 + sin^2\psi d\varphi^2). \tag{26}$$

3. COSMOLOGICAL PARAMETERS AND DISCUSSION

Equation (26), in conjunction with equation (25) illustrates the Kantowski-Sachs universe with Kaniadakis HDE in Brans-Dicke's theory of gravity. The following geometrical and physical factors are crucial to the debate of cosmology. The spatial volume (V) and average scale factor $(a(t))$ of the model are given by

$$V(t) = [a(t)]^3 = \left(\frac{t^2}{k-1} - A_1(k-1)\right)^{\frac{k+2}{2}}. \tag{27}$$

Mean Hubble's parameter (H) and expansion scalar (θ) are obtained as

$$H = \frac{\theta}{3} = \frac{(k+2)t}{3t^2 - 3A_1(k-1)^2}. \tag{28}$$

The shear scalar (σ^2) and anisotropic parameter (A_h) are

$$\sigma^2 = \frac{(k-1)^2 t^2}{3(t^2 - A_1(k-1)^2)^2}, \tag{29}$$

$$A_h = \frac{2(k-1)^2}{(k+2)^2}. \tag{30}$$

Eq. (26) indicates the spatially homogeneous and anisotropic Kantowski - Sachs KHDE cosmological model in the Brans-Dicke theory of gravity. There is no initial singularity in our model, i.e. at $t = 0$. From a finite volume when $t = 0$, the model's spatial volume increases with time. This indicates that the model's spatial expansion. At $t = 0$, the parameters $H(t),\ \theta(t)$, and σ^2 are finite and tend to infinity as $t \to \infty$. The mean anisotropic parameter A_h represents the deviation from isotropic expansion. It establishes the anisotropic or isotropic nature of the model. When $k = 1$, A_h equals 0. In this instance, the expansion of the universe is isotropic. In addition, if $V \to \infty$ and $A_h = 0$ as $t \to \infty$, The model steadily gets closer to isotropy.

According to the HDE, if DE is meant to regulate the universe's current, accelerated expansion, then, taking into account the Kaniadakis BH entropy equation (2), the total vacuum energy contained in a box of a certain size L^3 must not surpass the energy of a BH of the same mass. Next, one obtains

$$\Lambda^4 \equiv \rho_{kde} \propto \frac{S_{\mathcal{K}}}{\mathcal{L}^4}, \tag{31}$$

for the vacuum energy ρ_{kde}. Now, taking the Hubble horizon of the universe as the IR cutoff (i.e., $L = \frac{1}{H}, A = \frac{4\pi}{H^2}$),

$$\rho_{kde} = \frac{3\mathcal{C}^2 H^4}{\mathcal{K}} sinh\left(\frac{\pi\mathcal{K}}{H^2}\right), \tag{32}$$

where the constant $\mathcal{C}^2$ is unknown, $\mathcal{K}$ belongs to a set of real numbers, and $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Now, it's evident that we have $\rho_{kde} \to \frac{3\mathcal{C}^2 H^4}{\mathcal{K}}$ (the well-known Bekenstein entropy-based HDE) when $k \to 0$. Considering the pressureless fluid (with energy density ρ_m) and the DE candidate (with pressure p_{kde} and density ρ_{kde}). The fractional energy densities of matter (Ω_m) and DE (Ω_{kde}) are given as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3H^2} \quad \text{and} \quad \Omega_{kde} = \frac{\rho_{kde}}{\rho_{cr}} = \frac{\mathcal{C}^2 H^2}{\mathcal{K}} sinh\left(\frac{\pi\mathcal{K}}{H^2}\right), \tag{33}$$

ρ_{cr} is the critical energy density. The above equation can be written using Eq. (28) as

$$\rho_{kde}(t) = \frac{3d^2(k+2)^4 t^4}{(3t^2-3A_1(k-1)^2)^4 \delta} \sinh\left(\frac{\pi\,\delta\,\left(3t^2-3A_1(k-1)^2\right)^2}{(k+2)^2 t^2}\right). \tag{34}$$

From Eqs. (14) and (27), we get the energy density of matter as

$$\rho_m(t) = \rho_0 \left[\frac{t^2}{k-1} - A_1(k-1)\right]^{\frac{-(k+2)}{2}}. \tag{35}$$

Using Eqs. (23)-(25) in Eq. (14), we get the EoS parameter as

$$\omega_{kde}(t) = -\frac{\phi_0\left(3t^2-3A_1(k-1)^2\right)^4 \delta}{3d^2(k+2)^4 t^4}\left(\frac{t^2}{k-1} - A_1(k-1)\right)^{\frac{1}{6\,n(k+2)}} \left\{\frac{-3A_1(k-1)^2}{(t^2-A_1(k-1)^2)^2} + \frac{k(k-1)\left(t^2-A_1(k-1)\right)}{(t^2-A_1(k-1)^2)^2} + \frac{(k+1)t^2}{(t^2-A_1(k-1)^2)^2} + \frac{wn^2(k+2)^2 t^2}{(3t^2-3A_1(k-1)^2)^2} + \frac{2n(k+2)\left(n(k+2)t^2-3t^2-3A_1(k-1)^2\right)}{9(t^2-A_1(k-1)^2)^2} + \frac{n(k+3)(k+2)t^2}{3(t^2-A_1(k-1))^2} + \left(\frac{t^2}{k-1} - A_1(k-1)\right)^{-1}\right\}\left(\sinh\left(\frac{\pi\,\delta\,(3t^2-3A_1(k-1)^2)^2}{(k+2)^2 t^2}\right)\right)^{-1}. \tag{36}$$

Scalar field: We plotted a scalar field's behaviour against cosmic time for a range of parameter values. k in Fig. 1. One way to conceptualize the scalar field is as a positive, declining function that ultimately approaches a minimum positive value. Because of the scalar field's diminishing behaviour, the corresponding kinetic energy rises. This behaviour closely resembles that of scalar fields in DE models that have been developed by several writers and published in literature (Aditya and Reddy [30]). Moreover, it is evident that when parameter k rises, the scalar field contracts. Hence, in this work, Examining the additional dynamical parameters in the context of the BD scalar field is our goal.

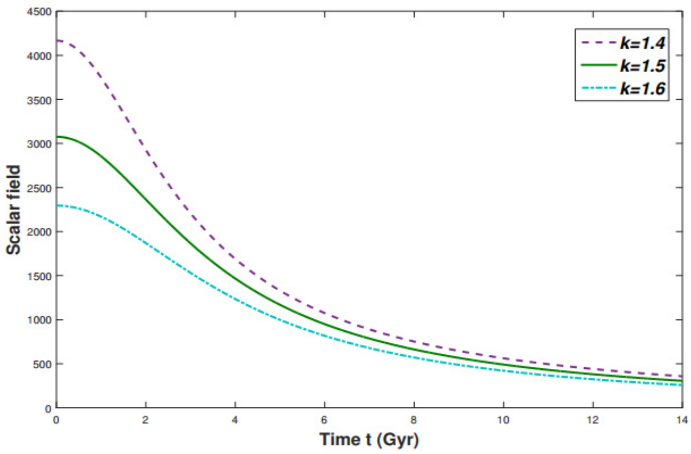

Figure 1. Plot of scalar field ϕ versus cosmic time t for $\phi_0 = 28000$ and $A_1 = -38$

Energy conditions: The Raychaudhuri equations provide the foundation for the study of energy conditions and are essential to any analysis of the congruence of time-like and null geodesics. Energy conditions are used to illustrate other general conclusions regarding the behaviour of powerful gravitational fields. These are the typical energy scenarios:

- Dominant energy condition (DEC): $\rho_{kde} \geq 0$, $\rho_{kde} \pm p_{kde} \geq 0$.
- Strong energy conditions (SEC) : $\rho_{kde} + p_{kde} \geq 0$, $\rho_{kde} + 3p_{kde} \geq 0$,
- Null energy conditions (NEC): $\rho_{kde} + p_{kde} \geq 0$,
- Weak energy conditions (WEC): $\rho_{kde} \geq 0$, $\rho_{kde} + p_{kde} \geq 0$,

Fig. 2 depicts the energy conditions for our KHDE model. It is clear that the NEC is violated, and the model results in a Big Rip. Furthermore, the WEC is observed to comply with the requirement $\rho_{de} \geq 0$. In addition, Fig. 2 shows that the DEC $\rho_{de} + p_{de}$ is not satisfied. Furthermore, our model appropriately violates the SEC. This tendency, Which results from the late-time acceleration of the universe, corresponds with current observational data.

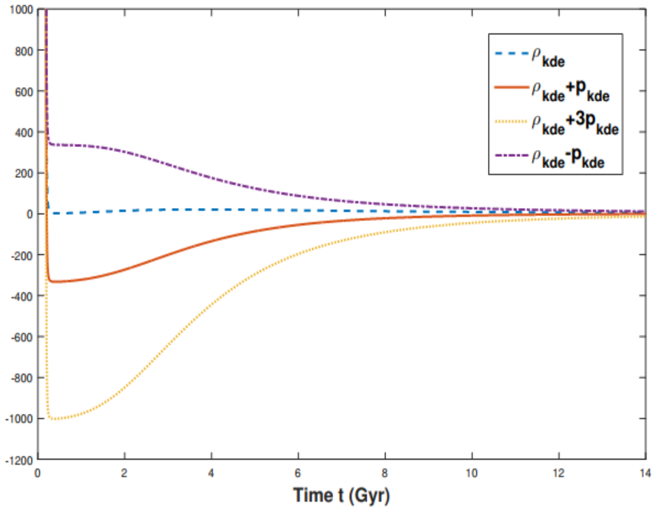

Figure 2. Plot of energy conditions versus cosmic time t for $\phi_0 = 28000$, $w = 0.025$, $\mathcal{K} = 0.001$, $\mathcal{C} = 9.2$ and $A_1 = -38$

EoS parameter: The definition of the EoS parameter is the correlation between DE's pressure p_{kde} and energy density ρ_{kde}, which is expressed as $\omega_{kde} = \frac{p_{kde}}{\rho_{kde}}$. The universe's accelerated and decelerated expansion are categorized using the EoS parameter, which separates epochs into the following groups: For $\omega = 1$ stiff fluid, $\omega = \frac{1}{3}$ radiation, and $\omega = 0$ matter commanded (dust) (decelerating phases). It symbolizes the quintessence $-1 < \omega < -1/3$, the cosmological constant $\omega = -1$, and the phantom $\omega < -1$.

The EoS parameter of our DE model is depicted in Fig. 3 for various values of $\mathcal{C}$. We note that the EoS parameter of our model starts in the aggressive phantom area ($\omega_{kde} << -1$) and finally attains ΛCDM model ($\omega_{kde} = -1$) and phantom region ($\omega_{kde} < -1$). According to current observational data, this behaviour is consistent, and the current value (at $t = 13.7\, Gyr$) of our DE model's EoS parameter is in close approximation with current Planck data (Aghanim et al. [31]).

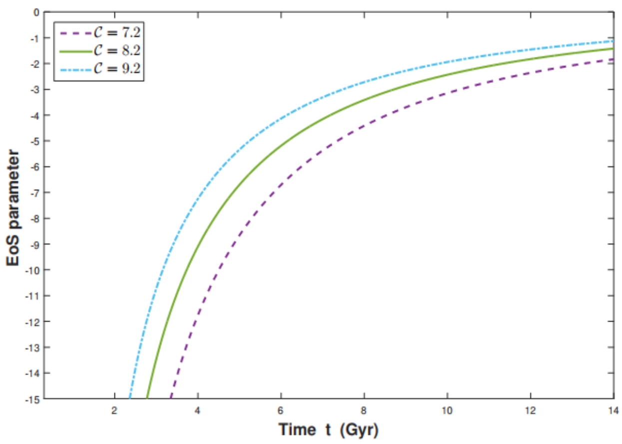

Figure 3. Plot of EoS parameter ω_{kde} versus cosmic time t for $\phi_0 = 28000$, $w = 0.025$, $\mathcal{K} = 0.001$and $A_1 = -38$

$\omega_{kde} - \omega'_{kde}$ plane: The dynamical characteristic of models of DE is examined through the $\omega_{kde} - \omega'_{kde}$ plane analysis, where prime ($'$) signifies derivative with regard to lna. Caldwell and Linder [32] proposed this approach to analyse the behaviour of the quintessence model. They divided the $\omega_{kde} - \omega'_{kde}$ plane into thawing ($\omega_{kde} < 0$ and $\omega'_{kde} > 0$)

and freezing ($\omega_{kde} < 0$ and $\omega'_{kde} < 0$) areas. Researchers have expanded the scope of this planar study to analyze the dynamic behaviour of several DE models and modified theories of gravity [33]. Our DE model's ω_{kde}-ω'_{kde} trajectory is depicted in Fig. 4 for distinct values of parameter $\mathcal{C}$ as the ω_{kde}-ω'_{kde} plane remains same for various values of $\mathcal{C}$. Both the thawing and freezing zones exhibit variation in the model; however, our model primarily fluctuates in the freezing region. The freezing region is where observational evidence indicates that the universe is expanding much more quickly. As a result, the behaviour of the ω_{kde}-ω'_{kde} plane is in line with the available observations.

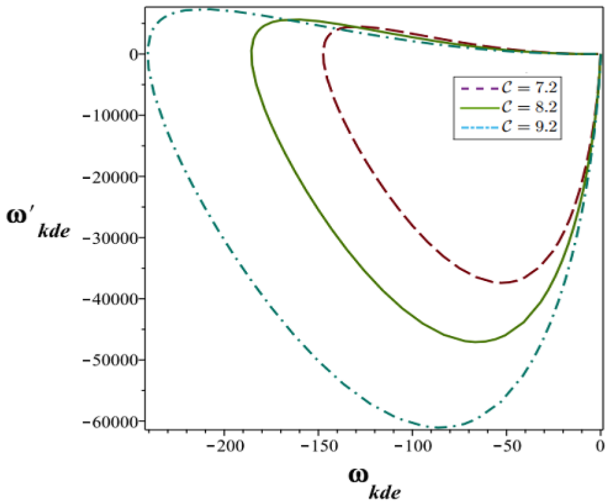

Figure 4. Plot of $\omega_{kde} - \omega'_{kde}$ plane for $\phi_0 = 28000$, $w = 0.025$, $\mathcal{K} = 0.001$, $\mathcal{C} = 9.2$ and $A_1 = -38$

Stability analysis: In this case, we evaluate our DE model's stability against minor perturbations using the squared speed of sound. The sign of the square of sound speed plays a vital role, as its negative ($v_s^2 < 0$) denotes instability and its positive ($v_s^2 > 0$) shows stability. It can be described as follows:

$$v_s^2 = \frac{\dot{p}_{kde}}{\dot{\rho}_{kde}}. \tag{37}$$

By differentiating the EoS parameter $\omega_{kde} = \frac{p_{kde}}{\rho_{kde}}$ about time t and dividing by $\dot{\rho}_{kde}$, we get

$$v_s^2 = \omega_{kde} + \frac{\rho_{kde}}{\dot{\rho}_{kde}} \dot{\omega}_{kde}. \tag{38}$$

We build the squared speed of sound trajectories in terms of cosmic time in the current scenario, as illustrated in Fig. 5 for various values of $\mathcal{C}$. We can witness from Fig. 5 that v_s^2 curve shows positive behaviour in the first epoch and changes in the negative section. As a result, our model is unstable at the present and in subsequent epochs of the universe, but stable at the beginning.

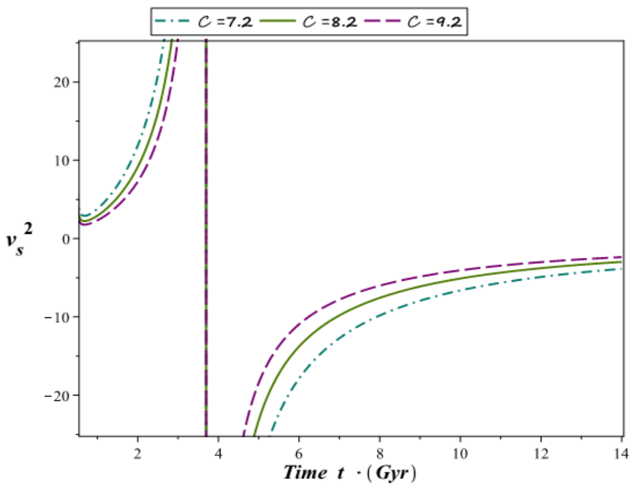

Figure 5. Plot of squared sound speed v_s^2 versus cosmic time t for $\phi_0 = 28000$, $w = 0.025$, $\mathcal{K} = 0.001$, $\mathcal{C} = 9.2$ and $A_1 = -38$

Deceleration parameter (DP): One crucial kinematical quantity is the deceleration parameter (q). This parameter shows the speed and slowness of the universe. There is an accelerating expansion if $-1 < q < 0$, a decelerating expansion if

$q > 0$, and a constant rate of expansion if $q = 0$. In addition, for $q = -1$, we get an exponential expansion and for $q < -1$, Our current growth is exponential. DP obtained for our model as

$$q(t) = \frac{3\,t + 3A_1(k-1)^2}{(k+2)t} - 1. \tag{39}$$

In Fig. 6, we have displayed the DP against cosmic time over a range of parameter k values. For all values of k, we observe that DP stays less than -1 and ultimately approaches -1 at late times. k, indicating that the universe is accelerating. As a result, The universe is expanding at an exponential rate.

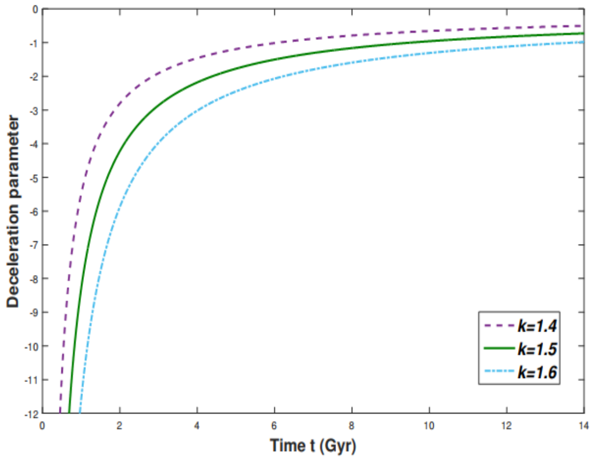

Figure 6. Plot of deceleration parameter q versus cosmic time t for $A_1 = -38$.

Statefinder parameters (r, s): The accelerated expansion of the universe has been explained by a variety of DE hypotheses. Sahni et al. [34] have presented statefinder parameters (r, s) to test the validity of these models. The $r - s$ plane is the cosmological plane corresponding to these parameters, and it indicates how far a certain DE model is from the ΛCDM limit. The cosmic planes of these parameters describe several well-known regions of the universe, e.g., $s > 0$ and $r < 1$ give the phantom and quintessence DE eras, respectively. $(r, s) = (1,0)$ is the ΛCDM limit, $(r, s) = (1,1)$ is the CDM limit, and $s < 0$ and $r > 1$ are the Chaplygin gas limits. Our models' statefinder parameters are provided by

$$r(t) = \frac{3\,t + 3A_1(k-1)^2}{(k+2)t} - 1 + 2\left(\frac{3\,t + 3A_1(k-1)^2}{(k+2)t} - 1\right)^2 + \frac{9A_1(k-1)^2\left(t^2 - A_1(k-1)^2\right)}{(k+2)^2 t^3}, \tag{40}$$

$$s(t) = \frac{\left\{\frac{3\,t+3A_1(k-1)^2}{(k+2)t} - 2 + 2\left(\frac{3\,t+3A_1(k-1)^2}{(k+2)t} - 1\right)^2 + 9\frac{A_1(k-1)^2\left(t^2 - A_1(k-1)^2\right)}{(k+2)^2 t^3}\right\}}{\left(3\frac{3\,t+3A_1(k-1)^2}{(k+2)t} - 4.5\right)}. \tag{41}$$

Plotting r versus s yields the statefinders plane, as shown in Fig. 7 for different values of k. The regions of quintessence and phantom models can be found in the $r - s$ plane for our model. Our model coincides with the ΛCDM model in its evolution.

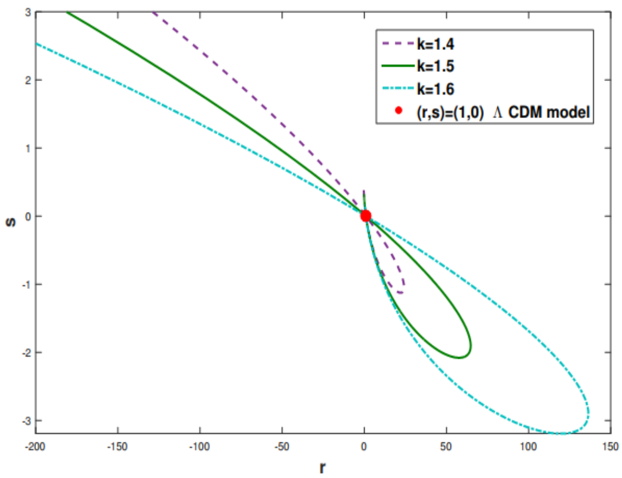

Figure 7. Plot of statefinder's plane for $A_1 = -38$.

4. DISCUSSION AND COMPARISON

In this section, we present a comparison of our work with the recent work on this subject and discuss the comparison with observational data.

Rao and Prasanthi [35] have discussed Bianchi type-I and III modified holographic Ricci DE models in Saez–Ballester theory of gravitation which evolve from the phantom region and ultimately reach the quintessence region. Rao et al. [36] have investigated a non-static plane-symmetric universe filled with matter and anisotropic modified holographic Ricci DE components within the framework of Saez–Ballester's theory of gravitation. In this model, the EoS parameter varies from matter-dominated to the phantom region by crossing the phantom divide line and then goes towards the quintessence region in the latter epoch. Sadri and Vakili [37] have studied the FRW new HDE model in the framework of the Brans–Dicke scalar–tensor theory of gravitation taking into account the interaction between dark matter and HDE. They have obtained an EoS parameter that can reach the phantom era without the necessity of interaction between DE and dark matter. Aditya and Reddy [38] have studied locally rotationally symmetric Bianchi type-I universe within the framework of the Saez–Ballester scalar–tensor theory of gravitation, where the models start in the matter-dominated era, varies in the quintessence region, cross phantom divided line and attains a constant value in the phantom region. Prasanthi and Aditya [39] have discussed Bianchi type-VI_0 RHDE models in general relativity where the model exhibits quintom as well as the phantom behaviour of the universe. Naidu et al. [40] have investigated the dynamical behaviour of Kaluza-Klein FRW-type DE cosmological models in the framework of a scalar-tensor theory of gravitation formulated by Saez and Ballester. Aditya [41] studied the Bianchi type-I RHDE model in the Saez-Ballester theory of gravitation, here the model displays quintom behaviour and consistent ranges with the observational data. Aditya and Prasanthi [42] have discussed the dynamics of SMHDE in the Brans-Dicke theory of gravity, here the model starts in the matter-dominated era, crosses the phantom division line, and finally reaches a constant value in the aggressive phantom region. Dasunaidu et al. [43] discussed Kaluza-Klein FRW type DE cosmological models in the context of Saez and Ballester's scalar-tensor theory of gravitation, where models begin in the matter-dominated era, evolves to the quintessence DE era, and finally approaches the vacuum DE and phantom era. In our KHDE model the study of the EoS parameter reveals that the model starts The EoS parameter analysis shows that the model starts in the aggressive phantom area ($\omega_{kde} << -1$) and finally attains ΛCDM model ($\omega_{kde} = -1$) and phantom region ($\omega_{kde} < -1$). This is quite in contrast with the models discussed above. Also, it is worthwhile to present, here, Planck's observational data given by Aghanim et al. [31] which gives the constraints on the EoS parameter of DE $\omega_{de} = -1.56^{+0.60}_{-0.48}$ (Planck + TT + lowE); $\omega_{de} = -1.58^{+0.52}_{-0.41}$(Planck+ TT, TE, EE + lowE); $\omega_{de} = -1.57^{+0.50}_{-0.40}$ (Planck + TT, TE, EE + lowE + lensing); $\omega_{de} = -1.04^{+0.10}_{-0.10}$ (Planck + TT, TE, EE + lowE + lensing + BAO) by implying different combinations of observational schemes at 95% confidence level. It can be observed from Fig. 3 that the EoS parameter of our model lies within the above observational limits which shows the consistency of our results with the above cosmological data. The above comparison shows that our KHDE model is more viable than the DE models obtained by several authors, in the BD scalar–tensor theory, discussed above.

5. SUMMARY AND CONCLUSIONS

In this work, we study the Kantowski-Sachs universe and the Kaniadakis holographic dark energy in the context of the Brans-Dicke scalar-tensor theory of gravity. Field equations are solved using a few physically possible circumstances. We may analyze the dynamical properties of the DE model by constructing the cosmological parameters of our models. The following are some conclusions:

- This model starts with a finite volume and extends from there with no initial singularity. As $t \to \infty$ approaches, the physical parameters H, θ, σ^2 diverge and all drop to constant values at $t = 0$. Our model also becomes isotropic (because $A_h = 0$) and shear free when $K = 1$. The scalar field of our models decreases with cosmic time and is positive (Fig. 1). This behaviour is comparable to various theories' scalar field models (Aditya and Reddy [30,38]).
- Based on the deceleration parameter, we conclude that our model exhibits a super-exponential expansion (Fig. 6). The trajectory of statefinder parameters varies in both quintessence and phantom zones (Fig. 7). There is an obvious breach of the NEC, which causes a Big Rip in the model. Similar to what is predicted, our model likewise breaks the other energy requirements. This is because fresh observational data supports the late-time acceleration of the universe.
- We produce the sound's squared speed v_s^2 trajectory for our DE model in this scenario (Fig. 5). That v_s^2 fluctuates fully in the negative area indicates that the model is unstable. The EoS parameter analysis shows that the model starts in the aggressive phantom area ($\omega_{kde} << -1$) and finally attains ΛCDM model ($\omega_{kde} = -1$) and phantom region ($\omega_{kde} < -1$). We looked into the $\omega_{kde} - \omega'_{kde}$ plane study and found that it happens during the freezing and thawing phases of the history of the universe (Fig. 4). We find observationally that the expansion of the universe is significantly faster in the freezing region. Thus, the behaviour of the $\omega_{kde} - \omega'_{kde}$ plane agrees with the available data.

Furthermore, we have examined how each dynamical parameter behaves for a range of C and k values. The pictures make it abundantly evident that the dynamics of cosmological parameters are significantly influenced by the BD scalar field $\phi(t)$.

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КОСМОЛОГІЧНА ДИНАМІКА АНІЗОТРОПНОЇ ГОЛОГРАФІЧНОЇ МОДЕЛЬ ТЕМНОЇ ЕНЕРГІЇ КАНІАДАКІСА В ГРАВІТАЦІЇ БРАНСА-ДІКЕ

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У цій роботі досліджується голографічна темна енергія Каніадакіса в контексті скалярно-тензорної теорії гравітації Бранса-Дікке (Phys. Rev. 124: 925, 1961). Ця стаття присвячена фону з анізотропним простором-часом Кантовського-Сакса, який є однорідним у просторі. За цих обставин скалярне поле Бренса-Дікке, позначене як ϕ, використовується як функція середнього масштабного коефіцієнта *a(t)*. Використання графічної моделі для аналізу фізичної поведінки моделі є частиною дослідження прискореного розширення Всесвіту. Ми оцінюємо космологічні параметри, такі як скалярне поле, параметр рівняння стану та параметр уповільнення. Крім того, стабільність моделей оцінюється за допомогою квадрата швидкості звуку (v_s^2). Для наших моделей ми виводимо загальноприйняті космічні площини, такі як $\omega_{kde} - \omega'_{kde}$ і площини вимірювача стану (r,s). Виявлено, що скалярне поле є спадною функцією космічного часу і, отже, відповідна кінетична енергія зростає. Параметр уповільнення демонструє прискорене розширення Всесвіту. Тут згадується, що рівняння параметра стану лежить у фантомній області і, нарешті, досягає моделі ΛCDM. Крім того, площина $\omega_{kde} - \omega'_{kde}$ забезпечує області замерзання і відтавання. Крім того, модель ΛCDM також відповідає площині вимірювача стану. Нарешті, зауважується, що всі вищезазначені обмеження космологічних параметрів узгоджуються з даними спостережень Планка.
Ключові слова: *скалярно-тензорна теорія; скалярне поле; голографічна темна енергія; модель Кантовського-Сакса*