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NONCLASSICALITIES OF THE SUPERPOSITION STATE OF COHERENT AND PHOTON-ADDED-COHERENT STATE

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The nonclassical properties of the hybrid coherent state (HCS), which is the superposition state of the coherent state and photon-added coherent (PAC) state, is investigated analytically. We evaluated the photon number statistics, the Wigner-Yanase skew information, the Mandel Q factor and the quadrature squeezing of the HCS to quantify its nonclassicality. This superposition state exhibits more nonclassical properties than the PAC state and even the superposition state of coherent state and single-photon-added coherent (SPAC) state. We reported that the addition of more photons to the PAC state part of the HCS generally quantifies more nonclassicalities. The nonclassical properties of the HCS also depend on the amplitudes of coherent state and the PAC state in the HCS.
Keywords: *Coherent state; Photon-added coherent state; Hybrid coherent state; Wigner-Yanase skew information; Mandel Q factor; Quadrature squeezing; Nonclassical effect*
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1. INTRODUCTION

The coherent state $|\alpha\rangle$ of light is the classical like state which exhibits the Poissonian photon number distribution with $|\alpha|^2$ average number of photons. On the contrary, the Fock state $|m\rangle$ is completely quantum mechanical and contains precisely m photons. The PAC state of order m and amplitude α is defined as $|\alpha, m\rangle = a^{\dagger m}|\alpha\rangle$ ($a^\dagger$ is the photon creation operator and the m times application of $a^\dagger$ on the coherent state $|\alpha\rangle$ results $|\alpha, m\rangle$) which has intermediate properties between the coherent state and Fock state. The state $|\alpha, m\rangle$ reduces to coherent state or Fock state for $m \to 0$ or $\alpha \to 0$, respectively. The nonclassical properties such as squeezing and sub-Poissonian photon statistics of PAC state were first described by Agarwal and Tara [1]. They also described theoretically how the PAC can be prepared. The experimental realization of SPAC state via parametric down-conversion in nonlinear crystals has been reported by Zavatta *et al.* [2]. Mattos and Vidiella-Barranco investigated the photon-added and photon-subtracted state using the optical amplifier and beam splitter [3-4]. Hu *et al.* proposed a scheme to prepare the SPAC state via three-wave mixing [5]. The SPAC state has large single photon probability as it has no vacuum part and also shows nonclassicalities. The nonclassical states have tremendous applications in various fields. Several applications of the nonclassical effects of the Bose-Einstein condensates system and Raman processes have been reported [6-8]. The nonclassical photon-added coherent states also have applications in various fields, such as it is essential in quantum communication [9], quantum key distribution [10], quantum state engineering [11], quantum metrology [12], quantum dense coding protocol [13], and to improve the perfection in quantum digital signature protocols [14]. The more nonclassicalities the quantum states have, the more effective it will be in practical applications. To improve the depth of nonclassicalities, scientists also prepare and study the superposition state of SPAC state and coherent state and in some cases, they achieved this [15-17]. But there is no such work on the HCS which is the superposition state of coherent state and PAC state $|\alpha, m\rangle$ with $m > 1$. In this paper we have investigated the nonclassical properties of such HCS in details. Turek *et al.* have theoretically shown that such hybrid coherent state can be prepared through the cross-Kerr interaction of coherent state with the single-photon state [17].

The HCS we have investigated is the superposition state of coherent state $|\alpha\rangle$ and PAC state $a^{\dagger m}|\alpha\rangle$. Such state can be written as

$$|\psi\rangle = \mathcal{N}\left[\sqrt{\varepsilon}\, e^{i\theta}|\alpha\rangle + \sqrt{1-\varepsilon}\, a^{\dagger m}|\alpha\rangle\right] \tag{1}$$

where ε is the superposition parameter that lies between $0 \le \varepsilon \le 1$ and θ is the phase difference between the coherent state and PAC state. Shringarpure and Franson showed that such a state can be generated from the output of an optical parametric amplifier by introducing the coherent state and the photon number state in the input signal mode and idler mode of the amplifier, respectively [15]. The superposition parameter can be controlled by varying the gain of the optical parametric amplifier. For $\varepsilon = 0$ or $\varepsilon = 1$ the HCS $|\psi\rangle$ will reduce to a PAC state or coherent state, respectively. Here, $\mathcal{N}$ is the normalization constant which is given by

$$\mathcal{N} = \left[\varepsilon + 2\sqrt{\varepsilon(1-\varepsilon)}\, Re\left[e^{i\theta}\alpha^m\right] + (1-\varepsilon)L_m(-|\alpha|^2)\, m!\right]^{-\frac{1}{2}} \tag{2}$$

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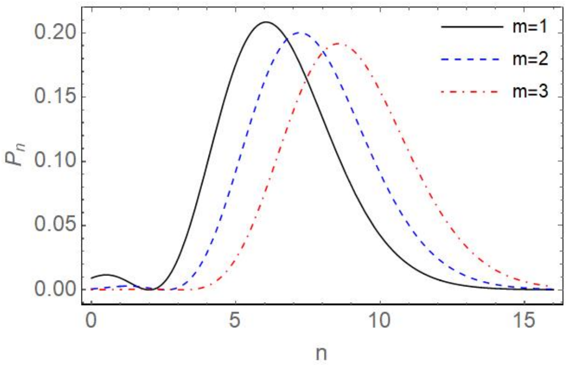
where $L_m(x)$ is the Laguerre polynomial of order m and $\alpha = |\alpha|e^{i\omega}$. We study various nonclassical properties of the state as described in Eq. (1) by varying $|\alpha|$, and the superposition parameter ε. We also investigate the nonclassical properties of the HCS by varying the photon number m.

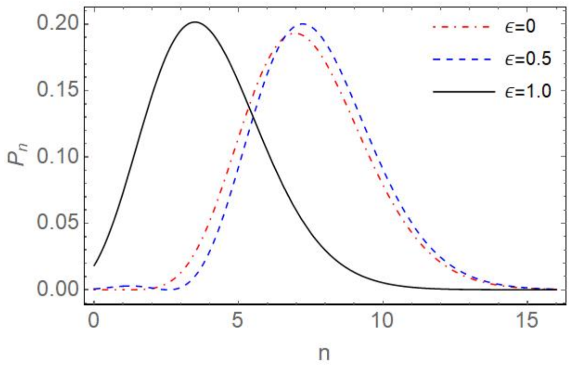
To study the photon number distribution of the HCS, we derive the probability of finding n photons in ψ, i.e. $P_n = |\langle n|\psi\rangle|^2$, which is given by

$$P_n = \left|\mathcal{N} e^{-\frac{|\alpha|^2}{2}}\left(\sqrt{\varepsilon}\, e^{i\theta} \frac{\alpha^n}{\sqrt{n!}} + \sqrt{1-\varepsilon} \frac{\sqrt{n!}}{(n-m)!} \alpha^{n-m}\right)\right|^2 \tag{3}$$

We plot the Eq. (3) with n for different values of m (Figure 1(a)) and ε (Figure 1(b)). Here we have taken $\theta = \pi, \omega = 0$, and $|\alpha| = 2$. For $\varepsilon = 1$, the state is a coherent state and it shows the Poisson distribution (Black line in Fig. 1). For $\varepsilon = 0$, the state is a photon-added coherent and for $0 \le \varepsilon \le 1$ the state is a HCS. For $\varepsilon = 0.5$ the HCS is the 50:50 superposition of the coherent state and PAC state. The different m values in Figure 1(a) correspond to the numbers of photons added with the coherent state to prepare the PAC state part of the HCS. For nonclassicalities, the width of the distribution curve requires to be narrower than that of the coherent state [18]. But it is difficult to quantifying the nonclassicality from the number distribution plot. To view this we derive the Mandel Q-factor and the Wigner-Yanase skew information [18-20].



(a)



(b)

Figure 1. Variation of P_n with n: (a) for different m with $\varepsilon = 0.5$; (b) for different ε with $m = 2$

This paper is organized as follows. We study the Mandel Q factor in Section 2. We study the Wigner-Yanase skew information in Section 3 and quantum squeezing in section 4. Finally, we concluded in Section 5.

2. MANDEL Q FACTOR

The sub-Poissonian statistical property of a quantum system is a nonclassical effect. Mandel Q factor (Q_m) efficiently quantifying the sub-Poissonian distribution of photons. The Q_m factor is defined as

$$Q_m = \frac{\langle a^{\dagger 2} a^2\rangle - \langle a^\dagger a\rangle^2}{\langle a^\dagger a\rangle}. \tag{4}$$

For coherent state, $Q_m = 0$; for Fock state, $Q_m = -1$. If $Q_m > 0$, the photon statistics is super-Poissonian. If $0 > Q_m \ge -1$, the number statistics of the state is sub-Poissonian and this is the sufficient condition of a quantum state to be nonclassical. For the state defined in Eq. (1), we evaluate

$$\langle a^{\dagger 2} a^2\rangle = |\mathcal{N}|^2 \Big[\varepsilon|\alpha|^4 + 2\sqrt{\varepsilon(1-\varepsilon)}\, Re[e^{i\theta}\alpha^m\{m(m-1) + 2m|\alpha|^2 + |\alpha|^4\}]$$
$$+(1-\varepsilon)\{(m+2)!\, L_{m+2}(-|\alpha|^2) - 4(m+1)!\, L_{m+1}(-|\alpha|^2) + 2m!\, L_m(-|\alpha|^2)\}\Big]. \tag{5}$$

The average photon number $\langle n\rangle$ is

$$\langle a^\dagger a\rangle = |\mathcal{N}|^2 \Big[\varepsilon|\alpha|^2 + 2\sqrt{\varepsilon(1-\varepsilon)}\, Re[e^{i\theta}\alpha^m(m + |\alpha|^2)]$$
$$+(1-\varepsilon)\{(m+1)!\, L_{m+1}(-|\alpha|^2) - m!\, L_m(-|\alpha|^2)\}\Big]. \tag{6}$$

Using Eq. (4)-(6) we plot Q_m with $|\alpha|$ for different values of m (Figure 2(a)) and ε (Figure 2(b)). Figure 2(a) shows that for $m = 1$, i.e., when a single photon is added to the PAC state part, the photon distribution of HCS is sub-Poissonian only for $0.3 > |\alpha| > 1.4$. But for $m > 1$, the distribution is sub-Poissonian for all values of $|\alpha|$ (Dashed blue line is for $m = 2$, and red dotdashed line for $m = 3$). More the negativity of Q_m indicates more nonclassicality. The higher the nonclassicality, it will be more useful in practical applications. The Q_m factor of the superposition state of coherent state and SPAC state was investigated by Turek *et al.* [17]. It is interesting to note that the HCS may have better useful applications as a nonclassical state when its PAC state part is prepared with the addition of more photons than the addition of a single photon. The depth of nonclassicality decreases with higher values of $|\alpha|$ because higher the values of $|\alpha|$, the

PAC state part will go toward the coherent state. Figure 2(b) shows that for $\varepsilon = 1$, i.e., when the HCS reduces to a coherent state, $Q_m = 0$ as usual (solid black line). For $|\alpha| > 1.7$, the HCS (blue dashed line) shows more nonclassicalities than the PAC state (dotdashed red line). This is a very important result because it reveals the necessity and importance of introduction of HCS over the PAC state.

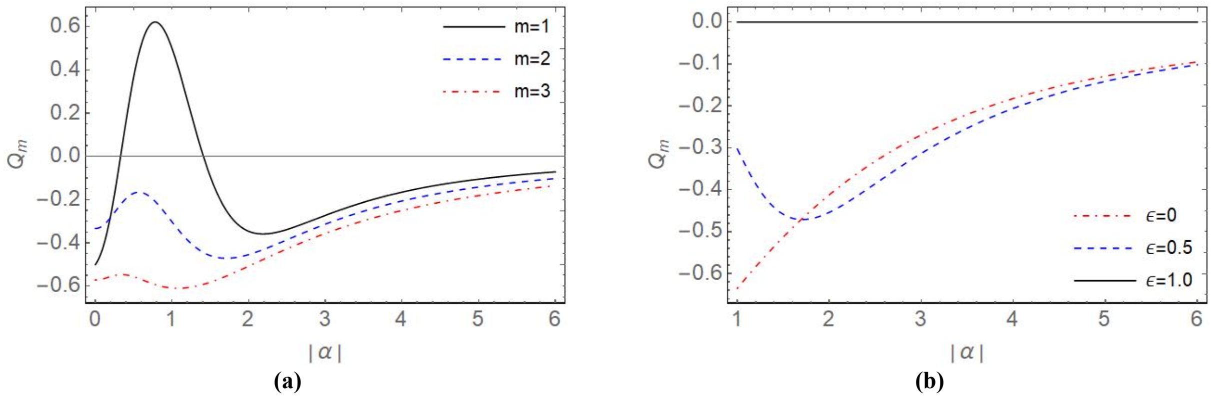


Figure 2. Variation of Q_m with $|\alpha|$: (a) for different m with $\varepsilon = 0.5$; (b) for different ε with $m = 2$. We take $\theta = \pi, \omega = 0$

3. Wigner-Yanase skew information

The Wigner-Yanase skew information is also a measure of the quantumness of a state and it is a special version of quantum Fisher information [21]. The skew information quantifies the nonclassicality present in a quantum state in terms of the ladder operators of the states involved. It contains the noticeable properties which have been found remarkable applications in quantum information theory [22, 23]. For single-mode radiation field the skew information (W) is defined as

$$W = 0.5 + \langle a^\dagger a \rangle - \langle a^\dagger \rangle \langle a \rangle. \tag{7}$$

The average of the field annihilation operator for the HCS defined in Eq. (1) is

$$\langle a \rangle = |\mathcal{N}|^2 \Big[\varepsilon\alpha + \sqrt{\varepsilon(1-\varepsilon)} \left\{ e^{-i\theta} \alpha^{*(m-1)} (m + |\alpha|^2) + e^{i\theta} \alpha^{(m+1)} \right\}$$
$$+ (1-\varepsilon) e^{-|\alpha|^2} \sum_{p=0}^{\infty} \frac{|\alpha|^{2p}\, \alpha\, (m+1+p)!}{p!\,(p+1)!} \Big]. \tag{8}$$

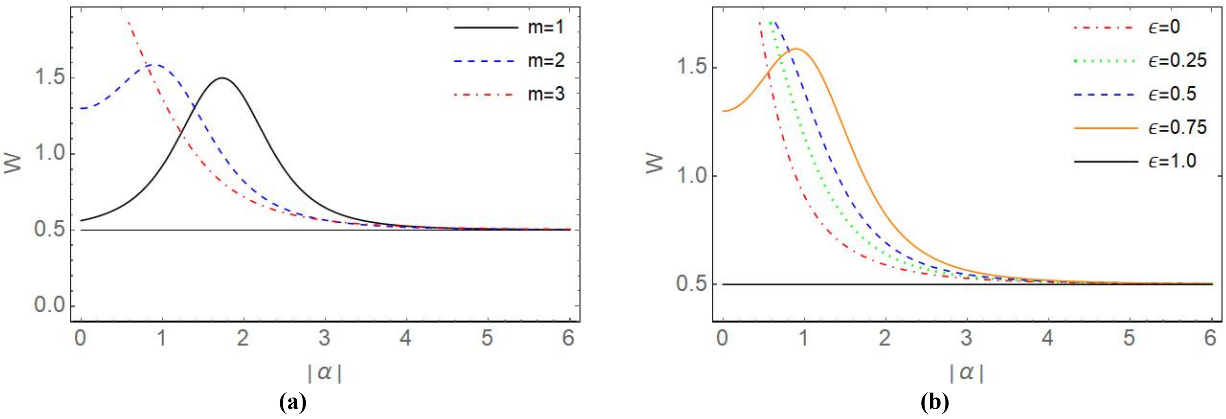


Figure 3. Variation of W with $|\alpha|$: (a) for different m with $\varepsilon = 0.75$; (b) for different ε with $m = 2$. We take $\theta = \pi, \omega = 0$

Using Eq. (6)-(8), we plot the skew information W with $|\alpha|$ for different values of m (Figure 3(a)) and ε (Fig. 3(b)). For coherent state, $W = 0.5$ (black line in Figure 3(b)). For nonclassical states, $W > 0.5$. The higher the values of W, the higher the nonclassicality. Figure 3(a) shows that for smaller values of $|\alpha|$, more photon addition in the PAC state part quantifies more nonclassicality. But for large $|\alpha|$ values, the quantumness of the HCS state is higher for smaller m values and finally the skew information W reduces to its coherent state value for more large values of $|\alpha|$. Figure 3(b) shows that the amount of nonclassicality of the HCS ($\varepsilon \neq 0, 1$) is more than that of the photon-added coherent state ($\varepsilon = 0$) (the only exception when $|\alpha| < 0.55$ for $\varepsilon = 0.75$). So, controlling the m-value and the superposition parameter in the HCS, we can magnify the depth of nonclassicality which is essential for practical applications of quantum state in quantum computation and quantum information. For large $|\alpha|$ the value skew information W reduces to its coherent state value irrespective of m and ε.

4. QUADRATURE SQUEEZING

Squeezed light has the fluctuation below the standard quantum limit and it is also a nonclassical quantum effect. It has various applications in sensitive measurements and quantum communications. The most nonclassical state is the single-photon state and the nonclassicality is also possible for photon number more than one [18]. The quadrature squeezing of PAC state is reported by Francis and Tame [16]. More the depth of squeezing below the minimum uncertainty has the better applications in quantum sensing. The quadrature operator X_φ and the squeezing parameter S_φ of a single mode field is defined as

$$X_\varphi = \frac{1}{\sqrt{2}}(ae^{-i\varphi} + a^\dagger e^{i\varphi}),\ S_\varphi = \left(\Delta X_\varphi\right)^2 - \frac{1}{2}, \tag{9}$$

where $\left(\Delta X_\varphi\right)^2 = \sqrt{\langle {X_\varphi}^2\rangle - \langle X_\varphi\rangle^2}$. Then, Eq. (9) can be written as

$$S_\varphi = \frac{1}{2}[e^{2i\varphi}(\langle a^{\dagger 2}\rangle - \langle a^\dagger\rangle^2) + e^{-2i\varphi}(\langle a^2\rangle - \langle a\rangle^2) + 2(\langle a^\dagger a\rangle - \langle a^\dagger\rangle\langle a\rangle)]. \tag{10}$$

The value of $\langle a^2\rangle$ for the HCS as defined in Eq. (1) is given by

$$\langle a^2\rangle = |\mathcal{N}|^2 \Big[\varepsilon\alpha^2 + \sqrt{\varepsilon(1-\varepsilon)}\left\{e^{-i\theta}\alpha^{*(m-2)}(m^2 - m + 2m|\alpha|^2 + |\alpha|^4) + e^{i\theta}\alpha^{(m+2)}\right\}$$
$$+ (1-\varepsilon)e^{-|\alpha|^2}\sum_{p=0}^{\infty}\frac{|\alpha|^{2p}\,\alpha^2\,(m+2+p)!}{p!\,(p+2)!}\Big] \tag{11}$$

For the coherent state, $S_\varphi = 0$. For quadrature squeezing in the X_φ quadrature, $-0.5 < S_\varphi < 0$. Using Eq. (6), (8)-(11), we plot the squeezing parameter S_φ with $|\alpha|$ in the $X_{\varphi=0}$ quadrature for different values of m (Figure 4(a)) and ε (Figure 4(b)). Interestingly, Figure 4(a) shows that the signature of squeezing appears at lower values of $|\alpha|$ for higher m values, i.e., when more photons are added to The PAC state part of the HCS. The amount of squeezing is also higher for higher m values indicating more reduction of quantum noise. Figure 4(b) shows that for $|\alpha| > 2.35$, the HCS with 50:50 superposition of coherent state and PAC state has higher squeezing than that of the PAC state part. So, the superposition of PAC state with coherent state will be more useful than the PAC state in practical applications.

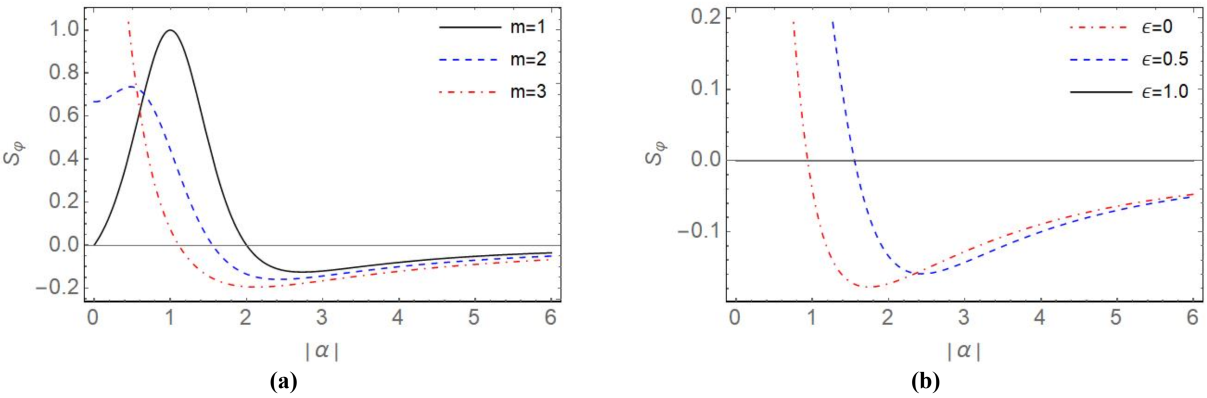


Figure 4. Variation of S_φ with $|\alpha|$: (a) for different m with $\varepsilon = 0.5$; (b) for different ε with $m = 2$. We take $\theta = \pi$, and $\omega = 0$

5. CONCLUSIONS

We have studied the photon number statistics, the Mandel Q factor, Wigner-Yanase skew information and the quadrature squeezing of the hybrid coherent state which is the superposition state of coherent state and photon-added-coherent state. Mandel Q factor indicates that for $|\alpha| > 0.2$, addition of more photons to the PAC state part of the HCS improved the depth of nonclassicality. Also, for $|\alpha| > 1.7$, the HCS manifested more nonclassicalities than what would have been achieved with PAC state alone. The Wigner-Yanase skew information reveals that the for $|\alpha| < 0.8$, the quantumness of the HCS increases with m values and the depth of nonclassicality of the HCS is more than that of the PAC state with the only exception for $|\alpha| < 0.55$ with $m = 2, \varepsilon = 0.75$. The quadrature squeezing is reported in the $X_{\varphi=0}$ quadrature for $|\alpha| > |\alpha|_{min}$. For $\varepsilon = 0.5$, the minimum values of $|\alpha|$ to appear squeezing are 1.1, 1.5, and 2 for $m = 3$, $m = 2$, and $m = 1$, respectively. The amount of squeezing is also higher for larger m. For $|\alpha| > 2.35$, the HCS with $m = 2$ and $\varepsilon = 0.5$ shows more squeezing than the PAC state in the $X_{\varphi=0}$ quadrature. All the nonclassicalities disappear for higher $|\alpha|$ values because for large $|\alpha|$ the PAC state reduces to a coherent state. Generally, the HCS with $m > 1$ quantifies more nonclassicalities than the PAC state and even the HCS with $m = 1$ that reveal the importance and necessity of this work.

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НЕКЛАСИЧНІСТЬ СУПЕРПОЗИЦІЙНОГО СТАНУ КОГЕРЕНТНОГО І ФОТОННО-ДОДАНОГО КОГЕРЕНТНОГО СТАНУ

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Аналітично досліджено некласичні властивості гібридного когерентного стану (HCS), який є станом суперпозиції когерентного стану та фотонно-доданого когерентного стану (PAC). Ми оцінили статистику кількості фотонів, інформацію про викривлення Вігнера-Янасе, Q-фактор Манделя та квадратурне стиснення HCS, щоб кількісно визначити його некласичність. Цей стан суперпозиції демонструє більше некласичних властивостей, ніж стан PAC і навіть стан суперпозиції когерентного стану та однофотонно-доданого когерентного стану (SPAC). Ми повідомляємо, що додавання більшої кількості фотонів до частини стану PAC HCS загалом кількісно визначає більше некласичностей. Некласичні властивості HCS також залежать від амплітуд когерентного стану та стану PAC у HCS.

Ключові слова: *когерентний стан; фотонно-доданий когерентний стан; гібридний когерентний стан; спотворена інформація Вігнера-Янасе; Q фактор Манделя; квадратурне видавлювання; некласичний ефект*