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EFFECT OF STRATIFICATION AND JOULE HEATING ON MHD DUSTY VISCOELASTIC FLUID FLOW THROUGH INCLINED CHANNELS IN POROUS MEDIUM IN PRESENCE OF MOLECULAR DIFFUSIVITY

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An analysis is carried out to study laminar MHD convection flow of a second order dusty viscoelastic fluid in porous medium through an inclined parallel plate channel in the presence of molecular diffusivity. The plates are maintained at two different temperatures that decay with time. The study is done under the consideration that viscosity and density of the fluid are variable to the extent that it causes stratification and joule heating effect in the process of the flow. The purpose of the study is to examine how stratification and joule heating affect the flow in relation to the physical quantities namely, Stratification factor, Hartmann number, Viscoelastic coefficient, Joule heating parameter, Prandtl number, Eckert number, Schmidt number and Porosity of the medium etc. The non-dimensional governing equations are solved analytically by using regular perturbation technique, and the graphs are plotted using MATLAB programming language. The mathematical expressions for fluid and particle velocity, fluid temperature, fluid concentration, skin friction for fluid and particle, flow flux for fluid and particle, Nusselt number, Sherwood number at the plates are evaluated and their nature of variations for different numerical values of physical parameters are shown graphically, discussed and conclusions are drawn.
Keywords: *Joule heating effect; Stratification effect; Inclined channel; Viscoelastic parameter; Mass diffusivity; Porous medium*
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INTRODUCTION

A non-Newtonian fluid embedded with symmetrically distributed uniform non-conducting solid spherical dust particles is commonly called 'dusty viscoelastic fluid'. Study of viscoelastic flow has been receiving a great attention among the researchers in recent years because of its numerous applications in various fields of science and technology. The presence of dust particles in a fluid flow, has influences on fluid motion, for instance, situations arises in the movement of dust-laden air, in case of fluidization, use of dust in gas cooling systems, in power technology, in petroleum industry on purification of crude oil, solid fuel rocket nozzles used in guided missile system, flow of polymer solutions in industry, construction of wet-bulb thermometer, and in many kinds of fluid flow relating to engineering and industrial fields. Such flow simultaneously exhibits both viscous and elastic properties, normal stresses and relaxation effects. Various mathematical models have been designed to simulate such hydrodynamics behaviour of the fluid. When such a conducting fluid flows in presence of a magnetic field, the flow is influenced by the magnetic field. There are interactions between the conducting fluid and the applied magnetic field that in turn modifies the flow pattern with attendant effects on flow properties. Thus, it is possible to control effectively the flow by adjusting the magnitude and direction of the applied magnetic field. Moreover, the nature of interaction strongly depends upon the orientation of the magnetic field; as a result, working viscosity the flow system is under influence of the induced magnetic field. Understating the dynamics of such flow has relevant applications in the advent of technology that involves in various MHD devices such as MHD power generator, thermonuclear power devices etc. Moreover, engineers and scientists are interested on studies of flow of binary mixtures of viscous fluid and dust particles because of their wide range of real-world uses, including industrial, geophysical, astronomical, and many more.

Several authors have been carried out research on dusty viscoelastic fluid under various physical situations and conditions. Chakraborty and Sengupta [1] have studied the MHD flow of two immiscible viscoelastic Rivlin-Ericksen fluids in a rectangular channel. Datti et al. [2] had studied MHD viscoelastic fluid flow over a non-isothermal stretching sheet. Khan et al. [3] examined MHD transient flows in a porous medium with rectangular cross-section. Inverse solutions of a second-grade MHD fluid flow in porous medium was explored by Islam and Zhou [4]. Sivaraj and Kumar [5] examined MHD heat and mass transfer in viscoelastic fluid flow over a vertical cone and flat plate. Akbar et al., [6] have investigated the MHD flow in a porous medium with prescribed vorticity. Sandeep and Sulochana [7] have examined the dual solutions of MHD micropolar fluid flow over a stretching/shrinking sheet. Verma and Singh [8] have studied MHD flow in a circular channel with porous medium. Reddy et al., [9] have studied the thermal radiation on MHD boundary layer flow over a stretching sheet with transpiration. Kiema and Wambua [10] have studied the steady and unsteady viscous incompressible MHD fluid flow. Ramadevi et. al., [11] have examined MHD Carreau fluid flow over a thickness melting surface with Cattaneo-Christov heat flux. Haq et. al., [12] have studied the MHD flow of Maxwell fluid in a channel with porous medium. Raghunath and Ravuri [13] have investigated the Hall effects, Soret effects and rotational

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effect on unsteady MHD fluid flow through a porous medium. Hall effects on the MHD flow of the Rivlin-Ericksen fluid in porous plate was explored by Krishna and Vajravelu [14]. Heat transfer of MHD dusty fluid flow over an inclined irregular porous channel has been examined by Kalpana and S. Saleem [15]. Recently, Kodi et al. [16] have examined MHD mixed convection flow of Maxwell nanofluid through a vertical cone and Raghunath et al. [17] have explored radiation absorption on MHD fluid flow through porous medium over a vertical plate with heat source.

Joule heating in magnetodynamics system is a consequence of the electrical resistance of the conducting medium. The energy dissipated as heat due to the resistance of the fluid to the electric current is known as Joule heating. The Joule heating effect contribute to the overall energy balance in MHD systems. It is crucial to consider this heat generation when analysing the efficiency and performance of MHD devices. Researchers have considerable interest in this field and give noteworthy conclusion by using this phenomenon. Zhang et al. [18] studied the Joule heating effects on natural convection participating MHD under different levels of thermal radiation. Mousavi et al. [19] examined 2D-3D analysis of Joule and viscous heating effect on MHD. Jamalabadi and Park [20] have investigated on thermal radiation, joule heating, and viscous dissipation effects on MHD with uniform surface temperature. Bhatti and Rashidi [21] have studied heat and mass transfer with Joule heating on MHD peristaltic blood flow under influence of Hall effect. Hayat et al. [22] studied the Joule heating effects on MHD flow of Burgers' fluid. Recently, Kheder et al. [23] investigated the effect of Joule heating and MHD on periodical analysis of current density and amplitude of heat transfer.

In many of the above studies the physical properties of the ambient fluid were supposed to be non-variable. However, practically, the physical properties of the ambient fluid e.g. density, viscosity varies with temperature that in turn causes fluid stratification and thereby influences the fluid flow and heat transfer. The impact of Stratification effect on MHD have been discussed by several researchers, for example, Daniel et al. [24], Mutuku and Makinde [25], Khashi'ie et al. [26], Waqas et al. [27], Khan et al. [28]. Motivating with the works mentioned above and the idea of stratification of fluid due to the variation of fluid density and viscosity we have tried to investigate the flow and heat transfer on magnetohydrodynamics viscoelastic fluid flow through an inclined parallel plate channel in presence of uniform magnetic field. Such kind of problem often arises in many situations e.g. in the field of power generation using MHD fluid (MHD generator), MHD controlled forced and heat exchange flow such as in gas turbine etc. Besides these, the influence of dust particles is also utilized many industrial purposes such as in the production of rayon and nylon, pulp and paper industry, powder technology, production of plastic, textile industry, treating environment pollution, petroleum industry, purification of rain water, chemical processing, nuclear processing in the production of plastic sheet and electronics bundles etc. In such flows, stratification effect developed due to the variation of fluid density and viscosity.

The present study tries to examine an unsteady flow of dusty viscoelastic MHD fluid in a porous medium down an inclined parallel plate channel in presence of uniform magnetic field and molecular diffusivity. The issue is investigated in terms of the stratification and joule heating effect caused by varying fluid density and viscosity in a porous medium. Expressions for fluid and dust particle velocity, temperature distribution, concentration distribution, skin friction for fluid and particle, flux flow for fluid and particle, Nusselt number and Sherwood number at the plates are obtained under the assumption that the plates are positioned at different temperatures that decay exponentially over time. The effects of aforementioned expressions are analysed graphically for various values of the non-dimensional physical quantities Stratification factor, Hartmann number, Viscoelastic parameters, Joule heating parameter, Prandtl number, Eckert number, Schmidt number and Porosity of the medium.

FORMULATION OF THE PROBLEM

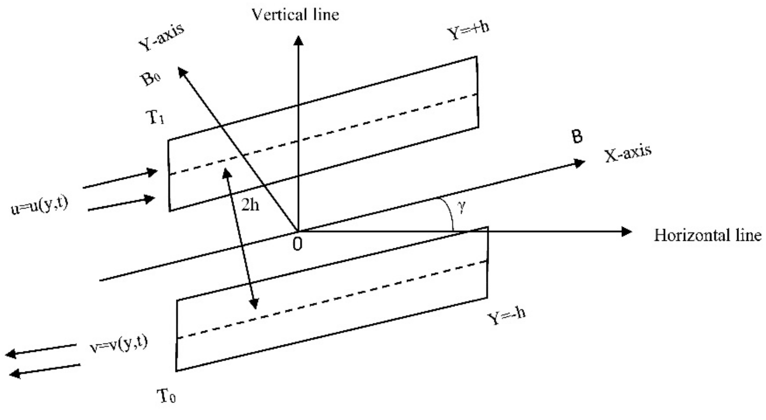

Figure 1. Diagram of the Physical problem

We take into account the laminar convection flow of a viscous, incompressible, electrically conducting fluid between two parallel plate channel that are spaced apart by 2h and inclined horizontally by an angle γ. The plates are kept at two different temperatures that exponentially decay with time. Let x-axis be the central line of the channel along the motion of the fluid and y-axis be perpendicular to it. The fluid velocity (u) and dust particle velocity (v) are equal and opposite in direction. A uniform magnetic field B_0 is applied normally to the parallel plates.

ASSUMPTIONS

The following assumptions are taken to write down the governing equations:

i. Because of the infinite length of the plates, the fluid velocity (u) and dust particle velocity (v) are functions of y and t only.
ii. During fluid motion, the number density of dust particles remains constant and is negligibly small.
iii. The dust particles are neither subject to chemical reactions nor heat radiation.
iv. The effects of the Hall, Polarization, Buoyancy are insignificant.
v. Magnetic Reynolds number is so small that the effect of induced magnetic field is negligible.
vi. Dust particles are identical, solid, elastic sphere, symmetric in size, also they are equally distributed within the fluid motion and electrically non-conducting.
vii. Initially(t=0) there is no flow and the parallel plates are at same temperature, and thereafter, (t>0) the plates are at two different temperatures i.e. t>0, $T=T_0$ at y=-h and t>0, $T=T_1$ at y=+h.
viii. Throughout the channel, the fluid's density and viscosity are varying along the y-axis as follows:

$$\rho = \rho_0 e^{-n(\frac{y}{h+1})}, \text{ and } \mu = \mu_0 e^{-n(\frac{y}{h+1})},$$

where n is fluid's stratification factor and ρ_0, μ_0 are coefficient of density and viscosity on the line of the channel at y=-h respectively so that the velocity and magnetic field distribution are V= [u(y,t), 0, 0] and B= [0, B_0, 0] respectively.

MATHEMATICAL ANALYSIS

The governing equations under the above assumptions are given as:
Momentum equations

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{n\nu_1}{h}\frac{\partial u}{\partial y} + \nu_1\frac{\partial^2 u}{\partial y^2} + \nu_2\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\nu_1}{k_1}u + g\sin\gamma + \frac{k_0 N}{\rho}(v-u) - \frac{\sigma}{\rho}(B_0^2 u). \tag{1}$$

$$m\frac{\partial v}{\partial t} - k_0(u-v) = 0. \tag{2}$$

Equation of fluid static

$$\frac{\partial p}{\partial y} + \rho g\cos\gamma = 0. \tag{3}$$

Energy equation

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu_1}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\nu_2}{C_p}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{\rho C_p}u^2. \tag{4}$$

Mass equation

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}, \tag{5}$$

where ν_1 is the kinematics coefficient of fluid viscosity, ν_2 is the kinematics coefficient of viscoelasticity, k_1 is the porosity of the medium, g is gravitational acceleration, k_0 is the proportionality constant, N is the number density of the dust particles, σ and κ are the electrical and thermal conductivity of the fluid, m is the mass of the dust particle, T and C are dimensional temperature and concentration respectively, C_p is specific heat at constant pressure, D is the mass diffusion coefficient and p is the fluid pressure defined as:

$$p = \rho g(x\sin\gamma - y\cos\gamma) + \rho x a(t) + A, \tag{6}$$

where a(t) is a function of t alone and A is constant value.
using (6), equation (1) can be written as

$$\frac{\partial u}{\partial t} = -a(t) - \frac{n\nu_1}{h}\frac{\partial u}{\partial y} + \nu_1\frac{\partial^2 u}{\partial y^2} + \nu_2\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\nu_1}{k_1}u + \frac{k_0 N}{\rho}(v-u) - \frac{\sigma}{\rho}(B_0^2 u). \tag{7}$$

The boundary conditions of the problem are

$$\left.\begin{array}{llllll} u = 0, & v = 0, & T = T_0 e^{-2nt}, & C = C_0 e^{-2nt} & at & y = -h \\ u = u_0 e^{-nt}, & v = v_0 e^{-nt}, & T = T_1 e^{-2nt}, & C = C_1 e^{-2nt} & at & y = +h \end{array}\right\} \tag{8}$$

where T_0 and C_0 are the fluid's temperature and concentration at y=-h respectively and T_1 and C_1 are the temperature and concentration of fluid at y=+h respectively.
Now we consider the following non-dimensional parameters

$$\left.\begin{array}{lllll} u^* = \frac{u}{u_0}, & v^* = \frac{v}{v_0}, & x^* = \frac{x}{h}, & y^* = \frac{y}{h}, & t^* = \frac{t u_0}{h} \\ a^* = \frac{ah}{u_0^2}, & T^* = \frac{T}{T_0}, & C^* = \frac{C}{C_0}, & K^* = \frac{h}{\sqrt{k_1}}, & \lambda = \frac{u_0}{v_0} \end{array}\right\} \tag{9}$$

Substituting (9) in the equations (7), (2), (4) and (5) and then removing the asterisks, we get

$$\frac{\partial u}{\partial t}=-a(t)-\frac{n}{R}\frac{\partial u}{\partial y}+\frac{1}{R}\frac{\partial^2 u}{\partial y^2}-\eta\left(\frac{\partial}{\partial t}\frac{\partial^2 u}{\partial^2 y}\right)-\frac{K^2}{R}u+\frac{C}{R_t}\left(\frac{v}{\lambda}-u\right)-\frac{M^2}{R}u, \tag{10}$$

$$R_t\frac{\partial v}{\partial t}-(\lambda u-v)=0, \tag{11}$$

$$\frac{\partial^2 T}{\partial y^2}=RP_r\frac{\partial T}{\partial t}-EP_r\left(\frac{\partial u}{\partial y}\right)^2+\eta ERP_r\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2-Ju^2\ , \tag{12}$$

$$\frac{\partial^2 C}{\partial y^2}=RS_c\frac{\partial C}{\partial t} \tag{13}$$

where,

$R=\frac{u_0 h}{v_1}$ is Reynolds number, $\eta=-\frac{v_2}{h^2}$ is viscoelastic parameter,
$C=\frac{mN}{\rho}$ is dust particle concentration, $P_r=\frac{\mu C_p}{k}$ is Prandtl number,
$M=\sqrt{\frac{B_0^2 h^2\sigma}{\rho v_1}}$ is Hartmann number, $E=\frac{u_0^2}{C_p T_0}$ is Eckert number,
$R_t=\frac{mu_o}{k_0 h}$ is relaxation time parameter of dust particles,
$S_c=\frac{v_1}{D}$ is Schmidt number, $J=\frac{\sigma B_0^2 u_0^2 h^2}{\kappa T_0}$ is Joule heating parameter.

The non-dimensional boundary conditions are

$$\left.\begin{array}{llllll} u=0, & v=0, & T=e^{-2nt}, & C=e^{-2nt} & at & y=-h \\ u=e^{-nt}, & v=e^{-nt}, & T=\chi e^{-2nt}, & C=\psi e^{-2nt} & at & y=+h \end{array}\right\} \tag{14}$$

where $\chi=\frac{T_1}{T_0}$ and $\psi=\frac{C_1}{C_0}$ are constant temperature and concentration respectively.

SOLUTIONS

To solve the equations (10) -(13) under the boundary conditions (14), we assume

$$u=f(y)e^{-nt},\quad v=g(y)e^{-nt},\quad T=\theta(y)e^{-2nt},\quad C=\phi(y)e^{-2nt},\quad a=a_0e^{-nt}\,. \tag{15}$$

Substituting (15) in equations (10) -(13), we get

$$f''(y)-A_1f'(y)+A_2f(y)-A_3=0 \tag{16}$$

$$g(y)=A_4f(y) \tag{17}$$

$$\theta''(y)+A_5\theta(y)+A_6\big(f'(y)\big)^2+A_7(f(y))^2=0 \tag{18}$$

$$\phi''(y)+A_8\phi=0 \tag{19}$$

where,

$$A_1=\frac{n}{1+n\eta R}\,,\quad A_2=\frac{R}{1+n\eta R}\left[n-\left(\frac{K^2+M^2}{R}\right)-C\frac{n}{1-nR_t}\right],\quad A_3=\frac{Ra_0}{1+n\eta R},$$
$$A_4=\frac{\lambda}{1-nR_t}\,,\quad A_5=2nRP_r\,,\quad A_6=EP_r(1+2n\eta R),\quad A_7=J=EM^2P_r\,,\quad A_8=2nRS_c$$

Now, the corresponding boundary conditions are

$$\left.\begin{array}{llll} f(-1)=0\,, & g(-1)=0\,, & \theta(-1)=1\,, & \phi(-1)=1 \\ f(+1)=1\,, & g(+1)=\frac{\lambda}{A_4}\,, & \theta(+1)=\chi, & \phi(+1)=\psi \end{array}\right\} \tag{20}$$

The solutions of the equations (16) -(19) with respect to the boundary conditions (20) are

$$f(y)=\left[C_1e^{m_1y}+C_2e^{m_2y}+\frac{A_3}{A_2}\right]. \tag{21}$$

$$g(y)=A_4\left[C_1e^{m_1y}+C_2e^{m_2y}+\frac{A_3}{A_2}\right], \tag{22}$$

$$\theta(y)=C_3\cos\sqrt{A_5}\,y+C_4\sin\sqrt{A_5}\,y-[(S_1+S_4)e^{2m_1y}+(S_2+S_5)e^{2m_2y}$$
$$+(S_3+S_6)e^{(m_1+m_2)y}+S_7e^{m_1y}+S_8e^{m_2y}+S_9] \tag{23}$$

$$\phi(y)=C_5\cos\sqrt{A_8}\,y+C_6\sin\sqrt{A_8}\,y \tag{24}$$

where,

$$m_1 = A_1 + \sqrt{\frac{(A_1^2 - 4A_2)}{2}}, \quad m_2 = A_1 - \sqrt{\frac{(A_1^2 - 4A_1)}{2}}, \quad S_1 = \frac{A_6 C_1^2 m_1^2}{4m_1^2 + A_5}, \quad S_2 = \frac{A_6 C_1^2 m_2^2}{4m_2^2 + A_5},$$

$$C_1 = \left[\frac{A_3}{A_2}\left(\frac{\sinh m_2}{\sinh(m_1 - m_2)}\right) + \frac{e^{-m_2}}{2\sinh(m_1 - m_2)}\right], \quad C_2 = -\left[\frac{A_3}{A_2}\left(\frac{\sinh m_1}{\sinh(m_1 - m_2)}\right) + \frac{e^{-m_1}}{2\sinh(m_1 - m_2)}\right],$$

$$S_3 = \frac{2A_6 C_1 C_2 m_1 m_2}{(m_1 + m_2)^2 + A_5}, \quad S_4 = \frac{A_7 C_1^2}{4m_1^2 + A_5}, \quad S_5 = \frac{A_7 C_2^2}{4m_2^2 + A_5}, \quad S_6 = \frac{2A_7 C_1 C_2}{(m_1 + m_2)^2 + A_5},$$

$$S_7 = \frac{2A_3 A_7 C_1}{A_2(m_1^2 + A_5)}, \quad S_8 = \frac{2A_3 A_7 C_2}{A_2(m_2^2 + A_5)}, \quad S_9 = A_7\left(\frac{A_3}{A_2}\right)^2, \quad C_5 = \frac{1 + \psi}{2\cos\sqrt{A_8}}, \quad C_6 = \frac{\psi - 1}{2\sin\sqrt{A_8}},$$

$$C_3 = \left(\frac{1}{\cos\sqrt{A_5}}\right)\left[\frac{(1+\chi)}{2} + (S_1 + S_4)\cosh 2m_1 + (S_2 + S_5)\cosh 2m_2 + (S_3 + S_6)\cosh(m_1 + m_2)\right.$$
$$\left. + S_7\cosh m_1 + S_8\cosh m_2 + S_9\right], \quad C_4 = \left(\frac{1}{\sin\sqrt{A_5}}\right)\left[\frac{(\chi - 1)}{2} + (S_1 + S_4)\sinh 2m_1 + (S_2 + S_5)\sinh 2m_2\right.$$
$$\left. + (S_3 + S_6)\sinh(m_1 + m_2) + S_7\sinh m_1 + S_8\sinh m_2\right]$$

Putting the solutions of f(y), g(y), θ(y) and ϕ(y) from equations (21) -(24) in equation (15), we get the final solutions

$$u(y,t) = \left[C_1 e^{m_1 y} + C_2 e^{m_2 y} + \frac{A_3}{A_2}\right]e^{-nt} \tag{25}$$

$$v(y,t) = A_4\left[C_1 e^{m_1 y} + C_2 e^{m_2 y} + \frac{A_3}{A_2}\right]e^{-nt} \tag{26}$$

$$T(y,t) = \{C_3\cos\sqrt{A_5}\,y + C_4\sin\sqrt{A_5}\,y - [(S_1 + S_4)e^{2m_1 y} + (S_2 + S_5)e^{2m_2 y} + (S_3 + S_6)e^{(m_1+m_2)y} + S_7 e^{m_1 y} + S_8 e^{m_2 y} + S_9]\}e^{-2nt} \tag{27}$$

$$C(y,t) = \left[C_5\cos\sqrt{A_8}\,y + C_6\sin\sqrt{A_8}\,y\right]e^{-2nt} \tag{28}$$

Skin-Friction

The viscous drag (Skin friction) for fluid (τ_f) and for dust particles (τ_p) acting at the plates are defined as:

$$\tau_f = \left[\left(\frac{1}{R} - \eta\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial y}\right)\right]_{y=\pm1} = \left[\left(\frac{1}{R} + n\eta\right)(C_1 m_1 e^{\pm m_1} + C_2 m_2 e^{\pm m_2})\right]e^{-nt}, \tag{29}$$

$$\tau_p = \left[\left(\frac{1}{R} - \eta\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial y}\right)\right]_{y=\pm1} = A_4\left[\left(\left(\frac{1}{R} + n\eta\right)(C_1 m_1 e^{\pm m_1} + C_2 m_2 e^{\pm m_2})\right]e^{-nt}. \tag{30}$$

Fluid and Particle Flux

The flux of flow for fluid (Q_f) and for particle (Q_p) throughout the channel is defined as:

$$Q_f = \int_{-1}^{1} u\,dy = 2\left[\frac{C_1}{m_1}\sinh m_1 + \frac{C_2}{m_2}\sinh m_2 + \frac{A_3}{A_2}\right]e^{-nt}$$

$$Q_p = \int_{-1}^{1} v\,dy = 2A_4\left[\frac{C_1}{m_1}\sinh m_1 + \frac{C_2}{m_2}\sinh m_2 + \frac{A_3}{A_2}\right]e^{-nt}$$

Nusselt Number

The rate of heat transfer in terms of Nusselt number (N_u) at the plates is defined as:

$$N_u = \left[\frac{\partial T}{\partial y}\right]_{y=\pm1} = \{\sqrt{A_5}(-C_3\sin\sqrt{A_5} + C_4\cos\sqrt{A_5}) - [2m_1(S_1 + S_4)e^{\pm 2m_1}$$
$$+ 2m_2(S_2 + S_5)e^{\pm 2m_2} + (m_1 + m_2)(S_3 + S_6)e^{\pm(m_1+m_2)} + m_1 S_7 e^{\pm m_1} + m_2 S_8 e^{\pm m_2}]\}e^{-2nt} \tag{31}$$

Sherwood Number

The rate of mass transfer in terms of Sherwood number (Sh) at the plates is given as:

$$Sh = \left[\frac{\partial C}{\partial y}\right]_{y=\pm1} = \left[\sqrt{A_8}(-C_5\sin\sqrt{A_8} + C_6\cos\sqrt{A_6})\right]e^{-2nt} \tag{32}$$

RESULT AND DISCUSSION

The aim of the study is to investigate the effect of stratification factor (n), Joule heating (J) and other physical parameters such as, Hartmann number (M), porosity of the medium (K), viscoelastic parameter (η), Prandtl number (Pr), Eckert number (Ec), Schmidt number (Sc) on fluid and particle velocity, fluid temperature, fluid concentration, Skin friction for fluid and particle, rate of heat transfer in terms of Nusselt number, rate of mass transfer in terms of Sherwood number. The numerical results for Skin friction, Nusselt number, Sherwood number have been carried out at the upper wall (y=+1) of the channel. This helped us comprehend the problem's physical importance better. The governing equations are solved by using regular perturbation method, and the graphs are plotted in MATLAB which has been described in Figures (2) -(22).

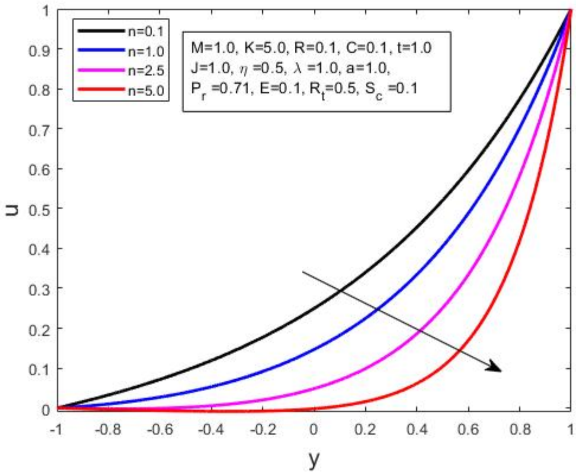

Figure 2. Effect of n on Fluid Velocity Profile

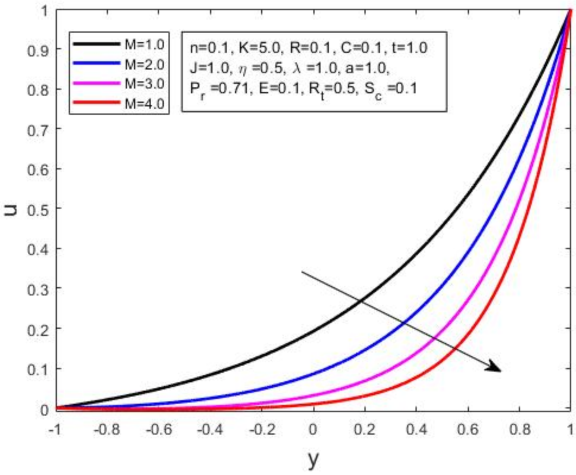

Figure 3. Effect of M on Fluid Velocity Profile

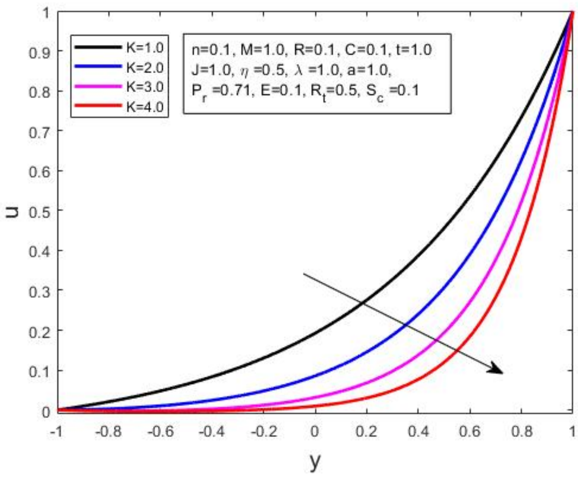

Figure 4. Effect of K on Fluid Velocity Profile

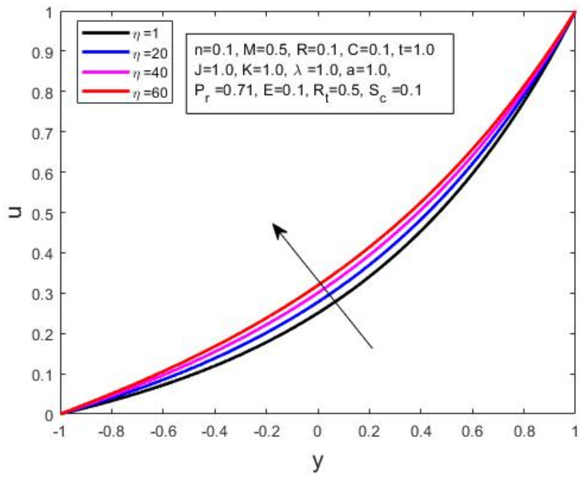

Figure 5. Effect of η on Fluid Velocity Profile

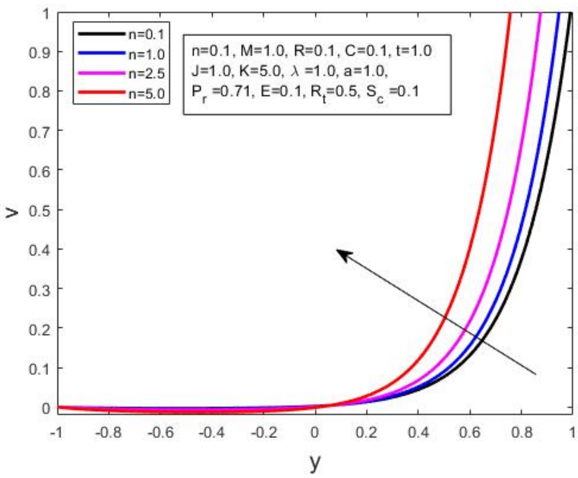

Figure 6. Effect of n on Particle Velocity Profile

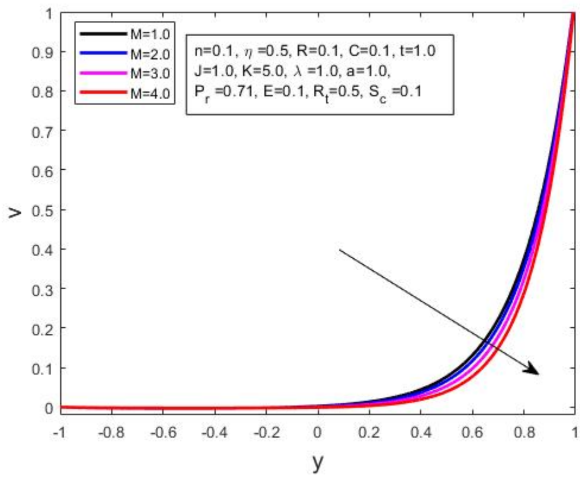

Figure 7. Effect of M on Particle Velocity Profile

Figures 2-5 show the impact of Stratification factor (n), Hartmann number (M), Porosity parameter (K) and Visco-elastic parameter (η) on fluid velocity. Figure 2 shows that the fluid velocity profile reduces with the increase in values of stratification factor. This arises due to the fact that increase in stratification factor increases the difference of density of momentum boundary layer, which illustrates a decrease in the fluid velocity profile. From Figure 3, it is clear that the fluid velocity reduces with the increase in values of Hartmann number. This is because the magnetic field creates a Lorentz force, which always produces resistance to the fluid flow and slows down the fluid velocity.

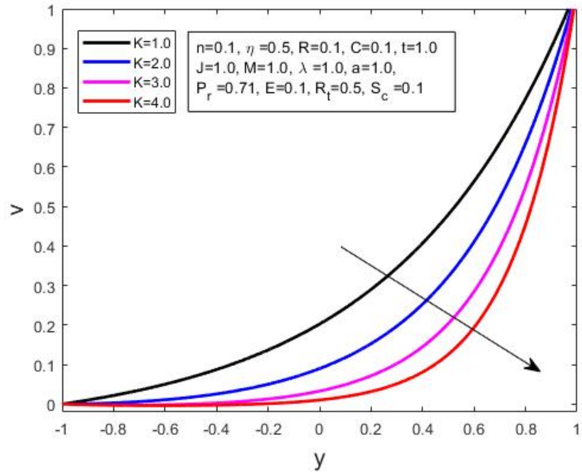

Figure 8. Effect of K on Particle Velocity Profile

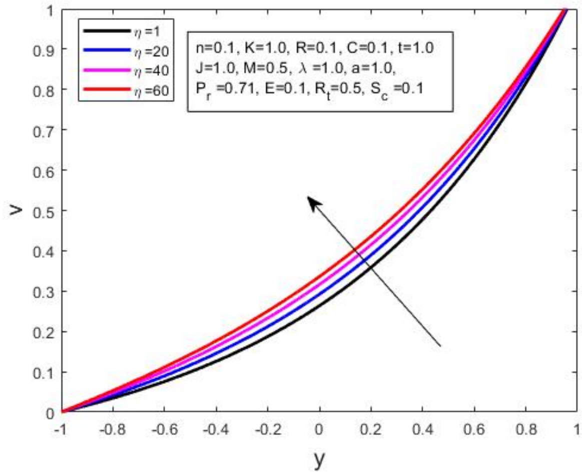

Figure 9. Effect of η on Particle Velocity Profile

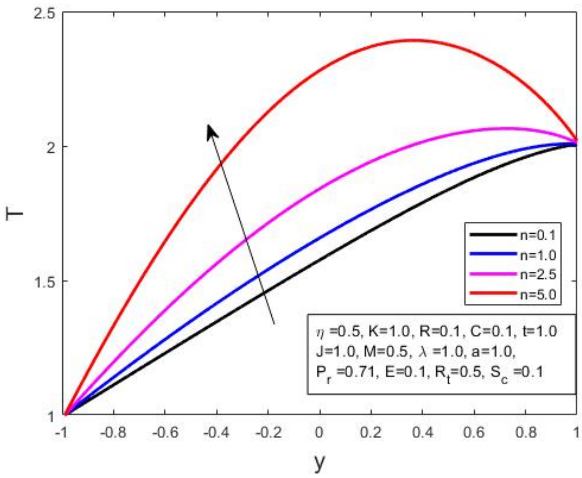

Figure 10. Effect of n on Fluid Temperature Profile

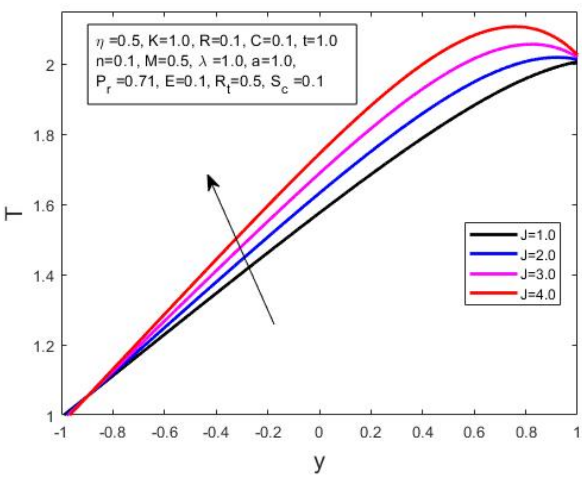

Figure 11. Effect of J on Fluid Temperature Profile

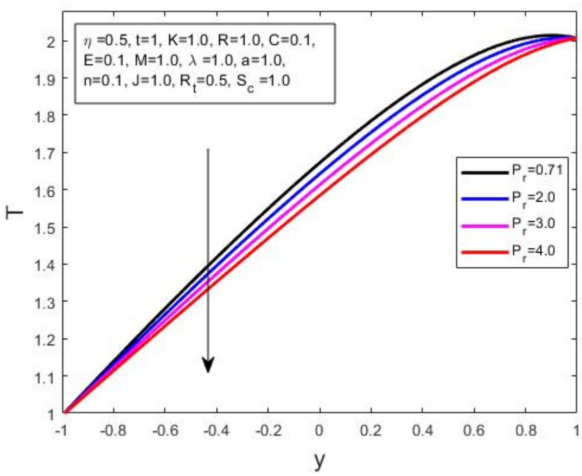

Figure 12. Effect of Pr on Fluid Temperature Profile

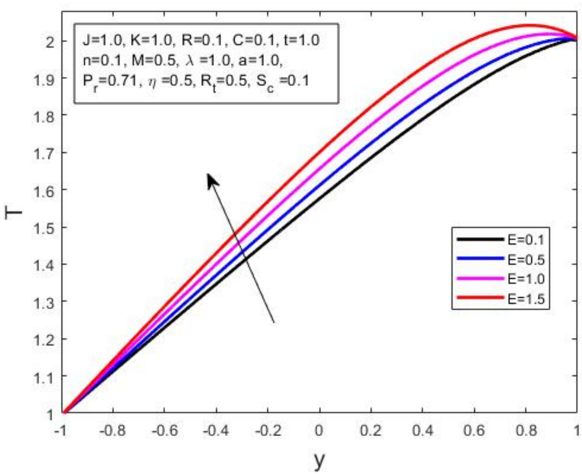

Figure 13. Effect of E on Fluid Temperature Profile

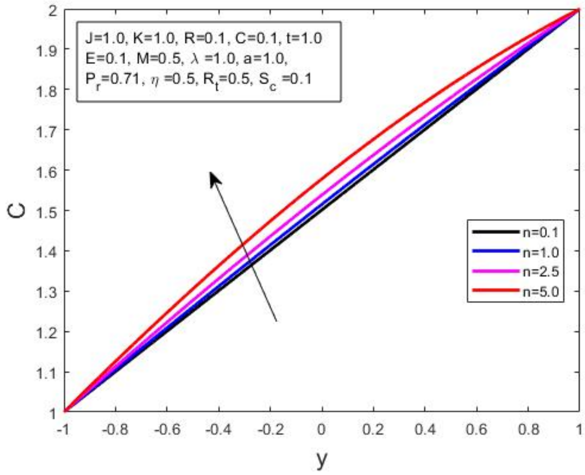

Figure 14. Effect of n on Fluid Concentration Profile

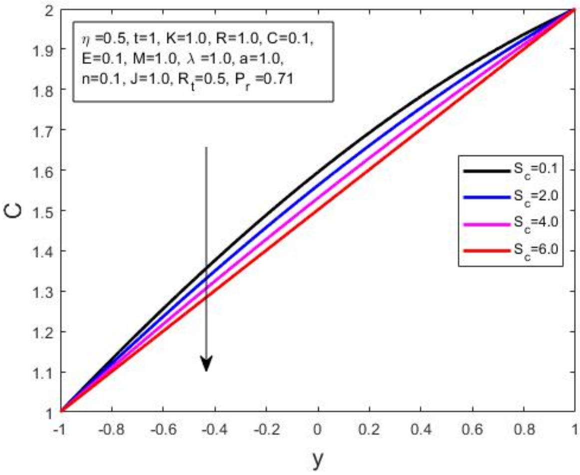

Figure 15. Effect of Sc on Fluid Concentration Profile

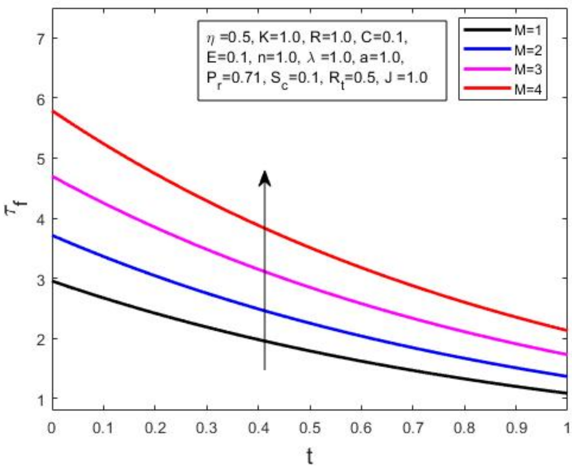

Figure 16. Effect of M on Fluid Skin friction

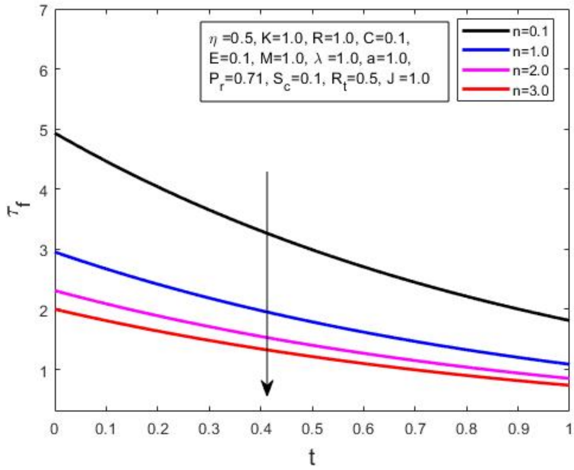

Figure 17. Effect of n on Fluid Skin friction

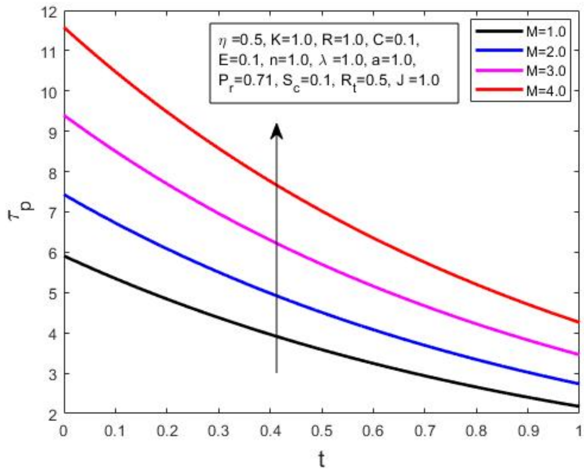

Figure 18. Effect of M on Particle Skin friction

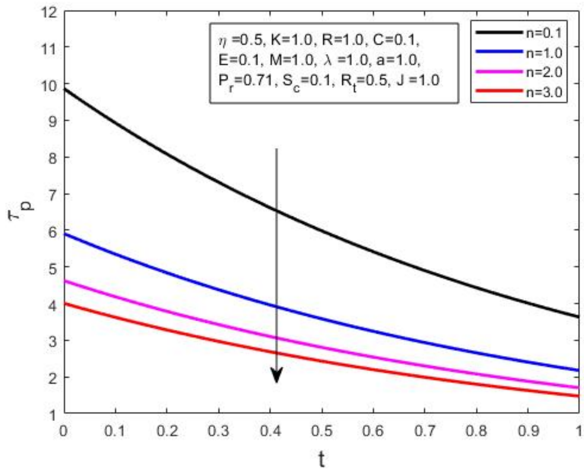

Figure 19. Effect of n on Particle Skin friction

Figure 4 depict that the fluid velocity profile decreases with the rise in values of porosity parameter. This occurs because the fluid flow is more restricted in the presence of porous medium, which slows the flow and lowers its velocity. Figure 5 elucidates that the fluid velocity profile increases with escalating values of viscoelastic parameter (η). Physically, η is negative to kinematic viscosity. Due to this low viscosity, enhancing values of η produce high fluid velocity.

Figures 6-9 represent the variation of particle velocity profile for different values of Stratification factor, Hartmann number, Porosity parameter and Viscoelastic parameter respectively. Figure 6 displays that the stratification factor increases the particle velocity profile, which shows an opposite behaviour for fluid velocity as mentioned above. However, figure 7, 8, 9 show almost similar behaviour as those for fluid velocity.

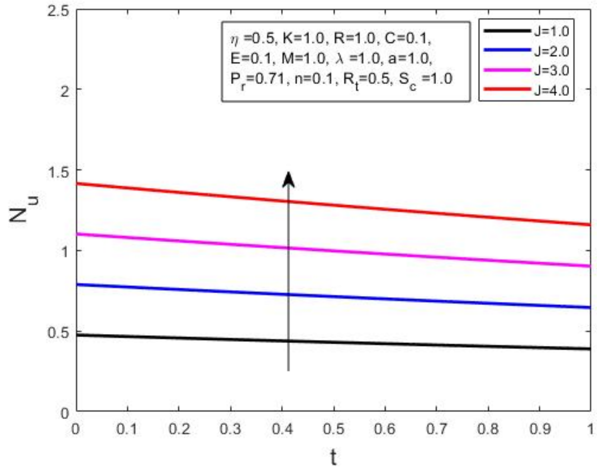

Figure 20. Effect of J on Nusselt number

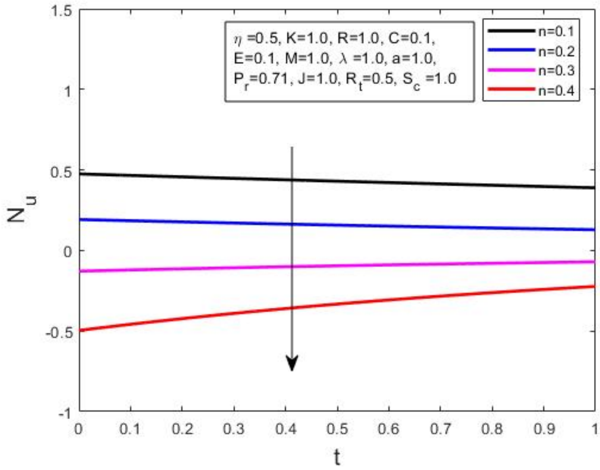

Figure 21. Effect of n on Nusselt number

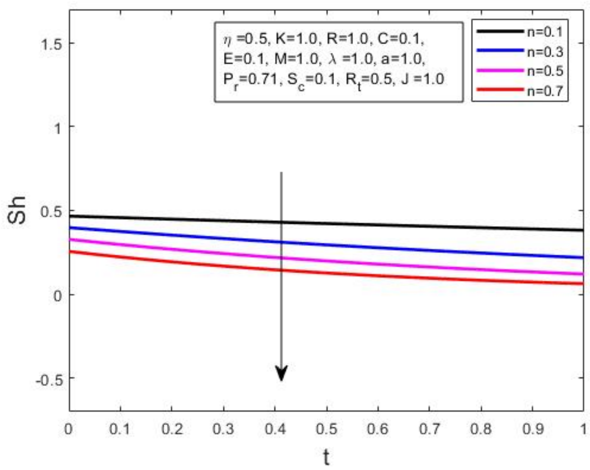

Figure 22. Effect of n on Sherwood number

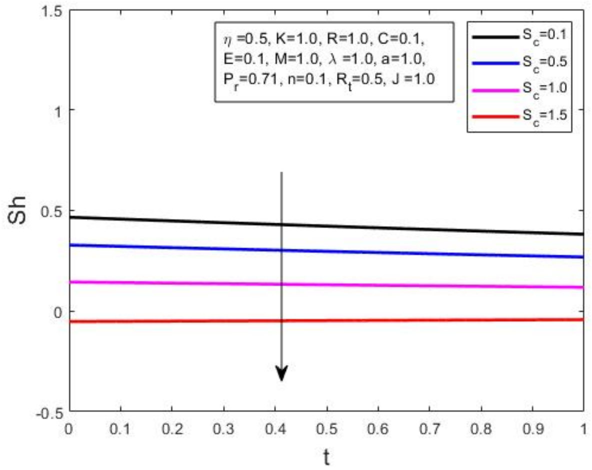

Figure 23. Effect of Sc on Sherwood number

Figures 10-13 illustrate the variation of fluid temperature for different values of Stratification factor, Joule heating parameter, Prandtl number and Eckert number respectively. Figure 10 and 11 admit that fluid temperature enhances due to the rise in stratification factor and Joule heating parameter respectively. Physically, with the increase in Joule heating, the heat generated is transferred to the surrounding of the channel, leading to an increase in temperature. Figure 12 shows an enhancement in Prandtl number causes a decrement in the temperature profile. It is due to the reason that the thermal conductivity declines with the enhancement in Prandtl number. On the other hand, figure 13 depicts that temperature increases with the escalating values of Eckert number. Physically, the thermal conductivity of the fluid rises as the dissipation is escalated which helps to enhance the thermal boundary layer thickness.

Figures 14 and 15 present the distribution of fluid concentration for different values of Stratification factor and Schmidt number respectively. Figure 14 reveals that the concentration exhibits an increasing trend with the escalating values of stratification factor. From figure 15, it is observed that the concentration profile de-escalates with the increase in Schmidt number. Physically, Schmidt number is the ratio of momentum diffusivity to mass diffusivity. So, with increasing Schmidt number, there is a strong reduction in concentration profile.

Figures 16 and 17 show how skin friction for fluid changes at the upper plate (y=+1) when a Hartmann number and Stratification factor change. It is observed that the skin friction exceeds with an increment as the Hartmann number increases. Further, skin friction falls as the stratification factor increases. Figure 18 and 19 registers the behaviour of skin friction for particle at the upper plate (y=+1) when Hartmann number and Stratification factor rises. A similar behaviour has been seen in figure 18 and 19 as those for figure 16 and 17 respectively.

Figures 20 and 21 show the variation of rate of heat transfer (N_u) at the upper plate (y=+1) with the rise in values of Joule heating parameter and Stratification factor respectively. It is observed that the Nusselt number increases with increasing Joule heating parameter and it decreases when stratification factor rises. The behaviour of Sherwood number at the upper plate (y=+1) has shown in figures 22 and 23. It is concluded that Sherwood number reduces for both increasing values of Stratification factor and Schmidt number.

CONCLUSIONS

The following conclusions can be made from the above results:

i. The velocity of fluid decreases with stratification factor and reverse in nature with particle velocity, fluid temperature and fluid concentration.
ii. Fluid and particle velocity enhance with raising value of viscoelastic parameter while opposite behaviour with Hartmann number and Porosity parameter.
iii. Fluid temperature increases with the rise in Joule heating parameter and Eckert number. However, the temperature tends to fall when the Prandtl number is raised.
iv. Increasing the Schmidt number decreases fluid concentration.
v. Fluid and particle viscous drag enhance with the rise of Hartmann number and opposite behaviour with stratification factor.
vi. The rate of heat transfer, in terms of Nusselt number, increases with the rise in Joule heating parameter but an opposite effect is observed with stratification factor.
vii. The rate of mass transfer, in terms of Sherwood number, reduces for higher values of stratification factor and Schmidt number.

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ВПЛИВ СТРАТИФІКАЦІЇ ТА ДЖОУЛЕВОГО НАГРІВУ НА МГД ПОТІК ПИЛОВОЇ В'ЯЗКОПРУЖНОЇ РІДИНИ КРІЗЬ ПОХИЛІ КАНАЛИ У ПОРИСТОМУ СЕРЕДОВИЩІ ЗА НАЯВНОСТІ МОЛЕКУЛЯРНОЇ ДИФУЗІЇ

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Проведено аналіз ламінарного МГД-конвекційного потоку запиленої в'язкопружної рідини другого порядку в пористому середовищі через похилий паралельний пластинчастий канал за наявності молекулярної дифузії. Пластини витримують при двох різних температурах, які з часом знижуються. Дослідження проводиться з урахуванням того, що в'язкість і щільність рідини є змінними в тій мірі, в якій це викликає розшарування і джоулевий ефект нагрівання в процесі потоку. Метою дослідження є вивчення того, як стратифікація та джоулеве нагрівання впливають на потік у зв'язку з фізичними величинами, а саме фактором стратифікації, числом Гартмана, коефіцієнтом в'язкопружності, параметром нагріву Джоуля, числом Прандтля, числом Еккерта, числом Шмідта та пористістю середовища. і т. д. Безвимірні керівні рівняння розв'язуються аналітично за допомогою методу регулярних збурень, а графіки будуються за допомогою мови програмування MATLAB. Оцінюються математичні вирази для швидкості рідини та частинок, температури рідини, концентрації рідини, поверхневого тертя для рідини та частинок, потоку рідини та частинок, числа Нуссельта, числа Шервуда на пластинах та їх характер варіацій для різних числових значень фізичних параметри показано графічно, обговорено та зроблено висновки.

Ключові слова: *Джоулевий ефект нагріву; ефект стратифікації; похилий канал; в'язкопружний параметр; масопровідність; пористе середовище*