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REINTERPRETATION OF FRIEDMANN-ROBERTSON-WALKER UNIVERSE WITH VARIABLE GRAVITATIONAL AND COSMOLOGICAL TERM IN BOUNCING COSMOLOGY

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This paper is devoted to investigate five dimensional homogeneous and isotropic FRW model with varying gravitational and cosmological constant with cosmic time. Exact solution of the Einstein field equations are obtained by using the equation of state $p = (\gamma - 1)\rho$ (gamma law), where γ which is an adiabatic parameter varies continuously as the universe expands. We obtained the solutions for different values of curvature $K = 0, 1, -1$ by using $a(t) = R_0(1 + \alpha^2 t^2)^n$, where α, n and R_0 are positive constants. Behaviour of the cosmological parameters are presented for different cases of the models. Physical interpretation of the derived model are presented in details. Interestingly the proposed model justified the current cosmological observations with dark energy.

Keywords: *Five dimension; FRW metric; Cosmological term; Bouncing scale factor*

PACS: 98.80.-K; 98.80Jk.

1. INTRODUCTION

The present universe is expanding with ever accelerating from the recent observational [1, 2, 3, 4] data on cosmic microwave background radiation(CMBR) and from WMAP data [5, 6]. Recent observations of type Ia supernovae (SNe Ia) at redshift $z < 1$ provide startling and puzzling evidence that the expansion of the universe at the present time appears to be accelerating, behavior attributed to "dark energy" with negative pressure[1, 2, 3, 7, 8, 9, 10, 11]. These observations strongly favour a significant and positive value of Λ. Some of the authors like [12, 13, 14] have studied the physics of the universe in higher dimensional space time.

Now-a-days peoples are more interested to study in higher dimensional space-time, since Einstein's field equations in higher dimensions may have physical relevance as early as a time before the universe underwent compactification transitions. The solutions of Einstein field equations may have physical relevance in these higher dimensional space times. The phase transitions in the early universe can also lead to topological knots in the vacuum expectation value of a scalar field, which are concentrated in a small region, and by using a suitable scalar field we can prove that phase transitions can result in such objects. Therefore, to unify gravity and other interactions, it is more feasible to study higher dimensional space-time to solve cosmological problems. Also, it is well known that Friedmann-Robertson-Walker (FRW) spatially homogeneous and isotropic cosmological models are widely considered as good approximation of the present and early stages of the universe. FRW line element fits best with the cosmological principle and consistent with the present day observational data. At its early stage of evolution, study of five dimensional space-time is important because of the fact that cosmos might have had a higher dimensional era. In order to unify gravitation with electromagnetism , a five dimensional space-time geometry was first proposed by Kaluza [15] and Klein [16]. In the context of the Kaluza-Klein theories[15, 16, 17, 18] the study of higher dimensional cosmological models have obtained much importance. Many researchers have studied the problems in the field of higher dimensions. Appelquist et al.[18], Rahaman et al. [19] formulated higher dimensional spherically symmetric perfect fluid model in Lyra geometry. Samanta et al.[20] investigated five dimensional Bianchi type-1 string cosmological model in Lyra manifold.

In FRW type of homogeneous cosmological model, the dimensionality has a marked effect on the time temperature relation of the universe and our universe appears to cool more slowly in higher dimensional space time as suggested by Chatterjee [21]. In the recent years, cosmological model with a relic cosmological constant have received considerable attention among researchers for various reasons [22, 23, 24, 25, 26, 27]. We should realize that the existence of a nonzero cosmological constant in Einstein's equations is a feature of deep and profound consequence. The recent observations indicate that $\Lambda \sim 10 - 55cm^2$ while particle physics prediction for Λ is greater than this value by a factor of order 10^{120}. This discrepancy is known as cosmological constant

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problem. Ratra and Peebles[28], Dolgov [29, 30, 31], Sahni and Starobinsky [32], Padmanabhan [33] and Peebles [34] are the some of the researchers who recently studied on the cosmological constant "problem" and consequence on cosmology with a time-varying cosmological constant. For earlier reviews on this topic, we can referred to Zeldovich [35], Weinberg [36] and Carrol et al. [37].

G.S. Khadekar et.al.[38] discuss the big-bounce cosmological model by assuming the cosmological constant $\Lambda = \alpha\rho$ and $\Lambda = \beta H^2$, where α and β are the constant and ρ and H, are the energy density and Hubble parameter respectively by considering a class of five-dimensional cosmological model. Adhav K.S. et.al.[39] studied Bianchi type-III cosmological models in the presence of the bulk viscous fluid with varying Λ. Mukhopadhyay U. et. al.[40] discuss about the time variable and the accelerating universe in the Einstein's field equations under the phenomenological assumption of $\Lambda = \alpha H^2$ for the full physical range of α. Shabani and Ziaie[42] studied about the classical bouncing behaviour of the Universe in terms of $f(R,T) = R + h(T)$ gravity theories. Minas et al. [43] examined the realization of bounces based on a modified gravity theory related to Finsler and Finsler-like geometries. Surendra and Kiranmala [44] studies the role of higher-dimensional FRW models with the framework of the particle creation in the context of variable cosmological and gravitational constants. Mahanta and Das [45] studied the spatially homogeneous and anisotropic Bianchi type-III universe filled with non interacting new holographic dark energy and cold dark matter with variable gravitational and cosmological constant terms. Further Agrawal et al.[46, 47, 48] developed a bouncing cosmology model with a suitable bouncing scale factor and studied its cosmic dynamics. Similarly, Singh et al.[49] studied the bouncing behaviour of the universe in modified gravity with higher-order curvature, finding that the extremal acceleration occurs at the bouncing point. And Zubair and Farooq[50] explore the bouncing models in the framework of 4D EGB with a flat, isotropic FRW universe.

Motivating from the above literatures here in this paper we investigate the spatially homogeneous and isotropic FRW universe in the context of variable cosmological and gravitational constants. We evaluate different cosmological parameters with the assumption that our universe is filled with distributions of matter. In sections 2.1-2.9, we have presented the solution of field equations and discussion for the cosmological parameters for the different values of curvature index parameters $K = 0, 1, -1$ and $\gamma = 1, \frac{4}{3}, 2$. In sec. 4 we also study the physical interpretation of the cosmological parameters in the context of solutions. Concluding remarks of the work is presented in sec. 5 .

2. FIELD EQUATIONS

Here we consider the five dimensional FRW metric[51, 52, 53] in the form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{(1-Kr^2)} + r^2(d\theta^2 + \sin^2\theta d\theta^2) + (1-Kr^2)d\psi^2\right] \tag{1}$$

where $a(t)$ is the scale factor $K = 0, -1$ or $+1$ is the curvature parameter for flat, open and closed universe respectively. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor of a perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu v_\nu - p g_{\mu\nu}, \tag{2}$$

where ρ is the energy density of the cosmic matter, p is its pressure and u_μ is the five-velocity vector such that $u_\mu u^\mu = 1$. The Einstein field equations with time-dependent cosmological and gravitational constants is given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(t)T_{\mu\nu} + \Lambda(t)g_{\mu\nu} \tag{3}$$

where $R_{\mu\nu}$ is the Ricci tensor, $G(t)$ and $\Lambda(t)$ being the variable gravitational and cosmological constants. The divergence of (3), taking into account the Bianchi identity, gives

$$(8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu})^{;\nu} = 0. \tag{4}$$

Equation (3) and (4) may be considered as the fundamental equations of gravity with G and Λ coupling parameters. Using comoving coordinates

$$u_\nu = (1, 0, 0, 0, 0), \tag{5}$$

in (2) and with the line element (1), Einstein's field equation (3), yields

$$8\pi G(t)\rho = \frac{6\dot{a}^2}{a^2} + \frac{6K}{a^2} - \Lambda(t), \tag{6}$$

$$8\pi G(t)p = -\frac{3\dot{a}}{a} - \frac{3\dot{a}^2}{a^2} - \frac{3K}{a^2} + \Lambda(t), \tag{7}$$

where dot denotes derivative w.r.t $'t'$.

In uniform cosmology $G = G(t)$ and $\Lambda = \Lambda(t)$ so that the conservation (4) is given by

$$\dot{\rho} + 4(\rho + p)H = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G}\right). \tag{8}$$

The usual energy momentum conservation relation $T^{\mu\nu}_{;\nu} = 0$ leads to

$$\dot{\rho} + 4(\rho + p)H = 0. \tag{9}$$

The field equations (6) - (7) can also be written as

$$\frac{3\ddot{a}}{a} = -4\pi G(t)\left[2p + \rho - \frac{\Lambda(t)}{8\pi G(t)}\right], \tag{10}$$

$$\frac{6\dot{a}^2}{a^2} = 8\pi G(t)\left[\rho + \frac{\Lambda(t)}{8\pi G(t)}\right]. \tag{11}$$

Equation (10) and (11) can be written in terms of the Hubble paramater $H = \frac{\dot{a}}{a}$ to give the below the equation respectively

$$\dot{H} + H^2 = -\frac{4\pi}{3}G(t)(2p + \rho) + \frac{\Lambda(t)}{6}, \tag{12}$$

$$H^2 = \frac{4\pi}{3}G(t)\rho + \frac{\Lambda(t)}{6} - \frac{K}{a^2}. \tag{13}$$

In order to solve the above field equations (9), (12) and (13) we used the bouncing scale factor as suggested by[41, 42, 50]

$$a(t) = R_0(1 + \alpha^2 t^2)^n, \tag{14}$$

where α, n and R_0 are positive constants.

The equation of state is

$$p = (\gamma - 1)\rho, \tag{15}$$

where γ is a constant $(1 \leq \gamma \leq 2)$.

Using equations (14) and (15), equation (9) yields

$$\rho = \frac{C_1}{\{R_0(1 + \alpha^2 t^2)^n\}^{4\gamma}} \tag{16}$$

where C_1 is a arbitrary constant.

From eqn (15) and (16), the pressure is

$$p = \frac{(\gamma - 1)C_1}{\{R_0(1 + \alpha^2 t^2)^n\}^{4\gamma}} \tag{17}$$

From eqn (14), Hubble Paramater is

$$H = \frac{2n\alpha^2 t}{1 + \alpha^2 t^2}. \tag{18}$$

From eqn (12) and (13), we get

$$\dot{H} = -\frac{8\pi}{3}G(t)\rho\gamma + \frac{K}{a^2}. \tag{19}$$

Using eqns (14), (16) and eqn (18), eqn. (19) becomes

$$G(t) = \frac{3R_0^{4\gamma}(1 + \alpha^2 t^2)^{4n\gamma}}{8\pi\gamma C_1}\left[\frac{K}{R_0^2(1 + \alpha^2 t^2)^{2n}} - \frac{2\alpha^2 n(1 - \alpha^2 t^2)}{(1 + \alpha^2 t^2)^2}\right] \tag{20}$$

Using eqns. (14), (16), (18) and (20) eqn. (13) becomes

$$\Lambda(t) = \frac{3}{\gamma}\left[\frac{2\alpha^2 n(1 + \alpha^2(4\gamma n - 1)t^2))}{(1 + \alpha^2 t^2)^2} + \frac{(2\gamma - 1)K}{R_0(1 + \alpha^2 t^2)^{2n}}\right] \tag{21}$$

The deceleration parameter becomes

$$q = \frac{-1 + \alpha^2 t^2(1 - 2n)}{2\alpha^2 n t^2}. \tag{22}$$

Now we can prove to study for the different values of curvature parameters K and γ as under

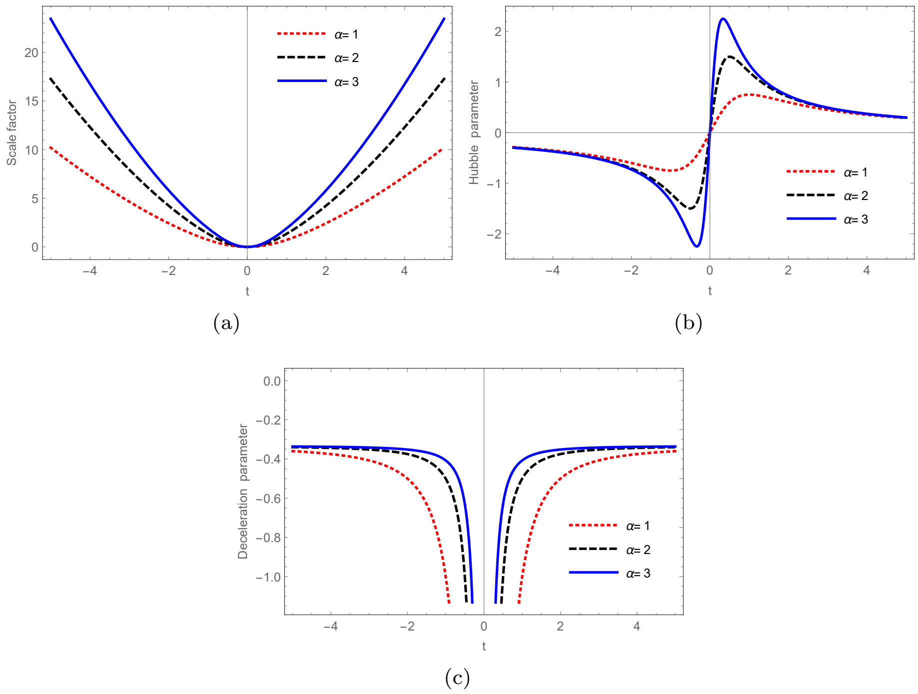


Figure 1. Behaviour of Scale factor $a(t)$, Hubble parameter $H(t)$ and Declaration parameter $q(t)$ against t for different values of α, $R_0 = 1$ and $n = \frac{3}{4}$.

2.1. When $K = 0$(flat) and taking $\gamma = 1$(Dust universe), we get

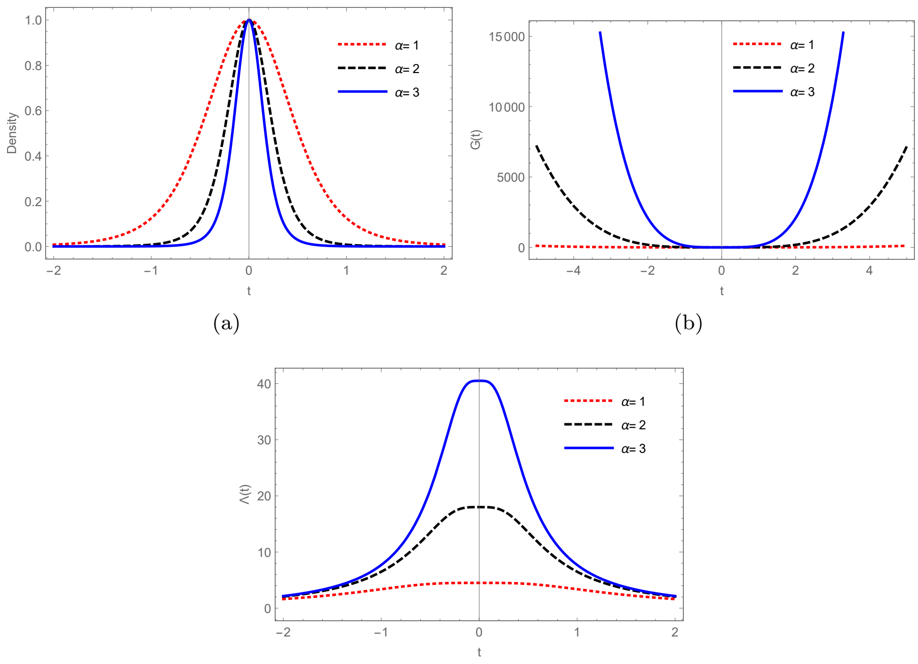
$$\rho = \frac{C_1}{R_0^4(1+\alpha^2t^2)^{4n}},$$
$$p = 0$$
$$G(t) = \frac{3\alpha^2 n R_0^4}{4C_1\pi}.(-1+\alpha^2t^2)(1+\alpha^2t^2)^{-2+4n},$$
$$\Lambda(t) = \frac{1}{(1+\alpha^2t^2)^2}6\alpha^2 n(1+(-1+4n)\alpha^2t^2).$$

2.2. When $K = 0$(flat) and taking $\gamma = \frac{4}{3}$(Radiation universe)

$$\rho = \frac{C_1}{R_0^{\frac{4}{3}}(1+\alpha^2t^2)^{\frac{16n}{3}}},$$
$$p = -\frac{2C_1}{3R_0^{\frac{4}{3}}(1+\alpha^2t^2)^{\frac{16n}{3}}},$$
$$G(t) = \frac{9\alpha^2 n(\alpha^2t^2-1)(R_0(1+\alpha^2t^2)^n)^{\frac{16}{3}}}{16\pi C_1(1+\alpha^2t^2)^2},$$
$$\Lambda(t) = \frac{9\alpha^2 n}{2(1+\alpha^2t^2)^2}\left(1+\left(-1+\frac{16n}{3}\right)\alpha^2t^2\right).$$

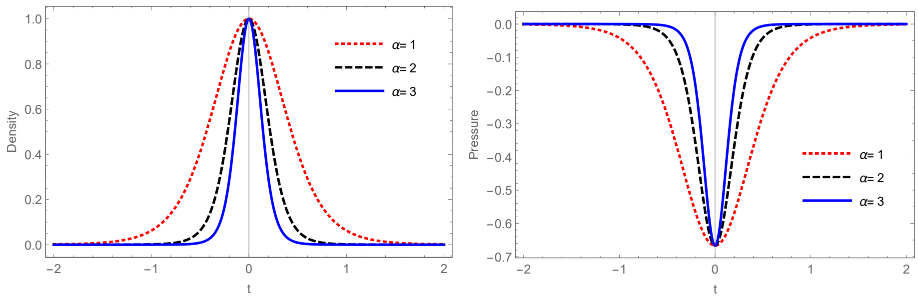
2.3. When $K = 0$(flat) and taking $\gamma = 2(Zel'dovichuniverse)$

$$\rho = \frac{C_1}{R_0^8(1+\alpha^2t^2)^{8n}},$$



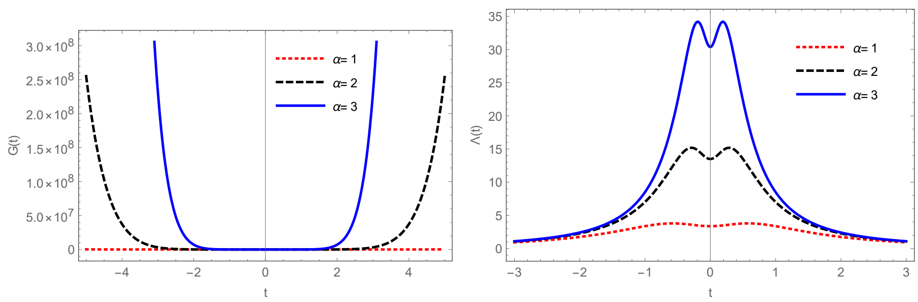
(a) (b) (c)

Figure 2. Plot (a) corresponds to the evolution of $\rho(t)$ against t, plot (b) corresponds to the behaviour of $G(t)$ against t, whereas plot (c) corresponds to the evolution of $\Lambda(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.



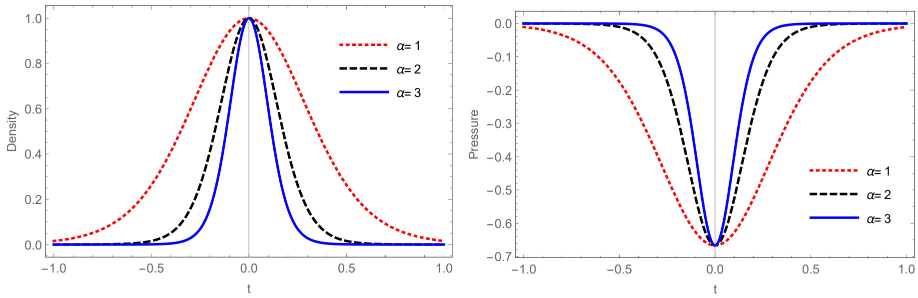
(a) (b)

Figure 3. Plot (a) corresponds to the evolution of $\rho(t)$ against t whereas plot (b) corresponds to the behaviour of $p(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.



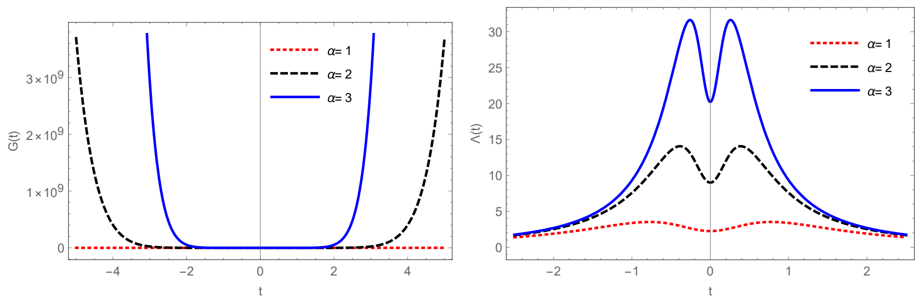
(a) (b)

Figure 4. Plot (a) depicts to the behaviour of $G(t)$ against t, whereas plot (b) depicts to the evolution of $\Lambda(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.



(a) (b)

Figure 5. Plot (a) corresponds to the evolution of $\rho(t)$ against t and plot (b) corresponds to the behaviour of $p(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.



(a) (b)

Figure 6. Plot (a) depicts to the behaviour of $G(t)$ versus t, whereas plot (b) depicts to the evolution of $\Lambda(t)$ versus t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.

$$
\begin{aligned}
p &= \frac{C_1}{R_0^8(1+\alpha^2t^2)^{8n}},\\
G(t) &= \frac{3\alpha^2 n R_0^8(\alpha^2t^2-1)(1+\alpha^2t^2)^{-2+8n}}{8\pi C_1},\\
\Lambda(t) &= \frac{3\alpha^2 n}{(1+\alpha^2t^2)^2}\left(1+\alpha^2\left(-1+8n\right)\alpha^2t^2\right).
\end{aligned}
$$

2.4. When $K = 1$(closed) and taking $\gamma = 1$(Dust universe)

$$
\begin{aligned}
\rho &= \frac{C_1}{R_0^4(1+\alpha^2t^2)^{4n}},\\
p &= 0\\
G(t) &= \frac{3R_0^4(1+\alpha^2t^2)^{4n}}{8\pi C_1}\left[\frac{1}{R_0(1+\alpha^2t^2)^{2n}} - \frac{2\alpha^2n(1-\alpha^2t^2)}{(1+\alpha^2t^2)^2}\right]\\
\Lambda(t) &= \frac{3}{R_0^2(1+\alpha^2t^2)^{2n}} + \frac{6\alpha^2n(1+(-1+4n)\alpha^2t^2}{(1+\alpha^2t^2)^2}.
\end{aligned}
$$

2.5. When $K = 1$(closed) and taking $\gamma = \frac{4}{3}$(Radiation universe)

$$
\begin{aligned}
\rho &= \frac{C_1}{R_0^{\frac{4}{3}}(1+\alpha^2t^2)^{\frac{16n}{3}}},\\
p &= -\frac{2C_1}{3R_0^{\frac{4}{3}}(1+\alpha^2t^2)^{\frac{16n}{3}}},
\end{aligned}
$$

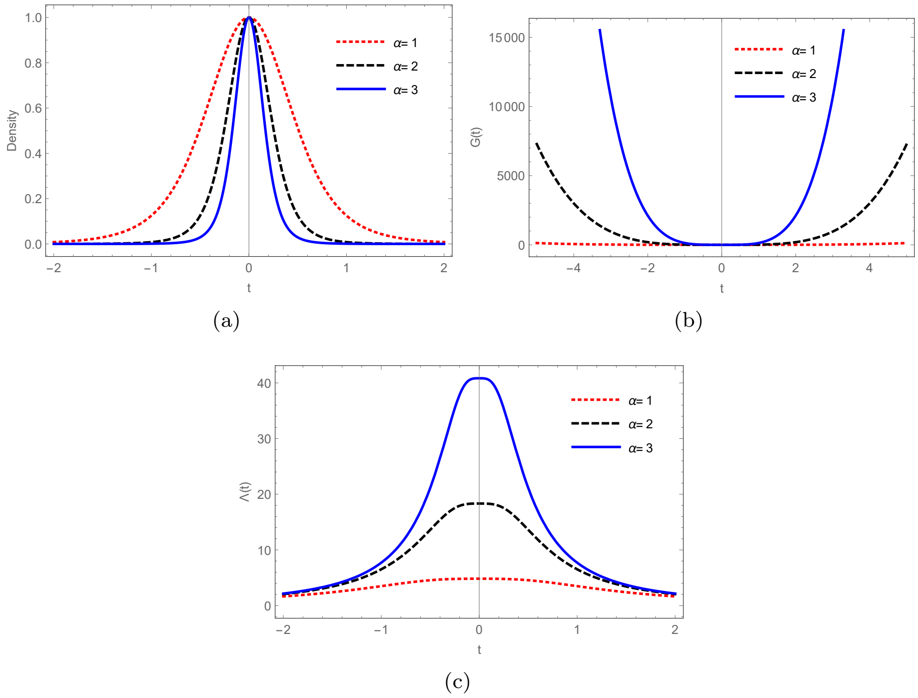


Figure 7. Plot (a) corresponds to the evolution of $\rho(t)$ versus t, (b) depicts to the behaviour of $G(t)$ against t, whereas plot (c) depicts to the evolution of $\Lambda(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.

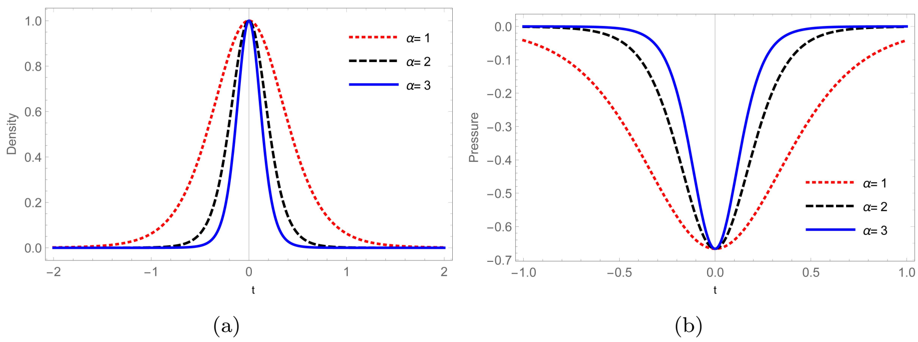


Figure 8. Plot (a) depicts to the behaviour of $\rho(t)$ against t, whereas plot (b) depicts to the evolution of $p(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.

$$G(t) = \frac{R_0^3(1+\alpha^2t^2)^{3n}}{2\pi C_1}\left[\frac{1}{R_0^2(1+\alpha^2t^2)^{2n}} - \frac{2\alpha^2n(1-\alpha^2t^2)}{(1+\alpha^2t^2)^2}\right]$$

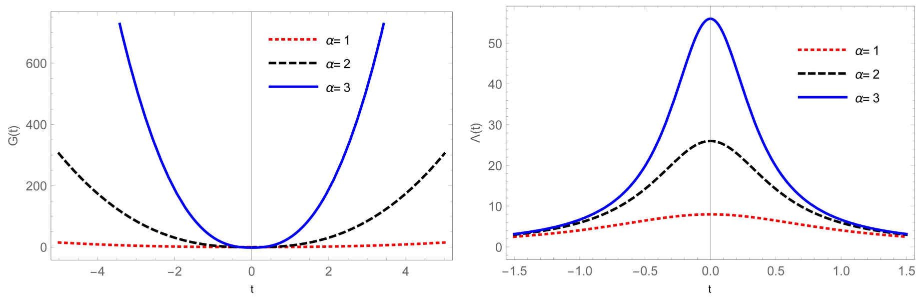
$$\Lambda(t) = \frac{2}{R_0^2(1+\alpha^2t^2)^{2n}} + \frac{8\alpha^2n(1+(-1+3n)\alpha^2t^2)}{(1+\alpha^2t^2)^2}.$$

2.6. When $K = 1$(closed) and taking $\gamma = 2$(Zel'dovich universe)

$$\rho = \frac{C_1}{R_0^8(1+\alpha^2t^2)^{8n}},$$

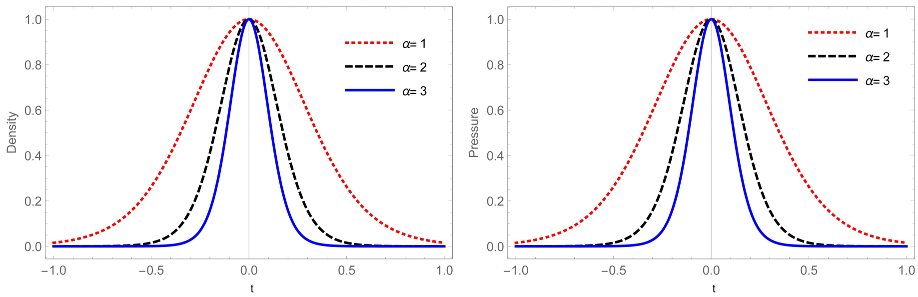
$$p = \frac{C_1}{R_0^8(1+\alpha^2t^2)^{8n}},$$

$$G(t) = \frac{3R_0^8(1+\alpha^2t^2)^{8n}}{16\pi C_1}\left[\frac{1}{R_0^2(1+\alpha^2t^2)^{2n}} - \frac{2\alpha^2n(1-\alpha^2t^2)}{(1+\alpha^2t^2)^2}\right]$$



(a) (b)

Figure 9. Plot (a) depicts to the behaviour of $G(t)$ versus t, whereas plot (b) depicts to the evolution of $\Lambda(t)$ versus t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.



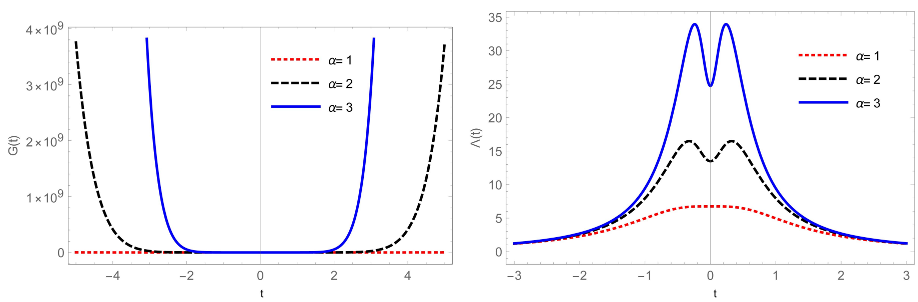
(a) (b)

Figure 10. Plot (a) depicts to the behaviour of $\rho(t)$ against t, whereas plot (b) depicts to the evolution of $p(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.

$$\Lambda(t) = \frac{9}{2R_0^2(1+\alpha^2 t^2)^{2n}} + \frac{3\alpha^2 n(1+(-1+8n)\alpha^2 t^2)}{(1+\alpha^2 t^2)^2}.$$

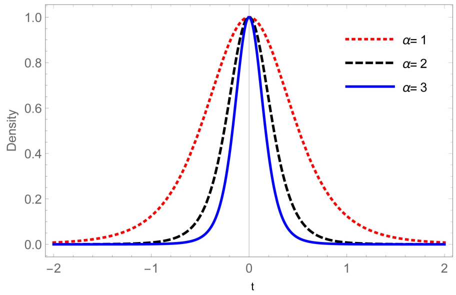
2.7. When $K = -1$(open) and taking $\gamma = 1$(Dust universe)

$$\rho = \frac{C_1}{R_0^4(1+\alpha^2 t^2)^{4n}},$$
$$p = 0,$$
$$G(t) = \frac{3R_0^4(1+\alpha^2 t^2)^{4n}}{8\pi C_1}\left[-\frac{1}{R_0^2(1+\alpha^2 t^2)^{2n}} - \frac{2\alpha^2 n(1-\alpha^2 t^2)}{(1+\alpha^2 t^2)^2}\right]$$

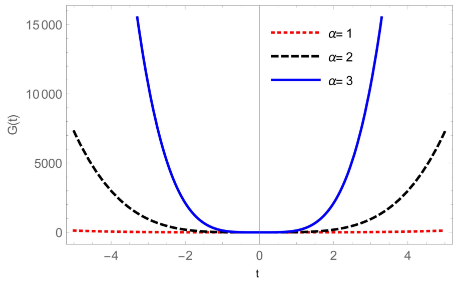


(a) (b)

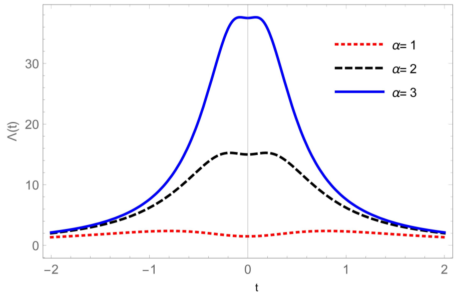
Figure 11. Plot (a) depicts to the behaviour of $G(t)$ against t, whereas plot (b) depicts to the evolution of $\Lambda(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.



(a)



(b)



(c)

Figure 12. Plot (a) corresponds to the evolution of $\rho(t)$ versus t, plot (b) depicts to the behaviour of $G(t)$ against t, whereas plot (c) depicts to the evolution of $\Lambda(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.

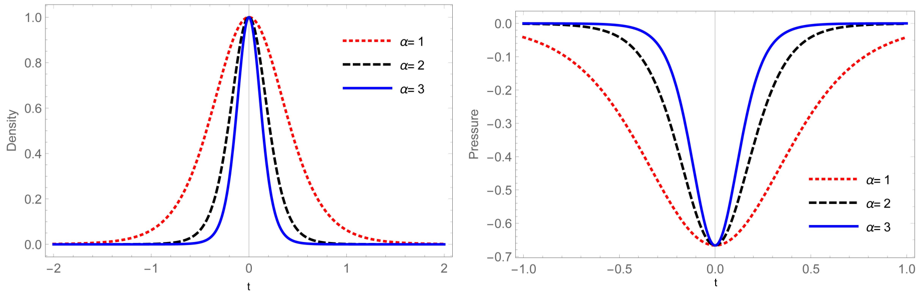
$$\Lambda(t) = -\frac{3}{R_0^2(1+\alpha^2t^2)^{2n}} + \frac{6\alpha^2 n(1+(-1+4n)\alpha^2t^2)}{(1+\alpha^2t^2)^2}.$$

2.8. When $K = -1$(open) and taking $\gamma = \frac{4}{3}$(Radiation universe)

$$\begin{aligned}
\rho =& \frac{C_1}{R_0^{\frac{4}{3}}(1+\alpha^2t^2)^{\frac{16n}{3}}}, \\
p =& -\frac{2C_1}{3R_0^{\frac{4}{3}}(1+\alpha^2t^2)^{\frac{16n}{3}}}, \\
G(t) =& \frac{9R_0^{\frac{16}{3}}(1+\alpha^2t^2)^{\frac{16n}{3}}}{32\pi C_1}\left[-\frac{1}{R_0^2(1+\alpha^2t^2)^{2n}} - \frac{2\alpha^2n(1-\alpha^2t^2)}{(1+\alpha^2t^2)^2}\right] \\
\Lambda(t) =& \frac{3}{4}\left[-\frac{5}{R_0^2(1+\alpha^2t^2)^{2n}} + \frac{2\alpha^2n(3+(-3+16n)\alpha^2t^2)}{(1+\alpha^2t^2)^2}\right].
\end{aligned}$$

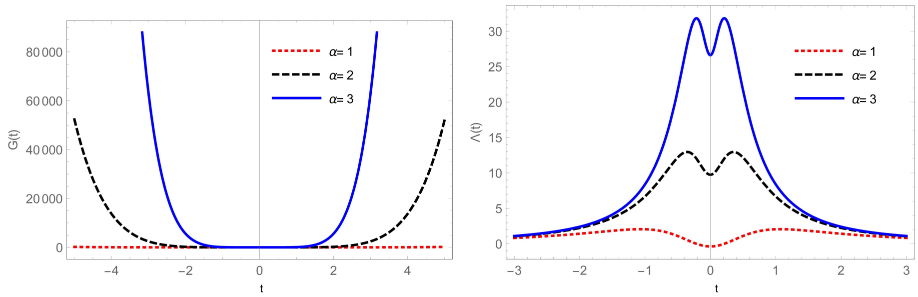
2.9. When $K = -1$(open) and taking $\gamma = 2$(Zel'dovich universe)

$$\begin{aligned}
\rho =& \frac{C_1}{R_0^8(1+\alpha^2t^2)^{8n}}, \\
p =& \frac{C_1}{R_0^8(1+\alpha^2t^2)^{8n}}, \\
G(t) =& \frac{3R_0^8n\alpha^2(-1+\alpha^2t^2)(1+\alpha^2t^2)^{-2+8n}}{8\pi C_1} - \frac{3R_0^6(1+\alpha^2t^2)^{6n}}{16\pi C_1} \\
\Lambda(t) =& \frac{3n\alpha^2(1+(-1+8n)\alpha^2t^2)}{(1+\alpha^2t^2)^2} - \frac{9}{2R_0^2(1+\alpha^2t^2)^{2n}}.
\end{aligned}$$



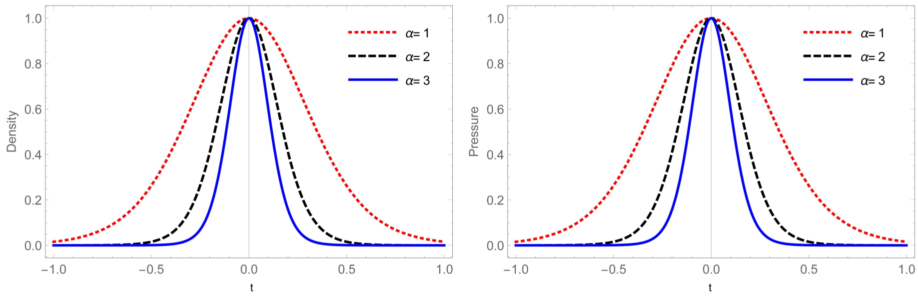
(a) (b)

Figure 13. Plot (a) corresponds to the evolution of $\rho(t)$ versus t, plot (b) corresponds to the evolution of $p(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.



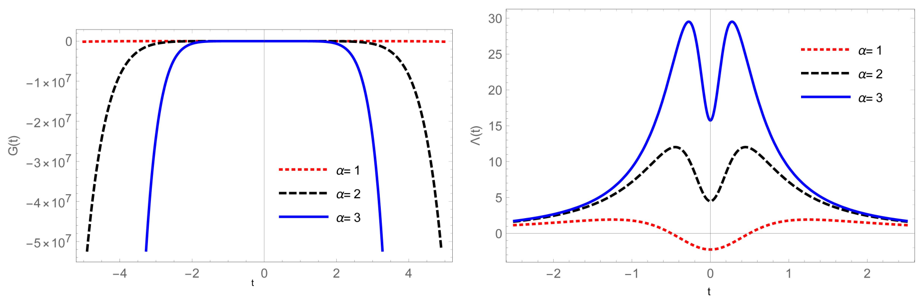
(a) (b)

Figure 14. Plot (a) depicts to the behaviour of $G(t)$ against t, whereas plot (b) depicts to the evolution of $\Lambda(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.



(a) (b)

Figure 15. Plot (a) corresponds to the evolution of $\rho(t)$ versus t, plot (b) depicts to the evolution of $p(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.



(a) (b)

Figure 16. Plot (a) depicts to the behaviour of $G(t)$ against t, whereas plot (b) depicts to the behaviour of $\Lambda(t)$ against t for different values of α, $C_1 = R_0 = 1$ and $n = \frac{3}{4}$.

3. BEHAVIOUR OF BOUNCING COSMOLOGY

We will discuss matter bounce scenarios in FRW model in the present analysis. The study focuses on the dynamic of energy density ρ, pressure p, and the EoS parameter ω. In general, bouncing models meet the following conditions.

Bouncing models go through a contracting phase before bouncing, resulting in non-singular bounces i.e., the expansion of universe $a(t)$ decreases with time as $\dot{a(t)} < 0$. Thus, Hubble parameter $H = \frac{\dot{a}}{a} < 0$ represents contracting era of universe. As a result, Hubble parameter $H = 0$ disappears at bouncing point. EoS ω and deceleration parameter q are identical for homogeneous and flat FRW. The bounce point shows singular behaviour for both expressions.

Whenever $a(t)$ increases with increase in cosmic time t, this implies $\dot{a(t)} > 0$, which implies $H > 0$. When accelerating near the bouncing point ($\dot{H} > 0$), we can predict that the derivative of H will be positive. When an EoS parameter is bouncing, it evolves in phantom form. Different values of α, $R_0 = 1$ and $n = \frac{3}{4}$ can be used to measure the contribution of theory, while the bouncing parameter is used to measure the bouncing effects. We have also studied cosmological parameters, energy densities, pressures, cosmological parameters, and gravitational parameters in terms of cosmic time.

4. PHYSICAL INTERPRETATION

Fig 1(a) shows the behaviour of scale factor against cosmic time for different values of bouncing parameter α. Fig 1(b) of equation (18) shows the evolution of Hubble parameter, $H(t)$ becomes 0(zero) as $t \to 0$ and ∞. Equation (22) suggests that the deceleration parameter versus time is plotted in Fig. 1(c), $q(t) \to -\infty$ at $t \to 0$ and q is negative quantities for sufficiently large values of t for different values of α, $R_0 = 1$ and $n = \frac{3}{4}$.

For different values of bouncing parameter, in the section 2.1 corresponds to fig.2 energy density becomes zero as t tends to infinity and at $t = 0$, $\rho = 1$ but for the flat universe the pressure is zero. The gravitational constant always increase when time increase whereas cosmological constant decreases when time increases and $G(t) \to 0$ when $t = 0$, but $\Lambda(t)$ is positive constant at $t = 0$. In fig. 3 and fig. 4 shows the dynamical behaviour of the section 2.2, energy density becomes zero as $t \to \infty$ and $\rho = 1$ when $t = 0$. The pressure is zero as $t \to \infty$ and $p = \frac{-2}{3}$ when $t = 0$. The gravitational constant always increase when time increase whereas cosmological constant decreases when time increases and $G(t) \to 0$ when $t = 0$, but $\Lambda(t)$ is positive constant at $t = 0$. In fig. 5 and fig. 6 shows the dynamical behaviour of the section 2.3, energy density becomes zero as $t \to \infty$ and $\rho = 1$ when $t = 0$. The pressure is zero as $t \to \infty$ and $p = \frac{-2}{3}$ when $t = 0$. The gravitational constant always increase when time increase whereas cosmological constant decreases when time increases and $G(t) \to 0$ when $t = 0$, but $\Lambda(t)$ is positive constant at $t = 0$.

For different values of α, $R_0 = 1$ and $n = \frac{3}{4}$, in the section 2.4 corresponds to fig.7 energy density becomes zero as t tends to infinity and at $t = 0$, $\rho = 1$ but for $K = 1$(closed) and $\gamma = 1(Dust\ universe)$ the pressure is zero. The gravitational constant always increase when time increases whereas cosmological constant decreases when time increases and $G(t) \to 0$ when $t = 0$, but $\Lambda(t)$ is positive constant at $t = 0$. In fig. 8 and fig. 9 shows the dynamical behaviour of the section 2.5, energy density becomes zero as $t \to \infty$ and $\rho = 1$ when $t = 0$. The pressure is zero as $t \to \infty$ and $p = \frac{-2}{3}$ when $t = 0$. The gravitational constant always increases when time increases whereas cosmological constant also increases when time increases and $G(t) \to 0$ when $t = 0$, but $\Lambda(t)$ is positive constant at $t = 0$. In fig. 10 and fig. 11 shows the dynamical behaviour of the section 2.6, energy density becomes zero as $t \to \infty$ and $\rho = 1$ when $t = 0$. The pressure is also zero as $t \to \infty$ and $p = 1$ when $t = 0$. When time increases the gravitational constant always increases whereas cosmological constant decreases and $G(t) \to 0$ when $t = 0$, but $\Lambda(t)$ is positive constant at $t = 0$.

The graphical behaviour of cosmological parameter in the section 2.7 corresponds to fig.12 energy density becomes zero as $t \to \infty$ and $\rho = 1$ at $t = 0$ but for $K = -1$(open) and $\gamma = 1(Dust\ universe)$ the pressure is zero. The gravitational constant always increase when time increase whereas cosmological constant decreases when time increases and $G(t) \to 0$ when $t = 0$, but $\Lambda(t)$ is constant at $t = 0$. In fig. 13 and fig. 14 shows the dynamical behaviour of the section 2.8, energy density becomes zero as $t \to \infty$ and $\rho = 1$ at $t = 0$. The pressure is zero as $t \to \infty$ and $p = \frac{-2}{3}$ at $t = 0$. The gravitational constant always increases when time increase whereas cosmological constant decreases when time increases and $G(t) \to 0$ when $t = 0$, but $\Lambda(t)$ is constant at $t = 0$. In fig. 15 and fig. 16 shows the dynamical evolution of the section 2.9, energy density becomes zero as $t \to \infty$ and $\rho = 1$ when $t = 0$. The pressure is zero as $t \to \infty$ and $p = 1$ when $t = 0$. The gravitational constant always increase when time increase whereas cosmological constant decreases when time increases and $G(t) \to 0$ when $t = 0$, but $\Lambda(t)$ is constant at $t = 0$.

5. CONCLUSION REMARKS

In the present contexts we attempt to reinterpret a mater bounce scenario with the framework of higher dimensional FRW model with variable G and Λ. Since now a days the study of bouncing cosmology becomes an interesting area to avoid the possible singularity occurring in the usual models under general theory of relativity. Our proposed model can provide some useful scenario for the bouncing model. We have presented the different model for different stages of the universe by calculating the physical parameters of the models with the use of bouncing parameters. Analysing the Scale factor, Hubble parameter, deceleration parameter, energy density, pressure, gravitational and cosmological constant have been extensively investigated for different values of α, $R_0 = 1$ and $n = \frac{3}{4}$. In general, the behaviour near bounce is influenced by the bouncing parameter. It is emphasized by the bouncing scale factors that the cosmos undergoes a contraction, a bounce, and an accelerating phase at late times. The parameter H indicates the contracting phase ($H < 0$) before the bounce, and the expanding phase ($H > 0$) after the bounce at $t \approx 0$. In the decelerating phase of the universe, all deceleration parameter values are negative and indicate accelerated expansion.

In our model for different values of K and γ, all the behaviour of energy density increase before bounce and decrease after bounce but $\rho = 1$ at bouncing point $t = 0$. While the pressure profile is negative in the section 2.2, 2.3, 2.5 and 2.8, whereas $p = \frac{-2}{3}$ at bouncing point $t = 0$ which justifies the current cosmic expansion with dark energy. Although $p = 1$ at bouncing point $t = 0$, the pressure is positive in sections 2.6 and 2.9, also satisfied a contracting phase before the bounce and an expanding phase after the bounce. The choice of model parameters is strictly determined by the evolution of cosmological parameters and in particular, the conservation equation.

The constant G and Λ are allowed to depend on the cosmic time t. The gravitational constant G decreases before the bounce, and increases after the bounce and $G(t) = 0$ at $t = 0$ in all the sections 2.1 - 2.9. The cosmological term Λ increases before the bounce, and decreases after the bounce and $\Lambda(t) = constant$ at bouncing point $t = 0$ in all the sections 2.1 - 2.9. But in the case of $\alpha = 1$, of the section 2.7 - 2.9 cosmological term Λ is negative at bouncing point $t = 0$. In our research, we have found that the explosion of the universe at the early stages of its creation was only a consequence of the creation of matter. Thus, studying the early evolution of the universe requires understanding the implications of time varying Λ and G. And also we hope to shed some light on the real universe. In addition to providing insights into cosmological structure formation, this study could also provide insight into the formation of universe. Through this approach, higher dimensional space time allows the unified description of early evolution of the universe with variables G and Λ. Generally, the models are scalar expansion, non-shear, and isotropic. According to the above points, our proposed model provides good bouncing solutions with the parameters chosen.

6. CREDIT AUTHORSHIP CONTRIBUTION STATEMENT

Asem Jotin Meitei: Writing - original draft, Investigation, Data curation, Software, Visualization, review and editing. **Kangujam Priyokumar Singh**: Supervision, Resources, Methodology, Validation, Formal analysis.**Syed Sabanam**: Investigation, Data curation, Visualization and editing and **S. Kiranmala Devi**: Investigation, Data curation, Visualization, review and editing.

7. DECLARATION OF COMPETING INTEREST

There are no known competing financial interests or personal relationships that could have affected the authors' work reported in this paper.

8. DATA AVAILABILITY

In this published article, no data is available.

9. APPENDIX-I

The Einstein field equations with time dependent cosmological and gravitational terms is given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(t)T_{\mu\nu} + \Lambda(t)g_{\mu\nu} \tag{23}$$

The Bianchi identities from eqn. (23) are given by

$$R^k_{\mu\nu i;j} + R^k_{\mu ij;\nu} + R^k_{\mu j\nu;i} = 0 \tag{24}$$

Apply antisymmetric property in the second term, we have

$$R^k_{\mu\nu i;j} - R^k_{\mu ji;\nu} + R^k_{\mu j\nu;i} = 0 \tag{25}$$

Contracting with respect to k and i, we get

$$R^k_{\mu\nu k;j} - R^k_{\mu jk;\nu} + R^k_{\mu j\nu;k} = 0 \tag{26}$$

But by the definition of Ricci tensor, we get

$$R^k_{\mu\nu k} = R_{\mu\nu} \; and \; R^k_{\mu jk} = R_{\mu j}$$

From eqn. (26) gives

$$R_{\mu\nu;j} - R_{\mu j;\nu} + R^k_{\mu j\nu;k} = 0$$

Since derivatives of fundamental tensors are 0(zero), we can expressed the above equation as:

$$(g^{\mu j}R_{\mu\nu})_{;j} - (g^{\mu j}R_{\mu j})_{;\nu} + (g^{\mu j}R^k_{\mu\nu j})_{;k} = 0$$

or

$$R^j_{\nu;j} - R_{;\nu} + R^k_{\nu;k} = 0$$

Changing the dummy indices j and k to μ, we obtain

$$R^\mu_{\nu;\mu} - R_{;\nu} + R^\mu_{\nu;\mu} = 0 \tag{27}$$

But

$$R_{;\nu} = \frac{\partial R}{\partial x^\nu}\frac{\delta}{\delta x^\mu}(\delta^\mu_\nu R) = (\delta^\mu_\nu R)_{;\mu}$$

Therefore eqn.(27) becomes

$$2R^\mu_{\nu;\mu} - (\delta^\mu_\nu R)_{;\mu} = 0$$

or

$$(R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R)_{;\mu} = 0$$

or

$$(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)^{;\mu} = 0$$

Hence eqn.(23) becomes

$$(8\pi G(t)T_{\mu\nu} + \Lambda(t)g_{\mu\nu})^{;\mu} = 0$$

10. APPENDIX-II

We have,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\delta_{\mu\nu}$$

Differentiation both side w.r.t. ν, we have

$$0 = [(\rho + p)u_\mu u_\nu - p\delta_{\mu\nu}]_{;\nu}$$

Multiplying both sides by u^μ, we have

$$\Rightarrow [(\rho + p)u_\mu u_\nu - p\delta_{\mu\nu}]_{;\nu}u^\mu = 0$$

$$\Rightarrow (\rho + p)_{;\nu}u_\mu u_\nu u^\mu + (\rho + p)(u_\mu u_\nu)_{;\nu}u^\mu - p_{;\nu}u^\mu = 0$$

$$\Rightarrow (\rho_{;\nu} + p_{;\nu})u_\nu + (\rho + p)(u_\nu)_{;\nu} - p_{;\nu}u^\mu = 0$$

$$\Rightarrow \rho_{;\nu}u_\nu + (\rho + p)[(u_\nu)_{,\nu} + u^\alpha\Gamma_{\alpha\nu,\nu}] = 0$$

$$\Rightarrow \dot{\rho} + 4(\rho + p)H = 0 \qquad \because \Gamma_{12,2} = \Gamma_{13,3} = \Gamma_{14,4} = \Gamma_{15,5} = \frac{\dot{a}}{a} = H$$

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ПЕРЕІНТЕРПРЕТАЦІЯ ВСЕСВІТУ ФРІДМАНА-РОБЕРТСОНА-ВОКЕРА ЗІ ЗМІННИМ ГРАВІТАЦІЙНИМ ТА КОСМОЛОГІЧНИМ ЧЛЕНОМ У КОСМОЛОГІЇ З ВІДСКОКОМ

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Ця стаття присвячена дослідженню п'ятивимірної однорідної та ізотропної моделі FRW зі зміною гравітаційної та космологічної постійної з космічним часом. Точний розв'язок рівнянь поля Ейнштейна отримується за допомогою рівняння стану $p = (\gamma - 1)\rho$ (гамма-закон), де γ, який є адіабатичним параметром, безперервно змінюється в міру розширення Всесвіту. Ми отримали рішення для різних значень кривизни $K = 0, 1, -1$, використовуючи $a(t) = R_0(1 + \alpha^2 t^2)^n$, де α, n і R_0 — додатні константи. Поведінка космологічних параметрів представлена для різних випадків моделей. Детально представлена фізична інтерпретація отриманої моделі. Цікаво, що запропонована модель виправдовує поточні космологічні спостереження темної енергії.

Ключові слова: *п'ятивимірний; FRW метрика; космологічний термін; масштабний коефіцієнт відскоку*