MODELING THE TEMPERATURE DEPENDENCE OF SHUBNIKOV-DE HAAS OSCILLATIONS IN LIGHT-INDUCED NANOSTRUCTURED SEMICONDUCTORS

Ulugbek I. Erkaboev*, Rustamjon G. Rakhimov, Jasurbek I. Mirzaev, Nozimjon A. Sayidov, Ulugbek M. Negmatov

Namangan Institute of Engineering and Technology, 160115, Namangan, Uzbekistan

*Corresponding Author e-mail: erkaboev1983@gmail.com

Received January 11, 2024; revised January 29, 2024; accepted February 17, 2024

In this work, the influence of light on the temperature dependence of transverse magnetoresistance oscillations is studied. A generalized mathematical expression that calculates the temperature and light dependence of the quasi-Fermi levels of small-scale p-type semiconductor structures in a quantizing magnetic field is derived. New analytical expressions have been found to represent the temperature dependence of transverse differential magnetoresistance oscillations in dark and light situations, taking into account the effect of light on the oscillations of the Fermi energy of small-scale semiconductor structures. A mathematical model has been developed that determines the light dependence of the second-order derivative of the transverse magnetoresistance oscillations of p-type semiconductors with quantum wells by magnetic field induction. A new theory is proposed, which explains the reasons for the significant shift of the differential magnetoresistance oscillations along the vertical axis measured in the experiment for dark and light conditions.

Keywords: Semiconductor; Heterostructure; Oscillation; Magnetoresistance; Quantum well
PACS: 73.63.Hs, 73.21.Fg, 73.21.--b

INTRODUCTION

Currently, the fundamental physical parameters of several new small-scale materials, including semiconductors and crystals, are being studied. Among them are thin films, nanotubes and two-dimensional (2D) structures [1-10]. In particular, quantum oscillation effects under the influence of a magnetic field and electromagnetic waves (light) have been observed in several new classes of narrow band quantum well heterostructures, for example, oscillations of transverse and longitudinal magnetoresistance, Shubnikov-de Haas oscillations and quantum Hall effects [11-20]. In works [21-23], the effect of temperature dependence of magnetoresistance oscillations, magnetic susceptibility oscillations, and quantum Hall effects in bulk and nano-scale semiconductor structures on external factors was investigated. In particular, a theory was developed based on the exact mathematical model of experimental results of quantum oscillation effects for bulk semiconductor materials as a function of temperature [24-26]. However, the effect of light on magnetoresistance oscillations for materials with quantum well heterostructures has not been studied in these works. In these works [13, 27, 28-34], the effect of light on the magnetoresistance oscillations of heterostructures with narrow band quantum wells was experimentally applied. That is, it was observed that magnetoresistance oscillations under the influence of light significantly shift compared to darkness. Even the dependence of these quantum oscillation effects on the frequency of light has been studied at different low temperatures. However, in these works, the reason why the experimental results (oscillation amplitudes) are fundamentally different in light and darkness has not been shown, that is, its perfect theory has not been developed.

The main aim of this work is to study the effect of light on the temperature dependence of transverse magnetoresistance oscillations in heterostructured semiconductors with p-type quantum wells.

MODEL

Calculation of the Influence of Light on Fermi Level Oscillations in Low-Dimensional Semiconductors in a Quantizing Magnetic Field

In quantum well heterostructures in a strong magnetic field, the continuous energy spectrum of free charge carriers becomes discrete Landau levels. As a result, the Landau levels in the conductivity field begin to be filled with free electrons, starting from the lower energy, based on the Pauli principle, that is, the distribution of free electrons by energy occurs. Increasing Landau levels filled with free electrons causes changes in the quantized Fermi level. This, of course, has a strong effect on the oscillation process of the transverse magnetoresistance. It follows that the light-induced shift of transverse electrical conductivity (or magnetoresistance) oscillations is explained by the appearance of quantized Fermi quasi-levels. The dependence of Fermi energy levels on temperature, magnetic field and concentration for 2D and 3D semiconductor materials has been thoroughly studied both theoretically and practically in several literatures. However, the dependence of Fermi quasi-levels on light in semiconductors with quantum wells as a function of temperature has not been considered at all.

© U.I. Erkaboev, R.G. Rakhimov, J.I.Mirzaev, N.A. Sayidov, U.M. Negmatov, 2024; CC BY 4.0 license
It is known that the quantized Fermi level in a semiconductor with a quantum well in the state of thermodynamic equilibrium under the influence of a quantizing magnetic field, that is, in the absence of illumination, at constant low temperatures, is determined by the concentration of equilibrium charge carriers in the conduction and valence zones. However, it is required to know the number of charge carriers that fill the Landau levels in the allowed zones with the illumination of the quantum well heterostructure. For this, it is necessary to use the Fermi-Dirac distribution function. Let the probability of a free electron being in a state with energy \( E \) in the conduction zone of a quantum well in a strong magnetic field be equal to \( f_0(E, \mu, T) \).

In that case, the number of charge carriers on a unit surface whose energy is in the range from \( E \) to \( E + dE \) is equal to the following:

\[
2 \int_{E}^{E+dE} f_0(E, \mu, T) dE = N^{2d}_E(E, B) dE
\]

Here, \( f_0(E, \mu, T) = \frac{1}{e^{(E-\mu)/kT} + 1} \) is the Fermi-Dirac distribution function, \( \mu \) is the Fermi energy, \( N^{2d}_E(E, B) \) is the energy density of states of the conduction band of the quantum well in the quantizing magnetic field. \( B \) – magnetic field induction.

Then, if we divide expression (1) by \( N^{2d}_E(E, B) dE \):

\[
\frac{1}{e^{(E-\mu)/kT} + 1} = \frac{1}{N^{2d}_E(E, B)} \frac{dn}{dE}
\]

We separately define the exponential expression given in (2):

\[
e^{(E-\mu)/kT} = \frac{N^{2d}_E(E, B)}{dn/dE} - 1
\]

We determine the Fermi energy \( \mu \) by taking the natural logarithm of both sides of the expression (3):

\[
\mu = E - kT \ln \left( \frac{N^{2d}_E(E, B)}{dn/dE} - 1 \right)
\]

In two-dimensional semiconductors under the influence of a quantifying magnetic field, we convert \( \frac{dn}{dE} \) to the following expression for the smallest energetic levels, according to the definition of the derivative:

\[
\frac{dn}{dE} = \lim_{\Delta E \to 0} \frac{\Delta n}{\Delta E} = \frac{n_2 - n_1}{E_2 - E_1} = \frac{\Delta n}{\hbar \omega}
\]

In that case, using (6) and (5) can be reduced to the following form:

\[
\mu = E - kT \ln \left( \frac{N^{2d}_E(E, B)}{\Delta n/\hbar \omega} - 1 \right)
\]

As can be seen from expression (7), the Fermi energy of two-dimensional materials depends on temperature, energy, density of two-dimensional energy states, concentration of charge carriers, and magnetic field induction.

However, often in non-equilibrium conditions, for example, when small-scale semiconductors are illuminated with light, the concentration of free electrons or holes in their allowed zones begins to change. If there is darkness (in the absence of light), the Fermi level of small semiconductors depends only on the concentration of equilibrium charge carriers in the conduction or valence band. However, when the small semiconductor is exposed to light, the concentration of charge carriers increases. It depends on the intensity of the light. From this, it is observed that the Fermi level moves up in the non-equilibrium state compared to the equilibrium state. At the same time, the concentration of holes also increases, and in this case, the Fermi level shifts downwards (Fig.1). However, the Fermi level cannot move up and down under the influence of light at the same time Consequently, it is not possible to introduce a single Fermi level under this
condition. Therefore, the state distribution of electrons and holes in non-equilibrium conditions cannot be described by the Fermi-Dirac equilibrium function with one Fermi level, the concept of a quasi-Fermi level is used.

As mentioned above, the concentration of charge carriers strongly depends on the light intensity. Let's consider a p-type quantum well heterostructure under the influence of light, as shown in Fig.1. In that case, the concentration of holes will be:

\[ p = p_0 + \Delta p \]  

(8)

Here, \( p_0 \) is the concentration of holes in the equilibrium state. \( \Delta p \) is the concentration of holes in the presence of external influence (in the presence of light).

![Energy diagrams of light and heavy holes of charge carriers together with the Fermi energy for \( B = 0 \) in p-type quantum well PbTe semiconductors in light and dark conditions [28]](image)

The change in the value of \( \Delta p \) strongly depends on the light intensity \( I \). When exposed to constant light, the generation process increases, in which case the concentration in lighting is determined by the following expression [35]:

\[ \Delta p = \alpha \beta I \tau_p \]  

(9)

Here, \( \alpha \) is the light absorption coefficient, \( \beta \) is the quantum yield that determines the number of electron-hole pairs created by one photon, and \( I \) is the light intensity. \( \Delta p \) is also called stationary concentration of non-equilibrium holes.

According to the theory of quantum physics, in accordance with the special absorption mechanism of light for small-scale semiconductor structures, the absorption coefficient [35]:

\[ \alpha^{2d}(\nu, B) = A_1 \left( \frac{\nu - E_\nu^{2d}(B, d)}{\nu} \right)^{\frac{1}{2}} \]  

(10)

For prohibited transitions, it is equal to:

\[ \alpha^{2d}(\nu, B, d) = A_2 \left( \frac{\nu - E_\nu^{2d}(B, d)}{\nu} \right)^{\frac{3}{2}} \]  

(11)

Here, \( A_1, A_2 \) are constant coefficients independent of frequency.

It follows that, using (8), (9), (10) and (11) to determine the Fermi quantum levels (7) of heterostructures with p-type quantum wells, the following new analytical expression is derived:

\[ \mu^{2d}(E_\nu, B, T, d, \nu) = E_\nu - kT \ln \left[ \frac{N^{2d}(E_\nu, B)}{p_0 + \alpha^{2d}(\nu, B, d)\beta_1 \tau_p} - 1 \right] \]  

(12)

The obtained equation (12), that is, \( \mu^{2d}(B, T, d, \nu) \), means that the quasi-Fermi level of semiconductors with a quantum layer heterostructure depends on the magnetic field, temperature, thickness of the quantum layer, and light energy.

**Calculation of the Influence of Light on Fermi Level Oscillations in Low-Dimensional Semiconductors in a Quantizing Magnetic Field**

In our previous works [36-43], a new mathematical model was proposed that determines the temperature dependence of the first-order differential transverse magnetoresistance for magnetic field induction for small-scale semiconductor structures. In these works, the temperature dependence of \( \frac{\partial \rho^{2d}(E, B, T, d)}{\partial B} \) was calculated by linear decomposition method of \( \frac{\partial \rho^{2d}(E, \mu, T)}{\partial E} \). In this case, the change of \( \mu \) Fermi level under the influence of external factors is not taken
into account. However, according to the equation (12), the Fermi level for small-scale p-type semiconductor structures is strongly dependent on the thickness of the quantum well, magnetic field induction, temperature, and the frequency of the light particle-photon.

Then, according to (12), the term there takes the following form:

$$\left( \frac{\partial f_0(E_0, T, B, \nu, d)}{\partial E} \right)$$

or, after a series of mathematical reductions:

$$\left( \frac{\partial f_0(E_0, T, B, \nu, d)}{\partial E} \right) = \left( \frac{1 + \exp \left[ kT \ln \left( \frac{N^{2d}(E_0, B, d)}{p_0 + \alpha^2(B, \nu, d)\beta I_T} \right) \right]}{\partial E} \right)^{-1}$$

According to the proposed new model taking into account the quasi-Fermi level of a small-scale p-type semiconductor structure in a quantizing magnetic field strongly dependent on light and temperature and taking into account the expression (14) oscillations it can also be observed that it changes significantly with respect to light. In particular, expression (14) becomes:

$$\rho_{2d}^{2d}(E_0, B, T, \nu, d) =$$

$$= \frac{e^2 B}{2\pi m^* c} \frac{1}{\sqrt{\frac{2}{\pi} G}} \sum \exp \left[ -2 \frac{E_0 - \frac{\hbar \omega}{2} \left( \frac{n + \frac{1}{2} + \frac{\pi^2 \hbar^2}{2m^* d^2 n^2} \right)}{G} \gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}} \left( \frac{\partial f_0(E_0, T, B, \nu, d)}{\partial E} \right) dE \right]$$

It follows that the vertical axis of the transverse magnetoresistance oscillations of small-scale p-type semiconductor structures under the influence of darkness and light, moving the quasi-Fermi levels (see Fig. 1) up or down in the non-equilibrium state leads to serious changes throughout. Of course, the shifts of these oscillations are given in the experiments, but their theoretical and physical meaning is justified for the first time using the Equation (15).

According to a series of experiments, the results of the second-order derivative of transverse magnetoresistance oscillations by magnetic field induction in light and dark conditions were compared. Therefore, according to (15):

$$\frac{\partial^2 \rho_{2d}^{2d}(E_0, B, T, \nu, d)}{\partial B^2} =$$

$$= \frac{e^2 B}{2\pi m^* c} \frac{1}{\sqrt{\frac{2}{\pi} G}} \sum \exp \left[ -2 \frac{E_0 - \frac{\hbar \omega}{2} \left( \frac{n + \frac{1}{2} + \frac{\pi^2 \hbar^2}{2m^* d^2 n^2} \right)}{G} \gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}} \left( \frac{\partial f_0(E_0, T, B, \nu, d)}{\partial E} \right) dE \right]$$

$$= \frac{e^2 B}{2\pi m^* c} \frac{1}{\sqrt{\frac{2}{\pi} G}} \sum \exp \left[ -2 \frac{E_0 - \frac{\hbar \omega}{2} \left( \frac{n + \frac{1}{2} + \frac{\pi^2 \hbar^2}{2m^* d^2 n^2} \right)}{G} \gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}} \left( \frac{\partial f_0(E_0, T, B, \nu, d)}{\partial E} \right) dE \right]$$

$$\gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}}$$

$$= \frac{e^2 B}{2\pi m^* c} \frac{1}{\sqrt{\frac{2}{\pi} G}} \sum \exp \left[ -2 \frac{E_0 - \frac{\hbar \omega}{2} \left( \frac{n + \frac{1}{2} + \frac{\pi^2 \hbar^2}{2m^* d^2 n^2} \right)}{G} \gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}} \left( \frac{\partial f_0(E_0, T, B, \nu, d)}{\partial E} \right) dE \right]$$

$$\gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}}$$

$$= \frac{e^2 B}{2\pi m^* c} \frac{1}{\sqrt{\frac{2}{\pi} G}} \sum \exp \left[ -2 \frac{E_0 - \frac{\hbar \omega}{2} \left( \frac{n + \frac{1}{2} + \frac{\pi^2 \hbar^2}{2m^* d^2 n^2} \right)}{G} \gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}} \left( \frac{\partial f_0(E_0, T, B, \nu, d)}{\partial E} \right) dE \right]$$

$$\gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}}$$

$$= \frac{e^2 B}{2\pi m^* c} \frac{1}{\sqrt{\frac{2}{\pi} G}} \sum \exp \left[ -2 \frac{E_0 - \frac{\hbar \omega}{2} \left( \frac{n + \frac{1}{2} + \frac{\pi^2 \hbar^2}{2m^* d^2 n^2} \right)}{G} \gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}} \left( \frac{\partial f_0(E_0, T, B, \nu, d)}{\partial E} \right) dE \right]$$

$$\gamma \sqrt{(k_B T)^{\beta} E_{r}^{-\alpha / 2}}$$
As can be seen from the Equation (16), \( \frac{\partial^2 \left[ \rho^{2d}(E, B, T, \nu, d) \right]}{\partial B^2} \) can be obtained in graphical form only with the help of a package of practical mathematical programs (Maple, Mathcad, Mathematica). Fig.2 shows graphs calculated based on the Equation (16) for dark and light conditions. These graphs were obtained for a p-GaAs/AlGaAs quantum well semiconductor structure, in which the bandgap of the hollow quantum well is 1.51 eV at \( T = 7 \) K, and the width of the quantum well is 3 nm [44] and the magnetic field induction was calculated in the range from 0.1 T to 3 T. The photon energy under the influence of light is taken to be equal to 1.4 eV. The graph in red is the \( \frac{\partial^2 \left[ \rho^{2d}(E, B, T, \nu, d) \right]}{\partial B^2} \) under the influence of light, while the oscillations in blue are the graph obtained in the dark. As can be seen from these graphs, it is observed that the amplitudes of \( \frac{\partial^2 \left[ \rho^{2d}(E, B, T, \nu, d) \right]}{\partial B^2} \) oscillations under the influence of light are greater than those in the dark.

Fig.3 shows the experimental results of \( \frac{\partial^2 \left[ \rho^{2d}(E, B, T, \nu, d) \right]}{\partial B^2} \) magnetoresistance oscillations of p-type PbTe quantum well in dark and light [28]. An infrared light-emitting diode with a wavelength of 940 nm and a power density of 12 mW/m^2 was used as light. The temperature is 1.9 K, the magnetic field induction range is from 0 T to 9 T.

Figure 2. Transverse magnetoresistance oscillations of p-GaAs/AlGaAs quantum well semiconductor structure under light and dark conditions. 1 – In light mode; 2 – In the dark

Figure 3. \( \frac{\partial^2 \left[ \rho^{2d}(E, B, T, \nu, d) \right]}{\partial B^2} \) magnetoresistance oscillations of p-type PbTe quantum well in dark and light conditions [28].

The thickness of the quantum well was equal to 10 nm. In Fig.4, theoretical graphs were obtained using the above-mentioned experimental values of the p-type PbTe quantum well and based on the Equation (16). \( \frac{\partial^2 \left[ \rho^{2d}(E, B, T, \nu, d) \right]}{\partial B^2} \) oscillations in both light and dark conditions are presented here. As you can see from these images, the experimental and theoretical graphs in light and dark conditions seem to be quite different. In fact, it is not
so, on the contrary, quantum oscillations are practically not visible in the range of magnetic field induction from 0.1 T to 4 T. We know theoretically that discrete Landau levels should be observed under the condition $kT \ll \hbar \omega$. It is at $T = 1.9$ K that the condition $kT \ll \hbar \omega$ is fulfilled since the magnetic field induction is 1.5 T. So, the equation (16) not only justifies the experimentally obtained shifts in Fig.3, but also allows to prove the existence of quantum oscillation effects, which are not noticed by the experiment. At the same time, based on the proposed model, it is possible to apply the temperature dependence of $\frac{\partial^2 \left[ \rho^{q2}(E, B, T, \hbar \nu, d) \right]}{\partial B^2}$ oscillations in light and dark conditions.

**CONCLUSIONS**

The following conclusions can be drawn from the important results of this research:

1. A new Equation (12) expressing the dependence of the quasi-Fermi level of quantum wound heterostructure semiconductors on the magnetic field, temperature and light energy has been derived.

2. Movement of the quasi-Fermi levels in the non-equilibrium state up or down leads to a significant change of transverse magnetoresistance oscillations along the vertical axis of small-scale p-type semiconductor structures under the influence of darkness and light (12) was proved based on the Equation. The displacements of these oscillations are given in experiments, but their theoretical physical meaning was justified using the equation (15).

3. A mathematical model has been developed that determines the dependence of $\frac{\partial^2 \left[ \rho^{q2}(E, B, T, \hbar \nu, d) \right]}{\partial B^2}$ oscillations of p-type semiconductors in the light and dark state on external factors.

4. A new theory was proposed that explains the reasons for the significant shift of the differential magnetoresistance oscillations measured in the experiment along the vertical axis for dark and light conditions.

**REFERENCES**


МОДЕЛЮВАННЯ ТЕМПЕРАТУРНОЇ ЗАЛЕЖНОСТІ ОСЦИЛЯЦІЙ ШУБНІКОВА-ДЕ ГААЗА У СВІТЛОІНДУКОВАНИХ НАНОСТРУКТУРНИХ НАПІВПРОВІДНИКАХ

Улугбек І. Еркабоєв, Рустамжон Г. Рахімов, Джасурбек І. Мірзаєв, Нозімжон А. Сайдов, Улугбек М. Негматов

Наманганський інженерно-технологічний інститут, 160115, Наманган, Узбекистан

У даній роботі досліджено вплив світла на температурну залежність коливань поперечного магнітоопору. Отримано узагальнений математичний вираз, який обчислює температурну та світлову залежність квазірівнів Фермі дрібномасштабних напівпровідникових структур p-типу в квантуваному магнітному полі. Було знайдено нові аналітичні вирази, які представляють температурну залежність коливань поперечного диференціального магнітоопору в темних і освітлених ситуаціях, враховуючи вплив світла на коливання енергії Фермі дрібномасштабних напівпровідникових структур. Розроблено математичну модель, яка визначає світлову залежність похідної другого порядку осциляцій поперечного магнітоопору напівпровідників p-типу з квантовими ямами від індукції магнітного поля. Запропоновано нову теорію, яка пояснює причини значного зсуву коливань диференціального магнітоопору вздовж вертикальної осі, виміряної в експерименті для темних і світлих умов.

Ключові слова: напівпровідник; гетероструктура; коливання; магнітоопір; квантовая яма