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PERFECT FLUID WITH HEAT FLOW IN $f(T)$ THEORY OF GRAVITY

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Bianchi Type-I cosmological models have been a subject of extensive research in cosmology due to their simplicity and relevance in understanding the dynamics of the early Universe. In this study, we investigate the dynamics of such models within the framework of $f(T)$ gravity, an alternative theory of gravity that extends teleparallel gravity by introducing a general function of the torsion scalar, T. We focus on the presence of a perfect fluid with heat flow in the cosmic medium. By solving the field equations of $f(T)$ gravity, we obtain exact solutions for the Bianchi Type-I cosmological models. These solutions provide valuable insights into the evolution of the Universe and how it is influenced by the modified gravity theory. Furthermore, we derive cosmological parameters in terms of redshift, offering a convenient way to interpret observational data and connect theoretical predictions to empirical measurements. Our findings not only contribute to a deeper understanding of the dynamics of Bianchi Type-I cosmological models but also provide a foundation for comparing $f(T)$ gravity with standard general relativity in the context of observational cosmology. This research paves the way for further exploration of alternative gravity theories and their implications for the early Universe's evolution and structure.

Keywords: *$f(T)$ theory; Perfect fluid with heat flow; Bianchi type-I*

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1. INTRODUCTION

The study of cosmological models has been instrumental in advancing our understanding of the evolution and structure of the Universe. In this context, Bianchi Type-I cosmological models have played a crucial role due to their inherent simplicity and applicability to various cosmological scenarios. These models assume a homogeneous and anisotropic distribution of matter and radiation in the early Universe, making them a valuable tool for investigating the dynamics of cosmic expansion.

Banerjee et.[1] al. conducted investigations on Bianchi I cosmological models that incorporate a fluid possessing both bulk and shear viscosity. These studies reveal the evolving significance of shear viscosity and fluid density dynamics throughout the cosmic evolution. Sharif and Rani [2] have studied spatially homogeneous and anisotropic Bianchi type I universe in the context of $F(T)$ gravity. Beesham and Aroonkumar [3] studied Bianchi type I cosmological models with variable gravitational constants (G) and cosmological constants (Λ). Jacobs and Kenneth [4] Explored cosmologies of Bianchi type I with a uniform magnetic field. Sahoo and Sivakumar [5] examined LRS (Locally Rotationally Symmetric) Bianchi type-I cosmological models in f(R, T) theory of gravity. Barrow [6] et al. analyzed the asymptotic stability of Bianchi type universes. Ashtekar[7] et al. Investigated loop quantum cosmology of Bianchi type I models. Akarsu [8] et al. explored LRS Bianchi type I models with anisotropic dark energy and constant deceleration parameters. Pawar [9] et al. investigated magnetized anisotropic dark energy by using Barber's self-creation theory. Saha [10] et al. investigated Bianchi type-I cosmology with scalar and spinor fields and they also Studied spinor fields in a Bianchi type-I universe, focusing on regular solutions. Jamil [11] et al. explored FRW and Bianchi type I cosmology of $f-$essence. Singh [12] et al. Explored Bianchi type-I cosmological models in Lyra's geometry. Singh [13] et al. analyzed Bianchi type-I cosmological models with variable G and Λ-term in general relativity. Arbab [14] has studied Bianchi type I viscous universe with variable G and Λ. Rodrigues [15] et al. have investigated Bianchi type-I, type-III, and Kantowski-Sachs solutions in $f(T)$ gravity. Shamir [16] has explored locally rotationally symmetric Bianchi type I cosmology in $f(R,T)$ gravity. Chirde [17] et al. Analyzed a Bianchi type I cosmological model with a perfect fluid and string in $f(T)$ theory of gravitation. Wanas [18] et al. have investigated Bianchi type I cosmological models in $f(T)$ gravitational theories. Fayaz [19] have studied f(T) theories from holographic dark energy models within a Bianchi type I universe. Rodrigues [20] have explored locally rotationally symmetric Bianchi type-I cosmological models in $f(T)$ gravity, spanning from early to dark energy-dominated universes. Aslam [21] et al. have investigated Noether gauge symmetry for the Bianchi type I model in $f(T)$ gravity. Qazi [22] et al. have explored conformal vector fields of Bianchi type-I perfect fluid solutions in $f(T)$ gravity. Hasmani

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and Al-Haysah [23] have provided exact solutions for Bianchi type-I cosmological models in $f(R)$ Theory of Gravity. Shekh [24] et al. have explored an accelerating Bianchi type dark energy cosmological model with a cosmic string in $f(T)$ gravity. Koussour and Bennai [25] have conducted a stability analysis of anisotropic Bianchi type-I cosmological model in teleparallel gravity. Dawande [26] et al. have investigated LRS Bianchi Type-I Universe in $f(T)$ Theory of Gravity. Shukla [27] et al. have explored a Bianchi type-I cosmological model in a modified theory of gravity. Van den Hoogen [28] et al. investigated Bianchi type cosmological models in $f(T)$ tele-parallel gravity. Shamir [29] et al. have explored locally rotationally symmetric Bianchi type I cosmology in $f(R)$ gravity. Rodrigues [30] et al. analyzed anisotropic universe models in $f(T)$ gravity. Pawar [31] et al. have investigated LRS Bianchi type-I cosmological models in $f(Q,T)$ theory of gravity with observational constraints. Dagwal [32] has explored tilted two forms of dark energy in $f(T)$ theory of gravity. Solanke [33] et al. have studied an accelerating dark energy universe with LRS Bianchi type-I space-time. Pradhan [34] et al. have investigated Bianchi type I anisotropic magnetized cosmological models with varying Λ. General Relativity (GR), formulated by Albert Einstein, has long been the cornerstone of our understanding of gravitational interactions in the cosmos. However, in recent decades, alternative theories of gravity have gained attention, offering different perspectives on the gravitational field equations. One such alternative is $f(T)$ gravity [35], which extends the concept of teleparallel gravity by introducing a general function of the torsion scalar, T. Pawar et al.[36] have studied anisotropic behaviour of perfect fluid in fractal cosmology. $f(T)$ gravity has been explored as a viable alternative to GR, providing a framework to study the gravitational dynamics of the Universe beyond the confines of Einstein's theory.

In this research, we delve into the dynamics of Bianchi Type-I cosmological models within the framework of $f(T)$ gravity[37] . Our primary focus is on the inclusion of a perfect fluid with heat[38], which is crucial in understanding the thermodynamic aspects of the early Universe. By solving the field equations derived from $f(T)$ gravity, we aim to obtain exact solutions for the evolution of the Universe in the presence of these additional components. Pawar et al. [39]-[40] have studied several aspects of Bianchi Type-I with Two fluid axially symmetric cosmological models in $f(R,T)$ theory of gravitation and Tilted congruences with stiff fluid cosmological models.

Furthermore, one of the essential aspects of cosmological models is their ability to provide a connection between theoretical predictions and empirical observations. To facilitate this connection, we derive cosmological parameters that are expressed in terms of redshift, a key observational quantity. This approach enables us to relate our theoretical findings to astronomical data, enhancing the applicability and relevance of our research to the broader field of observational cosmology.

In summary, this research presents a comprehensive exploration of Bianchi Type-I cosmological models in $f(T)$ gravity, incorporating a perfect fluid with heat. The obtained solutions and derived cosmological parameters contribute to our understanding of the early Universe's dynamics and offer a bridge between theoretical predictions and observational data.

2. FIELD EQUATION

The line element for a flat, homogeneous and anisotropic LRS Bianchi type-I space time[41] is

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)\left[dy^2 + dz^2\right] \tag{1}$$

Here, t represents time, x is one spatial coordinate, and y and z are the other two spatial coordinates. The functions $A(t)$ and $B(t)$ are scale factors that describe the expansion or contraction of the space in the x and y-z directions, respectively.

We obtain the tetrad components as follows:

$$h_i^\mu = \mathrm{diag}(1, A^{-1}, B^{-1}, B^{-1}) \tag{2}$$

The torsion scalar, denoted as "T," is a scalar derived from the torsion tensor. It quantifies the deviation of the Weitzenböck connection from the Levi-Civita connection, which is used in general relativity. In simple terms, the torsion scalar reflects the inhomogeneity in the spacetime geometry due to torsion. The torsion scalar has the form

$$T = S_\rho^{\mu\nu} T_{\mu\nu}^\rho \tag{3}$$

The formulation of this theory's action involves extending and building upon the foundational principles of the Teleparallel Theory of Gravity.

$$I = \int e[f(T) + L_{matter}]d^4x \tag{4}$$

In this context, $f(T)$ signifies a function concerning with the torsion scalar T. Meanwhile, L_{matter} stands for the Lagrangian density associated with matter. Additionally, "e" corresponds to the determinant of the tetrad field, which is intricately linked to the metric tensor through the relationship $e = \sqrt{-g}$. The non vanishing components of torsion tensor are defined as

$$T^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\nu\mu} - \Gamma^{\rho}_{\mu\nu} = h^{\rho}_{i}(\partial_{\mu}h^{i}_{\nu} - \partial_{\nu}h^{i}_{\mu}) \tag{5}$$

The elements of the corresponding contorsion tensor are characterized by:

$$K^{\mu\nu}_{\rho} = -\frac{1}{2}(T^{\mu\nu}_{\rho} - T^{\nu\mu}_{\rho} - T^{\mu\nu}_{\rho}) \tag{6}$$

The determination of the elements of the tensor $S_{\rho}{}^{\mu\nu}$ is carried out in the following manner:

$$S_{\rho}{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_{\rho} + \delta^{\mu}_{\rho}T^{\theta\nu}{}_{\theta} - \delta^{\nu}_{\rho}T^{\theta\mu}{}_{\theta}\right) \tag{7}$$

By using above components we have computed the torsion scalar, "T," as follows:

$$T = -2\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right) \tag{8}$$

The derivation of the modified field equation in the teleparallel theory of gravity involves obtaining it through the variation of the action concerning the vierbein components, denoted as h^{i}_{μ}. This is expressed as:

$$S^{\nu\rho}_{\mu}\partial_{\rho}Tf_{TT} + \left[e^{-1}e^{i}_{\mu}\partial_{\rho}(ee^{\alpha}_{i}S^{\nu\rho}_{\alpha} + T^{\alpha}_{\lambda\nu}S^{\nu}_{\lambda\alpha}\right]f_T + \frac{1}{4}\delta^{\mu}_{\nu}f = 4\pi T^{\mu}_{\nu} \tag{9}$$

Where T^{ν}_{μ} is the energy momentum tensor, $f_T = \frac{df}{dT}$ and $f_{TT} = \frac{d^2 f}{dT^2}$

The energy momentum tensor for perfect fluid with heat flow [42] is

$$T_{ij} = (\rho + p)u_i u_j + pg_{ij} + h_i u_j + h_j u_i \tag{10}$$

where ρ is the enrgy density, p is thermodynamic pressure, h_i is heat flow vector. The field equation corresponding to metric (1) are obtained by

$$f + 4f_T\left[\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB}\right] = -16\pi\rho \tag{11}$$

$$f + 4f_T\left[\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}\dot{B}}{AB}\right] + 4\frac{\dot{B}}{B}\dot{T}f_{TT} = 16\pi p \tag{12}$$

$$f + 2f_T\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 3\frac{\dot{A}\dot{B}}{AB}\right] + 2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\dot{T}f_{TT}\right) = 16\pi p \tag{13}$$

The crucial parameters in cosmological observations include the mean scale factor a, mean Hubble parameter H, scalar expansion θ, deceleration parameter q, shear scalar σ^2, and mean anisotropic parameter A_m. These quantities, derived from metric (1), are expressed as:

$$a = \left(AB^2\right)^{1/3} \tag{14}$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{15}$$

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{16}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H} - 1\right) \tag{17}$$

$$A_m = \frac{2}{9H^2}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{18}$$

3. SOLUTION OF FIELD EQUATION

Solving field equations (11), (12) and (13) we obtain

$$A(t) = \{(3+2m)(c_1 t + c_2)\}^{\frac{m}{3+2m}} \tag{19}$$

and

$$B(t) = \{(3+2m)(c_1 t + c_2)\}^{\frac{1}{3+2m}} \tag{20}$$

using equations (19) and (20) we get

$$T = \frac{-2(1+2m)}{(3+2m)^2 \cdot (c_1 t + c_2)^2} \tag{21}$$

$$f = -4K \frac{(1+2m)}{(5+3m)} (3+2m)^{\frac{2+m}{3+2m}} {c_1}^2 (c_1 t + c_2)^{-\frac{5+3m}{3+2m}} \tag{22}$$

$$\rho = \frac{K}{4\pi} \frac{(1+2m)(2+m)}{(5+3m)} {c_1}^2 \left((3+2m)(c_1 t + c_2)\right)^{-\frac{5+3m}{3+2m}} \tag{23}$$

$$p = \frac{K}{4\pi} \frac{(1+2m)}{(5+3m)} (3+2m)^{\frac{2+m}{3+m}} {c_1}^2 (c_1 t + c_2)^{-\frac{5+3m}{3+2m}} \tag{24}$$

The formulas for the Hubble parameter H, scalar expansion θ, shear scalar σ, and the mean anisotropic parameter A_m are obtained as follows:

$$\theta = 3H = c_1 \frac{2+m}{3+2m} (c_1 t + c_2)^{-1} \tag{25}$$

$$\sigma^2 = \frac{1}{3} \left(\frac{m-1}{3+2m} \right)^2 {c_1}^2 (c_1 t + c_2)^{-2} \tag{26}$$

$$A_m = \frac{2m^2 - 4m + 2}{(m+2)^2} \tag{27}$$

The value of the deceleration parameter is found to be

$$q = \frac{5m+7}{m+2} \tag{28}$$

which is constant.

By using $1 + z = \frac{a_0}{a}$ we get above physical parameters in terms of redshift z.

The torsion scalar is a geometric quantity associated with theories of gravity that incorporate torsion in addition to curvature. In the context of general relativity (which does not include torsion), the torsion tensor is assumed to be zero. However, in alternative theories of gravity, such as the Einstein-Cartan theory, torsion is considered. We have obtained torsion scalar in terms of redshift.

$$T = \frac{-2(1+2m)}{(3+2m)^2} c_1^3 \frac{(1+z)^{(\frac{3+2m}{m+2})}}{(3+2m)} \tag{29}$$

Figure 1 illustrates the correlation between the torsion scalar T and the redshift z across various constant m values in cosmological observations. The torsion scalar T, measuring spacetime geometry deviation from standard general relativity due to torsion, consistently exhibits negative values across all z and m, indicating a departure from general relativity. Redshift z, serving as a gauge of universe expansion and object distance, increases as the torsion scalar T decreases, suggesting a more pronounced deviation from general relativity in earlier cosmic epochs. Moreover, the value of m influences both the slope and magnitude of the torsion scalar T depicted in the figure, with higher m values corresponding to steeper slopes and smaller magnitudes of T.

$$f = \frac{-4K(1+2m)}{(5+3m)(3+2m)} (1+z)^{-3\frac{(5+3m)}{m+2}} \tag{30}$$

The relationship between pressure (P) and redshift (z) in cosmology is characterized by the equation of state parameter (ω). In the early universe dominated by radiation, ω is $\frac{1}{3}$, indicating positive pressure. As non-relativistic matter becomes dominant, ω for matter is 0, representing zero pressure. Dark energy, with a constant $\omega < 0$, contributes a negative pressure and is associated with the observed accelerated expansion of the universe.

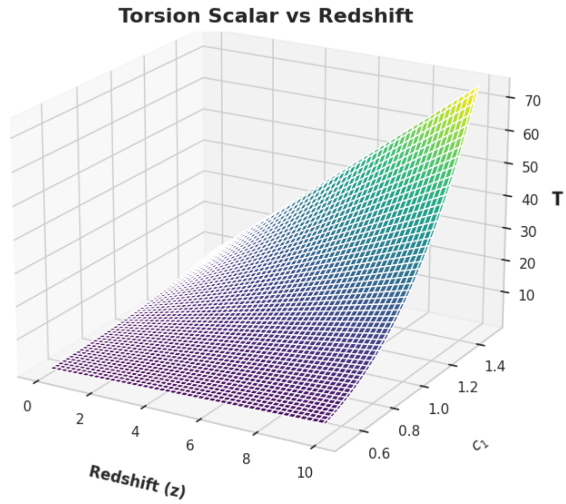
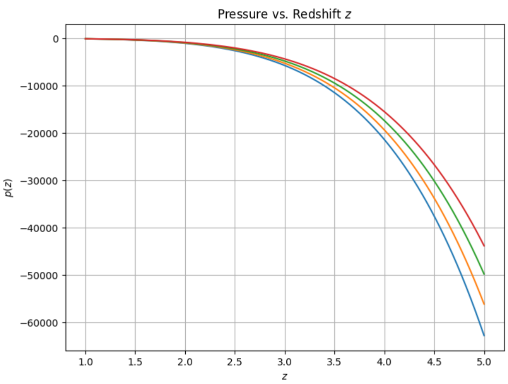
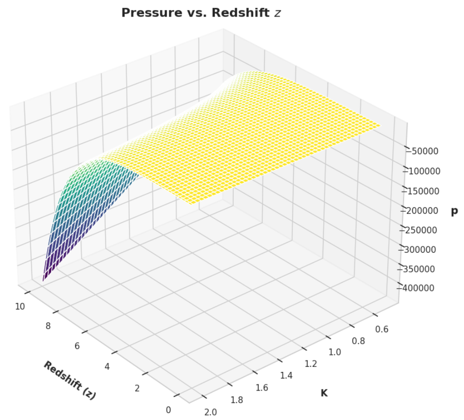


Figure 1. Torsion Scalar vs Redshift for $m = -1, 0.5 < c_1 < 1.5$



(a) 2D Plot



(b) 3D Plot

Figure 2. The plot of pressure vs cosmic redshift z

The pressure is calculated as follows:

$$p = \frac{K}{4\pi} \frac{(1+2m)(2+m)}{(5+3m)} (3+2m)(1+z)^{\frac{3(5+3m)}{m+2}} \tag{31}$$

In Figure 2, pressure is plotted against cosmic redshift z for different values of m, a constant in the Bianchi type-I cosmological model relating the expansion scalar and shear scalar. Pressure consistently exhibits negativity across all m and z values, indicative of a tension-like effect associated with dark energy, presumed to fuel the universe's accelerated expansion. With increasing redshift, pressure diminishes, reflecting a rise in dark energy density over time and its eventual dominance over matter and radiation in the late universe. Notably, smaller values of m correspond to more negative pressure, suggesting a heightened repulsive gravitational effect, where the anisotropy of the Bianchi type-I model enhances the impact of dark energy.

The relationship between energy density and redshift is influenced by the contributions from radiation, matter, dark energy, and possibly other components, and it is described by the evolving scale factor in the Friedmann equations of cosmology. Here we have obtained the energy density in terms of redshift for perfect fluid.

$$\rho = \frac{K}{4\pi} \frac{(1+2m)(2+m)}{(5+3m)} (1+z)^{-\frac{3(5+3m)}{m+2}} \tag{32}$$

From Figure 3 we observed that the density of the universe is positive and decreases with increasing redshift. Additionally, the density of the universe is higher for lower values of m, indicating a stronger influence

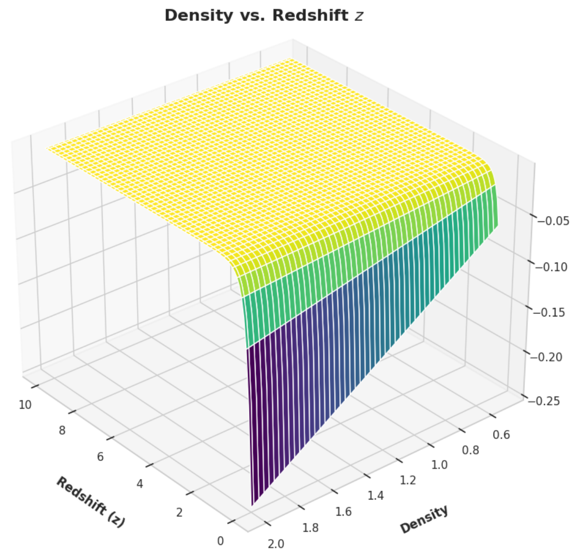


Figure 3. Density vs Redshift for $m = -1$

of matter on the cosmic expansion. Moreover, the density of the universe approaches zero as the redshift approaches infinity, implying a negligible contribution of matter in the early universe.

The equation of state parameter, typically denoted by ω, relates the pressure (p) to the energy density (ρ) of a substance. It's defined as $\omega = \frac{p}{\rho}$. The equation of state parameter (ω) characterizes the relationship between pressure and energy density in the universe. In the early universe, dominated by radiation, ω is $1/3$. As non-relativistic matter becomes dominant, ω for matter is 0, and for dark energy, assumed constant, ω is $<$ 0. The evolution of ω with redshift reflects the changing contributions of different cosmic components to the energy density over cosmic time. We have calculated the equation of state parameter in terms of redshift as follows:

$$\omega = (3 + 2m)(1 + z)^{\frac{6(5+3m)}{m+2}} \tag{33}$$

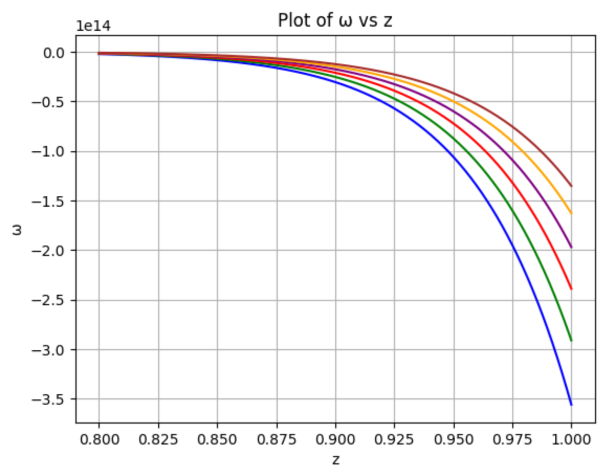


Figure 4. Equation of state parameter vs redshift

In Figure 4, we see how a parameter called the equation of state ω changes as the redshift z varies for different values of m. This parameter helps us understand the relationship between pressure and energy density in the universe. Some important points from Figure 4 include: all the curves have negative values for ω, indicating negative pressure, which is often associated with dark energy driving the universe's accelerated expansion; as the redshift increases, indicating earlier times in the universe, the negative pressure becomes stronger, suggesting that dark energy played a bigger role in the early universe; and the shape of the curves varies depending on the value of m, which reflects how the universe is structured (its anisotropy), indicating that the equation of state depends on the universe's structure.

The relationship between the Hubble parameter (H) and redshift (z) is often expressed in the context of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which describes the expanding universe in cosmology. The Hubble parameter is related to the rate of expansion of the universe.

Hubble parameter H is calculated as:

$$H = \frac{2+m}{3}(1+z)^{\frac{3(3+2m)}{m+2}} \tag{34}$$

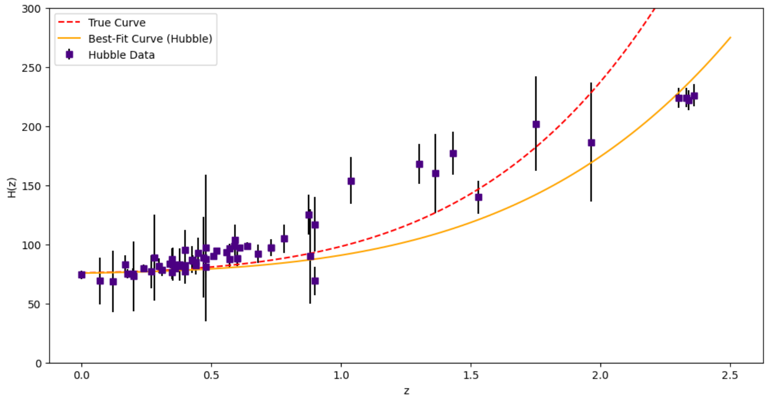


Figure 5. The plot of Hubbple parameter vs Redshift along with Hubble Data-Set

In Figure 5, we observe how the universe expands over time. It shows that as we look back in time, the universe expanded more slowly, measured by something called the Hubble parameter H at different redshifts z. The graph also indicates that the Hubble parameter is influenced by a constant called "m," which affects how the universe expands in a the model. Additionally, the graph compares theoretical expectations with real observations from telescopes like the Hubble Space Telescope and the Sloan Digital Sky Survey, and they seem to match up quite well, considering the uncertainties.

4. DISCUSSION AND CONCLUSION

Redshift in cosmology refers to the phenomenon where the light from distant galaxies or celestial objects appears to be shifted towards longer wavelengths, moving towards the red end of the electromagnetic spectrum. This is primarily due to the expansion of the universe.

As the universe expands, the space between galaxies also expands, causing the wavelengths of light emitted by these galaxies to stretch. This stretching of light results in a shift towards longer wavelengths, which is observed as a redshift. The greater the distance to a galaxy, the higher its redshift. Redshift is a crucial tool for astronomers in measuring the distances to and the velocities of objects in the universe.

In terms of redshift, as the universe expands, the effects of dark energy become more pronounced. If the pressure associated with dark energy remains negative, it can counteract the attractive force of gravity, leading to an accelerated expansion. This is consistent with the observations of distant supernovae and other cosmological data. The behavior of the universe is influenced when w is negative:

- **Accelerated Expansion:** The negative pressure associated with dark energy leads to an accelerated expansion of the universe. This is in contrast to matter, which has positive pressure and tends to slow down the expansion due to gravitational attraction.
- **Dominance at Late Times:** As the universe expands, the effects of dark energy become more pronounced over time. In the current epoch of the universe, dark energy is believed to be the dominant component, driving the observed acceleration.
- **Redshift of Distant Objects:** The acceleration of the universe affects the redshift of distant galaxies. Observationally, distant supernovae and other cosmological probes indicate that the rate of expansion is increasing with time.

The effect of the Hubble parameter on the universe can be summarized as follows:

- **Expansion Rate:** The Hubble parameter at a given redshift, $H(z)$, indicates the rate at which the universe is expanding at that particular cosmic time. A higher value of $H(z)$ implies a faster rate of expansion.

- **Historical Expansion:** Observing the Hubble parameter at different redshifts allows us to study the historical expansion of the universe. By looking at more distant objects, corresponding to higher redshifts, we are effectively looking back in time.
- **Cosmic Acceleration:** Changes in the Hubble parameter with redshift can provide insights into the cosmic acceleration. In a universe dominated by dark energy, the Hubble parameter may not decrease as much with increasing redshift as in a universe without dark energy.
- **Critical Density Determination:** The Hubble parameter is related to the critical density of the universe (ρ_{crit}). Understanding its behavior with redshift helps in determining the overall energy content and fate of the universe.

In this research, we explore the characteristics of the Bianchi type-I space-time within the framework of $f(T)$ gravity theory, where T represents the torsion scalar. The model is constructed based on specific assumptions. The first assumption posits a proportional relationship between the expansion scalar θ and the shear scalar σ, leading to the expression $A = B^m$, where A and B are metric coefficients, and m is a real constant. The second assumption sets equal pressure components in the x, y, and z directions, governed by an equation of state $p = \omega\rho$. Additionally, a power-law relation between F and the scale factor B is employed to derive the exact solution of the field equations.

Several key cosmological parameters, including the torsion scalar T pressure p, density ρ, equation of state parameter ω Hubble parameter Hin terms of cosmic time t and redshift z. The behaviour of the graph of the pressure vs redshift shows that the pressure is negative and constatly decreasing for the various values of $m = -1.03, -1.05, -1.07, -1.09$. If the pressure is negative (which corresponds to a situation where the substance has a tension-like effect rather than compressive), it can have significant implications for the evolution of the universe. A substance with negative pressure is often referred to as "exotic" or "dark energy."

The most well-known example of dark energy is the cosmological constant (Λ) associated with the vacuum of space. A negative pressure is a key component of dark energy because it's believed to be responsible for the observed accelerated expansion of the universe. The graph of density vs redshift we have plotted in 3D which shows that the density is positive. A positive density in terms of redshift typically refers to the energy density of matter in the universe. In cosmology, matter can have positive density, and its effects are often associated with the deceleration of the universe's expansion.

The pressure and density with the redshift shows the singularity at $m = -2$. Notably, the equation of state parameter ω shows the the negative behaviour and graph decreases rapidly for the values of $m = -2.201, -2.203, -2.205, -2.207, -2.209, -2.211$. When the equation of state parameter (w) in terms of redshift is negative, it implies that the substance in question has a negative pressure. This scenario is often associated with dark energy, which is believed to be responsible for the observed accelerated expansion of the universe.

The equation of state parameter is defined as $w = \frac{p}{\rho}$, where p is the pressure and ρ is the energy density. For dark energy, the negative pressure contributes to a repulsive gravitational effect, counteracting the attractive force of gravity caused by matter.

The Hubble parameter H plotted vs redshift z. The graph shows the values of Hubble parameter in the range of standard dataset wchich supports the current observational data.

The Hubble parameter in terms of redshift is a crucial observational quantity that informs us about the current state and past history of the universe's expansion. Studying its behavior with redshift provides valuable information about the underlying cosmological model and the influence of various components like matter, radiation, and dark energy on the evolution of the cosmos.

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ІДЕАЛЬНА РІДИНА З ТЕПЛОВИМ ПОТОКОМ У $f(T)$ ТЕОРІЇ ГРАВІТАЦІЇ

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Космологічні моделі Б'янкі типу I були предметом інтенсивних досліджень у космології через їхню простоту та актуальність для розуміння динаміки раннього Всесвіту. У цьому дослідженні ми досліджуємо динаміку таких моделей у рамках $f(T)$ гравітації, альтернативної теорії гравітації, яка розширює телепаралельну гравітацію шляхом введення загальної функції торсійного скаляра T. Ми акцентуємо увагу на наявності в космічному середовищі ідеальної рідини з тепловим потоком. Розв'язуючи рівняння поля гравітації $f(T)$, ми отримуємо точні розв'язки для космологічних моделей Б'янкі типу I. Ці рішення дають цінну інформацію про еволюцію Всесвіту та про те, як на неї впливає модифікована теорія гравітації. Крім того, ми виводимо космологічні параметри в термінах червоного зсуву, пропонуючи зручний спосіб інтерпретації даних спостережень і зв'язування теоретичних прогнозів з емпіричними вимірюваннями. Наші висновки не лише сприяють глибшому розумінню динаміки космологічних моделей Б'янкі типу I, але й створюють основу для порівняння $f(T)$ гравітації зі стандартною загальною теорією відносності в контексті спостережної космології. Це дослідження прокладає шлях для подальшого вивчення альтернативних теорій гравітації та їхнього впливу на еволюцію та структуру раннього Всесвіту.

Ключові слова: *теорія $f(T)$; ідеальна рідина з тепловим потоком; Б'янкі типу I*