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BIANCHI TYPE-III VISCOUS HOLOGRAPHIC RICCI DARK ENERGY COSMOLOGICAL MODEL IN BRANS-DICKE THEORY OF GRAVITATION

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This paper investigates and examines a wide range of findings related to the description of holographic Ricci dark energy (HRDE) with the bulk viscosity within Universe's late-time accelerated expansion in the framework of an anisotropic Bianchi type-III cosmological model with pressure less matter content in the Brans-Dicke theory of gravity. We are using the relationship between the metric potentials to obtain a precise conclusion to the field equations, resulting in a rapid expansion. To investigate the physical behaviour of our dark energy model, several major cosmological parameters, including Hubble, deceleration, matter energy density, Ricci dark energy density (RDE), and Equation of state (EoS), are used. We examined some of the viscosity of the holographic Ricci dark energy model using current cosmological observations. We describe how the models' physical and geometric properties are compatible with recent compilations.
Keywords: *Bianchi type- III metric; Holographic Ricci dark energy (HRDE); Bulk Viscosity; Brans-Dicke theory*
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1. INTRODUCTION

Recent experimental cosmology substantiates the idea that the Universe's expansion is accelerating [1-3]. The interaction between two galaxies caused by an unknown, dark element, that produces considerable negative pressure, is generally dark energy (DE). It is compatible with all cosmological substances inside the simplest model, that is, within a ΛCDM model. Still, it has problems similar to fine-tuning and thus, the cosmic coincidence problem, which has led to some indispensable styles to its description. Researchers have hypothecated that radiation and ordinary matter (baryons) account for 5% of the Universe's present energy, with the remaining 95% dominated by this dark component, to explain the Universe's ultimate-rapid growth.

Interestingly, dark energy explores as a possible explanation for the physical Universe's late-time rapid expansion [5]. Dark energy (DE) is considered to regard for 73% of the energy in our Universe, with Dark Matter (DM) accounting for 23% and baryonic matter accounting for the remaining 6% [6,7]. Two main approaches were proposed to explain this late time acceleration, the first is to investigate better DE possibilities, and the second is to investigate modified Einstein's theory of gravity. To characterize the behavior of DE events, the equation of state (EoS) parameter w ($= p/\rho$), where p represents pressure and ρ denotes energy density, is generally employed. The EoS parameter w with the range (-1, -1/3) describes the substance DE model, whereas $w = -1$explains the vacuum DE, also known as the cosmological constant or ΛCDM model, and $w < -1$ defines the phantom DE model. This phantom DE model will nearly clearly affect an ineluctable space-time singularity.

As a possible result of these difficulties, physicists worldwide have presented good dynamic DE models. These models classify into two categories: scalar-field models, which represent quintessence, phantom, k-essence, tachyon, and quintom [8-13], and interactions between the models, similar as the Chaplygin gas family, the brane-world Holographic DE (HDE), the age-graphic DE models [14-16], and so on. Changes in the quality of the Einstein-Hilbert reaction, for illustration, led to several modified gravity theories. Some modified gravity theories include the Brans-Dicke scalar-tensor theory [17], the f(R) and f(R, T) theories [18–22], (where R is the curvature scalar and T is the trace of the energy momentum tensor), and so on. Among the options, as preliminarily indicated, it is a good conception for a model to attract further, and further attention to the so-called "Holographic dark energy (HDE)". According to the holographic principle, the number of degrees of freedom of a physical system, the scope of the surrounding area L_2, rather than its volume, [23], will not be confined to an infrared (IR) cut-off point. Cohen et al. [24], confirm that the vacuum energy density is commensurable to the Hubble scale $L \approx H^{-1}$. The energy density of holographic dark energy is described by Li [25] as $\rho_\Lambda = 3c^2 M_{pl}^2 L^{-2}$, where L is the IR cut-off radius, C is constant, and $M_{pl}^2 = {}^1\!/_{8\pi G}$ is the Planck mass. This HDE model can explain the Universe's rapid expansion, and it is consistent with the data set available. Numerous investigations on the HDE models have been conducted to understand the Universe's accelerating nature. Later, Gao et al. [26] investigated the longer-term event horizon, which is supposed to be changed by inversion of the Ricci scalar curvature, i.e., $L \approx R^{-1/2}$. In this case, the model is called the Ricci Dark Energy Model. Granda and Oliveros [27, 28] proposed a new holographic Ricci Dark Energy Model with the energy density supplied by $\rho_\Lambda = \frac{3}{8\pi G}(\xi H^2 + \eta \dot{H})$, which was further meliorated by Chen and Jing [29] and termed modified holographic Ricci dark energy (MHRDE) with the energy density provided by

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$$\rho_h = \frac{3}{8\pi G}\left(\varepsilon_0 H^2 + \eta\dot{H} + \zeta\ddot{H}H^{-1}\right). \quad (1)$$

Sireesha et al. [30] examined modified holographic Ricci dark energy (MHRDE) cosmological models in f (R, T) gravity. In papers [31–34], the RDE is suitable for expressing the current acceleration of the Universe. During neutrino emulsion in the early cosmos, matter behaves like a viscous fluid. The coefficient of viscosity decreases as the Universe expands. The influence of viscosity on the expansion of the Universe, and Thus, the radiation anisotropy from a black body regenerated by Misner [35, 36] is greatly decreased by the extreme scattering produced on by viscosity. Rao et al. [37] investigated bulk viscous string cosmological models in the Saez – Ballester theory. Chakraborty et al. [38] derived the cosmology of an expanded form of HDE in the presence of bulk viscosity and the inflation dynamics using slow-roll parameters.

Anisotropic and spatially homogenous Universes have drawn a lot of attention in theoretical cosmology over the past few decades. Since the main observational data from CMBR (Bennett et al. [39]) has been thought to suggest the presence of an evolution from the anisotropic to the isotropic phase of the cosmos (Akarsu and Kilinc [40]). Additionally, it has been suggested that the isotropic FRW model might not provide an accurate and comprehensive description of matter in the early epochs of the universe. For a realistic analysis of cosmological models, it is necessary to assume spatially homogeneous and anisotropic space-times in order to examine whether they may evolve to the known level of homogeneity and isotropy. Many scientists have been interested in Bianchi type (BT) cosmological models, which are homogenous but not necessarily isotropic among different anisotropic space-times. Many scientists have developed innovative cosmological models in recent years that use DE against the context of anisotropic Bianchi space-times. Koussour et al., [41, 42] explored Anisotropic nature of space-time in **f(Q)** gravity and Late-time acceleration in **f(Q)** gravity: Analysis and constraints in an anisotropic background.

Modified theories of gravitation are generalized models that lead to the rise of changes in the force of gravity as a result of the Einstein-Hilbert action. Most importantly, in scalar-tensor gravity, Brans-Dick (BD) theory provides a practical approach to explore cosmic events in later ages of the Universe. This theory's significant features include variable gravitational constants, a scalar field and geometry combination, compliant with numerous physical laws, and solar launch restrictions. Variable-G theories developed from the Brans-Dicke (BD) scalar-tensor theory [17] and its generalization to other types of scalar-tensor theories, similar to the overall scalar-tensor theory with particular terms presented by Wagoner [43]. During this theory, the inverse of the gravitational constant G is substituted by a scalar field $'\varphi'$, which is the gravitational force coupling according to a new parameter. Within the Brans-Dicke field equations, the combined scalar and tensor fields are given by

$$G_{ij} = -8\pi\varphi^{-1}T_{ij} - \omega\varphi^{-2}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,k}\varphi^{,k}\right) - \varphi^{-1}\left(\varphi_{i;j} - g_{ij}\varphi_{;k}^{;k}\right), \quad (2)$$

and

$$\varphi_{;k}{}^{,k} = 8\pi(3+2\omega)^{-1}\mathrm{T} \quad (3)$$

We have also constructed an equation for energy conservation.

$$T^{ij}_{;j} = 0 \quad (4)$$

This equation results from the field equations (2) and (3).

Several authors have examined the different features of Brans-Dicke cosmology thoroughly. Rao and Sireesha [44, 45] studied several string cosmological models within the Brans-Dicke theory of gravity, while Rao et al. [46] anatomized the FRW holographic dark energy as a cosmological model. Dasu Naidu et al. [47] developed a new holographic DE cosmological model within the SB theory of gravity with an anisotropic background. Sandhya Rani et al. [48] investigated Bianchi type-III, V, and VI0 generalized ghost dark energy models with polytropic gas using the Brans-Dicke (BD) theory of gravity. Sireesha & Rao [49] examined Bianchi type II, VIII, and IX Holographic dark energy cosmological models within the Brans-Dicke theory of gravity.

According to the earlier, we should take into account the spatially homogenous anisotropic Bianchi type-III cosmological model with pressure less matter and holographic Ricci dark energy with bulk viscosity in the Brans – Dicke theory of gravity. We structured our efforts for this paper as follows: In section 2, we derived the Brans-Dicke theory field equations in the presence of pressure less matter and holographic Ricci dark energy with bulk viscosity within the Bianchi type-III cosmological model. In Section 3, we solved the field equations. In Section 4, we developed and examined numerous essential physical properties of the model. In section 5, we concluded our compliances.

2. METRIC AND FIELD EQUATIONS:

We consider a spatially homogeneous Bianchi type-III metric form

$$ds^2 = dt^2 - A^2x^2 - B^2e^{-2x}y^2 - C^2z^2 \quad (5)$$

whereA,B andC are functions of ‘t’ only.

The holographic Ricci dark energy (HRDE) energy momentum tensor with bulk viscosity is $T_{ij} = T_{ij}^{m} + T_{ij}^{h}$, T_{ij}^{m} is that the energy-momentum tensor for matter and T_{ij}^{h} is that the energy momentum tensor for holographic Ricci dark energy. They are

$$T_{ij}^{h} = (\rho_h + \overline{p_h})u_i u_j - \overline{p_h} g_{ij} \tag{6}$$

and

$$T_{ij} = \rho_m u_i u_j \tag{7}$$

Where ρ_m denotes the matter energy density, ρ_h indicates the holographic Ricci dark energy density, and $\overline{p_h}$ means the viscous holographic dark energy's pressure.

In the co-moving frame of reference, and that we will get

$$T_1^{m\,1} + T_1^{h\,1} = T_2^{m\,2} + T_2^{h\,2} = T_3^{m\,3} + T_3^{h\,3} = -\overline{p_h}\,\&\, T_4^{m\,4} + T_4^{h\,4} = \rho_m + \rho_h \tag{8}$$

where ρ_m, ρ_h and $\overline{p_h}$ are all functions of time 't' only.

The viscous holographic Ricci dark energy pressure satisfies the relation $\overline{p_h} = p_h - 3\varepsilon H$. In the late time evolution of the universe, dark energy with bulk viscosity generates phantom type rapid expansion. We have assumed that dark matter has no pressure, which the effective pressure of viscous holographic Ricci dark energy is suitable to the sum of HRDE pressure and bulk viscosity.

The field equations of Brans-Dicke theory (2), for the metric (5) with the backing of equations (6) - (8) are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{\omega\dot{\varphi}^2}{2\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{B}\dot{\varphi}}{B\varphi} + \frac{\dot{C}\dot{\varphi}}{C\varphi} = -8\pi\varphi^{-1}\overline{p_h}, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\omega\dot{\varphi}^2}{2\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{A}\dot{\varphi}}{A\varphi} + \frac{\dot{C}\dot{\varphi}}{C\varphi} = -8\pi\varphi^{-1}\overline{p_h} \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\omega\dot{\varphi}^2}{2\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{A}\dot{\varphi}}{A\varphi} + \frac{\dot{B}\dot{\varphi}}{B\varphi} - \frac{1}{A^2} = -8\pi\varphi^{-1}\overline{p_h} \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{\omega\dot{\varphi}^2}{2\varphi^2} + \frac{\dot{A}\dot{\varphi}}{A\varphi} + \frac{\dot{B}\dot{\varphi}}{B\varphi} + \frac{\dot{C}\dot{\varphi}}{C\varphi} - \frac{1}{A^2} = 8\pi\varphi^{-1}(\rho_m + \rho_h) \tag{12}$$

$$\frac{1}{A^2}\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) = 0 \tag{13}$$

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 8\pi(3+2\omega)^{-1}(\rho_m + \rho_h - 3\overline{p_h}) \tag{14}$$

$$\dot{\rho}_m + \dot{\rho}_h + 3\mathrm{H}(\rho_m + \rho_h + \overline{p_h}) = 0 \tag{15}$$

3. HOLOGRAPHIC RICCI DARK ENERGY COSMOLOGICAL MODELS

From equation (13), we get $\mathrm{B} = c_1\mathrm{A}$ without loss of generality, by taking $c_1 = 1$, we get

$$\mathrm{B} = \mathrm{A}. \tag{16}$$

The field equations (9) to (12) are four independent equations with six unknowns that are functions of 't': $\mathrm{B}, \mathrm{C}, \rho_m, \rho_h, \overline{P}_h$ & φ . Because these equations are very nonlinear, we use the following reasonable physical condition to obtain a deterministic result:

1. The connection between the scalar field φ and thus the average scale factor, a(t), is given by (Johri and Kalyani [50])

$$\varphi = a^m \tag{17}$$

where m is a constant.

2. The shear scalar is proportional to the expansion scalar, leading to the metric potentials relation (Collins et al. [51])

$$\mathrm{C} = B^n \tag{18}$$

where *n* is an arbitrary constant.

3. We assume that the parameterized kind of Bulk viscosity is (Reng and Meng [52])

$$\varepsilon = \left(\varepsilon_1 + \varepsilon_2\left(\frac{\dot{a}}{a}\right) + \varepsilon_3\left(\frac{\ddot{a}}{\dot{a}}\right)\right), \tag{19}$$

where $\varepsilon_0, \varepsilon_1$ & ε_2 are constants.

From equations (9) - (11), (17) and (18), we get

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)+\frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)-\frac{1}{B^2}=0. \tag{20}$$

Using equations (17 & 18), From equation (20), we get

$$\mathrm{A}=\mathrm{B}=\mathrm{K}\,\mathrm{Sinh}(k_2\mathrm{t}+k_3), \tag{21}$$

$$\mathrm{C}=[\mathrm{K}\,\mathrm{Sinh}(k_2\mathrm{t}+k_3)]^n. \tag{22}$$

From equations (17), (21) and (22), we get

$$\varphi=[K\,Sinh(k_2\mathrm{t}+k_3)]^{-(n+2)}. \tag{23}$$

where $\mathrm{K}=\frac{1}{k_{2\sqrt{n-1}}}$, k_2& k_3 are integrating constants

The Hubble parameter H is

$$\mathrm{H}=\frac{1}{3}\left(\frac{2\dot{B}}{B}+\frac{\dot{C}}{C}\right)=\frac{(n+2)K_2}{3}\,\mathrm{Coth}(k_2\mathrm{t}+k_3) \tag{24}$$

Now metric (5), with the assistance of metric potentials (21 & 22), are frequently written as

$$ds^2=dt^2-[K\,Sinh(k_2\mathrm{t}+k_3)]^2(dx^2+e^{-2x}dy^2)-[KSinh(k_2\mathrm{t}+k_3)]^{2n}dz^2 \tag{25}$$

The energy density of holographic Ricci dark energy within the universe could also be calculated using the equations (1) and (24) as

$$\rho_h=\frac{3k_2^2\varphi}{8\pi}\left[Coth^2(k_2t+k_3)\left\{\frac{\varepsilon_0(n+2)^2}{9}-\frac{\eta(n+2)}{3}+2\zeta\right\}+\frac{\eta(n+2)}{3}-2\zeta\right] \tag{26}$$

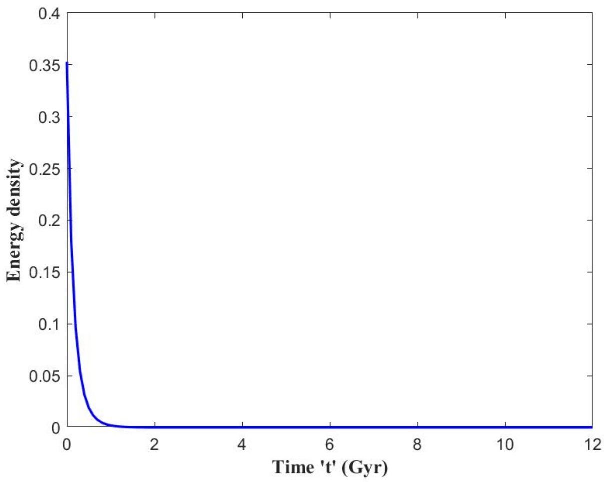

Figure 1. Plot of energy density versus time for $k_2=1, k_3=0.85, n=1.81, \varepsilon_0=1.015, \eta=0.025$ and $\zeta=0.15$

In terms of cosmic time 't', Fig. 1 describes the energy density of HRDE with bulk viscosity of our model. Throughout the development of the model, the energy density is positive. We also see that as cosmic time increases, the energy density ρ_h declines. As a result, we may infer that our model achieves the realistic energy requirements, $\rho_h \geq 0$.

We suppose that matter and viscous holographic dark energy factors interact minimally. Hence, they're conserved independently (Sarkar [53]). Therefore, as result of the conservation equation

$$\dot{\rho}_m+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\rho_m=0, \tag{27}$$

$$\dot{\rho}_m+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)(\rho_h+\overline{p_h})=0. \tag{28}$$

From eqs. (21), (22) and (27), we get the energy density of the matter is

$$\rho_m=k_1[K\,Sinh(k_2\mathrm{t}+k_3)]^{-(n+2)}, \tag{29}$$

where k_1 is an integrating constant.

The coefficient of bulk viscosity is given by

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \left[\left({}^{k_2(n+2)}/_{3}\right) \, Coth(k_2 t + k_3)\right] + \varepsilon_3 \left[k_2 \left({}^{(n-1)}/_{3}\right) Coth(k_2 t + k_3) + k_2 \, Tanh(k_2 t + k_3)\right]. \tag{30}$$

Using equations (21) – (23), we obtain the viscous pressure of the holographic Ricci dark energy as

$$\bar{P}_h = \frac{\varphi}{16\pi}\left[k_4 Coth^2(k_2 t + k_3) + k_5\right] \ , \tag{31}$$

where $k_4 = \left\{\frac{1}{K^2} - (n+3)k_2 - k_2^2[\omega(n+2)^2 - 2(3n+4) + 2(n+3)(n+2)]\right\}$
and $k_5 = \left\{2(n+2)k_2^2 - (n+3)k_2 - \frac{1}{K^2}\right\}$

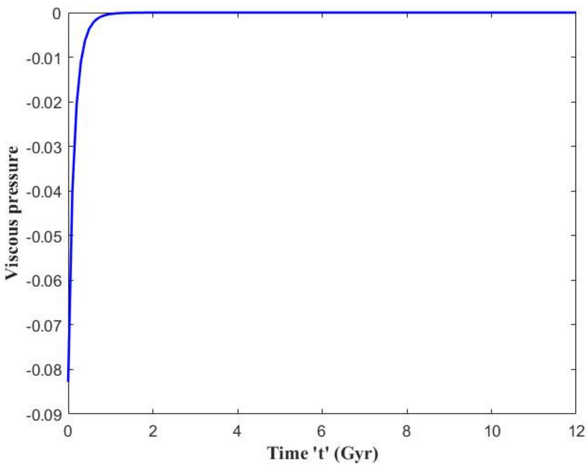

Figure 2. Plot of viscous pressure versus time for $k_2 = 1, k_3 = 0.85, n = 1.81, \varepsilon_0 = 1.015, \eta = 0.025, w = -1.25 \ and \ \zeta = 0.15$

According to the graph of Fig. 2, the viscous pressure of the HRDE is constantly negative throughout the elaboration and rises with time. Based on the substantiation of the Universe's rapid expansion, it is generally allowed that some energy-matter causes this cosmic acceleration with negative pressure known as DE.

The pressure of the holographic Ricci dark energy as

$$P_h = \frac{\varphi}{16\pi}\left[k_4 Coth^2(k_2 t + k_3) + k_5\right] + k_2(\mathrm{n}+2)\left\{\varepsilon_1 Coth(k_2 t + k_3) + \frac{k_2}{3}\mathrm{Coth}^2(k_2 t + k_3)\left[\varepsilon_2(n+2) + \varepsilon_3(n-1)\right] + \varepsilon_3 k_2\right\} \tag{32}$$

From equations (26) and (32), we obtain
The EoS parameter of the Ricci dark energy as

$$W_h = \frac{P_h}{\rho_h} = \frac{\frac{\varphi}{16\pi}\left[k_4 Coth^2(k_2t+k_3)+k_5\right]+k_2(n+2)\{\varepsilon_0 Coth(k_2t+k_3)+\frac{k_2}{3}Coth^2(k_2t+k_3)[\varepsilon_1(n+2)+\varepsilon_2(n-1)]+\varepsilon_2 k_2\}}{\frac{3\varphi k_2^2}{8\pi}\left[Coth^2(k_2t+k_3)\left\{\frac{\varepsilon_0(n+2)^2}{9}-\frac{\eta(n+2)}{3}+2\zeta\right\}+\frac{\eta(n+2)}{3}-2\zeta\right]}. \tag{32}$$

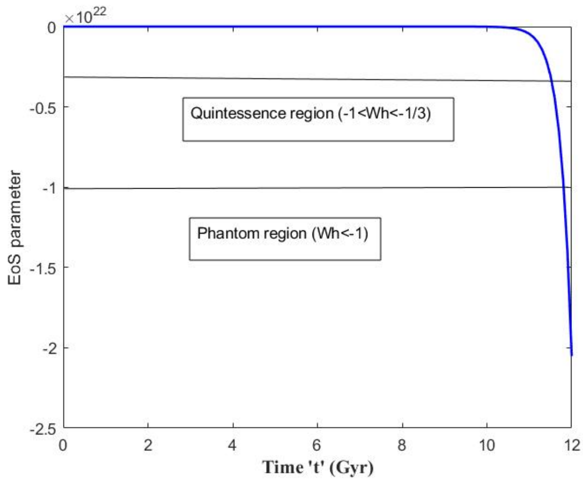

Figure 3. Plot of EoS parameter versus time for $k_2 = 1, k_3 = 0.85, n = 1.81, \varepsilon_0 = 1.015, \eta = 0.025, w = -1.25 \ and \ \zeta = 0.15$

The EoS parameter is defined as $w = p/\rho$, where p stands for pressure and ρ stands for energy density. Various EoS parameter values correlate to various epochs of the Universe's early decelerating and current accelerating expansion. The decelerating phase consists of stiff fluid (w = 1), radiation (w = 1/3), and matter-dominated (dust) (w = 0). The accelerating phase is made up of quintessence -1< w < -1/3, the cosmological constant w = -1, and the phantom case, w < -1.

The above Fig.-3 illustrates the EoS parameter as a function of cosmic time (t). We see that at the very beginning of the cosmos, the EoS parameter $w_h > 0$. As a result, at early epochs of the Universe, the EoS parameter of HRDE with bulk viscosity may play a significant role in representing the initial stage matter-dominated era of the universe. It is discovered that the EoS parameter reaches zero at some point, implying that our model reflects a dusty universe. Meanwhile, the EoS parameter of HRDE with bulk viscosity acts like quintessence (-1 < w_h < -1/3) and phantom (w_h < -1), resulting in an accelerated expansion phase of the cosmos.

The coincident parameter is

$$\mathrm{r} = \frac{\rho_h}{\rho_m} = \frac{\frac{3k_2^2\varphi}{8\pi}\left[Coth^2(k_2t+k_3)\left\{\frac{\varepsilon_0(n+2)^2}{9}-\frac{\eta(n+2)}{3}+2\zeta\right\}+\frac{\eta(n+2)}{3}-2\zeta\right]}{K_1[K\,inh(k_2\mathrm{t}+k_3)]^{-(n+2)}} \tag{33}$$

4. SOME ESSENTIAL PROPERTIES OF THE MODEL

The spatial volume is

$$\mathrm{V} = \mathrm{ABC} = [K\,Sinh(k_2\mathrm{t}+k_3)]^{(n+2)} \tag{34}$$

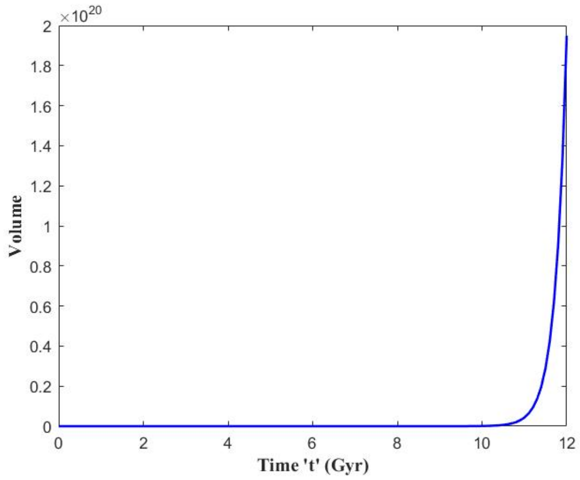

Figure 4. Plot of Volume versus time for $k_2 = 1, k_3 = 0.85$ & $n = 1.81$

Fig. 4 illustrates the volume concerning cosmic time. We can see that as the time 't' rises, the spatial volume 'V' increases and eventually becomes infinitely large, as displayed in the graph. It demonstrates that the cosmos expands as time goes.

The average scale factor is

$$\mathrm{a(t)} = V^{1/3} = [K\,Sinh(k_2\mathrm{t}+k_3)]^{(n+2)/3}. \tag{35}$$

For the flow vector u^i, the expansion and shear scalar expressions are calculated as

$$\theta = 3\mathrm{H} = k_2(\mathrm{n}+2)Coth(k_2\mathrm{t}+k_3). \tag{36}$$

$$\sigma^2 = \frac{7}{18}\theta^2 = \frac{7}{18}k_2^2(n+2)^2Coth^2(k_2\mathrm{t}+k_3) \tag{37}$$

Along the axes x,y, and z, the directional Hubble parameters H_1, H_2, and H_3 are given by

$$H_1 = H_2 = \frac{\dot{A}}{A} = \frac{\dot{B}}{B} = k_2Coth(k_2\mathrm{t}+k_3), \tag{38}$$

$$H_3 = \frac{n\dot{B}}{B} = nk_2Coth(k_2\mathrm{t}+k_3), \tag{39}$$

whereas the generalized Hubble parameter is given by

$$\mathrm{H} = \frac{1}{3}\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}\right) = \frac{k_2(n+2)}{3}Coth(k_2\mathrm{t}+k_3) \tag{40}$$

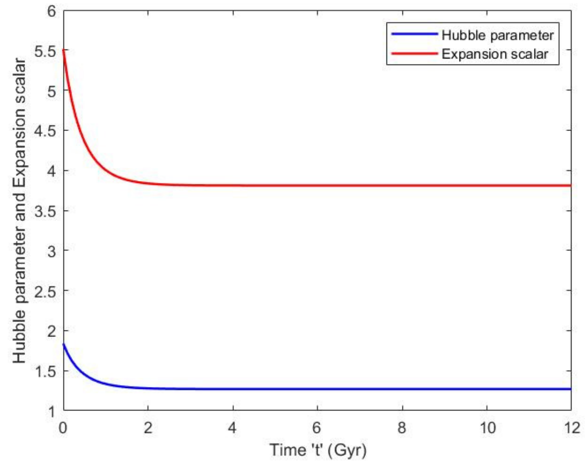

Figure 5. Plot of Hubble parameter & Expansion scalar versus time for $k_2 = 1, k_3 = 0.85$ & $n = 1.81$

The graph of Fig. 5 above depicts the variability of the Hubble parameter and the expansion scalar concerning cosmic time. These are two significant observable factors in cosmology that play an essential part in the Universe's expansion path. Both the expansion scalar graph and the Hubble parameter demonstrate that they are positive-valued, decreasing functions of cosmic time that approach zero over very long time periods. The Hubble parameter is generally known to be the rate of expansion of the Universe, and a measure of the fractional increase in the scale in unit time of the Universe. It is discovered that the expansion rate is quicker in the beginning and slower later on.

The average anisotropy parameter is defined by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2(n-1)^2}{(2n+1)^2} \tag{41}$$

The deceleration parameter (DP) q is given by

$$q = -\frac{3\dot{\theta}}{\theta^2} - 1 = \frac{3}{(n+2)} Sech^2(k_2 t + k_3) - 1 \tag{42}$$

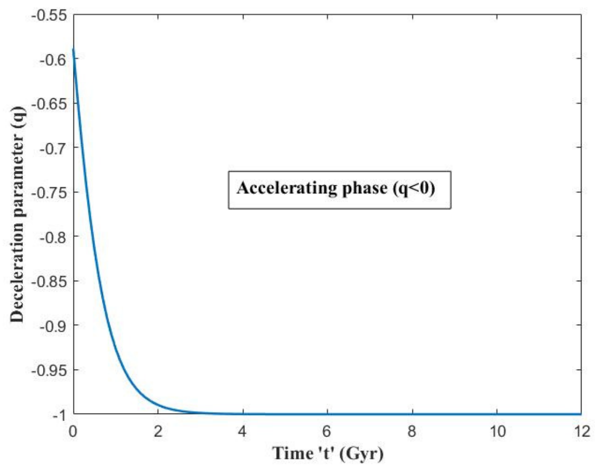

Figure 6. Plot of deceleration parameter versus time for $k_2 = 1, k_3 = 0.85, n = 1.81$

The model's acceleration or deceleration is determined by the sign of the deceleration parameter (DP) ‘q’. A positive deceleration parameter (DP) ‘q’ value implies model slowdown, whereas a negative sign suggests model acceleration. Fig. 6, depicts the elaboration of the deceleration parameter following cosmic time. We discover that our model displays a Universe expansion phase. It can also be shown that the current value (i.e., at $t_o = 13.8\ Gyr$) of the deceleration parameter is $q_o = -1$, which is consistent with recent observations.

5. CONCLUSIONS

This study investigates the spatially homogeneous anisotropic Bianchi type-III cosmological model with bulk viscosity in the presence of pressure less matter and holographic Ricci dark energy in the Brans – Dicke theory of gravity.

It is essential to remember that physical volume increases with time. Fig. 4 illustrates the volume concerning cosmic time. We can see that as the time 't' increases, the spatial volume V increases and ultimately becomes infinitely large, as displayed in the graph. It demonstrates that the Universe expands as time goes. It is frequently seen that the above-mentioned physical parameters, such as θ, σ^2 and H of the model, acquire a constant value at late intervals. This indicates that the cosmos expands consistently. Fig.5 depicts the variability of the Hubble parameter and the expansion scalar concerning cosmic time. These are two significant observable factors in cosmology that play an essential part in the Universe's expansion path. The Hubble parameter and the expansion scalar graphs show that both are positive-valued decreasing functions of cosmic time that tend to zero over long time scales. The Hubble parameter is generally acknowledged as the rate of expansion of the Universe, and a measure of the fractional increase in the scale in unit time of the Universe. It is discovered that the expansion rate is faster in the beginning and slower later on. We can also see that as time increases, the energy density ρ_m of matter increases, and therefore the Ricci dark energy (RDE) density ρ_h decreases. In terms of cosmic time 't', fig. 1 describes the energy density of HRDE with the bulk viscosity of our model. Throughout the development of the model, the energy density is positive. We also see that as cosmic time increases, the energy density ρ_h decreases. As a result, we may infer that our model achieves the actual energy needs, $\rho_h \geq 0$. Figure-2 shows the viscous pressure of the HRDE is constantly negative throughout the elaboration and increases with time. Based on the substantiation of the Universe's rapid expansion, it is generally allowed that some energy-matter causes this cosmic acceleration with negative pressure known as DE. We will see that Ricci Dark Energy has a negative pressure during the Universe's development, suggesting that the Universe is expanding faster. In our model, the EoS parameter starts within the phantom area ($W_h << -1$) and crosses the phantom dividing line ($W_h = -1$). Within the region of quintessence, the model progressively approaches the constant value ($-1 < W_h < {}^{-1}/_{3}$). Fig.3 illustrates the EoS parameter as a function of cosmic time (t). We see that at the very beginning of the cosmos, the EoS parameter $w_h > 0$. As a result, at early epochs of the Universe, the EoS parameter of HRDE with bulk viscosity may play a significant role in representing the initial stage matter-dominated era of the Universe. It is discovered that the EoS parameter reaches zero at some point, implying that our model reflects a dusty Universe. Meanwhile, the EoS parameter of HRDE with bulk viscosity acts like quintessence ($-1 < w_h < -1/3$) and phantom ($w_h < -1$), resulting in an accelerated expansion phase of the cosmos. The model's acceleration or deceleration is determined by the sign of the deceleration parameter (DP) 'q'. A positive deceleration parameter (DP) 'q' value implies model slowdown, whereas a negative sign suggests model acceleration. Fig. 6, depicts the evolution of the deceleration parameter in accordance to cosmic time. We discovered that our model displays a Universe expansion phase. It can also be shown that the current value (i.e., at $t_o = 13.8$ *Gyr*) of the deceleration parameter is $q_o = -1$, which is consistent with recent observations.

Data availability statement: This manuscript has no associated data or the data will not be deposited. [Authors' comment: All data analysed during this study are presented in this published article.] Derived data supporting the findings of this study are available from the corresponding author [***K.V.S. Sireesha***] on request.

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В'ЯЗКА ГОЛОГРАФІЧНА КОСМОЛОГІЧНА МОДЕЛЬ ТЕМНОЇ ЕНЕРГІЇ РІЧІ ТИПУ Б'ЯНКІ III В ТЕОРІЇ ГРАВІТАЦІЇ БРЕНСА-ДІКЕ

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У цій роботі досліджується та розглядається широкий спектр знахідок, пов'язаних з описом голографічної темної енергії Річчі (HRDE) з об'ємною в'язкістю в рамках прискореного розширення Всесвіту в пізній час у рамках анізотропної космологічної моделі Б'янкі типу III із вмістом матерії без тиску. в теорії гравітації Бренса-Дікке. Ми використовуємо співвідношення між метричними потенціалами, щоб отримати точний висновок рівнянь поля, що призводить до швидкого розширення. Щоб дослідити фізичну поведінку нашої моделі темної енергії, використовуються кілька основних космологічних параметрів, включаючи Хаббла, уповільнення, щільність енергії матерії, щільність темної енергії Річчі (RDE) і рівняння стану (EoS). Використовуючи поточні космологічні спостереження, ми виявили деяку в'язкість голографічної моделі темної енергії Річчі. Ми описуємо, як фізичні та геометричні властивості моделей сумісні з останніми компіляціями.

Ключові слова: *метрика Б'янкі III типу; голографічна темна енергія Річчі (HRDE); об'ємна в'язкість; теорія Бренса-Дікке*