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AXIAL STRUCTURE OF GAS DISCHARGE SUSTAINED BY THE EIGEN DIPOLAR WAVE OF THE METAL WAVEGUIDE WITH VARYING RADIUS FILLED BY MAGNETIZED NONUNIFORM PLASMA

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The article presents the results of the theoretical study of the plasma density axial distribution in a stationary gas discharge sustained by the eigen dipolar wave that propagates in a long cylindrical plasma-metal structure. The discharge structure consists of a column of magnetized non-uniform plasma placed in the metal waveguide of variable radius. The study of the gas discharge is carried out within the framework of the electrodynamic model, in which the main attention is paid to the electrodynamic part of the model. To describe the processes that take place in plasma, the model equations are used. The influence of the metal waveguide inhomogeneity along the structure and the plasma density radial non-uniformity on the phase characteristics of the dipolar wave, its spatial attenuation, the field components radial distribution, the axial distribution of the plasma density sustained by this mode are determined. It is also analysed the condition for the discharge stability and find the regions, where dipolar mode can sustain the stable discharge. The obtained results can be useful for various technological applications.

Keywords: *Gas discharge; Plasma-metal waveguide; Dipolar eigen wave; Phase and attenuation properties; Zakrzewski criterion*

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1. INTRODUCTION

Till now microwave gas discharge in stationary regime that is sustained by the eigen electromagnetic wave of the discharge structure is used as the effective plasma sources in different technological applications [1, 2, 3]. One of the important property of such discharges that take place in cylindrical structures and sustained by the waves with azimuthal wave numbers $m = 0, \pm1, \pm2$ is good uniformity of plasma density axial distribution [1, 2, 4]. Theoretical modeling of plasma density axial distribution of the eigen wave sustained discharges and the study of the stability conditions of such discharges, was carried out within the framework of the electrodynamic approach [2], which shows a good agreement with the experimental data [1]. One of the characteristic feature of this approach is the detailed study of the electrodynamics characteristics of the eigenwaves of the discharge structure, such as the dispersion (phase) properties, the spatial attenuation coefficient, and the wave field spatial structure. According to this approach plasma is described with rather simple model equations. Such model approach is simple but allow us to take into account the elementary processes occurring in plasma in proper manner, that relates, among other things, with the wave energy transfer to plasma in the diffusion and recombination gas discharge regimes [2, 5]. The number of articles were devoted to the study of the plasma density axial distribution in the stationary microwave discharges that take place in cylindrical plasma-metal structures with the fixed radius of metal enclosure [1, 6]. Besides it was shown that variable radius of the metal wall of plasma – metal cylindrical discharge structure can be used as one of the mechanism to control the plasma density value and its axial distribution in such discharges [7]. The work [8] was devoted to the study of plasma density axial distribution in the discharges, that are sustained by the eigen waves of such discharge structure taking into account the approximation of plasma density radial uniformity. The involving of a plasma density radial density nonuniformity to the model leads to more accurate desctiption of the plasma density axial distribution in the discharge. Let emphasize that long gas discharges can be sustained not only by symmetric ($m = 0$), but also by the dipolar ($m = \pm1$), eigen wave what significantly affect as the plasma density axial profile, as the stability region of such discharge. The experiments have shown that the use of eigen dipole ($m = \pm1$) waves give the possibility to obtain somewhat smaller plasma density values, but axial uniformity of plasma density profile is much better as compared with the discharges sustained by the symmetric wave [1, 2].

Previously, the study of the plasma density radial distribution for the discharges sustained by the symmetric ($m = 0$) mode was carried out in the work [9] with taking into account the radial inhomogeneity of the plasma density and changes in the radius of the metal waveguide along the discharge. The influence of the variable radius

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of the waveguide metal enclosure and the kind of plasma density radial profile on the dispersion properties, the attenuation coefficient and radial field structure of the eigen dipole ($m = \pm 1$) wave that sustains the discharge was studied in [10]. The aim of this work is to study the influence of the variable radius of the waveguide metal enclosure and the kind of plasma density radial profile on axial structure of the discharge sustained by the eigen dipole ($m = \pm 1$) waves of the discharge structure.

2. TASK SETTINGS

Let us study the plasma density axial distribution in a long cylindrical discharge structure that consists of non-uniform magnetized plasma column with radius R_p that surrounds by metal enclosure of radius $R_m > R_p$. The thin dielectric tube that separates plasma region from metal enclosure is not presented in this model. It is supposed that it's influence on wave phase and attenuation properties is not very strong [11]. Plasma region is separated form the waveguide metal wall by the vacuum (air) region with thickness $(R_m - R_p)$. The metal enclosure is supposed to be slightly varying in the axial direction. The studied discharge is sustained by the eigen dipolar ($m = \pm 1$) wave of this discharge structure that propagates along the discharge. External steady magnetic field $\vec{B_0}$ is directed along the axis of waveguide structure.

Plasma is described according to the hydrodynamic approach as cold and weakly absorbing media. The wave while propagating along the discharge damps due to the collisions in plasma. These collisions are characterized by the effective electron collision rate ν. The collision rate is supposed to be small as compared with wave frequency ω. It is also supposed that plasma density is non-uniform not only is axial, but also in radial directions. In the gas discharge model the plasma density radial radial distribution $n(r)$ models by the Bessel-like profile of the form $n(r) = n(0)J_0(\mu r)$, where $n(0)$ is plasma density at the axis of the plasma column ($r = 0$), J_0 is the Bessel function of the first kind and μ is the plasma density non-uniformity parameter. It's value can vary from $\mu = 0$ for the case of radial uniform plasma up to the $\mu = 2.405$ for the case of strong radially non-uniform plasma that corresponds to the discharge in the ambipolar diffusion regime. Such choice of plasma density radial profile give the possibility to model different gas discharge regimes [1, 9]. A detailed description of the procedure for deriving the electrodynamic equations is given in the previous work [1, 9]. Here we present the results that are important for the current study.

The dipolar wave (azimuth wave number $m = \pm 1$) propagates along the axis of the structure in the direction of the external magnetic field $\vec{B_0}$. It was supposed that wave damps slightly while propagates along the discharge and sustains plasma column. So, as wave field, as plasma density also slightly varies in along the discharge on the distances of wavelength order. Thus, the solutions of the Maxwell equations system that governs the wave propagation along the discharge structure can be found according to the WKB approach [12]:

$$E, H_{r,\varphi,z}(r, \varphi, z) = E, H(r, z) \exp\left(-i\omega t + im\varphi + \int\limits_{z_0}^{z} k_3(z')dz'\right), \tag{1}$$

here r, φ, z are coordinates in cylindrical coordinate system, k_3 is the axial wavenumber, E and H are the amplitudes of the electric and the magnetic wave field components, respectively. Let us suppose that changing any quantity A value, that varies in the axial direction, along the discharge at the distances of the wavelength order is small compared to the magnitude of this quantity $((A^{-1}(\partial A/\partial z) \ll k_3)$, where symbol A denotes E, H, k_3, or n. Thus, in all further equations all terms of order $O(k_3^{-1}(\partial \ln(A)/\partial z))$ are neglected [12].

Taking in to the account the expression (1) the equations for radial wave components in plasma region ($r < R_p$) can be obtained from the system of Maxwell equations [11] and can be written as:

$$\begin{cases} H_r^p(r) = -\dfrac{m}{\kappa r}E_z^p(r) - \dfrac{k_3}{k}E_\varphi^p(r), \\ E_r^p(r) = \dfrac{k_3 H_\varphi^p(r)}{k\varepsilon_1(r)} - \dfrac{mH_z^p(r)}{kr\varepsilon_1(r)} - \dfrac{i\varepsilon_2(r)E_\varphi^p(r)}{\varepsilon_1(r)}. \end{cases} \tag{2}$$

In the approximation of slight axial varying of the plasma density the system of ordinary differential equations for tangential wave field components in plasma region ($r < R_p$) can be written as:

$$\begin{cases} \dfrac{dE_z^p(r)}{dr} = \dfrac{k_3\varepsilon_2(r)}{\varepsilon_1(r)}E_\varphi^p(r) + i\dfrac{k_3^2 - k^2\varepsilon_1(r)}{k\varepsilon_1(r)}H_\varphi^p(r) - i\dfrac{mk_3}{kr\varepsilon_1(r)}H_z^p(r), \\ \dfrac{dE_\varphi^p(r)}{dr} = \left(\dfrac{m\varepsilon_2(r)}{r\varepsilon_1(r)} - \dfrac{1}{r}\right)H_\varphi^p(r) + ik\left(1 - \dfrac{m^2}{k^2r^2\varepsilon_1(r)}\right)H_\varphi(r) + i\dfrac{mk_3}{kr\varepsilon_1(r)}H_\varphi^p(r), \\ \dfrac{dH_z^p(r)}{dr} = \dfrac{k_3\varepsilon_2(r)}{\varepsilon_1(r)}H_\varphi(r) - i\dfrac{p(r)}{kr\varepsilon_1(r)}E_\varphi^p(r) + i\dfrac{mk_3}{kr}E_z^p(r) - \dfrac{m\varepsilon_2(r)}{r\varepsilon_1(r)}H_z^p(r), \\ \dfrac{dH_\varphi^p(r)}{dr} = -\dfrac{1}{r}H_\varphi^p(r) - i\dfrac{mk_3}{kr}E_\varphi^p(r) + i\dfrac{mk_3}{kr}E_\varphi^p(r) + ik\left(\dfrac{m^2}{k^2r^2} - \varepsilon_3(r)\right)E_z^p(r), \end{cases} \tag{3}$$

where $p(r) = \varepsilon_1(r)\left(k_3^2 - k^2\varepsilon_1(r)\right) + k^2\varepsilon_2^2(r)$, $\varepsilon_{1,2,3}$ are the components of the dielectric tensor cold collisional plasma [11].

It is possible to obtain analytic solutions for the wave field components in the vacuum region ($R_p < r < R_m$):

$$\begin{cases} E_z(r) &= A_1 I_m(\kappa r) + A_2 K_m(\kappa r), \\ E_\varphi(r) &= \dfrac{mk_3 A_1 I_m(\kappa r)}{\kappa^2 r} + \dfrac{mk_3 A_2 K_m(\kappa r)}{\kappa^2 r} + \dfrac{ikA_3 I_m'(\kappa r)}{\kappa} + \dfrac{imkA_4 K_m'}{\kappa}, \\ E_r(r) &= -\dfrac{ik_3 A_1 I_m'(\kappa r)}{\kappa} - \dfrac{imk_3 A_2 K_m'(\kappa r)}{\kappa r} + \dfrac{mkA_3 I_m(\kappa r)}{\kappa^2 r} + \dfrac{mkA_4 K_m(\kappa r)}{\kappa^2 r}, \\ H_z(r) &= A_3 I_m(\kappa r) + A_4 K_m(\kappa r), \\ H_\varphi(r) &= -\dfrac{ikA_1 I_m'(\kappa r)}{\kappa} - \dfrac{imkA_2 K_m'(\kappa r)}{\kappa} + \dfrac{mk_3 A_3 I_m(\kappa r)}{\kappa^2 r} + \dfrac{mk_3 A_4 K_m(\kappa r)}{\kappa^2 r}, \\ H_r(r) &= -\dfrac{mkA_1 I_m(\kappa r)}{\kappa^2 r} - \dfrac{mkA_2 K_m(\kappa r)}{\kappa^2 r} - \dfrac{ik_3 A_3 I_m'(\kappa r)}{\kappa} + \dfrac{imk_3 A_4 K_m'(\kappa r)}{\kappa}. \end{cases} \tag{4}$$

The expressions for A_{1-4} can be obtained from the boundary conditions at the plasma-vacuum interface (3), that consists of the continuity of tangential electric and magnetic wave field components when $r = R_p$:

$$\begin{cases} A_1 &= \kappa R_p K_m'(\kappa R_p) E_z^P(R_p) - i\dfrac{mk_3 K_m(\kappa R_p)}{k} H_z^P(R_p) + i\dfrac{\kappa^2 R_p K_m(\kappa R_p)}{k} H_\varphi^P(R_p), \\ A_2 &= \kappa R_p I_m'(\kappa R_p) E_z^P(R_p) - i\dfrac{mk_3 I_m(\kappa R_p)}{k} H_z^P(R_p) - i\dfrac{\kappa^2 R_p I_m(\kappa R_p)}{k} H_\varphi^P(R_p), \\ A_3 &= -\kappa R_p K_m'(\kappa R_p) H_z^P(R_p) + i\dfrac{mk_3 K_m(\kappa R_p)}{k} E_z^P(R_p) - i\dfrac{\kappa^2 R_p K_m(\kappa R_p)}{k} E_\varphi^P(R_p), \\ A_4 &= \kappa R_p I_m'(\kappa R_p) H_z^P(R_p) - i\dfrac{mk_3 I_m(\kappa R_p)}{k} H_z^P(R_p) - i\dfrac{\kappa^2 R_p I_m(\kappa R_p)}{k} E_\varphi^P(R_p), \end{cases} \tag{5}$$

here the quantities $E_z^P(R_p)$, $E_\varphi^P(R_p)$, $H_z^P(R_p)$, $E_\varphi^P(R_p)$ are the appropriate electric and magnetic wave filed components in plasma at plasma - vacuum interface $r = R_p$. These components are obtained by the numeric solution of the system of ordinary differential equations (3).

The vanishing of the tangential electric wave field components at the waveguide metal wall ($E_z(R_m) = 0$, $E_\varphi(R_m) = 0$) gives the following system that can be treated as local dispersion equation:

$$\begin{cases} A_1 I_m(\kappa R_m) + A_2 K_m(\kappa R_m) &= 0, \\ A_3 I_m'(\kappa R_m) + A_4 K_m'(\kappa R_m) &= 0. \end{cases} \tag{6}$$

The solution of the ordinary dispersion equation gives the relationship between the frequency ω of the eigenwave of the structure and its axial wave number k_3. When the analogue of the dispersion equation (so called phase equation) (6) is studied for the discharge modelling it is necessary to mention that the wave frequency ω is fixed and its value is set externally by the wave generator. This equation connects the local plasma density value $n(z)$ and the axial wave number k_3. The eigen wave propagates along the structure and sustains the discharge, the plasma density changes in axial direction, the wave frequency remains unchanged, while the complex value of the axial wave number also changes along the discharge. The real part of the normalized wave number $x = \mathrm{Re}(k_3)R_p$ determines the wavelength, and its imaginary part $\alpha(n) = \mathrm{Im}(k_3)R_p$ determines the spatial attenuation coefficient of the wave in the direction of its propagation. The dependence $\alpha(n)$ can be used to determine the density axial gradient dn/dz for the discharges in the diffusion controlled regime from the relation [2, 5, 6].

According to articled [2, 5, 6, 4], the dependence $\alpha(n)$ can be used to determine the axial gradient of plasma density dn/dz in discharges in the diffusion mode, using the following ratio:

$$\frac{dn}{dz} = -\frac{2n\alpha}{1 - \dfrac{n}{\alpha}\dfrac{d\alpha}{dn}}. \tag{7}$$

When studying the axial distribution of plasma density, it is important to control the conditions of gas discharge stability for the diffusion control regime. Such criterion was presented in one of the previous works [6]:

$$\frac{n}{\alpha}\frac{d\alpha}{dn} < 1. \tag{8}$$

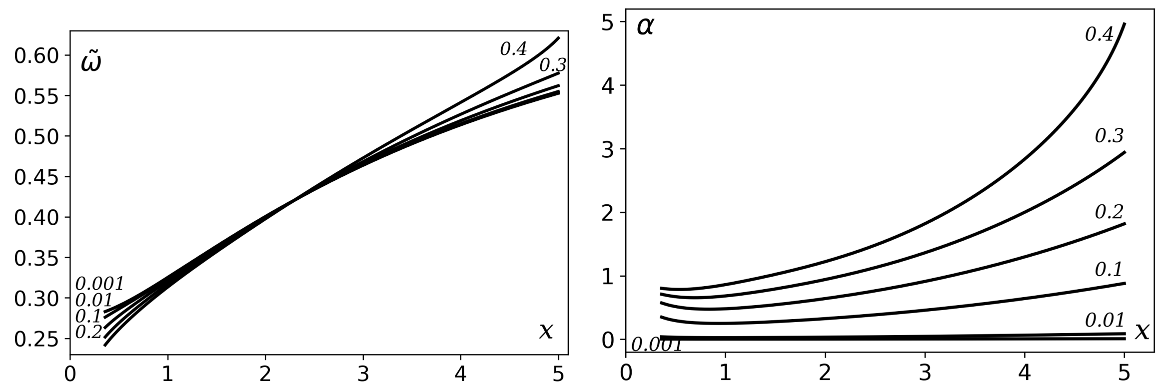
It is necessary to check this Zakrzewski stability criterion because there are possible the situations when the eigen wave has energy to sustain the discharge, but stability criterion is not fulfilled and the end of the discharge is determined by the determined by the ending of the stability region [6, 2].

3. BASIC RESULTS

The main aim of this work is to study the influence of the plasma density radial non-uniformity and the slight axial variation of the waveguide metal wall radius on the plasma density axial distribution in the discharge sustained by the eigen dipolar wave of the discharge structure. In the early work [6] it was noted that the dispersion properties and spatial attenuation of the eigenwaves of the discharge structure strongly determine both the axial distribution of the plasma density and the conditions for the stability of the discharge. So, the first step in the research is the study the phase properties and the spatial attenuation of the eigen wave considered [1, 2, 5, 8, 6, 9, 10]. The results of this step will help us to go to the next two steps: to determine the region of the stability according to the Zakrzewski stability criterion (8) and to choose appropriate parameters for axial plasma density profile calculation (7).

The dispersion equation (6) was solved with the help of numerical methods by introducing the following dimensionless parameters: normalized wave frequency $\tilde{\omega} = \omega/\omega_p$ and the complex normalized axial wavevector $k_3 R_p$, the real part of which $x = \mathrm{Re}(k_3)R_p$ determines wavelength, and the imaginary part $\alpha = \mathrm{Im}(k_3)R_p$ determines the normalized wave attenuation coefficient. The influence of the external magnetic field value on the attenuation coefficient α and the normalized wave frequency $\tilde{\omega}$ is taken into account by introducing the dimensionless parameter $\Omega = \omega_{Ce}/\omega$ (ω_{Ce} is the electron cyclotron frequency). The geometrical parameters of discharge structure is introduced into the model throw dimensionless plasma column radius $\sigma = R_p\omega/c$ and the dimensionless radius of the waveguide metal enclosure $\eta = R_m/R_p$. Such normalized parameters are very convenient for gas discharge modeling, where wave frequency ω is fixed but plasma density n varies along the discharge length. Thus let us determine such values of the parameters of the discharge structure for further gas discharge axial structure investigation that give good axial uniformity and large area of stability.

The Figure 1 presents the influence of the normalized effective collision plasma electron frequency ν/ω on the phase characteristics $\tilde{\omega}$ (Figure 1a) and spatial attenuation α (Figure 1b) of the $m = +1$ mode for the case of radially uniform plasma. The parameters of the discharge structure were chosen to be equal: normalized plasma radius $\sigma = 0.8$, normalized external magnetic field $\Omega = 0.2$. At a fixed frequency of the generator ω, an increase in the frequency of collisions of electrons ν in the range from 0.001 up to 0.1 practically does not affect the normalized frequency of the wave $\tilde{\omega} = \omega/\omega_p$. A further increase of ν/ω from 0.1 to 0.4 leads to a slight decrease in $\tilde{\omega}$ in the range of axial wavenumbers $x \leq 0.6$. The increasing of the normalize collisions frequency leads to a significant increase in the spatial attenuation of the $m = +1$ mode in the entire range of axial numbers both in the region of sufficiently small x and, especially, in the region of $x \geq 3.0$ (Figure 1b). As a result, for each studied value of ν/ω the dependence $\alpha(x)$ possesses some minimum value for some value of x.

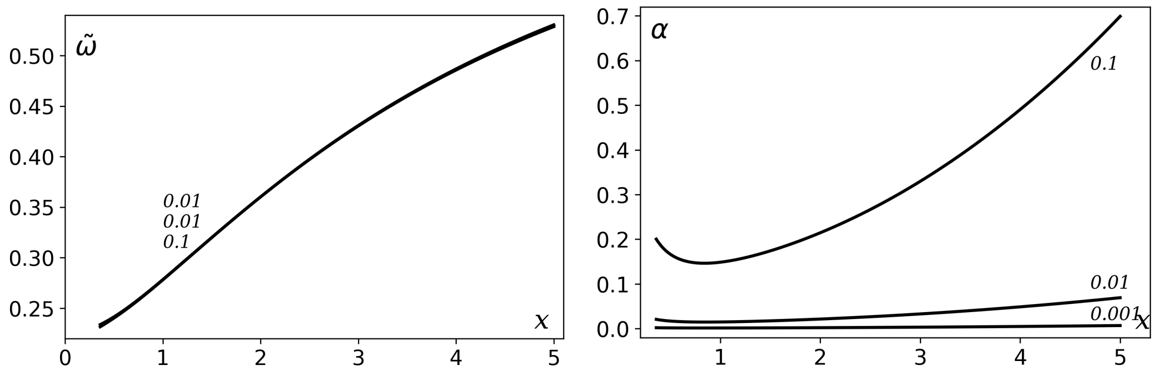


(a) Phase properties $\tilde{\omega}(x)$ **(b)** Attenuation coefficient $\alpha(x)$

Figure 1. The dependence of the dimensionless phase and attenuation properties of the $m = +1$ mode via the dimensionless wavenumber x for different parameter ν/ω values. The numbers near the curves are the parameter ν/ω values. Other parameters are equal: $\Omega = 0.2$, $\sigma = 0.3$, $\eta = 1.1$, $\mu = 0.0$

The influence of the normalized collision frequency ν/ω on the phase properties $\tilde{\omega}$ (Figure 2a) and spatial attenuation α (Figure 2b) of the eigen dipolar $m = -1$ mode of the discharge structure under the same parameter set as for the Figure 1 is presented in the Figure 2. Let us note that the disensionless collision frequency ν has a much smaller influence on the phase properties of the $m = -1$ eigen mode (Figure 2a) than on the $m = +1$ eigen mode (1a). At the same time, an increase in the dimensionlesss collision frequency ν/ω in the specified range (from 0.01 up to 0.1) leads to a significant increase in the spatial attenuation of the $m = -1$ mode in the entire range of x axial numbers (Figure 2b).

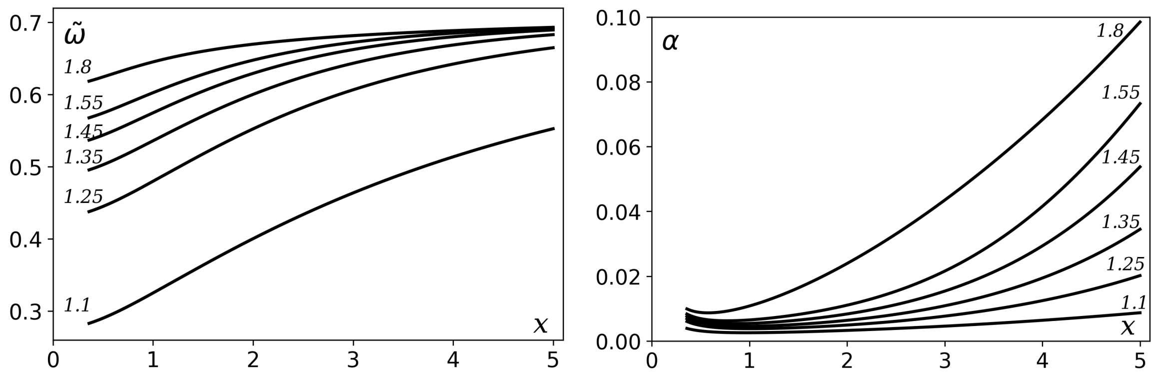
The influence of the normalized radius of the waveguide metal wall η on the phase properties $\tilde{\omega}$ and spatial attenuation coefficient α for the discharge structure with constant radius of metal enclosure for the $m = +1$ eigen



(a) Phase properties $\tilde{\omega}(x)$ **(b)** Attenuation coefficient $\alpha(x)$

Figure 2. The dependence of the dimensionless phase and attenuation properties of the $m = -1$ mode via the dimensionless wavenumber x for different parameter ν/ω values. Problem parameters and curve numbering are the same as for the Figure 1

mode is presented in the Figure 3. The parameters of the calculations were chosen to be equal $\Omega = 0.2$, $\sigma = 0.3$, $\nu = 0.001$, $\mu = 0.0$. The modelling shows that the increase of the vacuum gap size between the plasma column and the waveguide metal due to parameter η variation in the following interval of values $\eta \geq 1.1$ and $\eta \leq 1.8$ leads to the increase of the normalized wave frequency $\tilde{\omega}$ in the entire range of axial wavenumbers x, especially in the region when $x \leq 2$. A further increase of the parameter η up to $\eta \geq 1.3$ values leads to a weakening of the parameter η influence on the phase characteristics of the dipole mode $m = +1$. The Figure 3b presents the influence of the normalized radius of the waveguide metal wall η on the spatial attenuation coefficient α for the $m = +1$ mode. The increase of the parameter η leads to the increase of the coefficient α, especially in the wavenumber region $x \geq 3$ for the following range of parameter $\eta \geq 1.4$. It is necessary to mention the presence of the small region where the attenuation coefficient α growth when wavenumber x decrease in the region of small axial wavenumber values $x < 0.7$.

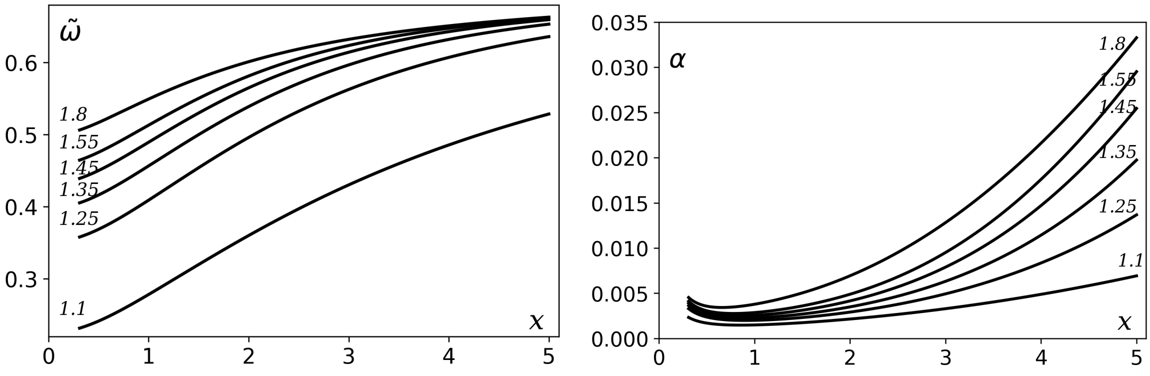


(a) Phase properties $\tilde{\omega}(x)$ **(b)** Attenuation coefficient $\alpha(x)$

Figure 3. The dependence of the dimensionless phase and attenuation properties of the $m = +1$ mode via the dimensionless wavenumber x for different parameter η values. The numbers near the curves are the parameter η values. Other parameters are equal: $\Omega = 0.2$, $\sigma = 0.3$, $\nu = 0.001$, $\mu = 0.0$

The Figure 4 shows the influence of the value of the normalized radius η of the waveguide metal wall on the phase characteristics (Figure 4a) and spatial attenuation (Figure 4b) of the eigen dipolar mode $m = -1$ for the waveguide with the constant radius of metal enclosure for the same parameters suite as for the Figure 3. The parameter η that characterises the radius of the waveguide metal wall has the same influence of the on the phase characteristics $\tilde{\omega}$ and attenuation coefficient α for the $m = -1$ mode as for the $m = +1$ mode. It should be noted that with the same parameters of the waveguide structure, the normalized wave frequency $\tilde{\omega}$ of the $m = -1$ mode is somewhat lower than the normalized frequency of the $m = +1$ mode for the same axial wavenuber x value (see Figure 4a and Figure 3a). At the same time the spatial attenuation coefficient α of the $m = -1$ mode is approximately in $2 - 3$ times smaller than attenuation coefficient of the $m = +1$ mode (see

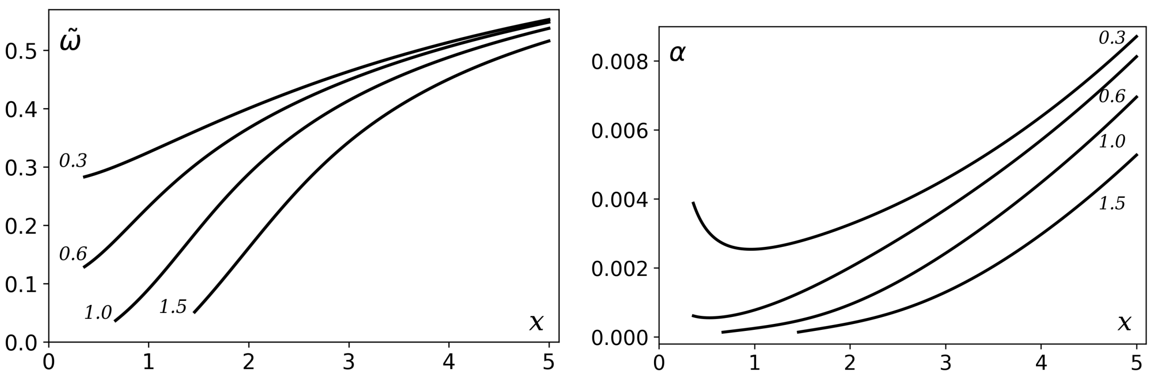
Figure 4b and Figure 3b). Let us mention that similar to the case of dipolar $m = +1$ mode the region where the attenuation coefficient α growth with the axial wavenumber x decrease also exists for the dipolar $m = -1$ mode in the region of small axial wavenumber values $x < 0.7$ (Figure 4b).



(a) Phase properties $\tilde{\omega}(x)$ **(b)** Attenuation coefficient $\alpha(x)$

Figure 4. The dependence of the dimensionless phase and attenuation properties of the $m = -1$ mode via the dimensionless wavenumber x for different parameter η values. Problem parameters and curve numbering are the same as for the Figure 3

For our further research, it is very important to determine the influence of the plasma column radius R_p on the phase characteristics and spatial attenuation of the eigen dipolar waves of the waveguide structure. The influence of the normalized parameter σ, that characterises the dimensionless plasma column radius, on the eigen wave charcteristics is shown in the Figure 5 and Figure 6 for the dipoar modes $m = +1$ and $m = -1$, respectively. It was obtained, that parameter σ has the same general influence on the phase characteristics $\tilde{\omega}(x)$ and spatial attenuation $\alpha(x)$ of the $m = +1$ and $m = -1$ waves, respectively. At the same time, the normalized frequency values $\tilde{\omega}$ of eigen dipolar modes $m = \pm 1$ for a given value of the axial wavenumber are quite close (see, Figure 5a and Figure 6a), but the $\tilde{\omega}$ value of the $m = -1$ mode is somewhat lower than that of the $m = +1$ mode. The spatial attenuation coefficient α of eigen modes $m = +1$ and $m = -1$ also possesses similar behavior (see, Figure 5b and Figure 6b), but eigen mode with $m = -1$ is attenuates somewhat weaker than the mode with $m = +1$, especially in the region of small axial wavenumbers $x \geq 3$. It is also necessary to mention the the existence of the region where attenuation coefficient α decreases with the axial wavenumber x increase ($x < 1.0$) for small parameter σ values ($\sigma \leq 0.6$). For the plasma columns with rather large radius ($\sigma > 1$) such region is not present.

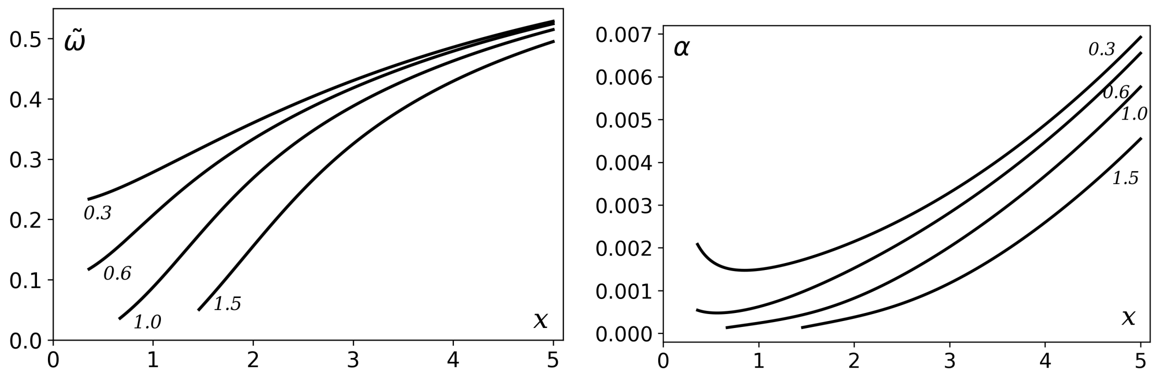


(a) Phase properties $\tilde{\omega}(x)$ **(b)** Attenuation coefficient $\alpha(x)$

Figure 5. The dependence of the dimensionless phase and attenuation properties of the $m = +1$ mode via the dimensionless wavenumber x for different parameter σ values. The numbers near the curves are the parameter σ values. Other parameters are equal: $\Omega = 0.2$, $\eta = 1.1$, $\nu = 0.001$, $\mu = 0.0$

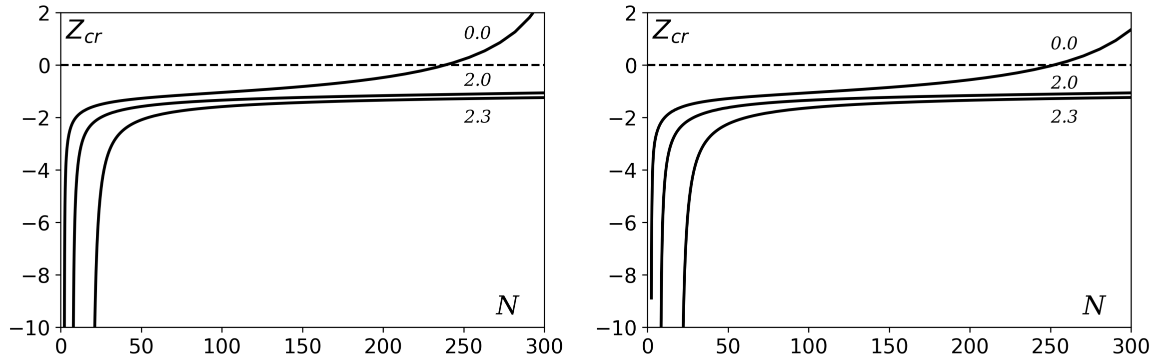
After the detailed study of the phase and attenuation properties of the eigen dipolar waves of the discharge structure (see results presented above in the Figures 1-6 and also in [10]) it is necessary to choose suitable parameters for further research and to move to the next study step. It is obvious that to obtain rather long discharge sustained by the eigen dipole waves with a high degree of plasma density axial homogeneity it is convenient to choose such parameters of the system, that give moderate values of the spatial attenuation coefficient α together with rather big value of dimensionless plasma density $N = \omega_p^2/\omega^2 = 1/\tilde{\omega}^2$. The analysis of the previously obtained results shows that small parameter η values ($\eta = 1.1$, see Figures 3, 4) leads to the quite small spatial attenuation coefficient value, but at the same time, in the future, it is quite possible to obtain the problems associated with gas discharge modeling in the waveguide with metal enclosure with decreasing radius along the discharge. The increase in the plasma column normalized radius σ leads to the decrease of the spatial attenuation coefficient α (see Figures 5, 6), but at the same time the value of the minimum permissible axial wave number x increases. In [10] it was shown that the decreasing of the external magnetic field value (the decrease of the parameter Ω value) leads to the decrease of the normalized frequency $\tilde{\omega}$ of the $m = +1$ wave and to the increase of its spatial attenuation coefficient α. In [10] it was also shown that the decreasing of the external magnetic field value leads to the increase of dimensionless frequency $\tilde{\omega}$ in the region of small values of the axial wave number x and to the decrease of $\tilde{\omega}$ in the region of sufficiently large x values for the dipolar wave $m = -1$. In contrast to the $m = +1$ dipolar wave, the attenuation coefficient α of the $m = -1$ dipolar mode in the region of small x values increases but decreases in the region of sufficiently large axial wavenumbers x with the decrease of the external magnetic field value Ω. As for the value of the parameter ν, the smaller is the normalized effective collision frequency of ν/ω, the smaller is the attenuation coefficient α (see Figure 1, 2).

Thus, taking into account the above considerations, the following normalized parameters of the waveguide structure were chosen for further gas discharge axial structure modeling: $\eta = 1.3$, $\sigma = 0.8$, $\Omega = 0.2$, $\nu = 0.001$.



(a) Phase properties $\tilde{\omega}$ **(b)** Attenuation coefficient α

Figure 6. The dependence of the dimensionless phase and attenuation properties of the $m = -1$ mode via the dimensionless wavenumber x for different parameter σ values. Problem parameters and curve numbering are the same as for the Figure 5



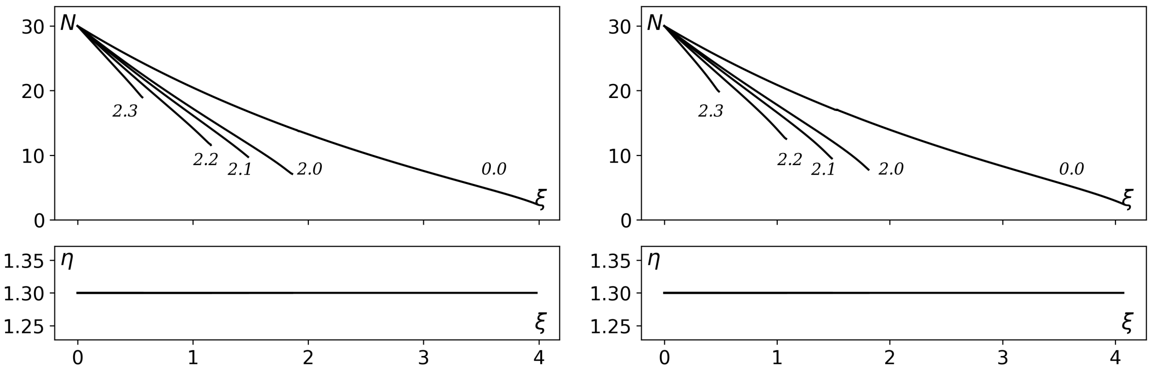
(a) The dipolar mode $m = +1$ **(b)** The dipolar mode $m = -1$

Figure 7. The Zakrzewski stability criterion (8). The numbers near the curves are the non-uniformity parameter μ values. Other parameters are equal: $\omega = 0.2$, $\sigma = 0.8$, $\eta = 1.3$, $\nu/\omega = 0.001$

The next step of the study is the determination of the $\tilde{\omega}$ region where the discharge can be sustained according to the Zakrzewski stability criterion (8). Some results of such study are presented in Figure 7. The

Figure 7a and Figure 7b shows the influence of plasma density radial non-uniformity (parameter μ varies in the range $\mu \in [0.0; 2.3]$) on the stability criterion (8) for dipolar mode with $m = +1$ and $m = -1$, respectively. Other parameters of calculations are equal to $\omega = 0.2$, $\sigma = 0.8$, $\eta = 1.3$, $\nu/\omega = 0.001$. The criterion in the Figure 7 is presented in the following equivalent to (8) form: $Z_{cr} = (n/\alpha) \cdot (d\alpha/dn) - 1 < 0$, so negative value of Z_{cr} corresponds to the regime of the stable gas discharge sustaining. When the parameter μ increases from zero (radially homogeneous plasma) up to $\mu = 2.3$ (the radial distribution of the plasma density is close to the ambipolar diffusion regime profile), the maximum possible plasma density value $N = {\omega_p}^2/\omega^2$ in the discharge that can be sustained by the dipolar mode $m = \pm 1$ becomes bigger and the corresponding density range becomes larger (see Figure 7a, 7b).

The final step of our study is the finding the plasma density axial distribution in the discharge sustained by the dipolar $m = \pm 1$ waves by solving ordinary differential equation (7). The Zakrzewski stability criterion (8) was also under the control. The dimensionless plasma density $N = \omega_p^2/\omega^2$ (ω_p is the electron plasma frequency) axial distributions in the discharge sustained by the eigen dipolar $m = \pm 1$ mode for different plasma density radial profiles (non-uniformity parameter $\mu = 0.0$, $\mu = 2.0$, $\mu = 2.1$, $\mu = 2.2$, $\mu = 2.3$) is presented in Figure 8. The normalized normalized axial coordinate is equal to $\xi = (\nu z)/(\omega R_p)$. At this modeling the radius of the waveguide metal enclosure is considered to be constant along the discharge. Other parameters are equal to $\Omega = 0.2$, $\sigma = 0.8$, $\eta = 1.3$, $\nu = 0.001$.



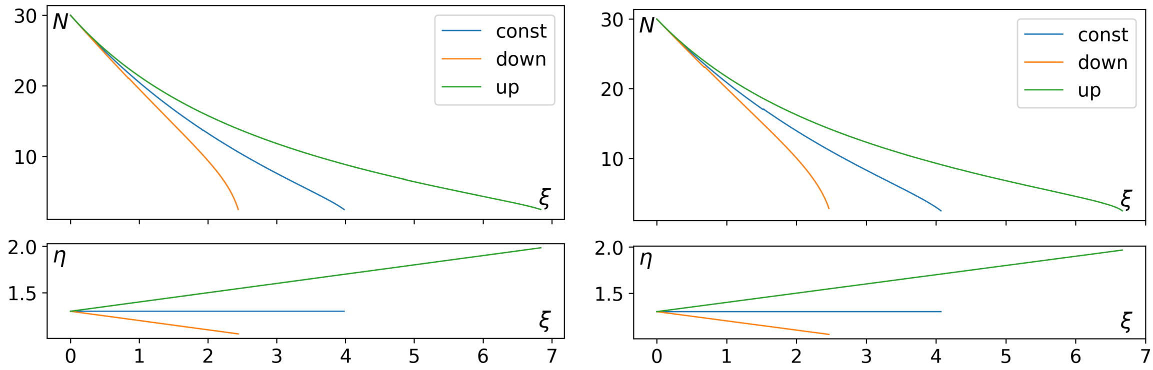
(a) The dipolar mode $m = +1$ **(b)** The dipolar mode $m = -1$

Figure 8. Plasma density axial distribution in dipolar mode sustained gas discharge. The numbers near the curves are the non-uniformity parameter μ values. Other parameters are equal: $\Omega = 0.2$, $\sigma = 0.8$, $\eta = 1.3$, $\nu = 0.001$

The Figures 8a, 8b present the normalized plasma density axial distribution $N = \omega^2/\omega^2$ along the discharge (normalized coordinate ξ) for a radially homogeneous plasma $mu = 0$ and for such values of nou-uniformity parameter μ at which the effect of radial non-uniformity becomes significant. The discharges, sustained by the $m = \pm 1$ waves in the considered discharge structure possesses the similar axial gradients and the discharge lengths.

The Figure 9 presents the plasma density axial distribution in the discharge sustained by the eigen dipolar mode in the case of radially uniform plasma for the metal waveguide with constant, increasing and decreasing radius in the direction of wave propagation. The other parameters are the same as in the Figure 8. The normalized plasma density value at the beginning of the discharge was chosen to be equal to $N = 30$. When the discharge is sustained in the metal waveguide with increasing radius one can obtain the discharge length approximately twice as long as for the discharge in a waveguide of constant radius. Besides one can obtain significantly smaller axial gradients of density non-uniformity, especially at the end of the discharge. When the discharge takes place in metal metal waveguide with the decreasing radius along the discharge the discharge length decreases approximately in half, as compared with the discharge in the waveguide of constant radius. At this case the axial gradients of density non-uniformity increases, especially at the end of the discharge. It is necessary to note the similarity of the axial profiles of the discharges sustained by the dipolar mode $m = \pm 1$.

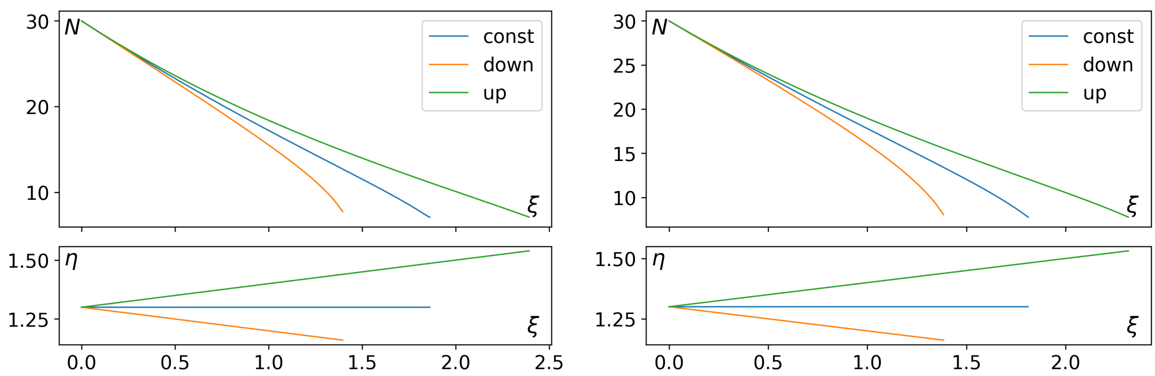
The Figure 10 shows the plasma density axial structure in the discharge sustained by the dipolar mode in the case when the non-uniformity parameter is equal to $\mu = 2.0$. The normalized plasma density value at the beginning of the discharge was also chosen equal to $N = 30$. The increase of waveguide metal radius along the discharge gives the possibility to obtain a longer discharge than in a waveguide of constant radius. In addition let us note that the plasma density decreases along the discharge almost linearly. In a waveguide with the decreasing radius along the discharge one can obtain a shorter discharge with a large axial density gradient at the end of the discharge. The model with such non-uniformity parameter possesses the similarity of the plasma



(a) The dipolar mode $m = +1$ **(b)** The dipolar mode $m = -1$

Figure 9. Plasma density axial distribution in dipolar mode sustained gas discharge for $\mu = 0.0$, $\Omega = 0.2$, $\sigma = 0.8$, $\eta = 1.3$, $\nu = 0.001$

density axial profiles for the discharges sustained by the dipolar mode $m = \pm 1$.



(a) The dipolar mode $m = +1$ **(b)** The dipolar mode $m = -1$

Figure 10. Plasma density axial distribution in dipolar mode sustained gas discharge for $\mu = 2.0$, $\Omega = 0.2$, $\sigma = 0.8$, $\eta = 1.3$, $\nu = 0.001$

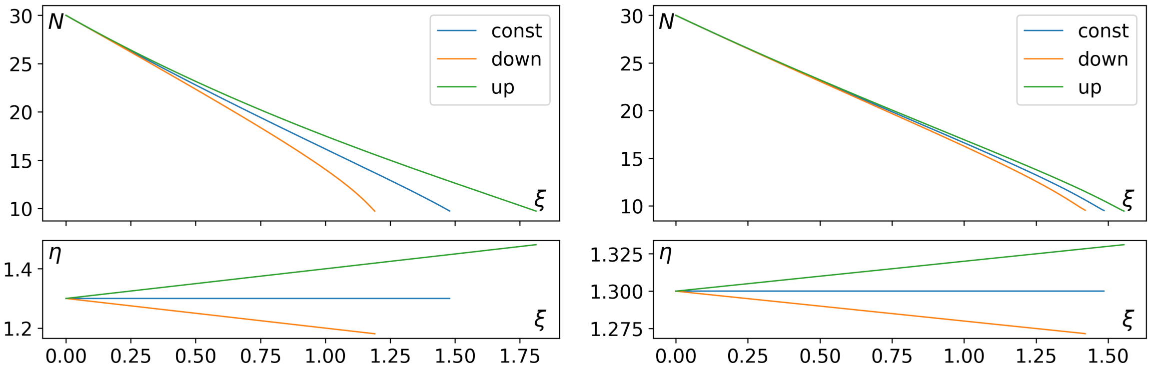
At the larger values of the non-uniformity parameter μ ($\mu = 2.1$), such similarity of the plasma density axial profiles in the discharges sustained by the dipolar mode $m = \pm 1$ starts to disappear (see Figure11). In the discharges sustained by the $m = +1$ mode, the influence of the variable radius of the metal waveguide is essential almost at the beginning of the discharge (Figure 11a). At the same time, in the discharges sustained by the $m = -1$ mode, the plasma density decreases along the discharge practically according to a linear law (Figure 11b). Besides the discharges length and the plasma density axial gradients for metal waveguides with variable radius somewhat differ from such quantities for the discharges in waveguide structure with constant radius $\eta = 1.3$. It is also should be noted that the length of discharge sustained by the mode $m = -1$ is slightly smaller as compared to the length of the discharge in the mode $m = +1$.

When the non-uniformity parameter increases up to $\mu = 2.2$, the axial profile of plasma density in the discharges sustained by the $m = \pm 1$ dipolar modes (Figure 12) become again similar, but since the discharges on the $m = +1$ mode are longer than the discharges on the mode $m = -1$, then the corresponding plasma density axial gradient for the discharge on the mode $m = +1$ is somewhat smaller.

The further growth of the non-uniformity parameter μ up to $\mu = 2.1$ leads to the further discharge decrease and to the increase of plasma density axial gradient.

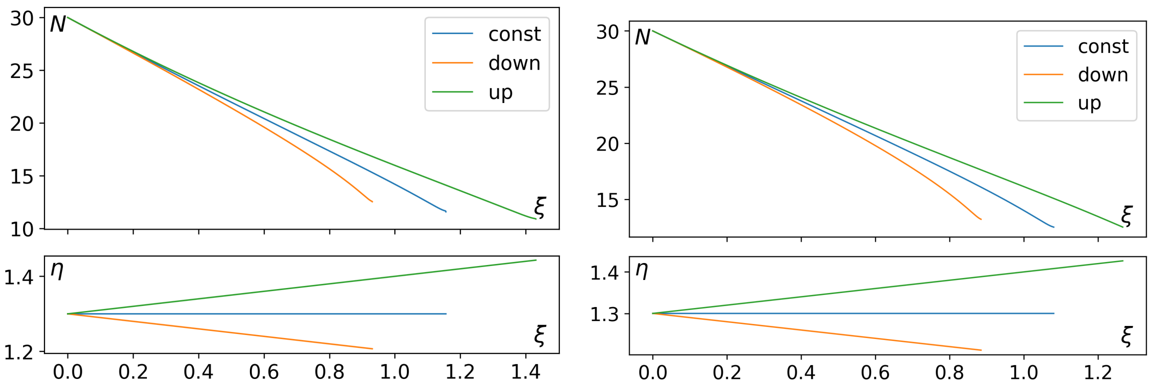
4. CONCLUSIONS

This article presents the results of the theoretical modelling of the axial structure of gas discharge sustained by the eigen dipolar ($m = \pm 1$) wave of the discharge structure that consists of cylindrical magnetized slightly collisional plasma column that surrounds by the vacuum region and enclosed by the metal wall of constant or



(a) The dipolar mode $m = +1$ **(b)** The dipolar mode $m = -1$

Figure 11. Axial structure of gas discharge for $\mu = 2.1$, $\Omega = 0.2$, $\sigma = 0.8$, $\eta = 1.3$, $\nu = 0.001$



(a) The dipolar mode $m = +1$ **(b)** The dipolar mode $m = -1$

Figure 12. Plasma density axial distribution in dipolar mode sustained gas discharge for $\mu = 2.2$, $\Omega = 0.2$, $\sigma = 0.8$, $\eta = 1.3$, $\nu = 0.001$

slightly varying along the discharge radius. The modelling was carried out in the framework of electrodynamics approach taking into account slightly axial and strongly radial non-uniformity of plasma density.

For the discharge structure with a constant radius of metal enclosure it was shown that the increase of the plasma density radial non-uniformity leads to the significant decrease of the discharge length and to the increase of the plasma density axial gradients. It was also obtained that plasma density radial non-uniformity has the essential influence on plasma density axial distribution starting from the non-uniformity parameter value $\mu \geq 2.0$. This influence is getting stronger when $\mu \to 2.3$. It was also obtained that gas discharges sustained in the waveguides with expanding along the discharge metal enclosure possesses a longer length with significantly smaller axial density gradients at the end of the discharge as compared to a constant-radius discharge structure. The use of a waveguide with narrowing along the discharge metal enclosure leads to the decrease of the discharge length and to the increase of the plasma density axial gradient, especially at the end of the discharge. The study of the simultaneous influence of the plasma density radial non-uniformity and the waveguide metal enclosure axial inhomogeneity on the plasma density axial structure in the discharge have showed that in the discharge structures with an expanding metal waveguide the length of the discharge sustained by the $m = +1$ mode is slightly bigger than for the discharge sustained by the $m = -1$ mode. It was obtained that plasma density decreases in the direction of wave propagation approximately according to a linear law when non-uniformity parameter value $\mu \geq 2.0$. The carried out simulation have shown that when the non-uniformity parameter μ increases from $\mu = 0.0$ (radially uniform plasma) up to $\mu = 2.3$ (the plasma density radial distribution that is close to the profile of the ambipolar diffusion regime), the plasma density value increases and also increases the corresponding density range of stable discharge that can be sustained by the $m = \pm 1$ dipolar modes. The obtained results can be useful for various different applications.

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REFERENCES

[1] I. Zhelyazkov, and V. Atanassov, Physics Reports, **255**(2-3), 79 (1995). https://doi.org/10.1016/0370-1573(94)00092-H.

[2] H. Schlüter, and A. Shivarova, Physics Reports, **443**(4-6), №4-6, 121 (2007). https://doi.org/10.1016/j.physrep.2006.12.006.

[3] E. Benova, P. Marinova, R. Tafradjiiska-Hadjiolova, Z. Sabit, D. Bakalov, N. Valchev, L. Traikov, et al., Appl. Sci. **12**, 969 (2022). https://doi.org/10.3390/app12030969

[4] M. S. Kovacevic, L. Kuzmanovic, M.M. Milosevic, and A. Djordjevich, Physics of Plasmas, **28**(2), (2021). https://doi.org/10.1063/5.0035035

[5] C.M. Ferreira, J. of Phys. D: Appl. Phys. **16**(9), 1673 (1983). https://doi.org/10.1088/0022-3727/16/9/013.

[6] M. Moisan, and Z. Zakrzewski, J. of Phys. D: Appl. Phys. **24**(7), 1025 (1991). https://doi.org/10.1088/0022-3727/24/7/001.

[7] J. Wolinska-Szatkowska, J. of Phys. D: Appl. Phys. **20**, 977 (1987). https://doi.org/10.1088/0022-3727/20/7/024.

[8] N.A. Azarenkov, V.P. Olefir, and A.E. Sporov, Problems of Atomic Science and Technology. Series ≪Plasma Physics≫, 25(6), 113 (2018). https://vant.kipt.kharkov.ua/ARTICLE/VANT_2018_6/article_2018_6_113.pdf.

[9] N.A. Azarenkov, V.P. Olefir, and A.E. Sporov, Problems of Atomic Science and Technology. Series "Plasma Physics", 130(6), 69 (2020). http://dx.doi.org/10.46813/2020-130-069

[10] N.A. Azarenkov, V.P. Olefir, and A.E. Sporov, Problems of Atomic Science and Technology. Series "Plasma Physics", 143(1) 1, 98 (2023). http://dx.doi.org/10.46813/2023-143-098

[11] D. Gary Swanson, *Plasma Waves*, 2nd Edition, (IOP Publishing Ltd, Bristol and Philadelphia, 2003).

[12] G.G. Lister, and T.R. Robinson, J. of Phys. D: Appl. Phys. **24**(11), 1993 (1991). https://doi.org/10.1088/0022-3727/24/11/013.

АКСІАЛЬНА СТРУКТУРА ГАЗОВОГО РОЗРЯДУ, ЩО ПІДТРИМУЄТЬСЯ ВЛАСНОЮ ДИПОЛЬНОЮ ХВИЛЕЮ МЕТАЛЕВОГО ХВИЛЕВОДУ ЗМІННОГО РАДІУСУ, ЗАПОВНЕНОГО МАГНІТОАКТИВНОЮ НЕОДНОРІДНОЮ ПЛАЗМОЮ

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В статті наведено результати теоретичного дослідження аксіального розподілу густини плазми в стаціонарному газовому розряді, який підтримується власною дипольною хвилею, що поширюється в довгій циліндричній плазмово-металевій структурі. Структура складається зі стовпа магнітоактивної неоднорідної плазми, що знаходиться всередині металевого хвилеводу змінного радіусу. Дослідження газового розряду проводиться в рамках електродинамічної моделі, в якій основна увага приділяється електродинамічній частині. Для опису процесів, що відбуваються в плазмі, використовуються модельні рівняння. Визначено вплив неоднорідності металевого хвилеводу вздовж структури та радіальної неоднорідності густини плазми на фазові характеристики дипольної хвилі, її просторове загасання, радіальний розподіл компонент поля, аксіальний розподіл густини плазми, що підтримується цією модою, проведено аналіз умов стаціонарності протікання розряду. Отримані результати можуть бути корисними для різних технологічних застосувань.

Ключові слова: *газовий розряд; плазмово - металевий хвилевід; дипольна власна хвиля; фазові властивості та просторове загасання; критерій Закревського*