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WEAKLY NONLINEAR BIOTHERMAL CONVECTION IN A POROUS MEDIA LAYER UNDER ROTATION, GRAVITY MODULATION, AND HEAT SOURCE

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In this paper, the influence of gravitational modulation on weakly nonlinear biothermal convection in a porous rotating layer is investigated. We consider a layer of porous medium saturated with Newtonian fluid, containing gyrotactic microorganisms, and subject to gravitational modulation, rotation, and internal heating. To analyze linear stability, it is sufficient to represent disturbances in the form of normal modes, while nonlinear analysis includes a truncated Fourier series containing a harmonic of the nonlinear interaction. A six-dimensional nonlinear Lorentz-type model is constructed, exhibiting both reflection symmetry and dissipation. We determined heat and mass transfer using a weakly nonlinear theory based on the representation of a truncated Fourier series. Additionally, the behavior of nonstationary Nusselt and Sherwood numbers was investigated by numerically solving finite amplitude equations. Applying the expansion of regular perturbations in a small parameter to a six-dimensional model of Lorentz equations with periodic coefficients, we obtained the Ginzburg-Landau (GL) equation. This equation describes the evolution of the finite amplitude of the onset of convection. The amplitude of convection in the unmodulated case is determined analytically and serves as a standard for comparison. The study examines the effect of various parameters on the system, including the Vadasz number, modified Rayleigh-Darcy number, Taylor number, cell eccentricity, and modulation parameters such as amplitude and frequency. By varying these parameters, in different cases, we analyzed heat and mass transfer, quantitatively expressed by the Nusselt and Sherwood numbers. It has been established that the modulation amplitude has a significant effect on the enhancement of heat and mass transfer, while the modulation frequency has a decreasing effect.
Keywords: *Darcy-Brinkman model; Bio-thermal convection; Gravity modulation; Porous rotating medium; Gyrotactic microorganism*
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1. INTRODUCTION

The study of fluid flow through porous media is of paramount importance in diverse practical applications such as soil mechanics, groundwater hydrology, oil production, and industrial filtration. In recent years, a novel research area called bioconvection in porous media has gained prominence. This field centers on investigating bacterial movement and biofilm growth, particularly in the context of microbiological oil production technologies. As a result, there is a compelling need for theoretical investigations to delve into the interactions between bioconvection and natural convection.

Ingham and Pop's monograph [1] and Nield and Bejan's work [2] serve as notable references extensively delving into the realm of thermal instability in fluid layers within porous media. Furthermore, Vadasz's comprehensive review [3] specifically focuses on fluid flow and heat transfer in rotating porous media. These monographs meticulously analyze and discuss various facets and challenges associated with thermal instability in these systems. Over time, the exploration of natural convection in fluid-saturated porous media has expanded to encompass additional factors, including rotation [3], magnetic fields [4]-[5], anisotropy [6], heterogeneity [7], gravity modulation [8], and other related effects. These investigations aim to unravel the influence and implications of these factors on the convective flow dynamics within porous media.

The classical Darcy's law, which describes flow in porous media, underwent modification by Brinkman to incorporate a Laplacian term in Stokes' equation. This modification accommodates the Darcy resistance within porous media and is commonly known as the Darcy-Brinkman equation. Widely employed for studying flow in porous media with high porosity [9], the Darcy-Brinkman model encounters several challenges and problems that demand careful consideration when applied to the investigation of bioconvection in highly porous media.

Several practical scenarios involve porous materials acting as their heat source. This presents an alternative approach to induce convective flow through localized heat generation within the porous material. This condition may arise from phenomena like radioactive decay or, in the present context, a relatively modest exothermic reaction occurring within the porous material. The internal heating of the Earth establishes a temperature gradient between the inner and outer layers of the Earth's crust, facilitating convective flow and transferring thermal energy to the planet's surface. Consequently, internal heat generation plays a crucial role in various applications, including geophysics, reactor safety assessments, the production of metal waste forms for used nuclear fuel, fire and combustion research, and the storage of radioactive materials. Despite its importance, the impact of internal heating on convective flow has only been explored in a limited number of investigations.

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Yadav et al. [10] explored the impact of an internal heat source on the initiation of Darcy-Brinkman convection in a porous layer saturated with nanofluid under various boundary conditions, including free-free, rigid-rigid, lower-rigid, and upper-free. Analyzing the onset of convection induced by internal heating, such as that generated by microwave heating or chemical reactions, in a horizontal layer of nanofluid, Nield and Kuznetsov [11] conducted an analytical investigation, considering Brownian motion and thermophoresis. Khalid et al. [12] performed a linear stability analysis in the presence of feedback control to study the effect of an internal heat source on the initiation of Rayleigh-Benard convection in a rotating nanofluid layer with double-diffusive convection. Jain and Solomatov [13] delved into the onset of convection in internally heated fluids with strongly temperature-dependent viscosity. Devy et al. [14] investigated the Darcy-Brinkman convective instability of a non-Newtonian nanofluid layer saturated in a porous medium, considering the presence of an internal heat source.

The term "bioconvection" denotes the phenomenon wherein convective patterns emerge due to the presence of self-propelled microorganisms that are denser than the surrounding fluid medium [15]-[17]. These microorganisms exhibit responsive movement to various stimuli, such as gravity, light, chemicals, or the presence of food, a behavior known as taxis. Taxis can be categorized based on the type of stimulus and the direction of movement of the organisms. Positive taxis, or attraction, occur when organisms move toward the stimulus source, while negative taxis, or repulsion, describe movement away from the source. Gravitaxis refers to the directional movement of organisms in response to gravitational forces, while magnetotaxis involves their ability to detect and respond to magnetic fields. Chemotaxis is the response to a gradient in chemical concentration, and phototaxis is movement in response to light. In this context, our emphasis is on gravitactic microorganisms.

Pioneering work by Childress et al. [16] established a comprehensive theory and mathematical model for the bioconvection of gravitactic microorganisms. Hill et al. [17] further developed a theoretical model specifically for gravitactic microorganisms, focusing on their bioconvective behavior. Pedley et al. [18] contributed to the field by developing a linear stability theory to analyze the stability of bioconvection involving gyrotactic microorganisms in a shallow layer of fluid. These studies have identified the conditions required for the initiation of bioconvective flow, providing insights into the behavior of gravitactic microorganisms in convective systems.

Numerous publications have delved into the impact of gyrotactic microorganisms on fluid flows in confined porous media, with notable contributions from Nield, Kuznetsov, and Avramenko [19]-[23] that significantly advance the understanding of biological processes in porous environments. In their work [19], it was established that if the permeability remains below a critical value, the system remains stable, and bioconvection does not manifest. Conversely, surpassing the critical permeability threshold leads to the development of bioconvection. Their subsequent study explored the occurrence of bioconvection in a horizontal layer filled with a saturated porous medium [20], determining critical Rayleigh numbers for various values of the Peclet number, gyrotaxis number, and cell eccentricity. The impact of vertical flow on the onset of bioconvection in a suspension of gyrotactic microorganisms within a porous medium was investigated in [21]. A linear analysis was employed to derive an equation for the critical Rayleigh number, revealing that vertical throughflow stabilizes the system. [22] presented a continuum model of thermobioconvection, focusing on oxytactic bacteria in a porous medium. This study examined the effect of heating microorganisms from below on the stability of a horizontally layered fluid saturated with a porous medium. Utilizing the Galerkin method to solve the linear stability problem, the study established a relationship between the critical value of the Rayleigh number and the thermal Rayleigh number. Avramenko [23] developed a nonlinear theory of bioconvection for gyrotactic microorganisms in a layer of ordinary liquid based on the Lorenz approach [24]. This work [23] delineated the boundaries of various hydrodynamic regimes observed in two-dimensional bioconvection.

Hwang and Pedley [25] explored the impact of uniform shear on the instability of bioconvection in a shallow suspension containing swimming gyrotactic cells. They introduced shear by implementing a flat Couette flow, counteracting the influence of gravity on the cells. The study identified three distinct physical processes contributing to bioconvection instability: gravitational overturning, cell gyrotaxis, and negative cross-diffusion flow. High shear velocities acted as a stabilizing factor, akin to Rayleigh-Benard convection. However, at low shear rates, it destabilized perturbations through the overstability mechanism discussed by Hill, Pedley, and Kessler [16]. Dmitrenko [26] provided a comprehensive review of bioconvection in nanofluids and porous media, presenting a mathematical model based on Darcy's law for porous media. Sharma and Kumar [27] investigated the influence of high-frequency vertical vibration on the onset of bioconvection in a dilute solution of gyrotactic microorganisms using analytical and numerical methods. Their findings revealed that high-frequency, low-amplitude vertical vibration and the bioconvection Peclet number had a stabilizing effect on the system. Kushwaha et al. [28] conducted a more detailed analysis of the stability of vibrational systems consisting of shallow layers filled with randomly swimming gyrotactic microorganisms. In a recent study, Garg et al. [29] examined the stability of thermo-bioconvection flow in an anisotropic porous medium, considering a Jeffery fluid containing gravitactic microorganisms.

The Darcy-Brinkman model, widely employed in porous media research, found an extension in its application by Zhao et al. [30] to investigate biothermal convection within a highly porous medium. Their stability analysis delved into the dynamics of biothermal convection influenced by bottom heating. In another exploration utilizing the Darcy-Brinkman model, Kopp et al. [31] delved into biothermal instability within a porous medium saturated with a water-based nanofluid containing gyrotactic microorganisms in a vertical magnetic field. Their findings revealed that an increase in the concentration of gyrotactic microorganisms amplifies the onset of magnetic convection. Additionally, the study highlighted the more significant role played by spherical gyrotactic microorganisms in developing biothermal instability. Moreover, Kopp and Yanovsky [32] investigated the impact of rotation, specifically the Coriolis force, on biothermal convection in a layer of porous medium saturated with a suspension containing gyrotactic microorganisms.

Controlling heat and mass transfer is paramount in engineering and technical applications, and one effective strategy involves manipulating convective processes through external perturbations or modulations. Understanding how these modulations influence the flow and transport phenomena within a system is crucial for optimizing performance. Common modulation techniques include temperature modulation, gravity modulation, rotation modulation, and magnetic field modulation. This study specifically focuses on a convection control method based on gravity modulation.

The use of gravity modulation to enhance the stability of a heated fluid layer from below was initially introduced by Gresho and Sani in their study [33]. Since then, numerous researchers have delved into the effects of gravity modulation on the onset of convection. Malashetty and Begum extended these investigations in their study [34], considering additional physical conditions and non-Newtonian fluids. They explored the impact of small-amplitude gravity modulation on the initiation of convection in both fluid layers and fluid-saturated porous layers. Kiran [35] conducted studies on the nonlinear thermal instability in a porous medium saturated with viscoelastic nanofluid under gravitational modulation. Over the years, Kiran et al. conducted several studies [36]-[38] to investigate the influence of gravity modulation on Rayleigh-Benard convection (RBC) and Darcy convection. Their focus was on the effect of g-jitter on RBC in nanofluids [39], using the Ginzburg-Landau (GL) model for nonlinear analysis. They calculated the thermal and concentration Nusselt numbers, considering various physical parameters. Additionally, Manjula et al. [40] studied the combined effects of gravity modulation and rotation on thermal instability in a horizontal layer of a nanofluid.

In the above literature review, there is a certain gap in studies of the effect of gravitational field modulation on biothermal convection in rotating porous media saturated with an aqueous solution containing gyrotactic microorganisms. Kopp and Yanovsky [41] were the first to explore the use of gravity modulation in controlling the development of bio-thermal convection in a layer of porous media that is saturated with Newtonian fluid and contains gyrotactic microorganisms. Their exploration focused on weakly nonlinear convective instability within a porous layer saturated with a Newtonian fluid containing gyrotactic microorganisms. The influence of gravitational modulation resulted in the gyrotaxis parameter becoming periodic over time. The study derived the non-autonomous Ginzburg-Landau equation to characterize heat transfer, quantified by the Nusselt number. Notably, the research extensively delved into the impact of various parameters on heat transfer, including Wadas numbers, the modified bioconvective Rayleigh-Darcy number, cell eccentricity, frequency modulation, and modulation amplitude. Their findings highlighted the remarkable efficiency enhancement of the heat transfer process due to the spherical shape of microorganisms, emphasizing the significant role of microorganism morphology in influencing convective heat transfer. In a related study, Kiran and Manjula [42] investigated the effects of thermal modulation and internal heating on Darcy-Brinkman bio-convection in a Newtonian porous medium containing gyrotactic microorganisms. In contrast to paper [41], the control of bio-thermal convection was specifically achieved through the modulation of the temperature gradient [42] without concurrent modulation of the gyrotaxis.

The motivation for this investigation stems from the significant roles of gravitaxis and gyrotaxis in bioconvection phenomena. Therefore, there is a need to deepen our understanding of the interplay between gravitaxis, gyrotaxis, and rotating thermal convection under gravity modulation to gain insights into the system's dynamics. The main objective of this article is to analyze the behavior of weakly nonlinear biothermal convection in a rotating porous medium with internal heating filled with a Newtonian fluid containing gyrotactic microorganisms.

2. DESCRIPTION OF THE PROBLEM AND MATHEMATICAL MODEL

We have an unbounded horizontal layer of a porous medium filled with a Newtonian fluid that contains gyrotactic microorganisms. This layer has a thickness of h and rotates steadily around a vertical axis with a constant angular velocity of Ω_0. The bottom boundary is where heating takes place, as shown in Fig. 1. The temperature at the lower boundary is T_d and at the upper boundary, it's T_u. To describe the problem's geometric configuration, we use a Cartesian coordinate system, denoted by (x, y, z), with the z-axis being vertically upward. Please refer to Fig. 1. To account for the influence of a time-periodic gravitational field, we introduce a vertically downward force given by $\vec{e}g_0(1+\varepsilon^2\delta\cos(\omega_g t))$, where δ and ω_g denote the amplitude and frequency of the gravity modulation, respectively. Our problem pertains to a physical model that is based on certain assumptions, which are as follows:

- We consider a Newtonian fluid containing a large number of gyrotactic microorganisms to be incompressible and a porous matrix incapable of absorbing microorganisms.
- We use the Darcy-Brinkman model for highly porous media.
- The heating from below and internal heating do not affect the cells' gyrotactic activity or viability.
- The pores of the material should be large enough to allow for the movement of biological organisms.
- The fluid phase and microorganisms are in a state of thermal equilibrium, so the heat flow may be described using a one-equation model.
- We assume that all thermophysical characteristics are constant, except for the density in the buoyancy force (Boussinesq approximation).

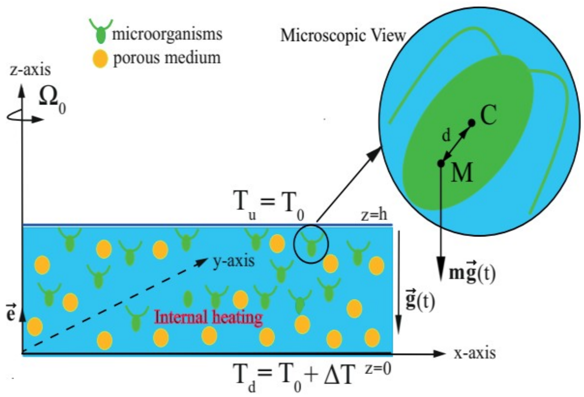

Figure 1. Scheme of swimming of gyrotaxic microorganisms (cells of green algae *Chlamydomonas Reinhardtii*) in a rotating layer of a porous medium. The center of buoyancy of the microorganism C is displaced by a distance d from the center of mass M

Under these assumptions, the mathematical model is based on the continuity, momentum, energy, and conservation equations for cells of the following form [2]-[3],[16],[19]:

$$\nabla\vec{V}_D = 0\,, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial\vec{V}_D}{\partial t} = -\nabla P + \tilde{\mu}\nabla^2\vec{V}_D - \frac{\mu}{K}\vec{V}_D + \frac{2\rho_0}{\varepsilon}[\vec{V}_D\times\vec{\Omega}] - \vec{e}g(t)\rho_0(1-\beta(T-T_0)) - \vec{e}g(t)(\delta\rho)\mathcal{V}n\,, \tag{2}$$

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f\vec{V}_D\cdot\nabla T = k_m\nabla^2 T + Q(T-T_0)\,, \tag{3}$$

$$\frac{\partial n}{\partial t} = -div\left(n\vec{V}_D + nW_c l - D_m\nabla n\right), \tag{4}$$

$$g(t) = g_0(1+\varepsilon^2\delta\cos(\omega_g t))\,,$$

where $\vec{V}_D = (u,v,w)$ is the Darcy velocity, which is related to the fluid velocity $\vec{V}$ as $\vec{V}_D = \varepsilon\vec{V}$, ε is the porosity of the porous medium, K is the permeability of the porous medium, ρ_0 is the fluid's density at the reference temperature, P is the pressure, β is the thermal expansion coefficient, $g(t)$ is the time-periodic gravitational acceleration, $\vec{e} = (0,0,1)$ is a unit vector in the direction of the axis z, $\tilde{\mu}$ is the Brinkman effective viscosity, μ is the viscosity of fluid, $(\rho c)_f$ is the heat capacity of fluid, $(\rho c)_m$ is the effective heat capacity, k_m is the effective thermal conductivity. Q is the internal heat source, n is the concentration of microorganisms, $\delta\rho$ is the density difference between microorganisms and a base fluid: $\rho_m - \rho_f$, $\mathcal{V}$ is the average volume of a microorganism, D_m is the diffusivity of microorganisms. We assumed that random motions of microorganisms are simulated by a diffusion process. $W_c\hat{l}(0)$ is the average microorganism swimming velocity (W_c is constant). The unit vector $\hat{l}(t)$ represents the direction of movement of the microorganisms, and it is a time-periodic quantity due to the modulation of the gravitational field.

At the boundaries, the temperature remains constant. Thus, the boundary conditions are: [20],[30]:

$$w = 0,\quad T = T_d,\quad \vec{J}\cdot\vec{e} = 0,\quad at\ z = 0, \tag{5}$$

$$w = 0,\quad T = T_u,\quad \vec{J}\cdot\vec{e} = 0,\quad at\ z = h, \tag{6}$$

where $\vec{J} = n\dfrac{\vec{V}_D}{\varepsilon} + nW_c\hat{l} - D_m\nabla n$ is the flux of microorganisms. To aid problem analysis, we introduce non-dimensional parameters:

$$(x^*,y^*,z^*) = \frac{(x,y,z)}{h},\vec{V}_D^* = \vec{V}_D\frac{h}{\alpha_m},t^* = \frac{t\alpha_m}{h^2\tilde{\sigma}},\quad T^* = \frac{T-T_u}{T_d-T_u},$$

$$P^* = \frac{PK}{\mu\alpha_m},\quad \tilde{\sigma} = \frac{(\rho c)_m}{(\rho c)_f},\quad n^* = n\mathcal{V},\quad \omega_g^* = \omega_g\frac{h^2\tilde{\sigma}}{\alpha_m}, \tag{7}$$

where $\alpha_m = k_m/(\rho c)_f$ is the coefficient of thermal diffusivity.

Applying transformation (7) to Eqs. (1)-(4) and next omitting the asterisks, we derive the following system of dimensionless equations:

$$\nabla \vec{V}_D = 0\,, \tag{8}$$

$$\frac{1}{\mathcal{V}_a}\frac{\partial \vec{V}_D}{\partial t} = -\nabla P + D_a \nabla^2 \vec{V}_D - \vec{V}_D - \vec{e} f_m \frac{R_b}{L_b} n + \vec{e} f_m Ra T + \sqrt{Ta}[\vec{V}_D \times \vec{e}]\,, \tag{9}$$

$$\frac{\partial T}{\partial t} + (\vec{V}_D \nabla) T = \nabla^2 T + \mathrm{R}_i T\ , \tag{10}$$

$$\frac{1}{\tilde{\sigma}}\frac{\partial n}{\partial t} = -\nabla\left(n\vec{V}_D + \frac{Pe}{L_b} n\hat{\vec{l}}(t) - \frac{1}{L_b}\nabla n \right), \tag{11}$$

where $f_m = 1 + \varepsilon^2 \delta \cos(\omega_g t)$. We have introduced the following dimensionless parameters in Eqs. (8)-(11): $\mathcal{V}_a = \dfrac{\varepsilon(\rho c)_m \tilde{\mu}}{\rho_0 k_m D_a} = \dfrac{\varepsilon\tilde{\sigma}\mathrm{Pr}}{D_a}$ is the modified Vadasz number, $\mathrm{Pr} = \dfrac{\tilde{\mu}}{\alpha_m \rho_0}$ is the Prandtl number, $D_a = \dfrac{\tilde{\mu} K}{\mu h^2}$ is the Darcy number, $Ta = \dfrac{4\Omega_0^2 K^2}{\varepsilon^2 \mu^2}\rho_0^2$ is the Taylor-Darcy number, $R_b = \dfrac{g(\delta\rho) h K}{\mu D_m}$ is the bioconvection Rayleigh-Darcy number, $L_b = \dfrac{\alpha_m}{D_m}$ is the bioconvection Lewis number, $Ra = \dfrac{\rho_0 g h K \beta \Delta T}{\mu \alpha_m}$ is the Rayleigh-Darcy number, $R_i = \dfrac{Q h^2}{\alpha_m}$ is the internal Rayleigh number, $Pe = \dfrac{W_c h}{D_m}$ is the bioconvection Peclet number.

If we assume that in a stationary state the liquid is at rest, then the main physical quantities can be written as

$$\vec{V}_D = \vec{V}_b = 0, \quad P = P_b(z), \quad T = T_b(z), \quad n = n_b(z)\ . \tag{12}$$

The ground state temperature profile $T_b(z)$ and microorganism concentrations $n_b(z)$ are obtained by solving the following equations:

$$\frac{d^2 T_b}{dz^2} + R_i T_b(z) = 0\,, \tag{13}$$

$$\frac{dn_b}{dz} = n_b(z) Pe\,. \tag{14}$$

Later on, we will come to realize that having knowledge of the pressure profile is not necessary. After solving equation (13) and imposing the boundary conditions (5)-(6), we can obtain the temperature distribution $T_b(z)$:

$$T_b(z) = \frac{\sin(\sqrt{R_i}(1-z))}{\sin(\sqrt{R_i})} \tag{15}$$

If there is no heat source $R_i \to 0$, then the base temperature profile has the form: $T_b(z) = 1 - z$. Next, we can obtain the solution for $n_b(z)$, which corresponds to the result presented in [20]:

$$n_b(z) = n_b(0)\exp(zPe)\,, \tag{16}$$

where $n_b(0)$ represents the value of the number density at the bottom of the layer. The constant $n_b(0)$ can be determined as follows:

$$n_b(0) = \frac{\langle n \rangle Pe}{\exp(Pe) - 1}, \quad \langle n \rangle = \int_0^1 n_b(z) dz\,. \tag{17}$$

Let the heating from below the mixed fluid layer cause small perturbations in the main flow as follows:

$$\vec{V}_D = \vec{V}'(u', v', w'), \quad T = T_b + T', \quad n = n_b + n', \quad P = P_b + P', \quad \hat{\vec{l}}(t) = \vec{e} + \hat{\vec{m}}'(t)\,. \tag{18}$$

We can derive the following expressions by analyzing Eq. (9):

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t} + 1 - D_a \nabla_\perp^2\right) u' = -\frac{\partial P'}{\partial x} + \sqrt{\mathrm{Ta}}\, v'\,, \tag{19}$$

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t} + 1 - D_a \nabla_\perp^2\right) v' = -\sqrt{\mathrm{Ta}}\, u'\,, \tag{20}$$

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t}+1-D_a\nabla_\perp^2\right)w'=-\frac{\partial P'}{\partial z}-f_m\frac{\mathrm{R}_b}{\mathrm{L}_b}n'+f_m\mathrm{R}aT', \tag{21}$$

$$\nabla_\perp^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}.$$

Here, eliminating the pressure term results in an equation of the form:

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t}+1-D_a\nabla_\perp^2\right)\left(\frac{\partial u'}{\partial z}-\frac{\partial w'}{\partial x}\right)=\sqrt{\mathrm{Ta}}\frac{\partial v'}{\partial z}+f_m\frac{\mathrm{R}_b}{\mathrm{L}_b}\frac{\partial n'}{\partial x}-f_m\mathrm{R}a\frac{\partial T'}{\partial x} \tag{22}$$

For convenience, we express the perturbations of velocities u' and w' in terms of the stream function:

$$u'=\frac{\partial\psi}{\partial z},\quad w'=-\frac{\partial\psi}{\partial x}. \tag{23}$$

Equations (20) and (22) can be written in the following form:

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t}+1-D_a\nabla_\perp^2\right)\nabla_\perp^2\psi=\sqrt{\mathrm{Ta}}\frac{\partial v'}{\partial z}+f_m\frac{\mathrm{R}_b}{\mathrm{L}_b}\frac{\partial n'}{\partial x}-f_m\mathrm{R}a\frac{\partial T'}{\partial x}, \tag{24}$$

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t}+1-D_a\nabla_\perp^2\right)v'=-\sqrt{\mathrm{Ta}}\frac{\partial\psi}{\partial z} \tag{25}$$

After reviewing previous research [19], [23] and considering gravity modulation, we formulate the equation of the unit vector perturbation that indicates the direction of microorganisms swimming in the form

$$\hat{\vec{m}}'(t)=\mathcal{B}_0(1-\varepsilon^2\delta\cos(\tilde{\omega}_g t))\zeta\vec{i}-\mathcal{B}_0(1-\varepsilon^2\delta\cos(\tilde{\omega}_g t))\xi\vec{j}+0\cdot\vec{e}. \tag{26}$$

Here, $\vec{i}$ and $\vec{j}$ are the unit vectors in the x- and y-directions, respectively. The parameter $\mathcal{B}_0$ represents the reorientation of microorganisms under the influence of a gravitational moment relative to viscous resistance in the absence of modulation, and it is defined as $\mathcal{B}_0=(\mu\alpha_\perp/\rho_0 g_0 d)(\alpha_m/h^2)$. The displacement of the center of mass of the cell from the center of buoyancy is denoted as d.

In Eq. (26), the parameters ζ and ξ in the x- and y-components of the vector $\hat{\vec{m}}'$ are given by:

$$\zeta=-(1-\alpha_0)\frac{\partial w'}{\partial x}+(1+\alpha_0)\frac{\partial u'}{\partial z},\quad \xi=(1-\alpha_0)\frac{\partial w'}{\partial y}-(1+\alpha_0)\frac{\partial v'}{\partial z}. \tag{27}$$

The parameter α_0, representing the cell eccentricity, can be calculated using the following equation [15]-[19]:

$$\alpha_0=\frac{r_{\max}^2-r_{\min}^2}{r_{\max}^2+r_{\min}^2}, \tag{28}$$

where $r_{\max}$ and $r_{\min}$ are the semi-major and semi-minor axes of the spheroidal cell.

The perturbation equations, in their nonlinear form, can be written as follows (without the asterisks):

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t}+1-D_a\nabla_\perp^2\right)\nabla_\perp^2\psi=\sqrt{\mathrm{Ta}}\frac{\partial v}{\partial z}+f_m\frac{\mathrm{R}_b}{\mathrm{L}_b}\frac{\partial n}{\partial x}-f_m\mathrm{R}a\frac{\partial T}{\partial x}, \tag{29}$$

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t}+1-D_a\nabla_\perp^2\right)v=-\sqrt{\mathrm{Ta}}\frac{\partial\psi}{\partial z}, \tag{30}$$

$$-\frac{\partial\psi}{\partial x}\frac{dT_b}{dz}-\nabla_\perp^2 T-\mathrm{R}_i T=-\frac{\partial T}{\partial t}+\frac{\partial(\psi,T)}{\partial(x,z)}, \tag{31}$$

$$\mathrm{Pe}G_0(2-f_m)n_0\hat{\alpha}\frac{\partial\psi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{dn_b}{dz}+\frac{Pe}{L_b}\frac{\partial n}{\partial z}-\frac{1}{L_b}\nabla_\perp^2 n=-\frac{1}{\tilde{\sigma}}\frac{\partial n}{\partial t}+\frac{\partial(\psi,n)}{\partial(x,z)}, \tag{32}$$

$$\hat{\alpha}=\nabla_\perp^2+\alpha_0\left(\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial x^2}\right),$$

where $G_0 = D_m \mathcal{B}_0 / h^2$ is a dimensionless orientation parameter in the absence of modulation [15]. We impose ideal boundary conditions for stream function, temperature, and concentration of microorganisms:

$$\psi = \nabla_\perp^2 \psi = \frac{dv}{dz} = T = n = 0 \quad at \quad z = 0 \quad and \quad z = 1 . \tag{33}$$

3. WEAK NONLINEAR ANALYSIS

In the study of weakly nonlinear convective instability, it is common to use a truncated Fourier series representation for perturbed quantities because it provides a convenient and effective way to analyze and understand the system's behavior. In the linear stability analysis of convective systems, one typically starts with a base state (often a steady state) and perturbs it with small disturbances. These disturbances can be expanded in a Fourier series, and by analyzing the growth rates of different Fourier modes, you can determine whether the system is linearly stable or unstable. Truncating the series keeps the analysis tractable while capturing the essential behavior. For nonlinear system described by equations (29)-(32), we employ Fourier series representations for key physical variables, including the stream function ψ, velocity component v, temperature T, and concentration of microorganisms n:

$$\psi(x,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn}(t)\sin(mkx)\sin(n\pi z) ,\ v(x,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} V_{mn}(t)\sin(mkx)\cos(n\pi z) ,$$

$$T(x,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{mn}(t)\cos(mkx)\sin(n\pi z) ,\ n(x,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{mn}(t)\cos(mkx)\sin(n\pi z) . \tag{34}$$

We then conduct a local nonlinear stability analysis, focusing on specific modes, namely (1,1) for the stream function and velocity component, and $(1,1)+(0,2)$ for temperature and microorganism concentration. Notably, we observe that nonlinearity introduces distortions in the temperature and concentration fields, resulting from the interactions among ψ, T, and n. Consequently, this interaction generates a sinusoidal component $\sin 2\pi z$, in the flow. Thus, we arrive at a simple expression that characterizes finite-amplitude convection in the form:

$$\psi = A_{11}(t)\sin(kx)\sin(\pi z) ,\ v = V_{11}(t)\sin(kx)\cos(\pi z) ,$$

$$T = B_{11}(t)\cos(kx)\sin(\pi z) + B_{02}(t)\sin(2\pi z) ,\ n = C_{11}(t)\cos(kx)\sin(\pi z) + C_{02}(t)\sin(2\pi z) , \tag{35}$$

where the amplitudes $A_{11}(t), B_{11}(t), B_{02}(t), C_{11}(t), C_{02}(t), V_{11}(t)$ are functions of time. After substituting (35) into equations (29)-(32) and considering the orthogonality condition, the resulting evolution equations for amplitudes are obtained:

$$\frac{\partial A_{11}}{\partial \tilde{t}} = -\frac{1 + D_a a^2}{a^2} A_{11} + \frac{\pi\sqrt{\mathrm{T}a}}{a^4} V_{11} - f_m \frac{kRa}{a^2} B_{11} + f_m \frac{kR_b}{a^2 L_b} C_{11} , \tag{36}$$

$$\frac{\partial V_{11}}{\partial \tilde{t}} = -\frac{1 + D_a a^2}{a^2} V_{11} - \frac{\pi\sqrt{\mathrm{T}a}}{a^4} A_{11} , \tag{37}$$

$$\frac{\partial B_{11}}{\partial \tilde{t}} = -\frac{(1-b)}{\mathcal{V}_a} B_{11} - \frac{k\theta_0}{a^2 \mathcal{V}_a} A_{11} - \frac{\pi k}{a^2 \mathcal{V}_a} A_{11} B_{02} , \tag{38}$$

$$\frac{\partial B_{02}}{\partial \tilde{t}} = -\frac{\gamma - b}{\mathcal{V}_a} B_{02} + \frac{k\pi}{2a^2 \mathcal{V}_a} A_{11} B_{11} , \tag{39}$$

$$\frac{\partial C_{11}}{\partial \tilde{t}} = -\frac{\tilde{\sigma}}{L_b \mathcal{V}_a} C_{11} - \frac{k\pi\tilde{\sigma}}{a^2 \mathcal{V}_a} A_{11} C_{02} + \frac{\tilde{\sigma}}{a^2 \mathcal{V}_a}((2 - f_m)\gamma_1 + \gamma_2) k A_{11} , \tag{40}$$

$$\frac{\partial C_{02}}{\partial \tilde{t}} = -\frac{\gamma\tilde{\sigma}}{\mathcal{V}_a L_b} C_{02} + \frac{k\pi\tilde{\sigma}}{2a^2 \mathcal{V}_a} A_{11} C_{11} , \tag{41}$$

where $a = \sqrt{k^2 + \pi^2}$ is the total wave number, and $\tilde{t} = (a^2 \mathcal{V}_a)t$ is the reduced time, $\gamma = 4\pi^2 / a^2, b = \mathrm{R}_i / a^2$, $\omega_g t \to (\omega_g / a^2 \mathcal{V}_a)\tilde{t} = \tilde{\omega}_g \tilde{t}$.

The system of equations (36)-(41) is dissipative and its solutions are bounded in the phase space. The impact of many parameters $(Ta, Ra, Pe, D_a, R_b, L_b, \mathcal{V}_a)$ on trajectories is to attract them to a set of measures zero or fixed points. The system of autonomous ordinary differential equations can be solved numerically, for example, in the Maple computer environment. However, in the steady state, Eqs. (36)-(41) have an exact solutions:

$$\frac{A_{11}^2}{8} = -\frac{w_1}{2} + \sqrt{\frac{w_1^2}{4} - w_2}\,, \tag{42}$$

$$V_{11} = -\frac{\pi\sqrt{\mathrm{T}a}}{1+D_a a^2}A_{11},\, B_{11} = -\frac{2k\theta_0(4\pi^2-\mathrm{R}_i)A_{11}}{2(a^2-\mathrm{R}_i)(4\pi^2-\mathrm{R}_i)+k^2\pi^2A_{11}^2}\,,$$

$$B_{02} = -\frac{\pi k^2\theta_0 A_{11}^2}{2(a^2-\mathrm{R}_i)(4\pi^2-\mathrm{R}_i)+k^2\pi^2A_{11}^2}\,, C_{11} = \frac{8(\gamma_1+\gamma_2)L_b k A_{11}}{8a^2+k^2L_b^2A_{11}^2}\,,\; C_{02} = \frac{(\gamma_1+\gamma_2)L_b^2k^2A_{11}^2}{\pi(8a^2+k^2L_b^2A_{11}^2)}\,,$$

where

$$w_1 = \frac{\xi_1+\xi_2}{\xi_1\xi_2} - \frac{1}{r\xi_1}\left(\mathrm{R}_b(\gamma_1+\gamma_2) + \frac{\mathrm{R}a\theta_0}{1-\dfrac{\mathrm{R}_i}{a^2}}\right), \xi_1 = \frac{k^2L_b^2}{a^2}, \xi_2 = \frac{4\pi^2k^2}{(a^2-\mathrm{R}_i)(4\pi^2-\mathrm{R}_i)}\,,$$

$$r = \frac{a^4(1+D_a a^2)}{k^2} + \frac{\pi^2a^2\mathrm{T}a}{k^2(1+D_a a^2)},\quad \theta_0 = \frac{4\pi^2}{4\pi^2-\mathrm{R}_i}\,,$$

$$\gamma_1 = \frac{4\pi^2\langle n\rangle}{4\pi^2+\mathrm{P}e^2}[\mathrm{P}eG_0((1-\alpha_0)k^2+(1+\alpha_0)\pi^2)], \gamma_2 = \frac{4\pi^2\langle n\rangle \mathrm{P}e}{4\pi^2+\mathrm{P}e^2}\,,\; w_2 = -\frac{(\mathrm{R}a-\mathrm{R}a_c)\theta_0}{r\xi_1\xi_2\left(1-\dfrac{\mathrm{R}_i}{a^2}\right)}\,.$$

The expression for amplitude A_{11} (42) was obtained from solving a quadratic equation of the form:

$$\left(\frac{A_{11}^2}{8}\right)^2 + w_1\left(\frac{A_{11}^2}{8}\right) + w_2 = 0 \tag{43}$$

It is important to note that the stream function's amplitude must be real. Therefore, in solution (42), only the positive sign in front of the radical is taken into account. If we determine the value of A_{11}, we can calculate the heat and mass transfer in a steady state.

In the small amplitude limit $(A_{11}\to 0)$, equation (43), becomes a dispersion equation for linear stationary bio-thermal convection:

$$Ra_c = \left[\frac{a^4(1+D_a a^2)}{k_c^2} + \frac{\pi^2a^2\mathrm{T}a}{k_c^2(1+D_a a^2)} - \mathrm{R}_b(\gamma_1+\gamma_2)\right]\left[\frac{4\pi^2}{(4\pi^2-\mathrm{R}_i)\left(1-\dfrac{\mathrm{R}_i}{a^2}\right)}\right]^{-1} \tag{44}$$

In order to determine the wave number k_c that corresponds to the onset of convection, we need to minimize the critical Rayleigh number Ra_c with respect to k_c^2. We can achieve this by differentiating Ra_c with respect to k_c^2 and setting the derivative to zero. Once we solve this equation, we will have the value of k_c that is associated with the onset of convection. In the limit of small Peclet numbers, expression (43) coincides with the result first obtained by Kopp and Yanovsky [41] when there is no rotation $(\mathrm{Ta}=0)$ and internal heat source $(\mathrm{R}_i=0)$. When there are no microorganisms present ($n_0=0$), the critical Rayleigh number is the same as the results for the Darcy-Brinkman model of a rotating porous medium (without nanoparticles) that was derived by Chand and Rana [43]-[44]. However, if there is no heating or rotation within the system, ordinary bioconvection is observed, which is caused by the motion of microorganisms. In this case, the bioconvection Rayleigh number R_b serves as the governing parameter for bioconvection. The critical value of R_b, denoted as R_b^{cr}, for bioconvection in the Darcy model of a porous medium was initially determined by Nield et al. [20].

Additionally, the amplitudes can be rescaled using the following relations:

$$A_{11}(\tilde{t}) = -\frac{a^2\sqrt{2}}{k\pi}X(\tilde{t}),\quad B_{11}(\tilde{t}) = \frac{\sqrt{2}}{\pi}Y(\tilde{t}),\quad C_{11}(\tilde{t}) = \frac{\sqrt{2}}{\pi}\tilde{Y}(\tilde{t}),\; B_{02}(\tilde{t}) = -\frac{Z(\tilde{t})}{\pi},\quad C_{02}(\tilde{t}) = -\frac{\tilde{Z}(\tilde{t})}{\pi},\quad V_{11}(\tilde{t}) = \frac{\sqrt{2}}{k}V(\tilde{t})\,. \tag{45}$$

Applying rescaling the amplitudes (45) in Eqs. (36)-(41), we obtain the following set of equations:

$$\begin{cases} \dot{X} = -\mathcal{D}X - \mathrm{T}V + f_m(RY - Rb\tilde{Y}) \\ \dot{Y} = \mathcal{V}_a^{-1}(-(1-b)Y + X - XZ) \\ \dot{Z} = \mathcal{V}_a^{-1}(-(\gamma - b)Z + XY) \\ \tilde{Y} = \tilde{\sigma}\mathcal{V}_a^{-1}(-\mathrm{L}_b^{-1}\tilde{Y} - ((2-f_m)\gamma_1 + \gamma_2)X - X\tilde{Z}) \\ \tilde{Z} = \tilde{\sigma}\mathcal{V}_a^{-1}(-\gamma \mathrm{L}_b^{-1}\tilde{Z} + X\tilde{Y}) \\ \dot{V} = -\mathcal{D}V + \sqrt{\mathrm{T}a}X \end{cases} \tag{46}$$

Here $(\cdot)$ represents the derivative with respect to time $\tilde{t}$; $\mathcal{D}, R, Rb, T$ are new dimensionless parameters of the following form:

$$\mathcal{D} = \frac{1 + D_a a^2}{a^2}, \quad \mathrm{R} = \frac{k^2 \mathrm{R}a}{a^6}, \quad \mathrm{R}b = \frac{k^2 \mathrm{R}_b}{a^6 L_b}, \quad \mathrm{T} = \frac{\pi^2 \sqrt{\mathrm{T}a}}{a^6}. \tag{47}$$

Equations (46) describe a model that can be simplified to the fundamental Lorenz model [24] for a regular fluid. The model is six-dimensional and uses variables X and V to represent the velocity field, while Y and Z represent the temperature changes, $\tilde{Y}$ and $\tilde{Z}$ are used to represent the changes in concentration of microorganisms. The Lorenz-like model in Eqs. (46) is invariant under the transformation $(X, Y, Z, \tilde{Y}, \tilde{Z}, V) \to (-X, -Y, Z, -\tilde{Y}, \tilde{Z}, -V)$.

Let's proceed to calculate the heat and mass transfer.

4. HEAT AND MASS TRANSPORTS

Determining the heat and mass transfer of microorganisms is important when studying the bio-thermal convection of liquids. This is due to the fact that the onset of convection with an increasing Rayleigh number is easier to observe because of its effect on heat and mass transfer. Therefore, thermal and mass flows of microorganisms are important for identifying thermo- and bioconvective movement in its early stages. Heat transfer can be calculated and described using the Nusselt number Nu:

$$Nu(\tilde{t}) = 1 + \left[\frac{\int_0^{2\pi/k_{cr}} \frac{\partial T}{\partial z} dx}{\int_0^{2\pi/k_{cr}} \frac{\partial T_b}{\partial z} dx} \right]_{z=0} \tag{48}$$

According to (15) and (35), we get from (48)

$$Nu(\tilde{t}) = 1 - 2\pi B_{02}(\tilde{t}) \left(\sqrt{\mathrm{R}_i} ctg(\mathrm{R}_i) \right) \tag{49}$$

By analogy with (48), we find a quantitative characteristic of the mass transfer (Sherwood number Sh) of the concentration of microorganisms:

$$Sh(\tilde{t}) = 1 + \left[\frac{\int_0^{2\pi/k_{cr}} \frac{\partial n}{\partial z} dx}{\int_0^{2\pi/k_{cr}} \frac{\partial n_b}{\partial z} dx} \right]_{z=0} = 1 + 2\pi C_{02}(\tilde{t}) \left(\frac{e^{\mathrm{Pe}} - 1}{\langle n \rangle \mathrm{Pe}^2} \right). \tag{50}$$

5. THE DERIVATION OF THE GINZBURG-LANDAU EQUATION FROM THE SIX-DIMENSIONAL LORENZ-LIKE MODEL

According to [46], we express all perturbed quantities in equations (46) in terms of a series expansion concerning the small supercritical parameter, ε:

$$\vec{X} = \varepsilon \vec{X}_1 + \varepsilon^2 \vec{X}_2 + \varepsilon^3 \vec{X}_3 + \ldots, \quad \vec{X} = [X, Y, Z, \tilde{Y}, \tilde{Z}, V]^{Tr}, \; R = R_0 + \varepsilon^2 R_2 + \ldots. \tag{51}$$

Here, the amplitudes of perturbed quantities only depend on slow time $\tau = \varepsilon^2 \tilde{t}$. Upon substituting the expansion (51) into (46) for first-order ε, we obtain a linear system of equations:

$$\mathcal{L}\vec{X}_1 = 0, \quad \vec{X}_1 = [X_1, Y_1, Z_1, \tilde{Y}_1, \tilde{Z}_1, V_1]^{Tr}, \tag{52}$$

where the matrix $\mathcal{L}$ has the form

$$\mathcal{L}=\begin{bmatrix} -\mathcal{D} & R_0 & 0 & -Rb & 0 & -T \\ \mathcal{V}_a^{-1}\theta_0 & (b-1)\mathcal{V}_a^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & (b-\gamma)\mathcal{V}_a^{-1} & 0 & 0 & 0 \\ -\widetilde{\sigma}\mathcal{V}_a^{-1}(\gamma_1+\gamma_2) & 0 & 0 & -L_b^{-1}\widetilde{\sigma}\mathcal{V}_a^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma L_b^{-1}\widetilde{\sigma}\mathcal{V}_a^{-1} & 0 \\ \sqrt{\mathrm{T}a} & 0 & 0 & 0 & 0 & -\mathcal{D} \end{bmatrix}.$$

For there to exist a nontrivial solution of the homogeneous linear system (52), a certain condition must be met:

$$R_0\left(\frac{\theta_0}{1-b}\right)=\mathcal{D}+\frac{T\sqrt{Ta}}{\mathcal{D}}-RbL_b(\gamma_1+\gamma_2)\,. \tag{53}$$

Expression (53) completely coincides with expression (44) for the critical Rayleigh number of stationary bio-thermal convection in the linear theory. The solutions to the system (52) are given by

$$Y_1=\frac{\theta_0X_1}{1-b},\, Z_1=0,\, \widetilde{Y}_1=-L_b(\gamma_1+\gamma_2)X_1,\, \widetilde{Z}_1=0,\, V_1=\frac{\sqrt{\mathrm{T}a}}{\mathcal{D}}X_1\,. \tag{54}$$

We can express the equations for the second order in ε as follows:

$$\mathcal{L}\vec{X}_2=[\mathcal{R}_{21},\mathcal{R}_{22},\mathcal{R}_{23},\mathcal{R}_{24},\mathcal{R}_{25},\mathcal{R}_{26}]^{Tr},\quad \vec{X}_2=[X_2,Y_2,Z_2,\widetilde{Y}_2,\widetilde{Z}_2,V_2]^{Tr}, \tag{55}$$

where the nonlinear terms $\mathcal{R}_{2j}$ $(j=1,2,3,4,5,6)$ are

$$\mathcal{R}_{21}=0,\, \mathcal{R}_{22}=\mathcal{V}_a^{-1}X_1Z_1,\, \mathcal{R}_{23}=-\mathcal{V}_a^{-1}X_1Y_1,\, \mathcal{R}_{24}=\widetilde{\sigma}\mathcal{V}_a^{-1}X_1\widetilde{Z}_1,\, \mathcal{R}_{25}=-\widetilde{\sigma}\mathcal{V}_a^{-1}X_1\widetilde{Y}_1,\, \mathcal{R}_{26}=0\,.$$

Solutions of the equations (55) have the form:

$$Y_2=\frac{\theta_0X_2}{1-b},\, Z_2=\frac{\theta_0X_1^2}{(b-\gamma)(b-1)},\, \widetilde{Y}_2=-L_b(\gamma_1+\gamma_2)X_2\,,\ \widetilde{Z}_2=-\frac{L_b^2}{\gamma}(\gamma_1+\gamma_2)X_1^2,\, V_2=\frac{\sqrt{\mathrm{T}a}}{\mathcal{D}}X_2\,. \tag{56}$$

We will now proceed to discuss third-order equations:

$$\mathcal{L}X_3=[\mathcal{R}_{31},\mathcal{R}_{32},\mathcal{R}_{33},\mathcal{R}_{34},\mathcal{R}_{35},\mathcal{R}_{36}]^{Tr},\quad X_3=[X_3,Y_3,Z_3,\widetilde{Y}_3,\widetilde{Z}_3,V_3]^{Tr}, \tag{57}$$

where

$$\mathcal{R}_{31}=\frac{\partial X_1}{\partial\tau}-R_2Y_1-R_0\delta\cos(\Omega\tau)Y_1+Rb\delta\cos(\Omega\tau)\widetilde{Y}_1,\, \Omega=\frac{\widetilde{\omega}_g}{\varepsilon^2}\,,$$

$$\mathcal{R}_{32}=\frac{\partial Y_1}{\partial\tau}+\mathcal{V}_a^{-1}(X_1Z_2+X_2Z_1),\, \mathcal{R}_{33}=0,\, \mathcal{R}_{34}=\frac{\partial\widetilde{Y}_1}{\partial\tau}-\widetilde{\sigma}\mathcal{V}_a^{-1}\gamma_1\delta\cos(\Omega\tau)X_1+\widetilde{\sigma}\mathcal{V}_a^{-1}(X_1\widetilde{Z}_2+X_2\widetilde{Z}_1)\,,$$

$$\mathcal{R}_{35}=0,\, \mathcal{R}_{36}=\frac{\sqrt{Ta}}{\mathcal{D}}\frac{\partial X_1}{\partial\tau}\,.$$

We utilized the Fredholm alternative condition [46] to obtain the Ginzburg-Landau amplitude equation in the following form:

$$\mathcal{A}_1\frac{\partial X_1}{\partial\tau}-\mathcal{A}_2X_1+\mathcal{A}_3X_1^3=0\,, \tag{58}$$

where the coefficients $\mathcal{A}_1,\mathcal{A}_2,\mathcal{A}_3$ are defined by:

$$\mathcal{A}_1=1+\frac{\mathrm{R}_0\theta_0\mathcal{V}_a}{(1-b)^2}+\mathrm{Rb}L_b^2\frac{(\gamma_1+\gamma_2)\mathcal{V}_a}{\widetilde{\sigma}}-\frac{\mathrm{T}\sqrt{Ta}}{\mathcal{D}^2}\,,\ \mathcal{A}_2=\frac{\mathrm{R}_2\theta_0}{1-b}+\frac{\mathrm{R}_0\theta_0}{1-b}\delta\cos(\Omega\tau)+\mathrm{Rb}L_b\gamma_2\delta\cos(\Omega\tau)\,,$$

$$\mathcal{A}_3=\frac{\mathrm{R}_0\theta_0}{(\gamma-b)(1-b)^2}+\frac{\mathrm{Rb}L_b^3}{\gamma}(\gamma_1+\gamma_2)\,. \tag{59}$$

Passing to the new rescaled derivative with respect to slow time $\frac{\partial}{\partial \tau_1} = (a^2 \mathcal{V}_a)\frac{\partial}{\partial \tau}$ and using relations (45), (47), we rewrite equation (58) in the following form:

$$A_1 \frac{\partial A}{\partial \tau_1} - A_2(\tau)A + A_3 A^3 = 0\,, \tag{60}$$

where

$$A_1 = \frac{a^2}{\mathcal{V}_a} + \frac{4\pi^2 k_c^2 Ra_c}{(4\pi^2 - \mathrm{R}_i)(a^2 - \mathrm{R}_i)^2} + \frac{k_c^2}{a^4}\frac{R_b L_b}{\widetilde{\sigma}}\frac{4\pi^2 \langle n\rangle}{4\pi^2 + \mathrm{Pe}^2}\left[\mathrm{Pe}G_0((1-\alpha_0)k^2 + (1+\alpha_0)\pi^2) + \mathrm{Pe}\right] - \frac{\pi^2 \mathrm{Ta}}{\mathcal{V}_a(1+D_a a^2)^2}\,,$$

$$A_2 = \frac{4\pi^2 k_c^2}{(4\pi^2 - \mathrm{R}_i)(a^2 - \mathrm{R}_i)}\left(Ra_2 + Ra_c \delta\cos(\Omega_1 \tau_1)\right) + \frac{k_c^2 R_b}{a^2}\frac{4\pi^2 \langle n\rangle \mathrm{Pe}}{4\pi^2 + \mathrm{Pe}^2}\delta\cos(\Omega_1\tau_1)\,,$$

$$A_3 = \frac{k_c^4}{8a^4}\left(\frac{16\pi^4 a^4 Ra_c}{(16\pi^4 a^4 - \mathrm{R}_i^2)(a^2 - \mathrm{R}_i)^2} + R_b \mathrm{L}_b^2(\gamma_1 + \gamma_2)\right), \Omega_1 = \frac{\omega_g}{\varepsilon^2}\,. \tag{61}$$

In the case where there is no rotation and internal heating, the GL equation (60) for small Peclet numbers coincides with the result obtained by Kopp and Yanovsky [41]. They obtained the GL equation by applying perturbation theory for the small supercritical parameter of the Rayleigh number to the nonlinear system of equations (29)-(32). Because nonlinearity near the critical state of convection is considered in this approach, it is assumed that $Ra_2 \approx Ra_c$. Thus, a small expansion parameter ε^2 is introduced in the weakly nonlinear theory of convective instability, which is the relative deviation of the Rayleigh number Ra from its critical value Ra_c:

$$\varepsilon^2 = \frac{Ra - Ra_c}{Ra_c} \ll 1\,.$$

For the unmodulated case, the analytical solution of the above equation (60) takes the following form:

$$\widetilde{A}(\tau_1) = \frac{A_0}{\sqrt{\frac{A_3}{A_2}A_0^2 + \left(1 - A_0^2\frac{A_3}{A_2}\right)\exp\left(-\frac{2\tau_1 A_2}{A_1}\right)}}\,, \tag{62}$$

where $\widetilde{A}(\tau_1)$ represents the amplitude of convection for the unmodulated case, and A_1 and A_3 have the same expressions as given in (37), while $A_2 = k_c^2 Ra_2 / a^2$.

6. RESULTS AND DISCUSSION

In our study, we utilized a truncated Fourier series to analyze the phenomenon of weakly nonlinear biothermal convection. We focused on a rotating layer of a porous medium that is saturated with Newtonian fluid and contains gyrotactic microorganisms. The medium is subjected to gravitational modulation and internal heating. To solve the resulting nonlinear system of equations (36)-(41), we employed numerical methods to determine the heat and mass exchange coefficients, Nu and Sh. We chose a reasonable order of modulation amplitude, i.e., $\delta = 0.1$, and suitable initial conditions for the disturbance amplitudes, i.e., about 0.5. Let the value of the modified bioconvective Rayleigh-Darcy number $\mathrm{Ra}_B = \mathrm{R}_b\langle n\rangle$ change in the range $[0, 5000]$, and the values of the parameters Pe and G_0 change in the vicinity of $\mathrm{Pe} = 0.1$ and $\mathrm{G}_0 = 0.01$ [34]. The cell eccentricity can vary in the range $\alpha_0 \in [0,1]$. Next, we started examining the impact of parameters $D_a, \mathrm{Ta}, \mathrm{Ra}_B, \mathrm{Pe}, \alpha_0$ and R_i on the development of in Fig. 2 illustrate the relationship between the stationary Rayleigh number (44) and the dimensionless wavenumber k_c in linear theory. From the graphs, it is clear that the stability curves divide the space into two regions: the region above the curve is unstable, and the region below the curve is stable. This provides a clear visual representation of the relationship between parameters and system stability.

Fig. 2a illustrates the relationship between the stationary Rayleigh-Darcy number Ra_c and k_c for different Darcy number values $D_a = (0.3, 0.5, 0.8)$. The plot has fixed parameters α_0 = 0.4, $\mathrm{Ra}_B = 3000$, $\mathrm{Pe} = 0.1$, $\mathrm{R}_i = 0.5$ and Ta = 100. The plot clearly shows that an increase in the Darcy number leads to a rise in the minimum Rayleigh-Darcy number, indicating that higher Darcy numbers stabilize the stationary convection and delay its onset.

Fig. 2b shows the relationship between the stationary Rayleigh-Darcy number $\mathrm{R}a_c$ and the dimensionless wavenumber k_c for different values of the Taylor number: $\mathrm{Ta} = (100, 200, 300)$. The fixed parameters in this plot are

$D_a = 0.3$, $\mathrm{Ra}_B = 3000$, $\mathrm{Pe} = 0.1$, $\mathrm{R}_i = 0.5$ and $\alpha_0 = 0.4$. The graph demonstrates that as the Taylor number increases, the minimum Rayleigh-Darcy number also increases. This means that higher Taylor numbers stabilize stationary convection and delay the onset of convection.

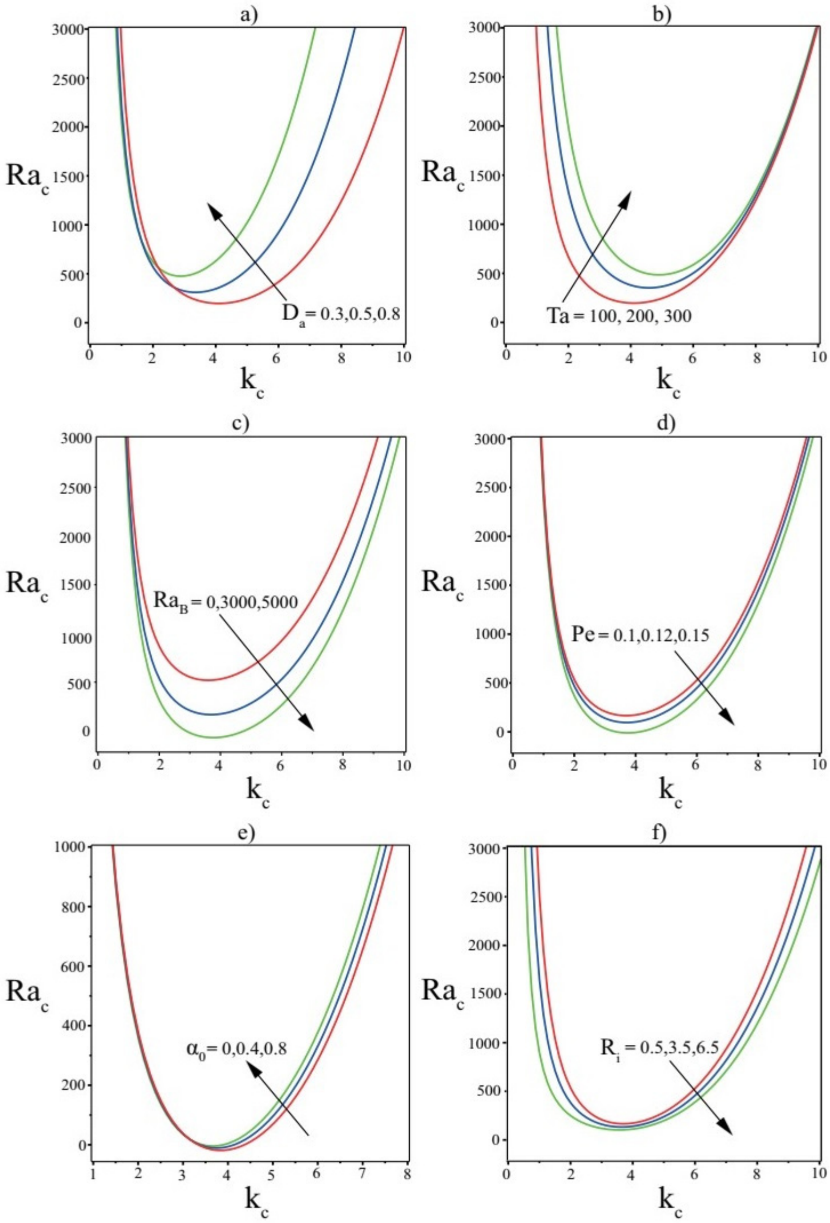

Figure 2. Plots of the stationary Rayleigh number Ra_c versus wave numbers k_c.

Fig. 2c shows the relationship between the stationary Rayleigh-Darcy number and the dimensionless wave number for various values of the modified bioconvection Rayleigh-Darcy number: $Ra_B = (0, 3000, 5000)$. The parameters $D_a = 0.3$, $\mathrm{Ta} = 100$, $\mathrm{Pe} = 0.1$, $\mathrm{R}_i = 0.5$ and $\alpha_0 = 0.4$ on this graph remain constant. The red curve represents the dependence of the stationary Rayleigh-Darcy number on the wave number in the absence of bioconvection of microorganisms ($Ra_B = 0$). Looking at Fig. 2c, it is clear that with increasing parameter Ra_B the threshold for the occurrence of biothermal convection decreases. This means that an increase in the concentration of microorganisms causes a redistribution of liquid density. Then, the swimming movement of microorganisms against gravity can destabilize the stationary process of biothermal convection.

Fig. 2d shows the relationship between the stationary Rayleigh-Darcy number and the dimensionless wave number for various values of the Peclet number: $Pe = (0.1, 0.12, 0.15)$. The parameters $D_a = 0.3$, $D_a = 0.3$, $\mathrm{R}a_B = 3000$, $\mathrm{R}_i = 0.5$ and $\alpha_0 = 0.4$ on this graph remain constant. From the graphs, it is evident that a decrease in the minimum Rayleigh numbers is correlated with an increase in the Peclet number or the swimming speed of cells. In other words, as Peclet numbers increase, the stability threshold of bio-thermal convection decreases.

Fig. 2e depicts the relationship between the stationary Rayleigh-Darcy number and the dimensionless wavenumber for different values of cell eccentricity: $\alpha_0 = (0, 0.4, 0.8)$. The parameters $D_a = 0.3$, $\mathrm{R}a_B = 3000$, $\mathrm{R}_i = 0.5$ and $\mathrm{Pe} = 0.15$ are fixed in this plot. By examining the graph, it becomes evident that as the cell eccentricity increases, the threshold for the onset of bio-thermal convection increases a bit. In simpler terms, microorganisms with a more elongated or non-spherical shape don't tend to facilitate the development of bio-thermal convection more than microorganisms with a spherical shape. This observation aligns with similar conclusions made in references [20],[23]. As a result, the shape of microorganisms influences the behavior and stability of the bio-thermal convection process within the studied system.

In Fig. 2f, the stationary Rayleigh-Darcy number $\mathrm{R}a_c$ is plotted as a function of the dimensionless wave number k_c for different values of the internal heating parameter R_i. The fixed parameters in this graph are $D_a = 0.3$, $\mathrm{R}a_B = 3000$, $\mathrm{Ta} = 100$, $\mathrm{Pe} = 0.1$ and $\alpha_0 = 0.4$. The graph shows that as the internal Rayleigh number R_i increases, the minimum Rayleigh-Darcy numbers decrease. This indicates that higher R_i numbers have a destabilizing effect on stationary convection.

For numerical analysis of equations (36)-(41) and (49)-(50), we use the standard Maple computer environment programs. The Runge-Kutta-Felberg method (rfk45) was used to solve nonlinear equations (36)-(41) with the following initial conditions: $A_{11}(0) = B_{11}(0) = C_{11}(0) = B_{02}(0) = C_{02}(0) = V_{11}(0) = 0.5$.

In Figs. 3a and 4a, it is shown that an increase in the Vadas number $\mathcal{V}_a$ contributes to an increase in heat and mass transfer over a short time interval. This observation is consistent with investigations carried out by Kiran et al. [38]-[40], Bhadauria and Agarwal [47], and Kopp and Yanovsky [42] where a comparable phenomenon was observed. The Vadasz number, which is directly proportional to the Prandtl number, assumes a pivotal role in determining the convective heat transfer characteristics. As the Vadasz number surges, it signifies an increased ratio of kinematic viscosity to thermal diffusivity in the fluid. This, in turn, leads to heightened fluid mobility. Consequently, elevating the Vadasz number results in a heightened rate of heat and mass transfer within the system.

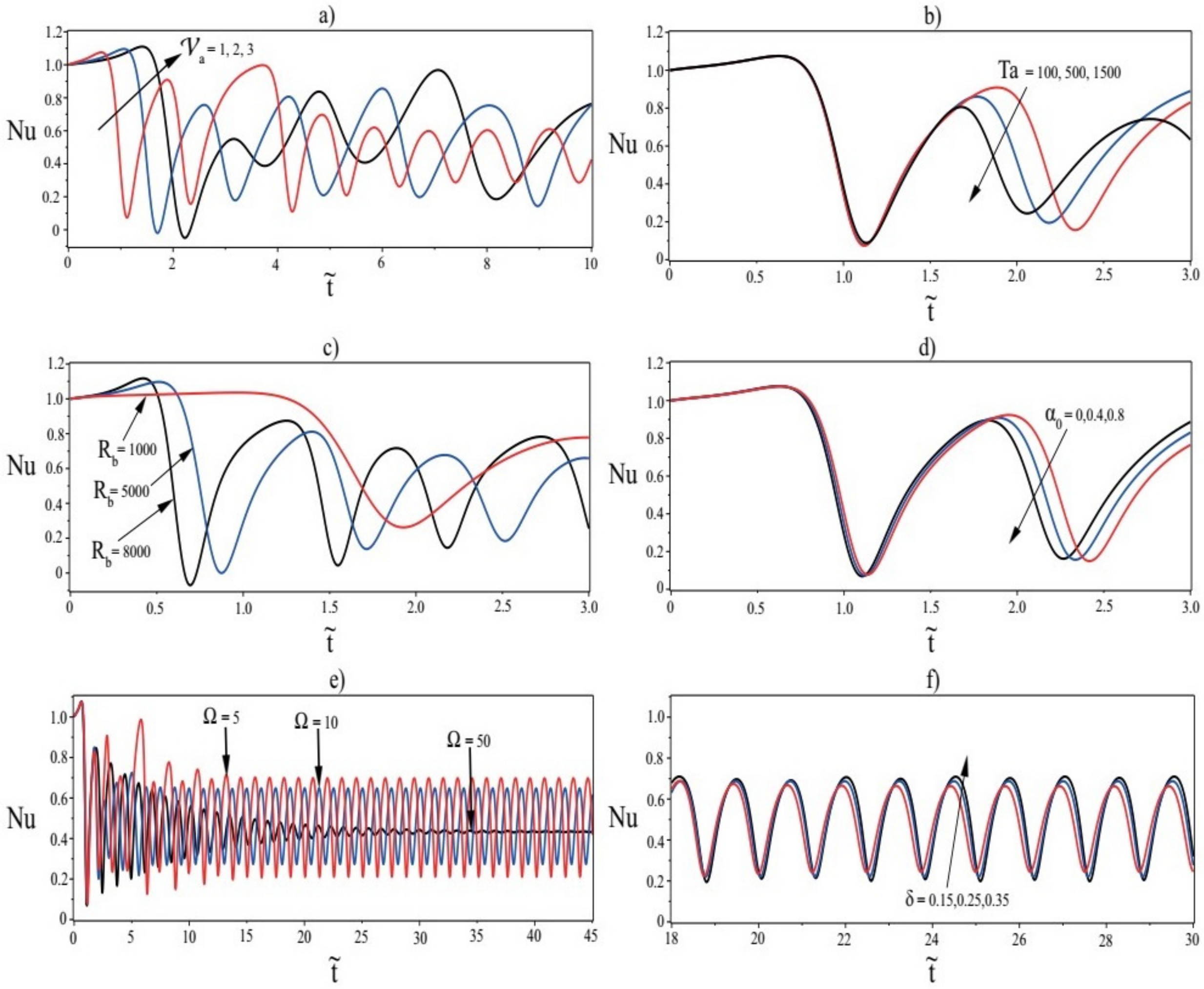

Figure 3. Dependence of the Nusselt number Nu on the time $\tilde{t}$ for a) $\mathcal{V}_a$, b) Ta, c) R_b, d) α_0, e) Ω, f) δ variations.

According to Figs. 3b and 4b, as the Taylor-Darcy number Ta increases, the value of the Nusselt Nu and Sherwood Sh numbers decreases, lowering the rate of heat and mass transfer and, as a result, stabilizing the system.

Figs. 3c and 4c show the effect of an increasing bioconvective Rayleigh-Darcy number R_b on heat and mass transfer. An increase in the bioconvective Rayleigh-Darcy number can be caused by an increase in the permeability of the porous medium or a decrease in the viscosity of the liquid medium. In both cases, this causes an increase in the velosity of convective flows, which promotes the transfer of heat and mass of microorganisms and therefore leads to destabilization of the system. This phenomenon eventually results in an increase in the Nusselt and Sherwood numbers over short time intervals, as shown in Figs. 3a and 4a.

Graphs in Figs. 3d and 4d demonstrate a significant influence of the shape of the microorganism on heat and mass exchange in the system, which is reflected in the temporary change in the Nusselt and Sherwood numbers. This physical phenomenon can be explained by considering how the shape of microorganisms affects their movement and interaction with liquid. In the case of spherical microorganisms $\alpha_0 = 0$ (represented by the red curve in Fig. 3d and 4d), their symmetrical shape ensures relatively unhindered movement in the liquid. This promotes improved heat transfer within the system, resulting in higher convective heat transfer rates and, therefore, higher Nusselt numbers. On the other hand, when microorganisms have a non-spherical or irregular shape $\alpha_0 = (0.4, 0.8)$ (represented by the blue and black curves in Fig. 3d and 4d), their movement in the liquid becomes more intense and complex. The presence of asymmetry in their shape can lead to a change in the flow regime. As a result, the process of convective heat transfer can be hindered, leading to lower heat transfer rates, lower Nusselt numbers, and, conversely, higher mass transfer rates (higher Sherwood numbers) compared to the case of spherical microorganisms.

Figs. 3e and 4e show the influence of the modulation frequency Ω on heat and mass exchange in the system. It can be noted that at lower modulation frequencies, especially in the low frequency case ($\Omega = 5$), higher heat and mass transfer rates are achieved compared to higher oscillation frequencies ($\Omega = 10$ and $\Omega = 50$). This suggests that lower modulation frequencies lead to more efficient heat transfer within the system. The observed trend is consistent with the results presented by Gresho and Sleigh [33] and Kopp et al. [48] in the context of conventional fluids, highlighting the importance of using low-frequency g-jitter to optimize the transfer process and improve heat transfer.

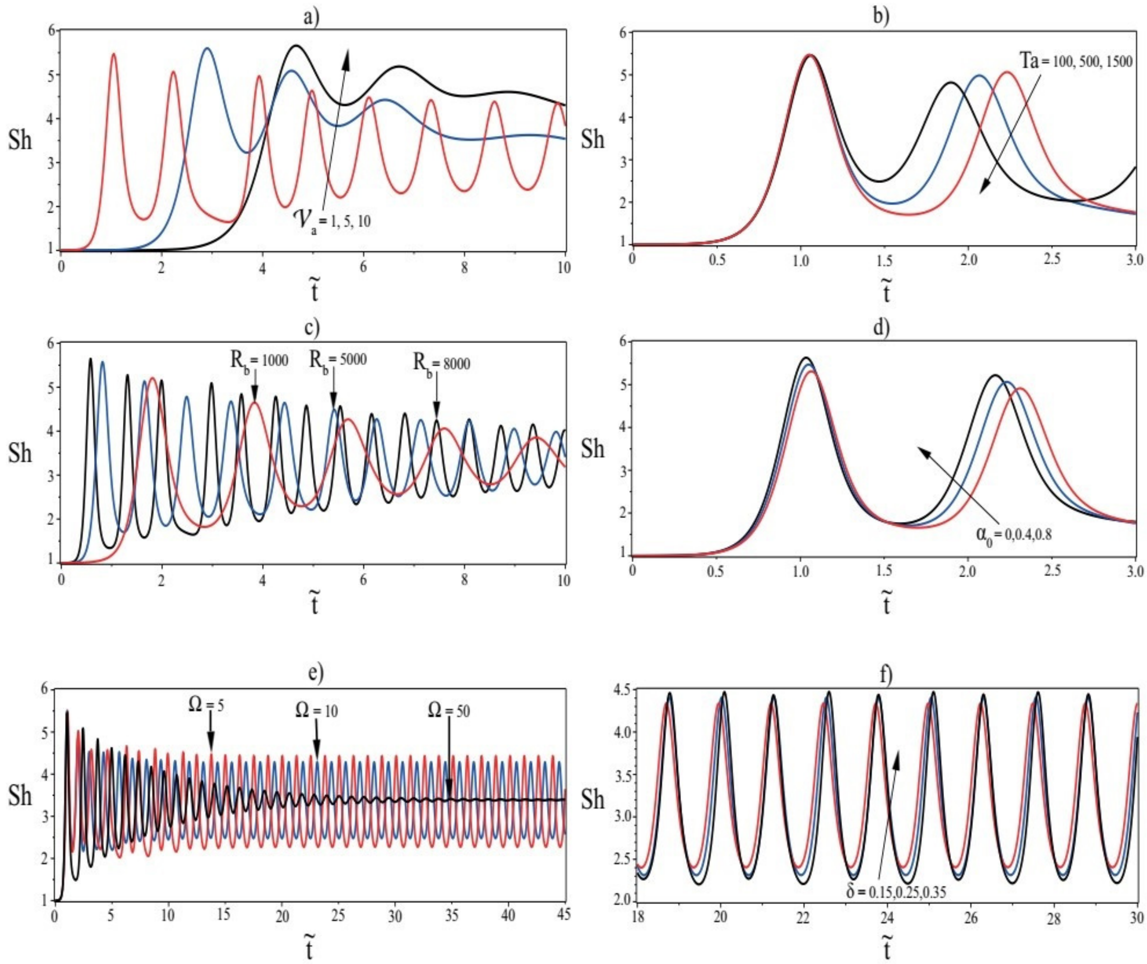

Figure 4. Dependence of the Sherwood number Sh on the time $\tilde{t}$ for a) $\mathcal{V}_a$, b) $\mathrm{T}a$, c) R_b, d) α_0, e) Ω, f) δ variations.

Figs. 3f and 4f show the influence of the modulation amplitude δ on heat and mass exchange in the system. The study considers a range of δ values, specifically chosen to improve heat transfer, from 0.15 to 0.35. The graphs show how changing the modulation amplitude affects the characteristics of heat and mass transfer. It is obvious that higher modulation amplitudes lead to an increase in the rate of heat and mass exchange. By appropriately selecting the modulation amplitude, heat and mass transfer can be optimized, thereby improving overall system performance.

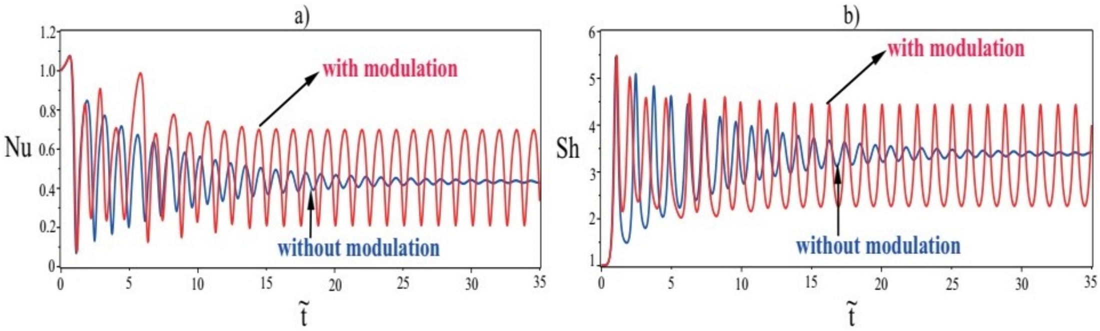

Figure 5. a) Variations of the Nusselt number $\mathrm{N}u(\tilde{t})$ in the absence of $\delta = 0$ and the presence of $\delta = 0.3, \Omega = 5$ modulation of the gravity field; b) variations of the Sherwood number $\mathrm{Sh}(\tilde{t})$ in the absence of $\delta = 0$ and the presence of $\delta = 0.3, \Omega = 5$ modulation of the gravity field.

In Figs. 5a and 5b, a comparison is presented between modulated and unmodulated systems. The graphs depict the characteristics of these two systems with regard to heat and mass transfer, as represented by the Nusselt and Sherwood numbers. In the unmodulated system, it is evident that the Nusselt number (Sherwood number) experiences a rapid increase for small values of the time parameter $\tilde{t}$. This initial surge in heat and mass transfer is followed by a phase of stabilization at higher $\tilde{t}$ values. This pattern suggests that, in the absence of gravitational field modulation, the process of heat and mass transfer eventually reaches a steady state. Conversely, the modulated system displays oscillatory behavior concerning Nusselt and Sherwood numbers. This signifies that heat exchange undergoes periodic fluctuations due to the modulation of the gravitational field. These fluctuations in the Nusselt and Sherwood numbers mirror the alternating enhancement and suppression of heat and mass exchange resulting from the gravitational field modulation.

7. CONCLUSIONS

In this research, we employed the Darcy-Brinkman model to explore the impacts of rotation, internal heating, and gravity modulation on biothermal convection within a porous medium saturated with a Newtonian fluid that includes gyrotactic microorganisms. Our analysis is based on the expansion of the perturbed parameters of the system using a truncated Fourier series. As a result of applying this approach, a non-autonomous system of nonlinear differential equations was obtained. An exact analytical solution to this system of equations was obtained for the stationary case. For small amplitudes of disturbances, i.e., within the framework of the linear theory of convective instability, we obtained the dispersion relation between the critical value of the Rayleigh number Ra_c and the wave number k_c. Based on the findings from the analysis of linear bio-thermal convective instability, the following conclusions can be drawn:

- Increasing the Darcy and Taylor numbers leads to an increase in the stability of the system.
- An increase in the modified bioconvective Rayleigh-Darcy number Ra_B, Peclet number Pe, and internal heating parameter R_i leads to destabilization of the system.
- Spherical gyrotactic microorganisms play a more significant role in fostering the onset of bio-thermal instability.

After conducting a numerical analysis of the resultant nonlinear system of equations, we drew several conclusions regarding the impact of gravitational modulation on biothermal convection in porous media:

- When the values of the parameters $\mathcal{V}_a$ and R_b are increased, a short-term growth in heat and mass transfer is observed.
- Increasing the Taylor number Ta has a stabilizing effect on the system, which delays the onset of convection and reduces heat and mass transfer.
- The spherical shape of the microorganisms contributes to a more efficient heat transfer process.
- Increasing the modulation frequency Ω leads to a decrease in the variations of the Nusselt and Sherwood numbers, resulting in suppressed heat and mass transfer.
- Increasing the modulation amplitude δ enhances heat and mass transfer.

Certainly, the examination of parameters like $\mathrm{Ta}, \mathrm{R}_b, \mathrm{Pe}, \alpha_0\ \mathcal{V}_a$, Ω, and δ offers valuable insights into their influence on the convection process. Through the manipulation of these parameters, it becomes feasible to amplify or control heat transfer in a porous medium saturated with a Newtonian fluid containing gyrotactic microorganisms. These conclusions provide a foundation for designing more efficient heat transfer systems, formulating strategies to regulate convection, and enhancing thermal management across diverse applications.

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СЛАБОНЕЛІНІЙНА БІОТЕРМАЛЬНА КОНВЕКЦІЯ В ШАРІ ПОРИСТОГО СЕРЕДОВИЩА ПІД ВПЛИВОМ ОБЕРТАННЯ, ГРАВІТАЦІЙНОЇ МОДУЛЯЦІЇ ТА ДЖЕРЕЛА ТЕПЛА

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В цій роботі досліджено вплив гравітаційної модуляції на слабонелінійну біотермальну конвекцію в пористому шарі, що обертається. Розглядається шар пористого середовища, насиченого ньютонівською рідиною, що містить гіротактичні мікроорганізми і схильної до гравітаційної модуляції, обертання і внутрішнього нагрівання. Для аналізу лінійної стійкості досить уявлення обурень як нормальних мод, тоді як нелінійний аналіз включає усічений ряд Фур'є, що містить гармоніку нелінійної взаємодії. Побудовано шестивимірну нелінійну модель типу Лоренца, яка виявляє як симетрію відображення, так і дисипацію. Тепло- і масоперенос ми визначили з використанням слабкої нелінійної теорії, заснованої на уявленні усіченого ряду Фур'є. Додатково було досліджено поведінку нестаціонарних чисел Нуссельта та Шервуда шляхом чисельного вирішення рівнянь кінцевої амплітуди. Застосовуючи розкладання регулярних збурень за малим параметром до шестивимірної моделі рівнянь Лоренца з періодичними коефіцієнтами, ми отримали рівняння Гінзбурга-Ландау (ГЛ). Це рівняння визначає еволюцію кінцевої амплітуди виникнення конвекції. Амплітуда конвекції у немодульованому випадку визначається аналітично і служить еталоном для порівняння. У дослідженні вивчається вплив різних параметрів на систему, включаючи число Вадасза, модифіковане число Релея-Дарсі, число Тейлора, ексцентриситет клітини та параметри модуляції, такі як амплітуда та частота. Варіюючи ці параметри, у різних випадках ми проаналізували тепло і масопередачу, яка кількісно виражається числом Нуссельта і Шервуда. Встановлено, що амплітуда модуляції істотно впливає на посилення тепло і масовіддачі, тоді як частота модуляції зменшує вплив.

Ключові слова: *модель Дарсі-Брінкмана; біотермальна конвекція; гравітаційна модуляція; пористе середовище, що обертається; гіротактичний мікроорганізм*