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EFFECTS OF TOPOLOGICAL DEFECTS AND MAGNETIC FLUX ON DISSOCIATION ENERGY OF QUARKONIUM IN AN ANISOTROPIC PLASMA

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In this paper, we investigate the effects of anisotropic parameters, topological defects, and magnetic flux on the dissociation energy of bottomonium in an anisotropic quark-gluon plasma. We use the three-dimensional Schrödinger equation and derive the energy eigenvalues. Our findings show that the dissociation energy decreases with increasing temperature, but there is a slight shift towards higher values when the magnetic flux is increased. Furthermore, the inclusion of topological defects causes further shifts in the dissociation energy at high temperatures. Additionally, we analyze the impact of anisotropic medium on dissociation energy, both with and without considering topological defects. We observe that including topological defects results in higher values for the dissociation energy across all temperatures, while ignoring them leads to lower values at all temperatures studied. Moreover, we consider the baryonic chemical potential and find that its effect on dissociation is negligible compared to temperature variations. These findings provide valuable insights into the behavior of heavy quarkonium systems under different physical conditions and contribute to our understanding of topological effects in anisotropic media.
Keywords: *Topological effects; Schrödinger equation; Nikiforov-Uvarov method; Finite temperature; Baryonic chemical potential*
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1. INTRODUCTION

The study contributes to the understanding of heavy quarkonium systems, specifically bottomonium mesons, in a hot and dense medium. It explores the effects of anisotropic parameters, topological defects, magnetic flux, and baryonic chemical potential on dissociation energy. By investigating these factors and their impact on quarkonium behavior, it adds valuable insights to this area of research. Dissociation of quarkonium in hot and dense media has been a topic of significant interest in the field of in the quark-gluon plasma. Quarkonium refers to a bound state of a heavy quark-antiquark pair, such as charm-anticharm (J/ψ) or bottom-antibottom (Υ). In the non-relativistic quark model, quarkonium are considered to be akin to a heavy particle moving in a Coulomb potential. When exposed to extremely high temperatures and densities, as found in heavy-ion collision experiments, quarkonium states may undergo dissociation due to the effects of the surrounding anisotropic plasma [1-4]. Further studies are extended to relativistic quark models to study the properties of hadron in high temperature [5-10]

The presence of an anisotropic plasma plays a crucial role in the dissociation process. Anisotropy refers to a situation in which the thermal motion of particles is not uniformly distributed in all directions. In the context of quarkonium dissociation, the anisotropic plasma can affect the screening properties of the medium. The ability of the plasma to screen the quark-antiquark potential depends on the direction of the motion of the heavy quark. Consequently, the dissociation rates of quarkonium states can exhibit a dependence on the direction of their relative motion through the anisotropic plasma [11].

Studying the dissociation of quarkonium in an anisotropic plasma requires theoretical frameworks that incorporate both non-relativistic quark models and the effects of the plasma. This is known as the non-relativistic QCD (NRQCD) framework. NRQCD provides a valuable tool to analyze the behavior of quarkonium in different plasma environments. It allows for the calculation of dissociation rates and other properties relevant to the study of quarkonium suppression in heavy-ion collisions. Understanding the dissociation of quarkonium in hot and dense media is crucial for unraveling the nature of the quark-gluon plasma and the properties of QCD matter under extreme conditions [12-13].

The study of point-like global monopoles has attracted significant interest in various branches of theoretical physics. These defects have implications in cosmology and astrophysics, as their existence could have left observable imprints on the early universe. Furthermore, their properties and interactions are of utmost importance in understanding the dynamics of field theories and the fundamental nature of particle physics. Experimental searches and theoretical investigations continue to shed light on the intriguing properties and implications of point-like global monopoles. Point-like global monopoles are hypothetical topological defects that may have formed during phase transitions in the early universe such as [14-15].

One area where point-like global monopoles have important implications is in the study of cosmic strings. Cosmic strings are linear topological defects that can form during phase transitions, and their interaction with point-like global monopoles can lead to the creation of cosmic junctions. These junctions serve as sources of gravitational radiation and

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can potentially leave imprints in the cosmic microwave background, providing valuable insights into the early universe's dynamics and supporting inflationary cosmology models [17].

Furthermore, the presence of point-like global monopoles can lead to the formation of cosmic texture. Cosmic textures are two-dimensional tangled structures that arise due to the evolution of coupled fields with non-trivial topology. The interaction between point-like global monopoles and cosmic textures can contribute to the generation of anisotropies in the cosmic microwave background, which can be probed through experiments such as the Planck satellite mission [18].

Beyond their impact on cosmic string evolution and cosmic texture formation, point-like global monopoles can also have implications on the large-scale structure of the universe. The presence of these topological defects can affect the distribution and clustering of matter over cosmological distances, potentially leaving detectable signatures in galaxy surveys and cosmological observables [19]. Their influence on structure formation is highly dependent on their initial conditions and properties, making their study crucial for understanding the dynamics of the universe at different scales.

The work explores the characteristics of heavy mesons in an anisotropic-plasma environment, paying attention to the impact of topological effects in space. Novelty: The incorporation of anisotropic parameters and topological defects in studying quarkonium dissociation energy sets this paper apart from previous works that primarily focus on classical cases or neglect these factors altogether. This novel approach provides a more comprehensive understanding of how different physical conditions affect heavy quarkonia.

The paper is structured as follows: Section 2 briefly describes the new method, Section 3 delves into the computation of energy eigenvalues and wave functions, Section 4 discusses the results obtained, and finally, Section 5 provides a summary and conclusion of the findings.

2. THE SCHRÖDINGER EQUATION IN POINT-LIKE GLOBAL MONOPOLEWITH POTENTIAL INTERACTION

In this section, we find the solution for the eigenvalues of non-relativistic particles in the presence of a quantum flux field, considering a point-like global monopole with potential.

For a detailed explanation of the two-particle system interacting through an electromagnetic spherically symmetric potential $V(r)$ in the framework of radial-Schrodinger equation, see Ref. [16].

$$\left[\frac{d^2}{dr^2}+\frac{1}{\alpha^2}\left(2\mu\left(E-V\left(r\right)\right)-\frac{L'(L'+1)}{r^2}\right)\right]\Psi(r)=0, \tag{1}$$

where $L'=L-\Phi$ and μ are the angular momentum quantum number and the reduced mass for the quarkonium particle (for charmonium $\mu=m_c/2$ and for bottomonium $\mu=m_b/2$), respectively, and $0<\alpha\leq 1$ characterize the topological defect parameter of point-like global monopole and Φ is the amount of magnetic flux which is a positive integer.

Real Part of The Potential in An Anisotropic Medium

Here, we aim to find the potential due to the presence of a dissipative anisotropic hot QCD medium. The in-medium modification can be obtained in the Fourier space by dividing the heavy-quark potential by the medium dielectric permittivity, $\varepsilon\left(K\right)$ as follows

$$\tilde{V}\left(k\right)=\frac{V\left(k\right)}{\varepsilon\left(K\right)}, \tag{2}$$

by taking the inverse Fourier transform, the modified potential is obtained as follows

$$V\left(r\right)=\int\frac{d^3k}{\left(2\pi\right)^{\frac{3}{2}}}\left(e^{ik\cdot r}-1\right)\tilde{V}\left(k\right), \tag{3}$$

where $V\left(k\right)$ is the Fourier transform of Cornell potential $V\left(r\right)=-\frac{\alpha}{r}+\sigma r$ that gives as follows

$$V\left(k\right)=-\sqrt{\frac{2}{\pi}}\left(\frac{\alpha}{k^2}+\frac{2\sigma}{k^4}\right), \tag{4}$$

$\varepsilon\left(K\right)$ may be calculated which found from the self-energy using finite temperature QCD. By applying hard thermal loop resummation technique as in Refs. [20, 21], the static gluon propagator which represents the inelastic scattering of an off-shell gluon to a thermal gluon is defined as follows

$$\Delta^{\mu\nu}(w,k)=k^2g^{\mu\nu}-k^{\mu}k^{\nu}+\Pi^{\mu\nu}(w,k), \tag{5}$$

the dielectric tensor can then be obtained in the static limit in Fourier space, from the temporal component of the propagator as

$$\varepsilon^{-1}(K) = -\lim_{w\to 0} k^2 \Delta^{00}(w,k), \tag{6}$$

to calculate the real part of the inter-quark potential in the static limit, one can obtain first the temporal component of real part of the retarded propagator in Fourier space at finite temperature and chemical potential as given in Ref. [20] as follows

$$\mathrm{Re}\left[\Delta_R^{00}\right](w=0,k) = -\frac{1}{k^2 + m_D^2(T,\mu)} - \xi\left(\frac{1}{3\left(k^2 + m_D^2(T,\mu)\right)}\right) - \frac{m_D^2(T,\mu)(3\cos 2\theta - 1)}{6\left(k^2 + m_D^2(T,\mu)\right)^2}, \tag{7}$$

the medium dielectric permittivity $\varepsilon(K)$ is then given

$$\varepsilon^{-1}(K) = \frac{k^2}{k^2 + m_D^2} + k^2\xi\left(\frac{1}{3\left(k^2 + m_D^2\right)}\right) - \frac{m_D^2(3\cos 2\theta - 1)}{6\left(k^2 + m_D^2\right)^2}. \tag{8}$$

Substituting Eqs. (4) and (8) into Eqs. $(2,3)$ and then taking its inverse Fourier transform, we can write the real part of the potential for $rm_D << 1$ as follows

$$V(r,\xi,T,\mu_b) = \sigma r\left(1+\frac{\xi}{3}\right) - \frac{\alpha}{r}\left(1+\frac{(rm_D)^2}{2}\right) + \xi\left(\frac{1}{3} + \frac{(rm_D)^2}{16}\left(\frac{1}{3} + \frac{(rm_D)^2}{16}\left(\frac{1}{3} + \cos(2\theta)\right)\right)\right), \tag{9}$$

where ξ is the anisotropic parameter. T and μ_b are the temperature and the baryonic chemical potential, respectively. In Eq. (9), the potential depends on θ which is the angle between the particle momentum and the direction of anisotropy. We note that the potential in Eq. (9) reduces to the Cornell potential for $\xi = 0$ and $m_D = 0$ {For details, see Ref. [20]}. In the present work, the Debye mass $D(T,\mu_b)$ is given as in Refs. [22, 23] by

$$D(T,\mu_b) = gT\sqrt{\frac{N_c}{3} + \frac{N_f}{6} + \frac{N_f}{2\pi^2}\left(\frac{\mu_q}{T}\right)^2}, \tag{10}$$

where, g is the coupling constant and μ_q is the quark chemical potential $\left(\mu_q = \frac{\mu_b}{3}\right)$, N_f is number of flavors, and N_c is number of colors. The NU method [24] is briefly given here to solve the form of the following equation

$$\Psi''(s) + \frac{\bar{\tau}(s)}{\sigma(s)}\Psi'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\Psi(s) = 0, \tag{11}$$

where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials of maximum second degree and $\bar{\tau}(s)$ is a polynomial of maximum first degree with an appropriate $s = s(r)$ coordinate transformation. We try to find a particular solution by separation of variables, if one deals with the transformation

$$\Psi(s) = \Phi(s)\chi(s). \tag{12}$$

Eq. (11) is written as

$$\sigma(s)\chi''(s) + \tau(s)\chi(s) + \lambda\chi(s) = 0, \tag{13}$$

where

$$\sigma(s) = \pi(s)\frac{\Phi(s)}{\Phi'(s)}, \tag{14}$$

and

$$\tau(s) = \bar{\tau}(s) + 2\pi(s); \quad \tau'(s) < 0, \tag{15}$$

$$\lambda = \lambda_n = -n\tau'(s) - \frac{n(n-1)}{2}\sigma''(s), n = 0,1,2,..., \tag{16}$$

$\chi(s)=\chi_n(s)$ is a polynomial of degree n which satisfies the hypergeometric equation, taking the form

$$\chi_n(s)=\frac{B_n}{\rho_n}\frac{d^n}{ds^n}(\sigma''(s)\rho(s)), \tag{17}$$

where B_n is a normalization constant and $\rho(s)$ is a weight function which satisfies the following equation

$$\frac{d}{ds}\omega(s)=\frac{\tau(s)}{\sigma(s)}\omega(s); \quad \omega(s)=\sigma(s)\rho(s), \tag{18}$$

$$\pi(s)=\frac{\sigma'(s)-\bar{\tau}(s)}{2}\pm\sqrt{(\frac{\sigma'(s)-\bar{\tau}(s)}{2})^2-\tilde{\sigma}(s)+K\sigma(s)}, \tag{19}$$

and

$$\lambda=K+\pi'(s), \tag{20}$$

$\pi(s)$ is a polynomial of the first degree. The values of K in the square root of Eq. (19) is possible to calculate if the function under the square is a square of a function. This is possible if its discriminant is zero. For r parallel to the direction of n of anisotropy at $\theta=0$, the potential is given by

$$V(r)=a_1r-\frac{b_1}{r}, \tag{21}$$

where

$$a_1=\sigma+\frac{1}{3}\sigma\xi-\frac{1}{2}\alpha m_D^2-\frac{1}{2}\alpha\xi m_D^2, \tag{22}$$

$$b_1=\alpha+\frac{\alpha\xi}{3}. \tag{23}$$

By applying the above method to the potential given in Eq. (21), we obtain the energy eigenvalues as follows

$$E_{nl}^{\parallel}=\frac{3a_1}{\delta}-\frac{2\mu_1(\frac{3a_1}{\delta^2}+b_1)^2}{[(2n+1)+\sqrt{1+\frac{8\mu_1a_1}{\delta^3}+\frac{4}{\alpha}L'(L'+1)}]^2}, \tag{23}$$

Similarly, for r perpendicular to the direction of n anisotropy at $\theta=\pi/2$, the potential is given by

$$V(r)=a_2r-\frac{b_2}{r}, \tag{24}$$

where

$$a_2=\sigma+\frac{1}{3}\sigma\xi-\frac{1}{2}\alpha m_D^2+\frac{1}{24}\alpha\xi m_D^2, \tag{25}$$

$$b_2=\alpha+\frac{\alpha\xi}{3}. \tag{26}$$

and the energy eigenvalues are given as follows

$$E_{nl}^{\perp}=\frac{3a_2}{\delta}-\frac{2\mu_1(\frac{3a_2}{\delta^2}+b_2)^2}{[(2n+1)+\sqrt{1+\frac{8\mu_1a_1}{\delta^3}+\frac{4}{\alpha}L'(L'+1)}]^2}, \tag{27}$$

where δ is a parameter will be determined as in Ref. [2].

DISCUSSION OF RESULTS

In this section, we calculate spectra of the heavy quarkonium system such as bottomonium mesons in the hot and dense medium. The mass of quarkonium is calculated in the 3-dimensional space. We apply the following relation as in Ref. [2].

$$M=2m+E_{nL}, \tag{28}$$

where m is quarkonium bare mass for the charmonium or bottomonium mesons. By using Eq. (23), we write Eq. (28) as follows:

$$M = 2m + \frac{2\mu_1(\frac{3a_1}{\delta^2} + b_1)^2}{[(2n+1) + \sqrt{1 + \frac{8\mu_1 a_1}{\delta^3} + \frac{4}{\alpha} L'(L'+1)}]^2}. \tag{29}$$

Eq. (29) represents the quarkonium masses in hot and dense medium with topological effects and magnetic flux in an anisotropic plasma. By taking $\alpha = 1$ and $\Phi = 0$, we obtain

$$M = 2m + \frac{2\mu(\frac{3a_1}{\delta^2} + b_1)^2}{[(2n+1) + \sqrt{1 + \frac{8\mu a_1}{\delta^3} + \frac{4}{\alpha} L(L+1)}]^2}. \tag{30}$$

Eq. (30) coincides with Ref. [2]. We discuss the effect of the anisotropic parameter on the quarkonium dissociation energy, specifically focusing on the bottomonium meson. The parameters used in this calculation are based on Ref. [2] which sets the mass of the bottom quark to be 4.686 GeV. Furthermore, we are specifically considering the 1S state of the bottomonium meson. The Eq. (23) is employed to calculate the dissociation energy, in which r parallel to the direction of n of anisotropy at $\theta = 0$. In Fig. (1), the dissociation energy is plotted against the temperature, ranging from 0.17 to 0.35 GeV, which corresponds to the quark-gluon plasma phase. In this figure, the effect of topological defect is ignored, and we observe that the dissociation energy decreases as the temperature increases. When we increase the magnetic flux, the curves representing the dissociation energy slightly shift towards higher values at the starting temperature. However, the effect of the temperature becomes more apparent at higher temperatures, indicating that the magnetic flux has a stronger impact in very hot mediums. It is important to note that the medium in this context is anisotropic, with a parameter value of $\zeta = 0.3$. Moving on to Fig. (2), we consider the incorporation of topological defect in the anisotropic medium.

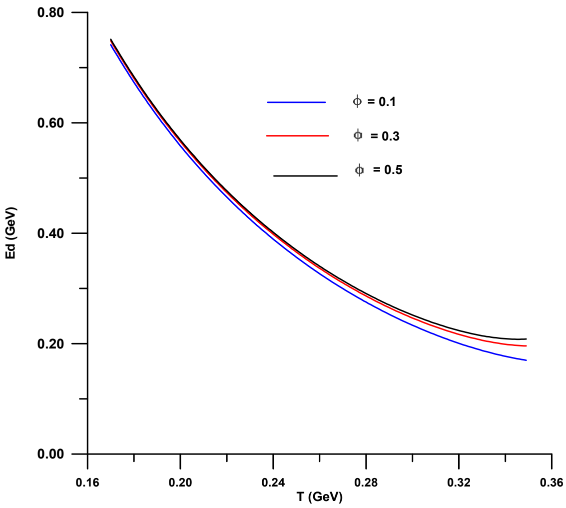

Figure 1. The dissociation energy is plotted with temperature for different values of magnetic flux at ζ=0.3, ∝= 1.0 and u_b=0

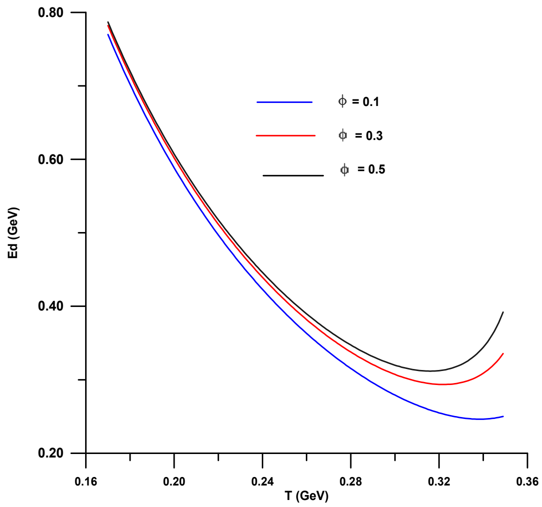

Figure 2. The dissociation energy is plotted with temperature for different values of magnetic flux at ζ=0.3, ∝= 0.5and u_b= 0

Here, we observe that the dissociation energy is shifted to larger values for all different magnetic flux values. This effect becomes more prominent at very high temperatures, indicating that topological defect plays a crucial role in hot anisotropic mediums. Now, if we turn our attention to Figures (3,4), we concentrate on the effect of anisotropic medium on dissociation energy. In Fig. (3), with the inclusion of topological defect, the dissociation energy is shifted to higher values in the anisotropic medium at every temperature. On the other hand, in Fig. (4), when the topological defect is ignored, we observe that the dissociation energy decreases to lower values at all temperatures. It is worth noting that recent works have not considered topological defect, as most of them focus on the study of quarkonium in the classical case, which refers to zero temperature and chemical baryonic potential. In Ref. [16], a similar effect of the magnetic flux on the eigenvalue of energy is observed, where the energy is shifted to higher values with increasing magnetic flux. The authors of the work also investigated the potential interaction under different values of topological parameter and magnetic flux.

Also, in the present work, we considered the effect of a dense medium by including the baryonic chemical potential in our study. In Figure 5 and Figure 6, we examined two cases: one where we ignored the topological defect (as shown in Figure 5), and another where we considered it (as shown in Figure 6). We observed that without the topological defect, the dissociation occurs more rapidly with increasing temperature and gradually very slightly decreases with the baryonic chemical potential. However, when we include the topological defect, we see the same behavior but with a shift towards higher energy values. Furthermore, in contour 7, we note that higher values of dissociation are observed at lower temperature and lower chemical potential, and these values gradually decrease as the temperature increases. We also observe that the effect of the baryonic chemical potential is negligible at every temperature point. A similar behavior is seen in contour 8 when the effect of the topological defect is considered.

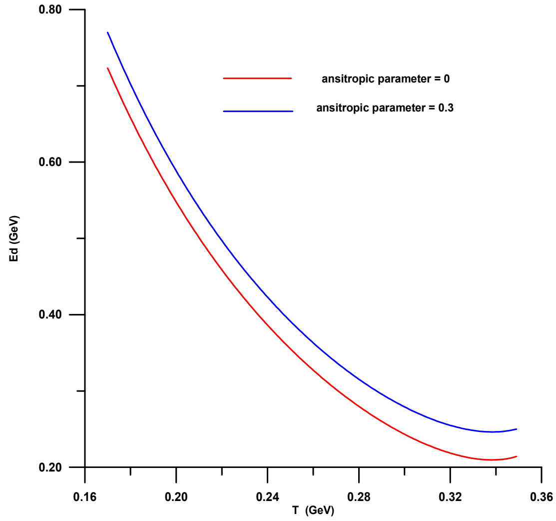

Figure 3. The dissociation energy is plotted with temperature for different values of ansitropic parameter at α = 0.5 and Φ = 0.1 and $u_b = 0$

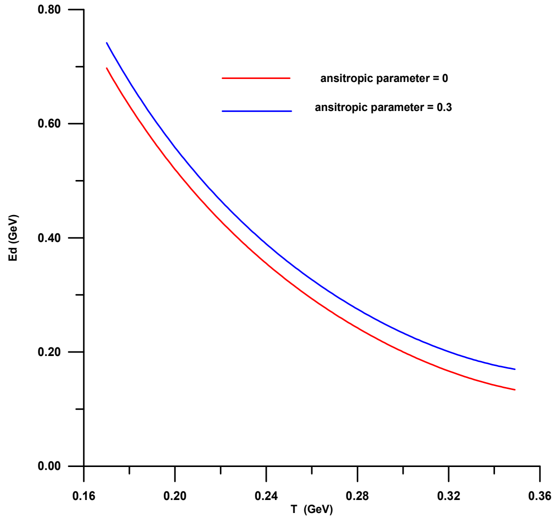

Figure 4. The dissociation energy is plotted with temperature for different values of ansitropic parameter at α = 1.0 and Φ = 0.1 and $u_b = 0$

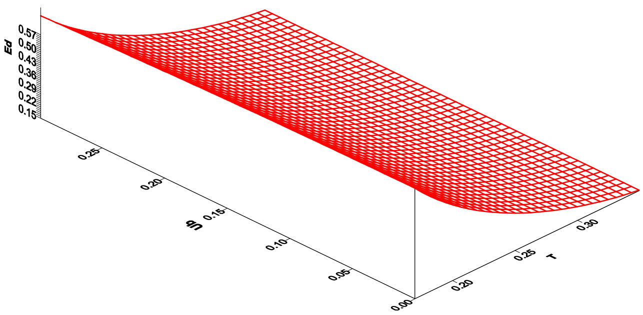

Figure 5. The dissociation energy is plotted as a function of temperature and baryonic chemical potential at α =1.0 and Φ = 0

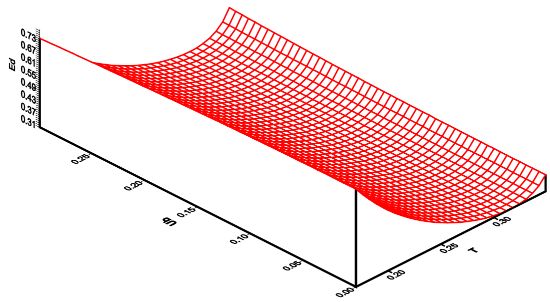

Figure 6. The dissociation energy is plotted as a function of temperature and baryonic chemical potential at α = 0.5 and Φ = 0.5

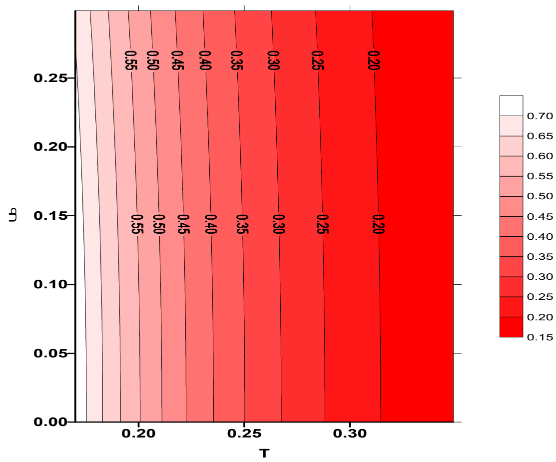

Figure 7. The dissociation energy is plotted as a countur of temperature and baryonic chemical potential at α=1.0 and Φ = 0

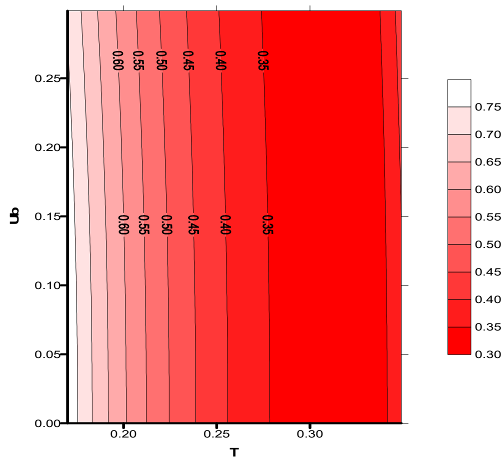

Figure 8. The dissociation energy is plotted as a countur of temperature and baryonic chemical potential at α=0.5 and Φ = 0.5

CONCLUSION

This study explores the effects of anisotropic parameters, topological defects, magnetic flux, and baryonic chemical potential on the dissociation energy of bottomonium mesons in a hot and dense medium. The results demonstrate that temperature plays a significant role in decreasing dissociation energy while magnetic flux slightly shifts it towards higher values. Incorporating topological defects further increases dissociation energy at high temperatures. Additionally, considering anisotropic medium leads to higher dissociation energies compared to isotropic conditions. The inclusion of baryonic chemical potential has negligible impact on dissociation compared to temperature variations. These findings provide valuable insights into the behavior of heavy quarkonium systems under different physical conditions and contribute to our understanding of topological effects in anisotropic mediums. We hope to extend this work as future works relativistic quark model as in Ref. [25] with fractional derivative as in Ref. [26] or extend to molecular structure as in Ref. [27].

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ВПЛИВ ТОПОЛОГІЧНИХ ДЕФЕКТІВ ТА МАГНІТНОГО ПОТОКУ НА ЕНЕРГІЮ ДИСОЦІАЦІЇ КВАРКОНІЮ В АНІЗОТРОПНІЙ ПЛАЗМІ

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У цій статті ми досліджуємо вплив анізотропних параметрів, топологічних дефектів і магнітного потоку на енергію дисоціації боттомонію в анізотропній кварк-глюонній плазмі. Ми використовуємо тривимірне рівняння Шредінгера та отримуємо власні значення енергії. Наші результати показують, що енергія дисоціації зменшується зі збільшенням температури, але є невеликий зсув у бік вищих значень, коли збільшується магнітний потік. Крім того, включення топологічних дефектів викликає подальші зрушення в енергії дисоціації при високих температурах. Крім того, ми аналізуємо вплив анізотропного середовища на енергію дисоціації, як з урахуванням, так і без урахування топологічних дефектів. Ми спостерігаємо, що включення топологічних дефектів призводить до більш високих значень енергії дисоціації при всіх температурах, тоді як їх ігнорування призводить до нижчих значень при всіх досліджуваних температурах. Крім того, ми розглядаємо баріонний хімічний потенціал і знаходимо, що його вплив на дисоціацію є незначним у порівнянні зі змінами температури. Ці висновки дають цінну інформацію про поведінку важких кварконієвих систем за різних фізичних умов і сприяють нашому розумінню топологічних ефектів в анізотропних середовищах.

Ключові слова: *топологічні ефекти; рівняння Шредінгера; метод Нікіфорова-Уварова; кінцева температура; баріонний хімічний потенціал*