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OSCILLATORY MAXWELL-CATTANEO FERROCONVECTION IN A DENSELY PACKED ROTATING POROUS MEDIUM SATURATED WITH A VISCOELASTIC MAGNETIC FLUID

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The combined effect of second sound and the viscoelasticity is examined using the classical stability analysis on the onset of rotating porous medium ferroconvection. Local thermal equilibrium is assumed between the solid matrix and fluid. Present problem is examined by an analytical approach by considering the pertinent boundary conditions. Normal mode analysis technique is utilized for obtaining the critical values for both instabilities namely stationary and oscillatory. We noticed that the oscillatory mode of instability is favored over the stationary mode of instability. We found that magnetic forces, second sound, nonlinearity in magnetization, Vadasz number, stress relaxation due to viscoelasticity and Taylor-Darcy number are in favour of advancing oscillatory porous medium ferroconvection whereas strain retardation postpone the outset of oscillatory porous medium ferroconvection. Convection cell size effects by different parameters and the oscillation's frequency are also noted. This problem shall have significant feasible technological applications wherein viscoelastic magnetic fluids are involved.
Keywords: *Convection; Rotation; Viscoelastic fluids; Maxwell equations; Porous media; Navier-Stokes equations for incompressible viscous fluids*
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1. INTRODUCTION

The dynamics of ferrofluids can be controlled by an externally acting applied magnetic field (Shliomis [1]). Rosensweig [2-4] was the first to synthesize ferrofluids. Considering both magnetic and buoyancy forces, a comprehensive analysis of *RBC* in ferrofluids was reported by Finlayson [5]. The findings of Finlayson [5] were examined both thereotically and experimentally by Schwab *et al*. [6] and Stiles and Kagan [7] respectively. Lalas and Carmi [8] reported the unique results on ferroconvection with energy stability approach. The impact of internal heating on the energy stability of magnetic fluids was documented by Mahajan and Sharma [9]. Nisha Mary and Maruthamanikandan [10] investigated a time-dependent body force effect on magnetic fluid convection. Soya Mathew *et al*. [11] studied porous medium ferroconvection with Maxwell-Cattaneo equation. Laroze and Pleiner [12] examined numerical and theoretical impact on ferroconvection in a viscoelastic carrier liquid. Recently Balaji *et al*. [13] worked on magnetic field modulation affected ferroconvection in a Brinkman porous medium. Vidyashree *et al*. [14] examined the combined effect of variable gravity and MFD viscosity on porous medium ferroconvection. Naseer *et al*. [15] analyzed the dual nature of Prandtl number in the presence and the absence of non-classical conduction.

When it comes to instabilities in viscoelstic fluids, Oldroyd model [16] gives the fundamental rheological equation describing the properties of viscoelastic realistically. In comparison the relaxational time in normal liquids is very short as that of viscoelastic liquids. Green [17] examined that for viscoelastic liquids the principle of exchange of stabilities is invalid when the restoring force is large. Malashetty *et al*. [18] and Jianhong Kang *et al*. [19] studied the rotating *RBC* in viscoelastic fluids by means of both linear and weakly non-linear techniques. Laroze *et al*. [20] presented theoretical and numerical results on ferroconvection in a viscoelastic carrier liquid. Several other researchers contributed to addressing the problem of convective instability of viscoelastic fluids with a variety of constraints techniques (Bhadauria and Kiran [21], Alves *et al*. [22], Sohail Nadeem *et al*. [23], Mahmud *et al*. [24], Sharma and Mondal [25] and Kaiyao *et al*. [26], Dhiman *et al*. [27]).

As for the convection due to porous medium, Saravanan and Sivakumar [28] made an investigation on the impact of vibrations on *RBC* in porous media with arbitrary amplitude and frequency. Very recently, Rudresha *et al*. [29] studied the theoretical influence of time-periodic electric field on electroconvection of Brinkman type. Malashetty and Mahantesh [30] investigated the linear stability of an Oldroyd type viscoelastic liquid filled horizontally asymmetric porous material warmed beneath and chilled from above. More recently, Rudresha *et al*. [31] reported a theoretical investigation of the combine effect of anisotropy and time-periodic electric field on Darcy-electroconvection. Lebon and Cloot [32] studied the effects of Maxwell-Cattaneo model in *RBC* and Marangoni instability. Maruthamanikandan and Smita [33] investigated Rayleigh-Benard instability taking into account second sound in a dielectric fluid. Soya and Maruthamanikandan [34] examined the porous medium ferroconvective instability subjected to the heat flux model.

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Recently, Naseer Ahmed and Maruthamanikandan [35] analyzed anisotropic porous medium under Brinkman Model on viscoelastic ferroconvective instability due to Maxwell-Cattaneo.

External rotation in regards with thermal convection has gained a high interest both theoretically and experimentally. Due to its general existence in oceanic flows and geophysical, it is crucial to realize how the Coriolis force ambience the transport properties and structure of thermal convection. The investigation on thermal convection stability in rotating porous media are done by many researchers. Friedrich [36] analyzed the porous layer stability with rotation warmed from underneath considering linear and a nonlinear numerical analysis. This problem with the variable viscosity impact has been addressed by Patil and Vaidyanathan [37]. A fascinating analogy have been well-established by Palm and Tyvand [38] among an anisotropic porous layer and a rotating porous layer. Various researchers have examined the rotation under different costraints as follows Jou and Liaw [39], Qin and Kaloni [40], Vadasz [41], Straughan [42], Govender [43,44], Desaive *et al.* [45], Straughan [46], Malashetty and Swamy [47], Dhiman and Sood [48] and Pulkit Kumar Nadian [49].

The present paper concentrates on examining the oscillatory convective instability of viscoelastic ferrofluid saturated in a rotating porous medium using extended Darcy model with second sound as we cannot find any study related to this from the literature review.

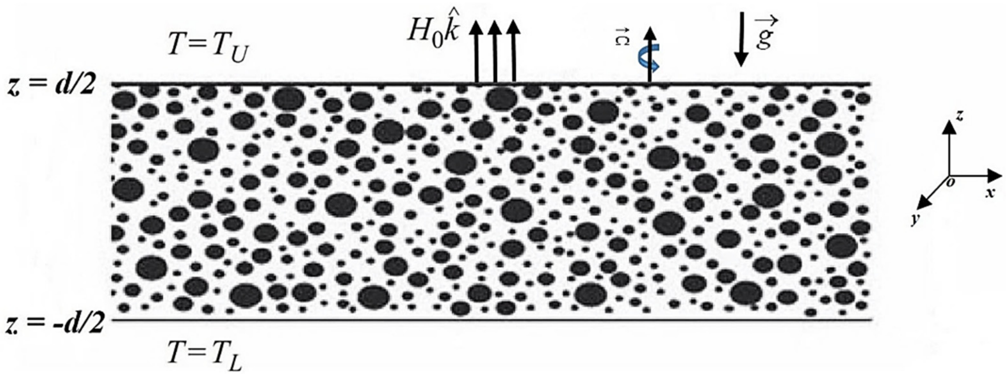

Figure 1. Physical Configuration

2. MATHEMATICAL FORMULATION

Let us consider a Boussinesq viscoelastic ferromagnetic fluid saturated densely distributed porous layer rotating with angular velocity $\vec{\Omega}(0,0,\Omega)$ restricted between two endless horizontal surfaces of height ‘d’. The viscoelastic behaviour is characterized by Oldroyd’s model (non-Newtonian). The above and bottom surface is maintained at T_U and T_L where $T_L > T_U$ (see Fig. 1). Magnetic field $\vec{H}_0$ acts parallel in the z-axis vertically and the force of gravity assisting vertically descending. The governing equations aiding the Boussinesq approximation are recorded as follows.

$$\nabla\bullet\vec{q}=0 \tag{2.1}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\varepsilon}\frac{\partial\vec{q}}{\partial t}+\frac{\rho_0}{\varepsilon^2}\left(\vec{q}\bullet\nabla\right)\vec{q}+\nabla p-\rho\vec{g}-\nabla\bullet\left(\vec{H}\vec{B}\right)+\frac{2}{\varepsilon}\left(\vec{\Omega}\times\vec{q}\right)\rho_0\right]=-\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left[\frac{\mu_f}{k}\vec{q}\right] \tag{2.2}$$

$$\varepsilon\left[\rho_0 C_{V,H}-\mu_0\vec{H}\bullet\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\right]\left[\frac{\partial T}{\partial t}+\vec{q}\bullet\nabla T\right]+(1-\varepsilon)\left(\rho_0 C\right)_s\frac{\partial T}{\partial t}+\mu_0 T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\bullet\left[\frac{\partial\vec{H}}{\partial t}+\left(\vec{q}\bullet\nabla\right)\vec{H}\right]=-\nabla\bullet\vec{Q} \tag{2.3}$$

$$\tau\left[\frac{\partial\vec{Q}}{\partial t}+\left(\vec{q}\bullet\nabla\right)\vec{Q}+\vec{\omega}\times\vec{Q}\right]=-\vec{Q}-k_1\nabla T \tag{2.4}$$

$$\rho=\rho_0\left[1-\alpha\left(T-T_a\right)\right] \tag{2.5}$$

$$M=M_0+\chi_m\left(H-H_0\right)-K_m\left(T-T_a\right) \tag{2.6}$$

All the terms above are defined in Naseer *et al.* [15].
Maxwell’s equations (Finlayson [5]).

$$\nabla \bullet \vec{B} = 0, \qquad \nabla \times \vec{H} = \vec{0}, \qquad \vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right). \tag{2.7}$$

We notice that for $\lambda_2 = 0$ the fluid scale down to Maxwell's fluid and also if $\lambda_2 = 0$ and $\lambda_1 = 0$ then the fluid scale down to Newtonian fluid.

The basic state equations are as follows

$$\left.\begin{aligned} &\frac{\partial}{\partial t} = 0,\ \vec{q}_b = (0,0,0),\ T = T_b(z), \\ &p = p_b(z),\ \rho = \rho_b(z),\ \vec{H} = H_b(z), \\ &\vec{M} = M_b(z),\ \vec{B} = B_b(z),\ \vec{Q} = \vec{Q}_b(0,0,k_1\beta) \end{aligned}\right\} \tag{2.8}$$

where $\beta = (T_1 - T_0)/2$

The basic state solution reads as follows

$$\rho_b = \rho_0 \left[1 + \alpha\beta z\right] \tag{2.9}$$

$$\vec{H}_b = \left[H_0 - \frac{K_m \beta z}{1+\chi_m} \right] \hat{k} \tag{2.10}$$

$$\vec{M}_b = \left[M_0 + \frac{K_m \beta z}{1+\chi_m} \right] \hat{k} \tag{2.11}$$

$$\vec{B} = \mu_0 \left[\vec{H} + \vec{M} \right] \hat{k} \tag{2.12}$$

3. STABILITY ANALYSIS

Due to small perturbations, we obtain dimensionless equations for stability analysis embracing normal modes (Finlayson [5]).

After an infinitesimally small perturbations the perturbed state equations are as follows

$$\left.\begin{aligned} &\vec{q} = \vec{q}_b + \vec{q}\,',\ T = T_b + T',\ p = p_b + p', \\ &\rho = \rho_b + \rho',\ \vec{H} = \vec{H}_b + \vec{H}',\vec{M} = \vec{M}_b + \vec{M}', \\ &\vec{B} = \vec{B}_b + \vec{B}',\ \vec{Q} = \vec{Q}_b + \vec{Q}',\ \phi = \varphi_b + \phi' \end{aligned}\right\} \tag{3.1}$$

where the perturbed quantities are indicated br primes. Therefore, the linearized equations due to small perturbed governing takes the form.

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t}\left(\nabla^2 w'\right) - \alpha g \rho_0 \nabla_1^2 T' + \mu_0 K_m \beta \frac{\partial}{\partial z}\left(\nabla_1^2 \phi'\right) - \frac{\mu_0 K_m^2 \beta \nabla_1^2 T'}{1+\chi_m} + \frac{2\rho_0}{\varepsilon}\Omega \frac{\partial \zeta}{\partial z}\right] = \left(1+\lambda_2 \frac{\partial}{\partial t}\right)\left[-\frac{\mu_f}{k}\nabla^2 w'\right] \tag{3.2}$$

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\varepsilon}\frac{\partial \zeta}{\partial t} - \frac{2\rho_0 \Omega}{\varepsilon}\frac{\partial w'}{\partial z}\right] = -\left(1+\lambda_2 \frac{\partial}{\partial t}\right)\left[\frac{\mu_f\, \zeta}{k}\right] \tag{3.2}$$

$$\left(\rho_0\, C\right)_1 \frac{\partial T'}{\partial t} - \mu_0\, T_a K_m \frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial z}\right) = -\nabla.\vec{Q}' + \left[\left(\rho_0\, C\right)_2 - \frac{\mu_0\, T_a K_m^2}{1+\chi_m}\right]\beta w' \tag{3.4}$$

$$\left(1+\tau \frac{\partial}{\partial t}\right)\vec{Q}' = -\frac{\tau k_1 \beta}{2}\left(\frac{\partial \vec{q}\,'}{\partial z} - \nabla w'\right) - k_1 \nabla T' \tag{3.5}$$

$$(1+\chi_m)\frac{\partial^2 \phi'}{\partial z^2} + \left(1+\frac{M_0}{H_0}\right)\nabla_1^2 \phi' - K_m \frac{\partial T'}{\partial z} = 0 \tag{3.6}$$

Solving equations (3.4) and (3.5) to eliminate $\vec{Q}'$. The linearized perturbed equations reduce to the following.

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t}\left(\nabla^2 w'\right)-\alpha g\rho_0\nabla_1^2 T'+\mu_0 K_m\beta\frac{\partial}{\partial z}\left(\nabla_1^2\phi'\right)-\frac{\mu_0 K_m^2\beta\nabla_1^2 T'}{1+\chi_m}+\frac{2\rho_0}{\varepsilon}\Omega\frac{\partial\zeta}{\partial z}\right]=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left[-\frac{\mu_f}{k}\nabla^2 w'\right] \quad (3.7)$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\varepsilon}\frac{\partial\zeta}{\partial t}-\frac{2\rho_0\Omega}{\varepsilon}\frac{\partial w'}{\partial z}\right]=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left[-\frac{\mu_f\,\zeta}{k}\right] \quad (3.8)$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\left(\rho_0 C\right)_1\frac{\partial T'}{\partial t}-\mu_0 T_a K_m\frac{\partial}{\partial t}\left(\frac{\partial\phi'}{\partial z}\right)-\left\{\left(\rho_0 C\right)_2-\frac{\mu_0 T_a K_m^2}{1+\chi_m}\right\}\beta w'\right]=-k_1\nabla^2 T'-\frac{\tau k_1\beta}{2}\nabla^2 w' \quad (3.9)$$

$$\left(1+\chi_m\right)\frac{\partial^2\phi'}{\partial z^2}+\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\phi'-K_m\frac{\partial T'}{\partial z}=0 \quad (3.10)$$

where

$\left(\rho_0 C\right)_1=\varepsilon\rho_0 C_{V,H}+\varepsilon\mu_0 H_0 K_m+(1-\varepsilon)\left(\rho_0 C\right)_s$, $\left(\rho_0 C\right)_2=\varepsilon\rho_0 C_{V,H}+\varepsilon\mu_0 H_0 K_m$, $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, $\nabla^2=\nabla_1^2+\frac{\partial^2}{\partial z^2}$,

$K_m=-\left(\frac{\partial M}{\partial T}\right)_{H_o,T_a}$, $\chi_m=\left(\frac{\partial M}{\partial H}\right)_{H_o,T_a}$ and $\zeta=\frac{\partial v'}{\partial x}-\frac{\partial u'}{\partial y}$ denotes the z-component of vorticity and ϕ' being magnetic potential.

Considering the normal mode as follows

$$\begin{bmatrix} w' \\ T' \\ \phi' \\ \zeta' \end{bmatrix}=\begin{bmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \\ \zeta(z) \end{bmatrix} e^{i(lx+my)+\sigma t} \quad (3.11)$$

along x and y directions wave numbers are l and m respectively and σ is the growth rate. Substitution of equation (3.11) into (3.7) to (3.10) leads to

$$\begin{aligned}\left(1+\lambda_1\sigma\right)&\left[\frac{\rho_0}{\varepsilon}\sigma\left(D^2-K_h^2\right)W+\alpha\rho_0 g K_h^2\,\Theta-\mu_0 K_m\beta K_h^2\,D\Phi+\frac{\mu_0 K_m^2\beta K_h^2\,\Theta}{1+\chi_m}+\frac{2\rho_0\,\Omega\,D\zeta}{\varepsilon}\right]\\ &=\left(1+\lambda_2\sigma\right)\left[-\frac{\mu_f}{k}\left(D^2-K_h^2\right)W\right]\end{aligned} \quad (3.12)$$

$$\left(1+\lambda_1\sigma\right)\left[\frac{\rho_0}{\varepsilon}\sigma\zeta-\frac{2\rho_0\Omega}{\varepsilon}DW\right]=\left(1+\lambda_2\sigma\right)\left[-\frac{\mu_f\,\zeta}{k}\right] \quad (3.13)$$

$$\begin{aligned}\left(1+\tau\sigma\right)&\left[\left(\rho_0 C\right)_1\sigma\,\Theta-\mu_0 T_a K_m\,\sigma\,D\Phi-\left\{\left(\rho_0 C\right)_2-\frac{\mu_0 T_a K_m^2}{1+\chi_m}\right\}\beta W\right]\\ &=k_1\left(D^2-K_h^2\right)\Theta-\frac{\tau k_1\beta}{2}\left(D^2-K_h^2\right)W\end{aligned} \quad (3.14)$$

$$\left(1+\chi_m\right)D^2\Phi-\left(1+\frac{M_0}{H_0}\right)K_h^2\,\Phi(z)-K_m\,D\Theta=0 \quad (3.15)$$

where $D=d/dz$ and $K_h^2=l^2+m^2$ is the overall horizontal wave number. Considering the following scaling to non-dimensionalize the equations (3.12) to (3.15)

$$\left.\begin{aligned} &W^*=\frac{Wd}{\kappa},\ \Theta^*=\frac{\Theta}{\beta d},\ \Phi^*=\frac{\Phi}{\frac{K_m\beta d^2}{1+\chi_m}},\\ &a=K_h d,\quad z^*=\frac{z}{d},\quad \sigma^*=\frac{\sigma}{\frac{\kappa}{d^2}},\ \zeta^*=\frac{\zeta}{\frac{\kappa}{d^2}}\end{aligned}\right\} \quad (3.16)$$

we get the following non dimensionless equations (for simplicity asterisks are neglected)

$$(1+F_1\sigma)\left[\frac{\sigma}{Va}\left(D^2-a^2\right)W+\left(1+M_1\right)R\,a^2\,\Theta-N\,a^2\,D\Phi+\sqrt{Ta_D}\;D\zeta\right]=-\left(1+F_2\,\sigma\right)\left[\left(D^2-a^2\right)W\right] \tag{3.17}$$

$$\left(1+F_1\,\sigma\right)\left[\frac{\sigma}{Va}\zeta-\sqrt{Ta_D}\;DW\right]=-\left(1+F_2\,\sigma\right)\zeta \tag{3.18}$$

$$\left(1+2\,G\sigma\right)\left[\lambda\sigma\Theta-M_2\,\sigma D\Phi-\left(1-M_2\right)W\right]=\left(D^2-a^2\right)\Theta-G\left(D^2-a^2\right)W \tag{3.19}$$

$$\left(D^2-M_3\,a^2\right)\Phi-D\Theta=0 \tag{3.20}$$

where $\lambda=\dfrac{\left(\rho_0 C\right)_1}{\left(\rho_0 C\right)_2}$, $M_2=\dfrac{\mu_0\,K_m^2 Ta}{(1+\chi_m)(\rho_0 C)_2}$ and $G=\dfrac{\tau\,\kappa}{2\,d^2}$.

Eliminating ζ by substituting ζ from equation (3.18) in (3.17) and then equations (3.17) and (3.18) reduces to one equation as mentioned in equation (3.21), also neglecting M_2 from Finlayson [5] and assuming $\lambda=1$ we have the following

$$\begin{aligned}&\left[\left(1+F_1\,\sigma\right)\frac{\sigma}{Va}+\left(1+F_2\,\sigma\right)\right]\left(1+F_1\,\sigma\right)\left[\frac{\sigma}{Va}\left(D^2-a^2\right)W+\left(1+M_1\right)R\,a^2\,\Theta-R\,M_1\,a^2D\Phi\right]\\&+\left(1+F_1\,\sigma\right)^2\,Ta_D\;D^2W=-\left[\left(1+F_1\,\sigma\right)\frac{\sigma}{Va}+\left(1+F_2\,\sigma\right)\right]\left(1+F_2\,\sigma\right)\left[\left(D^2-a^2\right)W\right]\end{aligned} \tag{3.21}$$

$$\left(1+2G\sigma\right)\left(\sigma\Theta-W\right)-\left(D^2-a^2\right)\Theta+G\left(D^2-a^2\right)W=0 \tag{3.22}$$

$$\left(D^2-M_3\,a^2\right)\Phi-D\Theta=0 \tag{3.23}$$

where $F_1=\dfrac{\lambda_1\kappa}{d^2}$ is the non-dimensional stress relaxation time, $F_2=\dfrac{\lambda_2\kappa}{d^2}$ is the non-dimensional strain retardation time, $Va=\dfrac{\varepsilon\,\mu_f\,d^2}{\rho_0\,\kappa k}$ is the Vadasz number, $R_D=\dfrac{\alpha g\rho_0\beta d^2k}{\mu_f\,\kappa}$ is the Rayleigh-Darcy number, $M_1=\dfrac{\mu_0 K_m^2\beta}{\left(1+\chi_m\right)\alpha g\rho_0}$ is the Magnetic number, $G=\dfrac{\tau\,\kappa}{2d^2}$ is the Cattaneo number, $Ta_D=\left(\dfrac{2\rho_0\Omega\,k}{\mu_f\,\varepsilon}\right)^2$ is the Taylor-Darcy number and $M_3=\left(\dfrac{1+\dfrac{M_0}{H_0}}{1+\chi_m}\right)$ is the non-buoyancy-magnetization parameter. Appropriate boundary conditions are $W=\Theta=D\Phi=0$ at $z=\pm 1/2$.

3.1. Stationary Instability

For the stationary mode equations from (3.21) - (3.23) turn out to be the following

$$\left(1+M_1\right)R\,a^2\,\Theta-R\,M_1\,a^2D\Phi+Ta_D\;D^2W+\left(D^2-a^2\right)W=0 \tag{3.24}$$

$$\left[G\left(D^2-a^2\right)-1\right]W-\left(D^2-a^2\right)\Theta=0 \tag{3.25}$$

$$\left(D^2-M_3\,a^2\right)\Phi-D\Theta=0 \tag{3.26}$$

Equations (3.24) – (3.26) embracing an eigenvalue problem along with the boundary conditions with R being eigen value. The forthright solution $W=A_1\cos\left(\pi z\right)$, $\Theta=A_2\cos\left(\pi z\right)$, $\Phi=\dfrac{A_3}{\pi}\sin\left(\pi z\right)$, where A_1, A_2 and A_3 are constants. On solving we obtain

$$R_D^{st}=\frac{p\left(a^2M_3+\pi^2\right)\left(p+\pi^2\,Ta_D\right)}{a^2\left(1+G\,p\right)\left[a^2\left(1+M_1\right)M_3+\pi^2\right]} \tag{3.27}$$

On substitution $G=0$ and $M_3 = 0$ in equation (3.27) exactly coincides with Kang *et al.*, [19] and Vadasz [41] and which is mentioned in equation (3.28). It should be noted that equation (3.27) is stationary Rayleigh-Darcy number is independent of viscoelastic parameters.

$$R_D^{st} = \frac{p^2 + p\pi^2 \, Ta_D}{a^2} \tag{3.28}$$

where superscript 'st' represents stationary convection.

3.2. Oscillatory Instability

$$\begin{bmatrix} (1+F_1\sigma)^2 \left\{ -\dfrac{\sigma^2}{Va^2}(\pi^2+a^2) - Ta_D\,\pi^2 \right\} \\ +(1+F_1\sigma)(1+F_2\sigma)\left(-\dfrac{2\sigma}{Va}\right)(\pi^2+a^2) \\ -(1+F_2\sigma)^2(\pi^2+a^2) \end{bmatrix} A_1 + (1+F_1\sigma)(1+M_1)Ra^2\left[(1+F_1\sigma)\frac{\sigma}{Va}+(1+F_2\sigma)\right]A_2$$
$$-(1+F_1\sigma)RM_1a^2\left[(1+F_1\sigma)\frac{\sigma}{Va}+(1+F_2\sigma)\right]A_3 = 0 \tag{3.29}$$

$$\left[1+2G\sigma+G(\pi^2+a^2)\right]A_1 - \left[(\pi^2+a^2)+(1+2G\sigma)\sigma\right]A_2 = 0 \tag{3.30}$$

$$\pi^2 A_2 - (\pi^2 + M_3 a^2)A_3 = 0 \tag{3.31}$$

On applying the solvability condition, we obtain

$$R = \frac{(a^2M_3+\pi^2)(p+\sigma+2G\sigma^2)\begin{bmatrix} \pi^2 Ta_D (Va)^2(1+F_1\sigma)^2 \\ +p\left(Va+\sigma(1+F_2Va+F_1\sigma)\right)^2 \end{bmatrix}}{a^2\left[a^2(1+M_1)M_3+\pi^2\right]Va(1+F_1\sigma)\begin{bmatrix} Va(1+F_2\sigma) \\ +\sigma(1+F_1\sigma) \end{bmatrix}\left[1+G(p+2\sigma)\right]} \tag{3.32}$$

where $p = \pi^2 + a^2$. Let $\sigma = i\omega$ where ω is frequency of oscillation and we retrieve R in the form $R = R_1 + iR_2$, both R_1 and R_2 are computed by MATHEMATICA SOFTWARE.

4. RESULTS AND DISCUSSION

The aim of the study is to uptight with rotating porous medium ferroconvection in a viscoelastic magnetic fluid with second sound. Conditions for the pair of the stationary as well as oscillatory convection utilizing linear theory, has been established by normal mode technique. Characterization of the system's stability is taken into account by the thermal Rayleigh number R, which is obtained as a function of the various parameters. By utilizing MATHEMATICA software, Eigen value expression and the corresponding critical number are found. Newtonian behavior of viscoelastic fluid in stationary convection can be noticed. In oscillatory mode Rayleigh-Darcy number is derived as a function of Vadasz number, viscoelastic parameters namely strain retardation time and stress relaxation time, non-buoyancy magnetization parameter, Cattaneo number, Taylor-Darcy number and magnetization parameter. The values of the various parameters are fixed as follows $F_1 = 1.5, Va = 2, F_2 = 0.3, G = 0.06, M_3 = 2$ *and* $Ta_D = 0.4$ and from the Figs. (2-8) critical Rayleigh-Darcy number $R_{D_c}^{osc}$ is expressed as a function of magnetic number M_1.

In Figure 2 as there is an increment in M_1, R_{D_c} decreases and destabilizes the system. We notice that the exchange principle of instabilities is invalid as stationary convection is not preferred over oscillatory convection as $R_{D_c}^{st}$ is higher than the $R_{D_c}^{osc}$. As the certain ranges of the governing parameters the fluid layer becomes overstable, i.e. the thermal instability gives rise to an oscillatory convective motion. Overstability is possible in the presence of rotation or a magnetic field because they lend an elastic-like behaviour to the fluid thereby enabling it to sustain appropriate modes of wave propagation. It is therefore expected that a layer of viscoelastic fluid can become overstable due solely to heating from below.

In Figure 3 we see that as and how F_1 and M_1 increases there is a decrement in $R_{D_c}^{osc}$ which conveys that the system destabilizes as oscillatory convection is hasten by the stress relaxation parameter F_1. It is due to the fact that the relaxation time parameter accelerates the convection flow and weakens the viscoelastic fluid elasticity.

In Figure 4 as the the values of F_2 and M_1 increases we note that there is an increment in $R_{D_c}^{osc}$ which reveals that the retardation parameter F_2 halts the onset of oscillatory convection as it enhance the effect of elastic. Hence, the system stabilizes.

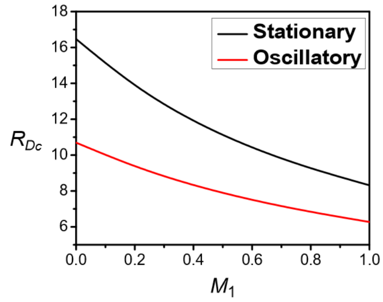

Figure 2. Variation of R with M_1 for $F_1=1.5$, $Va=2$, $F_2=0.3$, $G=0.06$, $M_3=2$ and $Ta_D=0.4$

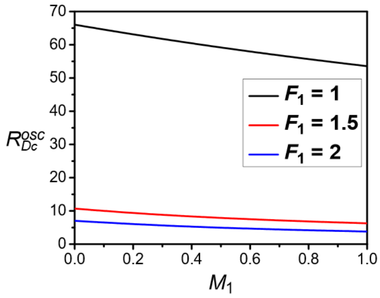

Figure 3. Variation of $R_{D_c}^{osc}$ with M_1 for $F_2=0.3$, $Va=2$, $G=0.06$, $M_3=2$ and $Ta_D=0.4$

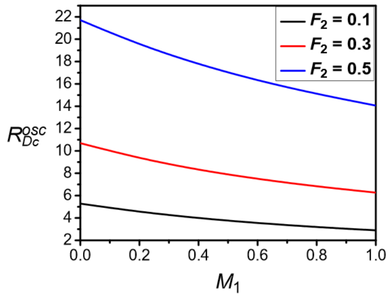

Figure 4. Variation of of $R_{D_c}^{osc}$ with M_1 for $F_1=1.5$, $Va=2$, $G=0.06$, $M_3=2$ and $Ta_D=0.4$

In Figure 5 as Va and M_1 increases there is a decrease in $R_{D_c}^{osc}$ and hence system destabilizes. As Vadasz number is the ratio of porosity, Prandtl number and Darcy number. In Figure 6 as there is an increment in G and M_1 we observe that there is an decrement in $R_{D_c}^{osc}$ due to the presence of dawn value of G and destabilizes the system. As parabolic equation is replaced by the hyperbolic equation in equation of temperature which guarantees the finite transmit of heat signals instead of infinite.

In Figure 7 the magnetic equation linear departure is expressed by the parameter M_3. We observe from figure 7, as there is an increment in M_1 and M_3 then R_c^{osc} decreases monotonically which conveys that the magnetic equation of state grows larger and larger to nonlinear owed to which ferroconvection is threshold in porous layer with second sound is hastened.

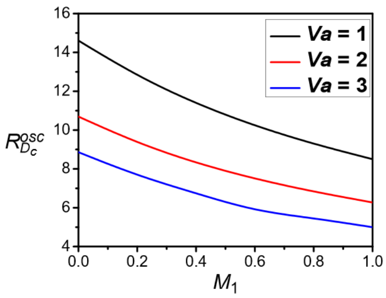

Figure 5. Variation of $R_{D_c}^{osc}$ with M_1 for $F_1=1.5$, $G=0.06$, $F_2=0.3$, $M_3=2$ *and* $Ta_D=0.4$

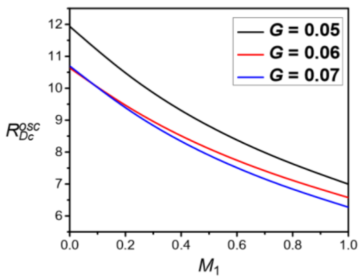

Figure 6. Variation of $R_{D_c}^{osc}$ with M_1 for $F_1=1.5$, $Va=2$, $F_2=0.3$, $M_3=2$ *and* $Ta_D=0.4$

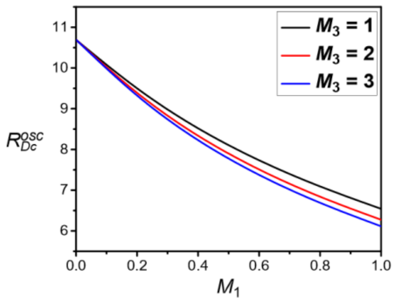

Figure 7. Variation of $R_{D_c}^{osc}$ with M_1 for $F_1=1.5$, $Va=2$, $F_2=0.3$, $G=0.06$ *and* $Ta_D=0.4$

We notice from Figure 8, as M_1 and Ta_D increases the $R_{D_c}^{osc}$ monotonically decreases which implies that the system destabilizes as observed in Pérez *et el*. [50]. From Figure 9 through 13 we can observe that all parameters increase ω_c^2 also increases whereas noted from Fig. 14 as parameter increases ω_c^2 decreases. Hence, we can conclude from Figs. (9-14) that for all parameter ω_c is sensitive.

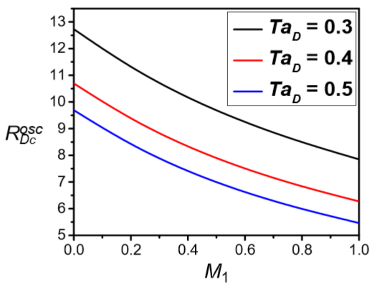

Figure 8. Variation of $R_{D_c}^{osc}$ with M_1 for $F_1=1.5$, $Va=2$, $F_2=0.3$, $G=0.06$ and $M_3=2$

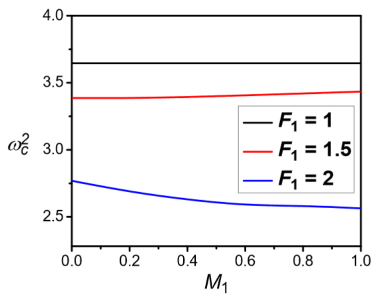

Figure 9. Variation of ω_c^2 with M_1 for $F_2=0.3$, $Va=2$, $G=0.06$, $M_3=2$ *and* $Ta_D=0.4$

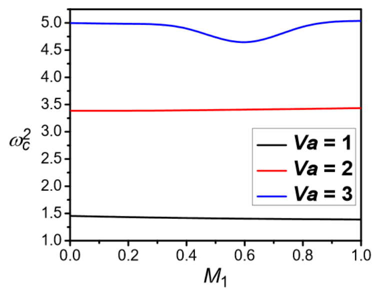

Figure 10. Variation of ω_c^2 with M_1 for $F_1=1.5$, $G=0.06$, $F_2=0.3$, $M_3=2$ *and* $Ta_D=0.4$

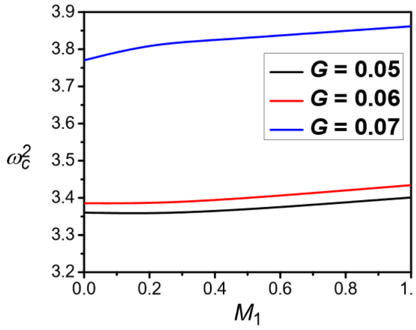

Figure 11. Variation of ω_c^2 with M_1 for $F_1=1.5,\ Va=2,\ F_2=0.3,$ $M_3=2$ and $Ta_D=0.4$

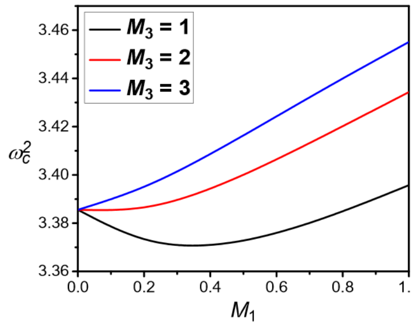

Figure 12. Variation of ω_c^2 with M_1 for $F_1=1.5,\ Va=2,\ F_2=0.3,$ $G=0.06$ and $Ta_D=0.4$

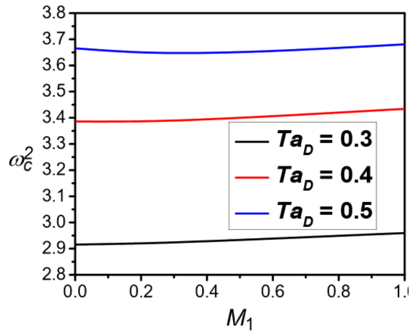

Figure 13. Variation of ω_c^2 with M_1 for $F_1=1.5,\ Va=2,\ F_2=0.3,$ $G=0.06$ and $M_3=2$

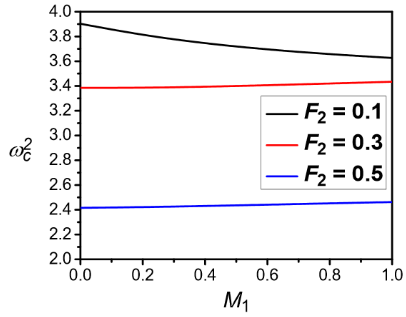

Figure 14. Variation of ω_c^2 with M_1 for $F_1=1.5,\ Va=2,\ G=0.06,\ M_3=2$ and $Ta_D=0.4$

From Table 1 through 10, we can analyze the effect of $M_1, F_1, F_2, Va, M_3, G$ and Ta_D on wave number which represents the shape and size of the convection cell. If we observe closely α_c increases with an increase in F_1, Va, and Ta_D which implies that the convection cell size is contracted and decrement of α_c with an increment in F_2 and M_3 which implies that the convection cell size is enlarge.

Table 1. Rayleigh-Darcy number and wavenumber critical values for $M_3=2,\ G=0.06$ and $Ta_D=0.4$.

M_1	**Stationary**		**Oscillatory** $(F_1=1.5,\ F_2=0.3$ and $Va=2)$	
	$R_{D_c}^{st}$	α_c^{st}	$R_{D_c}^{osc}$	α_c^{osc}
0	16.4701	10.37	10.6941	2.95604
0.2	13.7853	11.8764	9.33656	2.95659
0.4	11.8449	13.3015	8.30025	2.9467
0.6	10.3798	14.6747	7.48287	2.93229
0.8	9.23542	16.0167	6.82082	2.91624
1.0	8.31733	17.3428	6.27287	2.89995

Table 2. Rayleigh-Darcy number and wavenumber critical values with variation in F_1 by fixing $F_2=0.3, Va=2,$ $G=0.06,\ M_3=2$ and $Ta_D=0.4$

M_1	$F_1=1$		$F_1=1.5$		$F_1=2$	
	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c
0	66.0416	1.0	10.6941	2.95604	7.02019	4.08829
0.2	63.0998	1.0	9.33656	2.95659	6.01014	4.23206
0.4	60.4089	1.0	8.30025	2.9467	5.24903	4.33444
0.6	57.9381	1.0	7.48287	2.93229	4.65661	4.41049
0.8	55.6615	1.0	6.82082	2.91624	4.18356	4.41592
1.0	53.557	1.0	6.27287	2.89995	3.79694	4.44985

Table 3. Rayleigh-Darcy number and wavenumber critical values with variation in F_2 by fixing $F_1 = 1.5$, $Va = 2$, $G = 0.06$, $M_3 = 2$ *and* $Ta_D = 0.4$

M_1	$F_2 = 0.1$		$F_2 = 0.3$		$F_2 = 0.5$	
	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c
0	5.28459	3.73769	10.6941	2.95604	21.7013	2.10033
0.2	4.54376	3.88231	9.33656	2.95659	19.5045	2.09634
0.4	3.98017	3.98809	8.30025	2.9467	17.7445	2.08973
0.6	3.5386	4.06835	7.48287	2.93229	16.2999	2.08205
0.8	3.18399	4.13112	6.82082	2.91624	15.0903	2.07412
1.0	2.89327	4.18153	6.27287	2.89995	14.0605	2.06634

Table 4. Rayleigh-Darcy number and wavenumber critical values with variation in Va by fixing $F_1 = 1.5$, $F_2 = 0.3$, $G = 0.06$, $M_3 = 2$ *and* $Ta_D = 0.4$

M_1	$Va = 1$		$Va = 2$		$Va = 3$	
	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c
0	14.6089	2.86345	10.6941	2.95604	8.85992	3.42225
0.2	12.7803	2.88742	9.33656	2.95659	7.6532	3.43747
0.4	11.3542	2.90293	8.30025	2.9467	6.74327	3.43444
0.6	10.2125	2.91381	7.48287	2.93229	5.83093	3.86296
0.8	9.27831	2.92172	6.82082	2.91624	5.4638	3.40712
1.0	8.50012	2.92776	6.27287	2.89995	4.99651	3.38998

Table 5. Rayleigh-Darcy number and wavenumber critical values with variation in G by fixing $F_1 = 1.5$, $F_2 = 0.3$, $Va = 2$, $M_3 = 2$ *and* $Ta_D = 0.4$

M_1	$G = 0.05$		$G = 0.06$		$G = 0.07$	
	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c
0	11.9367	2.93641	10.6352	2.51388	10.6941	2.95604
0.2	10.4218	2.93976	9.42077	2.5064	9.33656	2.95659
0.4	9.26372	2.93204	8.47648	2.49413	8.30025	2.9467
0.6	8.3496	2.91933	7.71939	2.48015	7.48287	2.93229
0.8	7.60895	2.90462	7.09719	2.46593	6.82082	2.91624
1.0	6.99591	2.8894	6.57551	2.45216	6.27287	2.89995

Table 6. Rayleigh-Darcy number and wavenumber critical values with variation in M_3 by fixing $F_1 = 1.5$, $F_2 = 0.3$, $Va = 2$, $G = 0.06$ *and* $Ta_D = 0.4$

M_1	$M_3 = 1$		$M_3 = 2$		$M_3 = 3$	
	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c
0	10.6941	2.95604	10.6941	2.95604	10.6941	2.95604
0.2	9.45654	2.97246	9.33656	2.95659	9.26594	2.94614
0.4	8.48826	2.97412	8.30025	2.9467	8.18962	2.92977
0.6	7.71149	2.96775	7.48287	2.93229	7.34809	2.91153
0.8	7.07459	2.95709	6.82082	2.91624	6.6709	2.89336
1.0	6.54253	2.94429	6.27287	2.89995	6.11327	2.87608

Table 7. Rayleigh-Darcy number and wavenumber critical values with variation in Ta_D by fixing $F_1 = 1.5$, $F_2 = 0.3$, $Va = 2$, $G = 0.06$ *and* $M_3 = 2$

M_1	$Ta_D = 0.3$		$Ta_D = 0.4$		$Ta_D = 0.5$	
	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c	$R_{D_c}^{osc}$	α_c
0	12.7272	2.4949	10.6941	2.95604	9.69108	3.39212
0.2	11.2701	2.49011	9.33656	2.95659	8.37313	3.41198
0.4	10.1351	2.47993	8.30025	2.9467	7.37744	3.4126
0.6	9.22452	2.46759	7.48287	2.93229	6.59972	3.40376
0.8	8.47612	2.45467	6.82082	2.91624	5.9755	3.3904
1.0	7.84879	2.44195	6.27287	2.89995	5.46319	3.37507

CONCLUSIONS

The onset of thermal ferro-convection in a viscoelastic fluid saturated rotating porous layer with second sound is examined analytically using linear stability analysis. The linear theory provides the onset criteria for both stationary and oscillatory convection. The following conclusions are drawn:

- The most favorable mode of thermal instability is the oscillatory mode.
- Ferro-convective viscoelastic fluid coincides with the ferro-convective Newtonian fluid saturated rotating porous layer with second sound in stationary case. It is due to the fact that the base state has no flow and any viscoelastic fluid of simple fluid type becomes Newtonian when the flow is steady and weak.
- Magnetic parameters M_1 and M_3, viscoelastic stress relaxation parameter F_1, Vadasz number Va and Cattaneo number G strengthens the destabilizing effect of Taylor-Darcy number Ta_D in the oscillatory mode.
- Viscoelastic strain retardation parameter F_2, advances the oscillatory mode.
- Critical frequency and wavenumber of oscillatory motions are determined as functions of all the parameters of the problem. For all the parameters they are sensitive.

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ОСЦИЛЯЦІЙНА ФЕРОКОНВЕКЦІЯ МАКСВЕЛЛА-КАТТАНЕО В ЩІЛЬНОУПАКОВАНОМУ ОБЕРТОВОМУ ПОРИСТОМУ СЕРЕДОВИЩІ, НАСИЧЕНОМУ В'ЯЗКОПРУЖНОЮ МАГНІТНОЮ РІДИНОЮ

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За допомогою класичного аналізу стабільності на початку фероконвекції обертового пористого середовища досліджено комбінований ефект другого звуку та в'язкопружності. Передбачається локальна теплова рівновага між твердою матрицею та рідиною. Поточна проблема розглядається за допомогою аналітичного підходу з урахуванням відповідних граничних умов. Техніка аналізу нормального режиму використовується для отримання критичних значень для обох видів нестабільностей, а саме стаціонарної та коливальної. Ми помітили, що коливальний режим нестабільності має перевагу над стаціонарним режимом нестабільності. Ми виявили, що магнітні сили, другий звук, нелінійність намагніченості, число Вадаша, релаксація напруги через в'язкопружність і число Тейлора-Дарсі сприяють розвитку осцилюючої пористої фероконвекції середовища, тоді як затримка деформації відкладає початок коливальної пористої фероконвекції середовища. Також відзначено вплив розміру конвекційної комірки за різними параметрами та частотою коливань. Ця проблема матиме значні можливі технологічні застосування, у яких задіяні в'язкопружні магнітні рідини.

Ключові слова: *конвекція; обертання; в'язкопружні рідини; рівняння Максвелла; пористі середовища; рівняння Нав'є-Стокса для нестисливих в'язких рідин*