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# INSTABILITY OF ION CYCLOTRON WAVES(ICWS) AT THE EXPENSE OF LOWER HYBRID DRIFT WAVES (LHDWS) TURBULENCE ENERGY

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Instability of ion cyclotron waves(ICWs) is investigated in presence of lower hybrid drift waves(LHDWs) turbulence. Plasma inhomogeneity in the Earth's magnetopause region supports a range of low frequency drift wave turbulent fields due to gradients in density in different regions of the media. One of these drift phenomena is identified as lower hybrid drift waves(LHDWs)which satisfies resonant conditions $\omega - \mathbf{k}\cdot\mathbf{v} = 0$. We have considered a nonlinear wave-particle interaction model where the resonant wave that accelerates the particle in magnetopause may transfer its energy to ion cyclotron waves through a modulated field. In spite of the frequency gaps between the two waves, energy can be transferred nonlinearly to generate unstable ion cyclotron waves which always do not satisfy the resonant condition $\Omega - \mathbf{K}\cdot\mathbf{v} \neq 0$ and the nonlinear scattering condition $\Omega - \omega - (\mathbf{K}-\mathbf{k})\cdot\mathbf{v} \neq 0$. Here, $\omega$ and $\Omega$ are frequencies of the resonant and the nonresonant waves respectively and $\mathbf{k}$ and $\mathbf{K}$ are the corresponding wave numbers. We have obtained a nonlinear dispersion relation for ion cyclotron waves (ICWs) in presence of lower hybrid drift waves (LHDWs) turbulence. The growth rate of the ion cyclotron waves using space observational data in the magnetopause region has been estimated.

**Keywords:** *Ion Cyclotron Waves; Lower Hybrid Drift Waves; Wave Amplification; Density Gradient; Nonlinear wave-particle interaction*

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## 1. INTRODUCTION

Plasma covers nearly 99.9% of the universe. A wide range of electrostatic waves and electromagnetic waves dominate the nature of plasma.But, in general a real plasma can never be entirely homogeneous. Plasma is made up of different boundary layers which are very dynamical and active regions comprising many waves. One of them is the LHDWs which are supported by the free energy reserved in density gradients. The LHDWs are strong plasma waves found in Earth's Magnetosphere. According to some scientists, LHDWs cause anomalous resistivity and thus initiate magnetic reconnection (MR).It transfers the energy stored in the magnetic field to particles, further heating and accelerating them[1]. Many observations of the LHDWs have been made in the magnetosphere [2, 3] as well as in laboratory plasmas [4, 5].

For many decades,electrostatic and electromagnetic ion cyclotron waves have been observed, [cf. Gurnett and Frank, 1972; Kintner *et al.,* 1978] in the terrestrial auroral zones.Recently from the data, [cf. McFadden *et al.*,, 1998; Carlson *et al.,* 1998; Lund *et al.,* 1998; Cattell *et al.,* 1998; Chaston *et al.,* 1998] it was found that these waves are the source ion heating (conics)and parallel electron acceleration in the auroral zone. Ion cyclotron frequency is common in the terrestrial magnetosphere. Broughton *et al.,* has also reported the observation of ion cyclotron harmonically related waves in the vicinity of the plasma sheet boundary layer [6].

In Tokamak, LHDWs heating has attracted maximum attention for heating and toroidal current drive efficiency. Here, the ions are directly heated at the lower hybrid resonance layer where they can convert the ion waves into fast ion waves and the latter are strongly Landau damped on ions. Several experiments of tokamaks have disclosed that LHDWs give rise to parametric excitation of ion cyclotron modes [7].

Huba *et al.,* [2] proposed that in various confinement systems of magnetic fusion [e.g., Davidson *et al.,* 1976; Comrnisso and Griem, 1976], the lower hybrid-drift instability [Krall and Liewer, 1971], operates over a large area of magnetotail.Furthermore, it plays a significant role in the development of field line reconnection as a source of anomalous resistivity.Gurnett *et al.,* carried out some experimental observations [1976] considering the theoretical studies of the lower-hybrid-drift instabilities .He found some strong evidence for the existence of lower hybrid drift instability in terms of existence criteria, spectral characteristics, and amplitude of fluctuations.

In the magnetosphere and plasmasphere very often different wave modes are observed at the same time. For example, in the Freja mission,the data analysis of the waves shows a close relationship between Alfven wave activity and ion acoustic wave activity within auroral energization regions [Wahlund *et al.,* 1994]; similarly,lower hybrid wave (LHW) activity has been observed at the same time with ultralow-frequency waves [Olsen *et al.,*

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1987; LaBelle and Treumann, 1988; Pottelette *et al.,* 1990]. Again, simultaneous wave activity has also been observed in the active ionospheric sounding rocket experiments [Arnoldy, 1993; Bale *et al.,* 1998]. Colpitts [2015] reported that the Van Allen Probes observations show very strong modulation of whistler, magnetosonic, and lower hybrid waves by EMIC or ultra low-frequency waves. He relates this modulation to wave-wave and/or wave-particle interactions [8].

The inhomogeneous plasmas like the solar corona or planetary magnetospheres one can observe drift waves to propagate extensively. The electrostatic drift waves propagate perpendicular both to the ambient magnetic field and to the gradient due to density gradients or temperature gradients. But, they become unstable during collisions or electron Landau damping. Hence these waves may play an important role in the destabilization process of the magnetotail before a substorm. Fruit *et al.,* 2017 proposed a kinetic model with trapped bouncing electrons for the electrostatic instabilities in the resonant interactions. Thus, in the period of electron bounce period a linearized Vlasov equation is solved for electrostatic fluctuations and through the quasineutrality condition, a dispersion relation is obtained [9].

Singh and Deka (2005) studied the plasma maser effect in inhomogeneous plasma in the presence of drift wave turbulence. Here,they studied the growth rate of the high frequency Bernstein mode in presence of the spatial density gradient parameter [10].

Borgohain and Deka (2010) studied the instability of electrostatic waves in inhomogeneous plasma in presence of drift wave turbulence. Here, they studied the interaction of ion acoustic waves with drift waves [11]. Deka and Senapati (2018) studied the amplification of upper hybrid waves in presence of lower hybrid waves in an inhomogeneous plasma through a non linear wave particle interaction. Here,they studied the energy transfer from the accelerated electrons which are in phase relation with the LHDWs turbulent field to the unstable upper hybrid through a modulated field nonlinearly. On the other hand, dissipation of unstable upper hybrid wave energy is possible through radiation phenomenon after conversion while propagating through inhomogeneous plasma [12].

Deka and Deka (2022) [13] studied the growth rate of whistler mode in presence of kinetic alfven wave turbulence through nonlinear wave–particle interaction. Here, in this model he considered an external force $\mathbf{F}$ which helps to create a drifting motion.

Kumar *et al.,* (2022) [14] studied the effect of dust charge fluctuations on the parametric upconversion of a lower hybrid wave into an ion cyclotron wave and a side band wave in a two-ion species tokamak plasma. Here, the lower hybrid wave becomes unstable and decays into two modes: an ion cyclotron wave mode and a low frequency lower hybrid side band wave. Here, the growth rate decreases with the increase in the size of dust grains and electron cyclotron frequency.

Our present investigation is based on the lowest order mode-mode coupling process in a turbulent plasma which was proposed by Nambu [19]. This mode-mode coupling and wave energy conversion process was also suggested by Tsytovich [16] simultaneously. This process suggests that even though there is a large frequency difference, wave energy exchange may be possible. Nambu and Tsytovich proposed that if in a plasma both resonant and non-resonant waves are present, wave energy from resonant mode may be transferred to non-resonant waves. By resonant wave, we mean that the Cherenkov resonant condition $\omega - \mathbf{k}\cdot\mathbf{v} = 0$ is satisfied whereas for non- resonant waves both the resonant condition and nonlinear scattering conditions are not satisfied, i.e. $\Omega - \mathbf{K}\cdot\mathbf{v} \neq 0$ and $\Omega - \omega - (\mathbf{K}-\mathbf{k})\cdot\mathbf{v} \neq 0$ . Here, $\omega$ and $\Omega$ are frequencies of the resonant and the nonresonant waves respectively and $\mathbf{k}$ and $\mathbf{K}$ are the corresponding wave numbers.The LHDWs operates in the frequency range[1] where both ion and electron dynamics play a crucial role: $\omega_{ci} \ll \omega \ll \omega_{ce}$ . The wave oscillate at a frequency in between the ion and electron gyroradius.

In this present paper we have studied the interaction of high frequency ICWs with low frequency LHDWs turbulence. Here, we have used a zeroth- order distribution function that satisfies the time independent Vlasov equation. There is a non zero current associated with the drifts; this current represents a free energy that can drive instabilities and further we obtain the dispersion relation as well as estimate the growth rate of ICWs.

## 2. FORMULATION

We consider a non-uniform electrostatic LHDWs turbulence to be present in the system with propagation vector $k = (k_\perp, 0, k_{//})$ . We consider a weak density gradient perpendicular to $B_0$ of the form [17]

$$n_j^{(0)}(y) = n_j(1 + \epsilon_n y) \tag{1}$$

So, the density gradient is taken along the $\mathbf{y}-direction$ and the external magnetic field $B = B_0(y)$ is taken in the $\mathbf{z}-direction.$ $\epsilon_n$ is the density gradient scale length n.

The particle distribution function is considered as

$$f_{0j}(y,v) = \frac{n_j}{(2\pi v_j^2)^{\frac{3}{2}}}\Big[1 + \epsilon_n\Big(y + \frac{v_x}{\Omega_j}\Big)\Big]exp\Big(\frac{-v^2}{2v_j^2}\Big) \tag{2}$$

The first and second velocity moment of the above distribution are

$$\Gamma_j^{(0)} = n_j v_{nj} \hat{y} \tag{3}$$

$$n_j T_j = n_j T_j (1 + \epsilon_n y) + o(\epsilon_n^2) \tag{4}$$

Where the density gradient drift speed of the $j^{th}$ species [17]is

$$v_{nj} = \frac{\epsilon_n v_j^2}{\Omega_j} \tag{5}$$

Here, $\Omega_j = \frac{eB_0}{mc}$ is the cyclotron frequency.Here the subscript $j$ refers to $j = i$ for ions and $j = e$ for electrons.
Now,

The interaction of the high frequency ICWs with low frequency LHDWs turbulence is well explained by the Vlasov-Poisson's equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial \mathbf{r}} - \frac{e}{m}(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}).\frac{\partial}{\partial \mathbf{v}}\right] F_{0i}(\mathbf{r},\mathbf{v},\mathbf{t}) = 0 \tag{6}$$

$$\nabla \cdot \mathbf{E} = -4\pi e n_i \int f_{0i}(\mathbf{r},\mathbf{v},\mathbf{t}) d\mathbf{v} \tag{7}$$

The non-perturbed distribution function and fields are considered according to the linear response theory of the plasma.

$$F_{0i} = f_{0i} + \epsilon f_{1i} + \epsilon^2 f_{2i} \tag{8}$$

$$\mathbf{E}_{0i} = \epsilon \mathbf{E}_l + \epsilon^2 \mathbf{E}_2 \tag{9}$$

$$B_l = B_0 \tag{10}$$

where $\epsilon$ is a small parameter associated with LHDWs turbulence field $E_l = (E_{l\perp}, 0, E_{l//})$. $f_{0i}$ is the space and time average parts, $f_{1i}$ , $f_{2i}$ are the fluctuating parts of the distribution function. $\mathbf{E}_2$ is the second order electric field. $B_l$ is the total magnetic field in the system in presence of LHDWs turbulence.But LHDWs is an electrostatic turbulent that doesnot contribute turbulent to the system.
From eq.(6), we have

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial \mathbf{r}} - \frac{e}{m}\left(\epsilon\mathbf{E}_l + \epsilon^2\mathbf{E}_2 + \frac{\mathbf{v}\times\mathbf{B}_0}{c}\right).\frac{\partial}{\partial \mathbf{v}}\right]\left[f_{0i}(\mathbf{r},\mathbf{v},t) + \epsilon f_{1i}(\mathbf{r},\mathbf{v},t) + \epsilon^2 f_{2i}(\mathbf{r},\mathbf{v},t)\right] = 0 \tag{11}$$

To the order of $\epsilon$,we have

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial \mathbf{r}} - \frac{e}{m}\left(\frac{\mathbf{v}\times\mathbf{B}_0}{c}\right).\frac{\partial}{\partial \mathbf{v}}\right] f_{1i}(\mathbf{r},\mathbf{v},t) = \frac{e}{m}\mathbf{E}_l.\frac{\partial}{\partial \mathbf{v}} f_{0i}(\mathbf{r},\mathbf{v},t) \tag{12}$$

To find $f_{1i}$, we use Fourier transforms of the form

$$H(\mathbf{r},\mathbf{v},t) = \sum_{\mathbf{k},w} H(\mathbf{k},\omega,\mathbf{v}) exp[i(\mathbf{k}.\mathbf{r} - \omega t)] \tag{13}$$

The Fourier component of $f_{1i}(\mathbf{k},\omega)$ is given by-

$$f_{1i}(\mathbf{k},\omega)=\Big(\frac{ie}{m}\Big)\left[\frac{m}{T_ik_\perp}E_{l\perp}\Big\{1+\Big(\omega-k_{//}v_{//}-\frac{\epsilon_nT_ik_\perp}{m\Omega_i}\Big)P_{a,b}\Big\}f_{0i}-E_{l//}\frac{\partial f_{0i}}{\partial v_{//}}P_{a,b}\right] \tag{14}$$

*where*

$$P_{a,b}=\sum_{a,b}\frac{J_a(\alpha')J_b(\alpha')exp[i(b-a)\theta]}{\omega-k_{//}v_{//}-a\Omega_i} \tag{15}$$

$$\alpha'=\frac{k_\perp v_\perp}{\Omega_i} \tag{16}$$

To this quasi-steady state, we consider high frequency electrostatic ICWs with propagating vector $K=(K_\perp,0,0)$ with electric field $\delta E=(\delta E_h,0,0)$ and a frequency $\Omega$ . So, we have $\Omega\approx\Omega_i$ This high frequency non resonant ICWs acts as the perturbation to the system.

Thus the total perturbed electric field and the distribution function are

$$\delta f=\mu\delta f_h+\mu\epsilon\delta f_{lh}+\mu\epsilon^2\Delta f \tag{17}$$

$$\delta\mathbf{E}=\mu\delta\mathbf{E}_h+\mu\epsilon\delta\mathbf{E}_{lh}+\mu\epsilon^2\Delta\mathbf{E} \tag{18}$$

$$\delta\mathbf{B}=0 \tag{19}$$

where $\delta\mathbf{E}_{lh},\Delta\mathbf{E}$ are the modulation fields, $\delta f_h$ is the fluctuating part, $\delta f_{lh},\Delta f$ are the particle distribution function due to modulating field and $\mu$ is the smallest parameter for the perturbed field, which is also smaller in compared to $\epsilon$.

Linearizing the Vlasov-Poisson equation to the order $\mu,\mu\epsilon,\mu\epsilon^2$, we have

$$P\delta f_h=\frac{e}{m}\delta\mathbf{E}_h.\frac{\partial}{\partial\mathbf{v}}f_{0i} \tag{20}$$

$$P\delta f_{lh}=\frac{e}{m}\delta\mathbf{E}_{lh}.\frac{\partial}{\partial\mathbf{v}}f_{0i}+\frac{e}{m}\delta\mathbf{E}_h.\frac{\partial}{\partial\mathbf{v}}f_{1i}+\frac{e}{m}\mathbf{E}_l.\frac{\partial}{\partial\mathbf{v}}\delta f_h \tag{21}$$

$$P\Delta f=\frac{e}{m}\mathbf{E}_l.\frac{\partial}{\partial\mathbf{v}}\delta f_{lh}+\frac{e}{m}\delta\mathbf{E}_{lh}.\frac{\partial}{\partial\mathbf{v}}f_{1i} \tag{22}$$

where the operation P is given by-

$$P\equiv\Big[\frac{\partial}{\partial t}+\mathbf{v}.\frac{\partial}{\partial\mathbf{r}}-\frac{e}{m}\left(\frac{\mathbf{v}\times\mathbf{B}_0}{c}\right).\frac{\partial}{\partial\mathbf{v}}\Big] \tag{23}$$

Now, we evaluate the various fluctuating parts of the perturbed distribution function using the Fourier transform and integrating along the unperturbed orbits to obtain the nonlinear dielectric function of ICWs in presence of the LHDWs turbulence.
Now,

$$\delta f_h=\frac{ie}{m}\delta\mathbf{E}_h\sum_a\sum_b\frac{J_a(\alpha)J_b(\alpha)exp\{i(b-a)\theta\}}{\Omega-a\Omega_i}.\frac{\partial}{\partial v_\perp}f_{0i} \tag{24}$$

where $\alpha=\dfrac{K_\perp v_\perp}{\Omega_i}$

Again

$$\delta f_{lh}=I^1_{lh}+I^2_{lh}+I^3_{lh} \tag{25}$$

where

$$I^1_{lh}=\Big(\frac{ie}{m}\Big)\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-(\Omega-\omega)-a\Omega_i}\Big[E_{l\perp}\Big(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp}\Big)\cdot\frac{\partial\delta f_h}{\partial v_\perp}+E_{//}\frac{\partial\delta f_h}{\partial v_{//}}\Big] \tag{26}$$

$$I_{lh}^2 = \Big(\frac{ie}{m}\Big)\sum_a\sum_b \frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-(\Omega-\omega)-a\Omega_i}\frac{\delta\overrightarrow{E_{lh}}(\overrightarrow{K}-\overrightarrow{k})}{|\overrightarrow{K}-\overrightarrow{k}|}\Big[K_\perp\Big(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp}\Big)\cdot\frac{\partial f_{0i}}{\partial v_\perp}+k_{//}\frac{\partial f_{0i}}{\partial v_{//}}\Big] \quad (27)$$

$$I_{lh}^3 = \Big(\frac{ie}{m}\Big)\delta\mathbf{E}_h\frac{\partial f_{1i}}{\partial v_\perp}\sum_a\sum_b\frac{J_a(\alpha)J_b(\alpha)exp\{i(b-a)\theta\}}{(\Omega-\omega)-a\Omega_i} \quad (28)$$

Here,
$\alpha'' = \frac{(K_\perp-k_\perp)v_\perp}{\Omega_i}$ , $\alpha = \frac{K_\perp v_\perp}{\Omega_i}$
Again,

$$\Delta f = (\frac{ie}{m})\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-a\Omega_i-\Omega}\Big[E_{l\perp}\Big(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp}\Big)\cdot\frac{\partial\delta f_{lh}}{\partial v_\perp}+E_{l//}\frac{\partial\delta f_{lh}}{\partial v_{//}}\Big]$$

$$+\Big(\frac{ie}{m}\Big)\frac{\delta\overrightarrow{E_{lh}}(\overrightarrow{K}-\overrightarrow{k})}{|\overrightarrow{K}-\overrightarrow{k}|}\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-a\Omega_i-\Omega}\Big[K_\perp\Big(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp}\Big)\cdot\frac{\partial f_{1i}}{\partial v_\perp}+k_{//}\frac{\partial f_{1i}}{\partial v_{//}}\Big] \quad (29)$$

Here, from Poisson's equation we find the modulated field $\delta\mathbf{E}_{lh}(\mathbf{K}-\mathbf{k})$ ,

$$\nabla.\delta\mathbf{E}_{lh} = -4\pi e n_i\int\delta f_{lh}d\mathbf{v} \quad (30)$$

$$\therefore\delta\mathbf{E}_{lh}(\mathbf{K}-\mathbf{k}) = \frac{-\omega_{pi}^2}{|\mathbf{K}-\mathbf{k}|\cdot L(\mathbf{K}-\mathbf{k})}\Big[\int\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-(\Omega-\omega)-a\Omega_i}\Big\{E_{l\perp}(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp})\cdot\frac{\partial\delta f_{lh}}{\partial v_\perp}$$
$$+E_{l//}\frac{\partial\delta f_{lh}}{\partial v_{//}}\Big\}+\int\delta\mathbf{E}_h\sum_a\sum_b\frac{J_a(\alpha)J_b(\alpha)exp\{i(b-a)\theta\}}{(\Omega-\omega)-a\Omega_i}\cdot\frac{\partial f_{1i}}{\partial v_\perp}\Big]d\mathbf{v} \quad (31)$$

where

$$L(\mathbf{K}-\mathbf{k}) = 1+\frac{\omega_{pi}^2}{|\mathbf{K}-\mathbf{k}|}\int\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-(\Omega-\omega)-a\Omega_i}\Big\{K_\perp(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp})\cdot\frac{\partial f_{0i}}{\partial v_\perp}+k_{//}\frac{\partial f_{0i}}{\partial v_{//}}\Big\}d\mathbf{v} \quad (32)$$

Again, using the Poisson equation we obtain the dielectric response function :

$$\nabla.\delta\mathbf{E}_h = -4\pi e n_i\int[\delta f_h+\Delta f]d\mathbf{v} \quad (33)$$

we have

$$\delta E_h(\mathbf{K},\Omega)\epsilon_h(\mathbf{K},\Omega) = 0 \quad (34)$$

The dispersion relation $\epsilon_h(\mathbf{K},\Omega)$ of ion cyclotron wave is evaluated using the equation given as-

$$\epsilon_h(\mathbf{K},\Omega) = \epsilon_0(\mathbf{K},\Omega)+\epsilon_d(\mathbf{K},\Omega)+\epsilon_p(\mathbf{K},\Omega) \quad (35)$$

where $\epsilon_0(\mathbf{K},\Omega)$is the linear part, $\epsilon_d(\mathbf{K},\Omega)$ is the direct coupling part and $\epsilon_p(\mathbf{K},\Omega)$ is the polarization coupling part.

So, we have

$$\epsilon_0(\mathbf{K},\Omega) = 1+\frac{\omega_{pi}^2}{|K_\perp|^2}\int \frac{n_j}{v_j^2}[1+(\Omega-\frac{v_{nj}K_\perp}{n_j})f_{oi}]\sum_a\sum_b \frac{J_a(\alpha)J_b(\alpha)exp\{i(b-a)\theta\}}{\Omega-a\Omega_i}d\mathbf{v} \tag{36}$$

Here, $v_{nj}=\frac{\epsilon_n v_j^2}{\Omega_j}$ is the density gradient drift speed of the $j^{th}$ species. The subscript $j$ represents ions, $j=i$ . $v_{nj}$ bears the sign of $e_j$ , there is a non-zero current $J_0$ associated with these drifts; this current represents a free energy that can drive instabilities.

$$\epsilon_d(\mathbf{K},\Omega) = -\frac{\omega_{pi}^2}{|K_\perp|^2}(\frac{e}{m})^2|E_{l\perp}|^2 P_{a,b}(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp})\frac{\partial}{\partial v_\perp}\Big[Q_{s,t}(\frac{s\Omega_i}{(K_\perp-k_\perp)v_\perp})\frac{\partial}{\partial v_\perp}\{R_{u,v}\frac{n_j}{v_j}[1+(\Omega-\frac{v_{nj}K_\perp}{n_j})$$
$$f_{oi}]\}+S_{a,b}\frac{\partial}{\partial v_\perp}\{\frac{m}{T_ik_\perp}\{1+(\omega-k_{//}v_{//}-\frac{\epsilon T_iK_\perp}{m\Omega_i})\}\}T_{p,q}f_{0i}\Big]d\mathbf{v}-\frac{\omega_{pi}^2}{|K_\perp|^2}(\frac{e}{m})^2|E_{l//}|^2P_{a,b}\frac{\partial}{\partial v_{//}}$$
$$\Big[Q_{s,t}\frac{\partial}{\partial v_{//}}\{R_{u,v}\frac{n_j}{v_j}[1+(\Omega-\frac{v_{nj}K_\perp}{n_j})f_{oi}]\}+S_{a,b}\frac{\partial}{\partial v_\perp}\{T_{p,q}\frac{\partial f_{0i}}{\partial v_{//}}\}\Big]d\mathbf{v} \tag{37}$$

where

$$P_{a,b}=\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-a\Omega_i-\Omega} \tag{38}$$

$$Q_{s,t}=\sum_s\sum_t\frac{J_s(\alpha'')J_t(\alpha'')exp\{i(t-s)\theta\}}{k_{//}v_{//}-s\Omega_i-(\Omega-\omega)} \tag{39}$$

$$P_{u,v}=\sum_u\sum_v\frac{J_u(\alpha)J_v(\alpha)\frac{u\Omega_i}{K_\perp v_\perp}exp\{i(v-u)\theta\}}{\Omega-u\Omega_i} \tag{40}$$

$$S_{a,b}=\sum_a\sum_b\frac{J_a(\alpha)J_b(\alpha)\frac{a\Omega_i}{K_\perp v_\perp}exp\{i(b-a)\theta\}}{(\Omega-\omega)-a\Omega_i} \tag{41}$$

$$T_{p,q}=\sum_p\sum_q=\frac{J_p(\alpha)J_q(\alpha)exp\{i(q-p)\theta\}}{\omega-k_{//}v_{//}-a\Omega_i} \tag{42}$$

Again

$$\epsilon_p=-\frac{\omega_{pi}^4}{K_\perp^2}\frac{(\frac{e}{m})^2}{|\mathbf{K}-\mathbf{k}|^2L(\mathbf{K}-\mathbf{k})}[(A+B)\times(C+D)] \tag{43}$$

where

$$A=|E_{l\perp}|^2\int\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp})exp\{i(b-a)\theta\}}{k_{//}v_{//}-a\Omega_i-\Omega}\times\frac{\partial}{\partial v_\perp}\Big[\frac{J_s(\alpha'')J_t(\alpha'')exp\{i(t-s)\theta\}}{k_{//}v_{//}-s\Omega_i-(\Omega-\omega)}$$
$$\{K_\perp(\frac{p\Omega_i}{(K_\perp-k_\perp)v_\perp})\frac{\partial f_{0i}}{\partial v_\perp}+k_{//}\frac{\partial f_{0i}}{\partial v_{//}}\}\Big]d\mathbf{v}+\int\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-a\Omega_i-(\Omega-\omega)}$$
$$\Big[\sum_u\sum_v\frac{J_u(\alpha)J_v(\alpha)exp\{i(v-u)\theta\}}{\Omega-u\Omega_i}\frac{\partial f_{0i}}{\partial v_\perp}\{|E_{l\perp}|^2(\frac{u\Omega_i}{(K_\perp-k_\perp)v_\perp})\frac{\partial}{\partial v_\perp}+|E_{l//}|^2\frac{\partial}{\partial v_{//}}\}\Big]d\mathbf{v} \tag{44}$$

$$B=|E_{l//}|^2\int\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-a\Omega_i-\Omega}\times\frac{\partial}{\partial v_{//}}\Big[\frac{J_s(\alpha'')J_t(\alpha'')exp\{i(t-s)\theta\}}{k_{//}v_{//}-s\Omega_i-(\Omega-\omega)}$$
$$\{K_\perp(\frac{p\Omega_i}{(K_\perp-k_\perp)v_\perp})\frac{\partial f_{0i}}{\partial v_\perp}+k_{//}\frac{\partial f_{0i}}{\partial v_{//}}\}\Big]d\mathbf{v}+\int\sum_a\sum_b\frac{J_a(\alpha)J_b(\alpha)exp\{i(b-a)\theta\}}{(\Omega-\omega)-a\Omega_i}\Big[(|E_{l\perp}|^2+|E_{l//}|^2)$$

$$\frac{\partial}{\partial v_\perp}\Big\{\frac{m}{T_i k_\perp}1+(\omega-k_{//}v_{//}-\frac{\epsilon_n T_i K_\perp}{m\Omega_i})\sum_p\sum_q\frac{J_p(\alpha)J_q(\alpha)exp\{i(q-p)\theta\}}{\omega-k_{//}v_{//}-a\Omega_i}\}f_{0i}\Big\}$$
$$+|E_{l//}|^2\frac{\partial}{\partial v_\perp}\{\sum_p\sum_q\}\frac{J_p(\alpha)J_q(\alpha)exp\{i(q-p)\theta\}}{\omega-k_{//}v_{//}-a\Omega_i}\frac{\partial f_{0i}}{\partial v_{//}}\Big]d\mathbf{v} \tag{45}$$

$$C=\int\sum_s\sum_t\frac{J_s(\alpha'')J_t(\alpha'')exp\{i(t-s)\theta\}}{k_{//}v_{//}-s\Omega_i-(\Omega-\omega)}\Big[\{(\frac{s\Omega_i}{(K_\perp-k_\perp)v_\perp})(\frac{\partial}{\partial v_\perp}+\frac{\partial}{\partial v_{//}})$$
$$\sum_u\sum_v\frac{J_u(\alpha)J_v(\alpha)exp\{i(v-u)\theta\}}{\Omega-u\Omega_i}\frac{\partial f_{0i}}{\partial v_\perp}\Big]d\mathbf{v}+K_\perp\int\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp})exp\{i(b-a)\theta\}}{k_{//}v_{//}-a\Omega_i-\Omega}$$
$$\frac{\partial}{\partial v_\perp}(\sum_p\sum_q\frac{J_p(\alpha)J_q(\alpha)exp\{i(q-p)\theta\}}{\omega-k_{//}v_{//}-a\Omega_i}\frac{\partial f_{0i}}{\partial v_{//}})d\mathbf{v} \tag{46}$$

$$D=\int\sum_a\sum_b\frac{J_a(\alpha)J_b(\alpha)exp\{i(b-a)\theta\}}{(\Omega-\omega)-a\Omega_i}\cdot\frac{\partial}{\partial v_\perp}\Big[\frac{m}{T_iK_\perp}\{1+(\omega-k_{//}v_{//}-\frac{\epsilon T_iK_\perp}{m\Omega_i})$$
$$\sum_p\sum_q\frac{J_p(\alpha)J_q(\alpha)exp\{i(q-p)\theta\}}{\omega-k_{//}v_{//}-a\Omega_i+i0}\}f_{0i}-\sum_p\sum_q\frac{J_p(\alpha)J_q(\alpha)exp\{i(q-p)\theta\}}{\omega-k_{//}v_{//}-a\Omega_i+i0}\frac{\partial f_{0i}}{\partial v_{//}}\Big]d\mathbf{v}$$
$$+\int\sum_a\sum_b\frac{J_a(\alpha'')J_b(\alpha'')exp\{i(b-a)\theta\}}{k_{//}v_{//}-a\Omega_i-\Omega}\Big[\{K_\perp(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp})\cdot\frac{\partial}{\partial v_\perp}+k_{//}\frac{\partial}{\partial v_{//}}\}$$
$$\frac{m}{T_ik_\perp}\{1+(\omega-k_{//}v_{//}-\frac{\epsilon_nT_iK_\perp}{m\Omega_i})\sum_p\sum_q\frac{J_p(\alpha)J_q(\alpha)exp\{i(q-p)\theta\}}{\omega-k_{//}v_{//}-a\Omega_i}\}f_{0i}\Big]d\mathbf{v} \tag{47}$$

## 3. GROWTH RATE

We have obtained the growth rate by using the formula:

$$\frac{\gamma_h}{\Omega}=-[\frac{Im\epsilon_p+\frac{1}{2}\frac{\partial^2\epsilon_0}{\partial\Omega\partial t}}{\Omega(\frac{\partial\epsilon_0}{\partial\Omega})}]_{\Omega=\Omega_i} \tag{48}$$

The second part of the expression of the growth rate is due to the reverse absorption effect, which is in our case is given by-

$$\frac{\partial^2\epsilon_0}{\partial\Omega\partial t}=\frac{\omega_{pi}^2}{K_\perp^2}(\frac{n_j}{v_j})\int\Big[\sum_a\sum_b\frac{J_a(\alpha)J_b(\alpha)exp\{i(b-a)\theta\}}{\Omega-a\Omega_i}\times\{(1-\frac{v_{nj}K_\perp}{n_j})\frac{1}{\Omega-a\Omega_i}\}\frac{\partial f_{0i}}{\partial t}\Big]d\mathbf{v} \tag{49}$$

After partial integration,we find that the contribution of $\frac{\partial^2\epsilon_0}{\partial\Omega\partial t}$ in the growth rate becomes zero due to the reverse absorbtion effect.

Now,we consider the plasma maser interaction between the ICWs and LHDWs turbulence. The condition for the plasma maser is $\omega=k_{//}v_{//}$ and assuming $\Omega<Kv_{//}$ .Here,firstly we estimate the linear part of the dielectric function of the ICWs from $eq^n(36)$ . Considering the fact that for the ICWs, the most dominant contribution to Bessel sums come from the term $a=b=1, s=t=u=v=p=q=1$ .

We have evaluated the linear part of the dispersion relation of the ion cyclotron wave as-

$$\epsilon_0=1+\frac{\omega_{pi}^2}{K_\perp^2}\frac{\Omega_i\Lambda_1}{\Omega-\Omega_i} \tag{50}$$

So, we have

$$\frac{\partial\epsilon_0}{\partial\Omega}=\frac{\omega_{pi}^2}{K_\perp^2}\Big(\frac{-\Lambda_1\Omega_i}{(\Omega-\Omega_i)^2}\Big) \tag{51}$$

The imaginary part of the direct coupling term after partial integration, we have obtained-

$$Im\epsilon_d(\mathbf{k},\Omega) = 0 \tag{52}$$

Again,
For evaluating the imaginary part of the polarization coupling term, we have-

$$Im\epsilon_p(\mathbf{K},\Omega) = -\frac{\omega_{pi}^4}{K_\perp^2}\frac{(\frac{e}{m})^2}{|\mathbf{K}-\mathbf{k}|^2 L(\mathbf{K}-\mathbf{k})}\int[A\times ImD + C\times ImB]d\mathbf{v} \tag{53}$$

Now,

$$\begin{aligned} A = \Big[|E_{l\perp}|^2\int P_{a,b}(\frac{a\Omega_i}{(K_\perp-k_\perp)v_\perp})\frac{\partial}{\partial v_\perp}[Q_{s,t}\{K_\perp(\frac{s\Omega_i}{(K_\perp-k_\perp)v_\perp})\frac{n_j}{v_j}[1+(\Omega-\frac{v_{nj}K_\perp}{n_j})f_{0i}]+k_{//}\frac{\partial f_{0i}}{\partial v_{//}}\}] \\ +\int Q_{s,t}[\frac{n_j}{v_j}[1+(\Omega-\frac{v_{nj}K_\perp}{n_j})\{|E_{l\perp}|^2(\frac{u\Omega_i}{(K_\perp-k_\perp)v_\perp})\frac{\partial R_{u,v}}{\partial v_\perp}+|E_{l//}|^2\frac{\partial R_{u,v}}{\partial v_{//}}\}]\Big]2\pi dv_\perp dv_{//} \end{aligned} \tag{54}$$

Here, after partial integration the first part of the product is contributing zero. So, we have-

$$A = 0 \tag{55}$$

Again,

$$\begin{aligned} ImB = |E_{l//}|^2\frac{\Lambda_2 n_j}{(K_\perp-k_\perp)v_j}\frac{\sqrt{\pi}}{v_e^2|k_{//}|}\Big[\{K_\perp\frac{\epsilon}{\Omega_i}+k_{//}\frac{2v_d}{v_e^2}\}\{1+\frac{(K_\perp-k_\perp)^2v_e^2}{4\Omega_i^2(\Omega_i-\Omega+(K_\perp-k_\perp))}(\Omega-\frac{v_{nj}K_\perp}{n_j})\}\Big] \\ exp\{-(\frac{v_d}{v_e})^2\}-\frac{\sqrt{\pi}}{v_d|k_{//}|}\frac{\Lambda_2}{v_e^2}\frac{k_{//}v_d-\Omega}{(k_{//}v_d-\Omega)^2-\Omega_i^2}\{(|E_{l\perp}|^2+|E_{l//}|^2)\frac{\epsilon}{\Omega_i}+2|E_{l//}|^2\frac{2v_d}{v_e^2}\} \end{aligned} \tag{56}$$

$$C = \frac{4k_{//}}{K_\perp v_e^4}\frac{\Omega_i^2}{(K_\perp-k_\perp)}\frac{(\Omega^2+\Omega_i^2)}{(\Omega^2-\Omega_i^2)^3}\Lambda_2\frac{n_j}{v_j}(\Omega-\frac{\epsilon v_j^2K_\perp}{n_j\Omega_i})+\frac{\sqrt{\pi}}{v_d|k_{//}|}\frac{4K_\perp}{v_e^4}\Lambda_2\frac{k_{//}v_d-\Omega}{(k_{//}v_d-\Omega)^2-\Omega_i^2} \tag{57}$$

Where,
$\Lambda_1 = \int_0^\infty 2\pi v_\perp J_0^2(\alpha_i)f_{0e}(v_\perp)dv_\perp$ , $\Lambda_2 = \int_0^\infty 2\pi v_\perp^2 J_0^2(\alpha_i)f_{0e}(v_\perp)dv_\perp$

$v_d = \frac{\omega}{k_{//}}$ is the phase velocity.

$v_e$ is the electron thermal velocity.

$Im\int_{-\infty}^{\infty}\frac{\frac{\partial f_{0e}(v_{//})}{\partial v_{//}}}{-\omega+k_{//}v_{//}+i0^+}dv_{//} = -\int_{-\infty}^{\infty}\pi\delta(\omega-k_{//}v_{//})\frac{\partial f_{0e}(v_{//})}{\partial v_{//}} = \frac{2\sqrt{\pi}}{v_e^3}\frac{\omega}{k_{//}|k_{//}|exp\{-(\frac{\omega}{k_{//}v_e})^2\}}$

So, we have the growth rate as-

$$\frac{\gamma_p}{\Omega} = \frac{a}{b} \tag{58}$$

where

$$a = -\frac{\omega_{pi}^4}{K_\perp^2}\frac{(\frac{e}{m})^2}{(\mathbf{K}-\mathbf{k})^2|k_{//}|^2}(ImB\times C) \tag{59}$$

$$b = \Omega\frac{\partial\epsilon_0}{\partial\Omega} = \Omega\frac{\omega_{pi}^2}{K_\perp^2}\Big(\frac{-\Lambda_1\Omega_i}{(\Omega-\Omega_i)^2}\Big) \tag{60}$$

$$L|\mathbf{K}-\mathbf{k}|^2 \sim k_{//}^2 \tag{61}$$

## 4. DISCUSSIONS AND CONCLUSIONS

In earlier studies, Tang *et al.,* (2015) [28] in the THEMIS observation of electrostatic ion cyclotron(EIC) waves and associated ion heating found that the gradient in plasma density are the possible sources of free energy for the EIC waves. Rosenberg *et al.,* (2009) [29], in his paper discussed that wave frequency increases, the growth rate of higher harmonic EIC waves tends to increase within certain parameter ranges. Khaira *et al.,* (2015) [30], considered the Kappa distribution function and discussed the growth rate with respect to wave vector and its effects.

Here, we have considered the inhomogeneous plasma model for which the particle distribution function is constructed on a zeroth order distribution function that satisfies the time independent Vlasov equation. We assume a weak density gradient perpendicular to $B_0$ and we may take the magnetic field to be uniform. Here, we have considered $v_{nj}$ is independent of $m_j$ . We have considered the instabilities driven by the ion density drift. $v_{nj}$ carries a nonzero current $J_0$ which represents free energy that can drive instabilities. The ion density drift wave satisfies $\omega_r \simeq k_y v_{ni}$ at $k_y a_i \leq 1$ and $\omega_r$ remains less than $\Omega_i$ . At perpendicular propagation, we have ICWs at $\omega_r > \Omega_i$ .

Here, we have the effect of density gradient parameter in all the fluctuating parts of the particle distribution function. The study have been performed space plasma and investigated the amplification process of the non-resonant wave by estimating the growth rate considering only the dominant terms, neglecting other terms. Though the wave amplification process is mostly affected by the dominant term but we cannot neglect the influence of other terms involved in the calculation of growth rate.

Now, we consider the observational data in magnetopause of magnetosphere [32], [33]-
$K_\perp = 1.71m^{-1}$ , $\omega_{pi} = 1.32 \times 10^3 Hz$ , $\omega_i = 2.1 \times 10^6 Hz$ , $v_d \sim 10^6 ms^{-1}$ , $E_{l\perp} = 1.4 \times 10^{-4} vm^{-1}$ , $E_{l//} = 10^5 vm^{-1}$ , $\frac{e}{m} = 1.75 \times 10^{11} Ckg^{-1}$ , $k_{//} = 2\pi \times 10^{-5} m^{-1}$ , $k_\perp \sim 10^{-3} m^{-1}$ , $v_e = 4.19 \times 10^5 ms^{-1}$ , $\Omega = 5 \times 10^6 Hz$ , $\Omega_i = 5.6 \times 10^6 Hz$ , $\Lambda_1 = 1$ , $\Lambda_2 = \frac{\sqrt{\pi}}{2} v_e$

For the regions of plasma with very weak density gradient $(\epsilon_n = 0)$ as-

$$\frac{\gamma_p}{\Omega} \sim 10^{-6}. \tag{63}$$

The growth rate with a gradient $(\epsilon_n \neq 0)$ , we have
For $(\epsilon_n = 0.1)$ ,

$$\frac{\gamma_p}{\Omega} \sim 10^{-3}. \tag{64}$$

So, this shows that due to the presence of density gradient, free energy become apparent and thus influences the amplification process of ICWs in presence of LHDWs turbulence.

Now, we have plotted the graph of $\frac{\gamma_p}{\Omega}$ vs $\frac{\omega_{pi}}{\Omega_i}$ for ICWs with different values of the density gradient parameter.

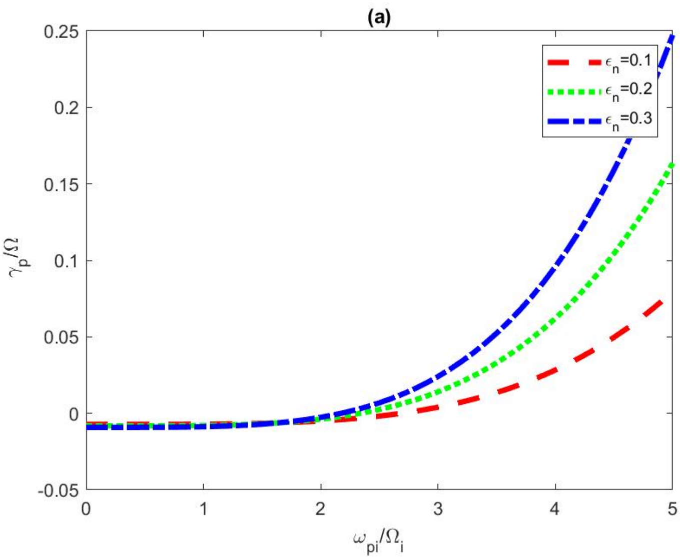


Fig.A.1: *Growth Rate*

Again, we have plotted the graph of $\frac{\gamma_p}{\Omega}$ vs $v_{nj}$ for ICWs with different values of the density gradient parameter.

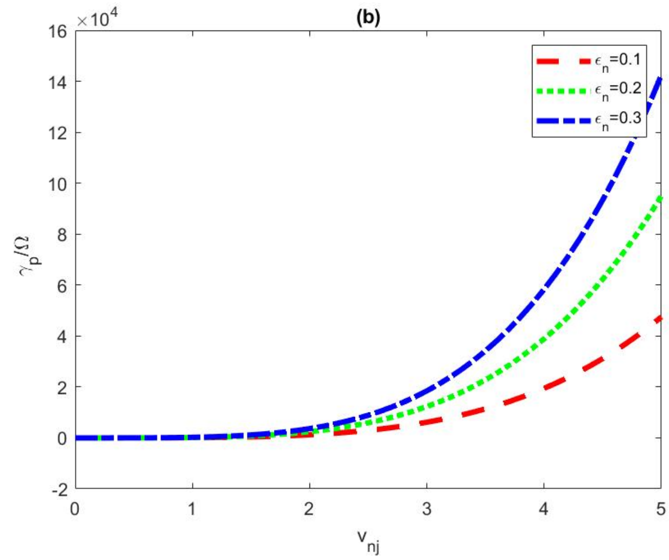


Fig.A.2: *Growth Rate vs density drift speed*

So, it has been clear from the Fig.A.1, that for different values of $\epsilon_n$ , we have growth rate increases with the increase in the value of the density gradient parameter. The growth rate is faster with the increase in value of $\epsilon_n$ . We can also say from Fig.A.2, that the growth rate increases for different density drift speed. Here, drift speed increases, frequency of the drift wave increases with successively higher ICWs and thus yields larger growth rate with the help of non-linear approach and these agrees with the earlier results of Gary in terms of linear approach [31]. Thus, we can say that there is an amplification of waves due to the interaction of ICWs and LHDWs. So, for the study of ICWs instability, we have identified that the density gradient and the density drift speed may play a significant role.

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## Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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## НЕСТАБІЛЬНІСТЬ ІОННИХ ЦИКЛОТРОННИХ ХВИЛЬ (ICW) ЗА РАХУНОК ЕНЕРГІЇ ТУРБУЛЕНТНОСТІ НИЖНІХ ГІБРИДНИХ ДРЕЙФОВИХ ХВИЛЬ (LHDW)

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Досліджено нестабільність іонних циклотронних хвиль (ІЦХ) за наявності турбулентності нижньогібридних дрейфових хвиль (НГДХ). Неоднорідність плазми в області магнітопаузи Землі підтримує діапазон низькочастотних хвиль дрейфових турбулентних полів через градієнти густини в різних областях середовища. Одне з цих явищ дрейфу ідентифіковано як нижчі гібридні дрейфові хвилі (LHDWs), які задовольняють умові резонансу $\omega - \mathbf{k}\cdot\mathbf{v} = 0$.ю Ми розглянули нелінійну взаємодію хвилі та частинок модель, де резонансна хвиля, яка прискорює частинку в магнітопаузі, може передавати свою енергію іонним циклотронним хвилям через модульоване поле. Незважаючи на частотні проміжки між двома хвилями, енергія може передаватися нелінійно для генерації нестабільних іонних циклотронних хвиль, які завжди не задовольняють умові резонансу $\Omega - \mathbf{K}\cdot\mathbf{v} \neq 0$ та умові нелінійного розсіювання $\Omega - \omega - (\mathbf{K} - \mathbf{k})\cdot\mathbf{v} \neq 0$. Тут $\omega$ і $\Omega$ — це частоти резонансної та нерезонансної хвиль відповідно, а $\mathbf{k}$ та $\mathbf{K}$ — відповідні хвильові числа. Отримано нелінійне дисперсійне співвідношення для іонних циклотронних хвиль (ICW) за наявності турбулентності нижніх гібридних дрейфових хвиль (LHDWs). Оцінено швидкість зростання іонних циклотронних хвиль з використанням даних космічних спостережень в області магнітопаузи.

**Ключові слова:** *іонні циклотронні хвилі; нижні гібридні дрейфові хвилі; посилення хвиль; градієнт густини; нелінійна взаємодія хвиля-частинка*