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IMPACTS OF TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY AND VISCOSITY ON SLIPPED FLOW OF MAXWELL NANOFLUID

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The mathematical model to inspect the effects of changeable thermo-physical properties such as thermal conduction, slip effects and viscosity on Maxwellian nanofluid is proposed. The thermal conductivity increases rapidly due to presence of nanoparticles such as metals, carbides, oxides etc. in base fluid. The flow occurs from the stagnated point pass a stretched sheet with slipped conditions. The characteristics of the Brownian motion as well as the thermophoresis processes are also taken into consideration. By means of similarity transformations, the ODEs are reduced from the equations influencing the fluid flow. A built-in solver of MATLAB namely bvp4c which is a collocation formula implementing the Lobatto IIIa finite difference numerical method is applied to solve these transformed equations numerically. The graphs of the numerical outcomes representing impacts of variations in different parameters on the fluid movement, transfer of heat along with mass are analyzed. This investigation leads to an important aspect that as the thermal conductivity in the flow is intensified, the temperature of the fluid reduces with high aggregation of the nanoparticles near the sheet's surface. Also, the rates of heat and mass transferral depletes due to the relaxation of Maxwellian fluid. Furthermore, the effectiveness of the present numerical computations is determined by carrying out comparisons of heat and mass transferred rates against the previous analytical results for several values of thermophoresis and Prandtl parameters. The effectiveness of its outcomes can be applied in nanoscience technology and polymeric industries for their developments.
Keywords: *Heat transfer; Variable fluid viscosity; Slip Effects; Variable thermal conductivity; Maxwell fluid*
PACS: 47.50.-d, 47.15.Cb, 47.11.-j, 44.20.+b, 65.80.-g, 82.60.Qr, 47.57.Ng, 82.35.Np, 83.50, 65.20.-w, 83.60.Bc, 83.60.Df

Nomenclature

T_W	Temperature on the sheet	T	Fluid temperature
C_W	Nanoparticle fraction on the wall	τ	Ratio of heat capacity of a nanoparticle to heat capacity of an ordinary liquid/gas
T_∞	Free stream temperature	c	Rate at which the sheet is stretched
C_∞	Free stream Nanoparticles concentration	$\upsilon = a(T_W - T_\infty)$	Dimensionless reference temperature corresponding to viscosity
u	velocity along $x$axis	$\beta = k_0 c$	Maxwell parameter known as Deborah number
v	velocity along $y$axis	$\epsilon = b(T_W - T_\infty)$	Dimensionless reference temperature corresponding to thermal conductivity
ρ_∞	density of the base fluid	$A = \frac{k}{c}$	Ratio of rates of velocities
$\nu(T) = \frac{\mu(T)}{\rho_\infty}$	fluid kinematic viscosity	$Pr = \frac{\rho_\infty \nu_\infty C_P}{K_\infty} = \frac{\nu_\infty}{\alpha}$	Prandtl number
C_P	specific heat at constant pressure	$Nt = \frac{\tau D_T}{\nu_\infty T_\infty}(T_W - T_\infty)$	Thermophoresis parameter
D_B	Brownian diffusion coefficient	$Nb = \frac{\tau D_B}{\nu_\infty}(C_W - C_\infty)$	Brownian motion parameter
D_T	Thermophoresis diffusion coefficient	$Le = \frac{\alpha}{D_B} = \frac{K_\infty}{\rho_\infty C_P D_B}$	Lewis number
κ_0	relaxation time of the upper-convected Maxwell fluid	$S = \frac{V_W}{\sqrt{c\nu_\infty}}$	suction parameter
$K(T)$	variable thermal conductivity	$\lambda = F\sqrt{\frac{c}{\nu_\infty}}$	Velocity slip parameter
C	Nanoparticles volume fraction	$\delta = G\sqrt{\frac{c}{\nu_\infty}}$	Thermal slip parameter
P	Pressure	$\gamma = H\sqrt{\frac{c}{\nu_\infty}}$	Solutal slip parameter
ψ	Stream function	η	Similarity variable

INTRODUCTION

Over the past few decades, inspection of nonnewtonian fluids flow passing a stretched sheet is a topic of immense curiosity amongst researchers, engineers and scientists because of its vast utilizations in biomechanics and engineering fields such as in extrusion process, annealing, extraction of metals, etc. Changes in shear stress and some properties of fluid lead to further classification of non-Newtonian fluids into Bingham plastic, Pseudoplastic, Viscoelastic. Poisson [1] and Maxwell [2] gave rigorous arguments and stated that all fluids must have some degree of elasticity. In fact,

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constitutive theories such as Maxwell and Boltzmann [3] were based on the thoughts that there will be momentary elastic responses in fluids. This concept includes the idea that fluids such as water and glycerin have viscous response while others (like less strong solutions) are somewhat elastic and some viscous. Viscoelastic fluid has both the property of viscosity and elasticity. On sudden removal of stress of viscoelastic fluid, the fluid strain doesn't disappear at once but relaxes quite slowly. The simplest model of viscoelastic fluids is the Maxwell model which has very small dimensionless relaxation time. In order to investigate the role of the parameters such as electrical conductivity in heterogeneous solid particles, J. C. Maxwell proposed a model in 1867. Wide range of applications of Maxwell fluid in technical, engineering and industrial areas has fascinated many researchers.

In 1989, Barnes et al. [4] in their research had conferred that in rheology even a solid-like material would start flowing after a given large amount of time. Keeping this in mind, a nondimensional number is required which is associated with both viscosity and elasticity of material. Thus, a non-dimensional number named as Deborah number was introduced which was well described by Poole [5]. Sadeghy et al. [6] in their work have investigated numerically the stagnated point flow of Maxwellian fluid. Wang and Tan [7] analysed the Maxwellian flow in a porous media. The wide utilizations of stagnated point flow in industrial and engineering applications such as electrochemical engineering, storage devices, waste water management, paper production, aero-engineering, etc. have embarked the interests of numerous investigators. Hiemenz [8] was the first researcher to model the stagnated point flow problem. He used similarity variables to obtain its solution in a precise manner. Chiam [9], [10] followed his work and continued the study under the consideration of the geometry as the stretchable sheet. Later on, Ahmad et al. [11] utilized Buongiorno model of nanofluid to carry on his investigation on stagnated flow of Maxwell nanofluid past a disk. Recently, various researchers such as Sunder Ram et al. [12], Reddy et al. [13], Dessie [14], Reddy and Mangamma [15] and so on, investigated the porosity effects on stagnated point flow of convective magnetized flow of microfluids, Casson fluids and Newtonian fluids. They used several numerical methods such as Shooting method, Runge kutta fourth order method and Keller box method to obtain the numerical solutions.

Recently, a current topic of heat transfer medium has emerged named as Nanofluid coined by Choi [16] which contains nanoparticles of size 01 – 100 nm. These nanoparticles, generally a metal or metal oxide, are stably and uniformly distributed over base fluid which tremendously intensifies the nanofluid thermal conduction, enhances coefficients of conduction and convection thereby favouring more heat transfer. Compared with millimeters and micrometers, Nanoparticles have great potential for enlarging thermal transport facilities. Nanofluids have an extensive applicability for engineering in heat transfer systems, automotive applications, electronic systems and biomedical applications etc. Buongiorno [17] examined various theories explaining the advanced features of heat transfer of nanofluids. He proposed an analytical solution of convection in nanofluids that looked at thermophoresis and Brownian propagation. Kuznetsov and Nield [18] extended the study of Boungiorno and researched on the nanofluid flow passed a vertical surface. Khan & Pop [19] followed them to explore thoroughly the nanofluid flow across an expanded surface with a surface temperature. Makinde and Aziz [20] investigated the condition of the transmission limit to assess the nanofluid flow over the expandable surface. An investigation on the flow over an extended sheet of Oldroyd-B fluid was accomplished by Sajid et al. [21] taking magnetic effects into consideration. Assuming geometry as a sheet with tendency to stretch, the flow of the nanofluid from a stagnated point is investigated numerically by Yasin Abdela et al. [22]. Ramesh et al. [23] inquired on flow of Maxwell fluid from a stagnated point in the existence of nanoparticles. Under various physical conditions, researchers such as Mishra [24] and El-Aziz and Afify [25] investigated the effects of slipped conditions on MHD flow in various non-Newtonian models such as Casson fluid and Jeffrey fluid. Ibrahim and Negera [26] have investigated the stagnation point flows and slip effects of MHD Maxwell nanofluid past a stretched sheet.

Factors of variation in viscosity and thermal conductivity in liquid flow have the use of geothermal energy, the underground storage system and many other areas. Changes in viscosity in liquid or gas flows help predict flow patterns and heat transfer rates, while in heat transfer problems, changes in thermal conductivity help communicate the accuracy of the energy transfer. Makinde et al. [27, 28] and Ali et al. [29] investigated the impacts of variation in viscosity on the stable and unstable flow of nanofluid by assuming different conditions and different geometries. For MHD flow of fluid containing dust particles, the change in fluid viscosity and thermal conduction was inspected by Manjunatha and Gireesha [30]. In their work they inferred that a reduction in the fluid velocities and dust phase occur due to the increased viscousness parameter of the fluid. Borgohain [31] in her work investigated on the radiative Maxwell Nanofluid and numerically forecast the flow rate, temperature and concentration features of the flow. Iranian et al. [32] inspected on impacts of suction/ injection and slip conditions on flow of Maxwell fluid number and concluded that heat transferral rate is boosted by thermal and momentum slip conditions.

In earlier problems of upper-convected Maxwell fluid flow from the stagnation point, the effects of changeable thermal conduction and viscosity were not discussed in the presence of nanoparticles under the slip effects. Owing to the importance of variation in thermal conduction and viscosity in the thermal engineering works of insulation, energy production, devices enhancing thermal power, computer storage devices, cooling systems and polymeric industries, study on this topic has become relevant. The originality of this study is therefore, the modelling of the UCM flow problem under the constraint of changing thermal conduction and viscousness of the fluid along with the slip conditions and presence of nanoparticles in the fluid from a stagnated point.

The central objective of the current article is to investigate the effects of variation in viscousness, slip effects and thermal conduction on the flow of Maxwell fluid past a stretchable surface along with nanoparticles present in the fluid via programming in MATLAB. Present study can be helpful in the processes that involve engineering, polymeric, insulations and nanofluid operations involving the works of storage systems, extrusion processes, cooling systems, paper production, thermal power generation, etc.

FORMULATION OF THE PROBLEM

The problem is considered as a slip flow problem of an upperconvected Maxwell (UCM) fluid in positive y-axis region. At x-axis, a stretching surface is placed with stagnation point fixed at $x = 0$. Towards its perpendicular direction, the y-axis is assumed. This flow model of the assumed problem is physically illustrated in Figure 01. The fluid containing nanoparticles passes over this stretched surface. From the stagnation point, the free stream velocity $U(x)$ and fluid velocity on stretched/ shrinked velocity $U_W(x)$ are considered to vary linearly i.e. the free stream velocity is taken as $U(x) = kx$ and fluid velocity on the stretched/ shrinked sheet is taken as $U_W(x) = cx$ where $c > 0$ is the stretching sheet velocity, $c < 0$ is the shrinking sheet velocity and $k > 0$ are constants. The two-dimensional laminar fluid flow is considered to be incompressible and steady. The thermal conduction and viscousness of the fluid, being dependent on temperature, are taken into account as variables. All the other thermophysical properties of the fluid are considered to be constant.

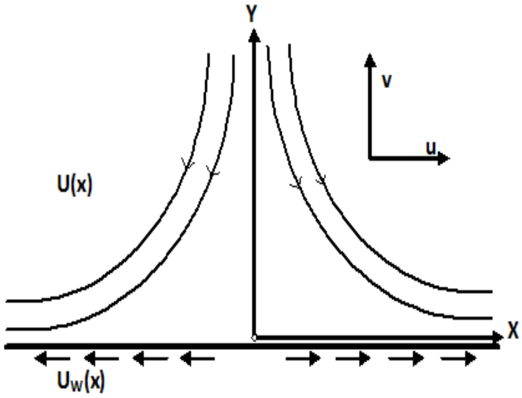

Figure 1. Physical illustration

Taking all these factors into account the Maxwell nanofluid flow governing boundary layer equations [27] over stretched sheet with changeable fluid viscousness and thermal conduction takes the following forms:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\nu(T)\frac{\partial u}{\partial y}\right) + U\frac{dU}{dx} - \kappa_0\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho_\infty C_P}\frac{\partial}{\partial y}\left(K(T)\frac{\partial T}{\partial y}\right) + \frac{\tau D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 + \tau D_B\frac{\partial C}{\partial y}.\frac{\partial T}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

The boundary conditions (bcs) suggested by Ibrahim et al. [26] are

$$\left.\begin{aligned} &u = cx + F\frac{\partial u}{\partial y}, v = V_W(x), T = T_W + G\frac{\partial T}{\partial y}, C = C_W + H\frac{\partial C}{\partial y} \text{ at } y = 0, \\ &u \to kx, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty, \end{aligned}\right\} \tag{5}$$

where $c > 0$ for the stretched sheet and $c < 0$ for the shrinking sheet.

Introduce the similarity transformations as follows:

$$\eta = \sqrt{\frac{c}{\nu_\infty}}y, \psi = \sqrt{c\nu_\infty}xf, \theta = \frac{T - T_\infty}{T_W - T_\infty}, \phi = \frac{C - C_\infty}{C_W - C_\infty}. \tag{6}$$

The dynamic viscosity of Maxwell nanofluid is taken as an exponential decreasing function of temperature [28] defined as

$$\mu(T) = \mu_\infty e^{-a(T - T_\infty)}, \tag{7}$$

where μ_∞ signifies fluid viscosity at free stream and a signifies viscosity variation exponent.

Similarly, the Maxwell nanofluid thermal conductivity [31] is taken as

$$K(T) = K_\infty e^{-b(T-T_\infty)}, \tag{8}$$

where K_∞ signifies ambient fluid conductivity and b signifies thermal dependence conductivity constant.

Using equations (6) - (8), equations (2) - (4) reduce to

$$(1-\beta e^{\upsilon\theta} f^2)f''' - \upsilon\theta' f'' + e^{\upsilon\theta}(ff'' - f'^2 + 2\beta f f' f'' + A^2) = 0, \tag{9}$$

$$\theta'' - \epsilon\theta' + Pre^{\epsilon\theta}(Nb\theta'\phi' + Nt\theta'^2 + f\theta') = 0, \tag{10}$$

$$Nb\phi'' + Nt\theta'' = -LePrNbf\phi'. \tag{11}$$

The corresponding non-dimensional bcs are

$$\left.\begin{aligned} &\eta = 0: \ f = S, f'' = \tfrac{1}{\lambda}(f'-1), \theta = 1 + \delta\theta', \phi' = \tfrac{1}{\gamma}(\phi - 1), \\ &\eta \to \infty: f' \to A, \theta \to 0, \phi \to 0. \end{aligned}\right\} \tag{12}$$

The other important physical quantities are the Sherwood (Sh) and Nusselt (Nu) numbers which are given as

$$Sh = \frac{-xC_y]_{y=0}}{(C_W - C_\infty)}, \qquad Nu = \frac{-xT_y]_{y=0}}{(T_W - T_\infty)}.$$

These are dimensionless numbers representing the effectiveness of heat and mass convection at the surface, respectively. Sherwood number represents the ratio of convective mass transfer rate to the mass diffusivity whereas Nusselt number represents the ratio of convective heat transfer to conductive heat transfer coefficient across the boundary layer. Thus, reduced Sherwood number signifies the mass transferral rate and reduced Nusselt number signifies the heat transferral rate for the convective flows at the surface.

These physical quantities under the similarity transformations can be written as:

$$Nu = -\sqrt{Re_x}\theta'(0), Sh = -\sqrt{Re_x}\phi'(0),$$

where $Re_x = \frac{xU_W}{v_\infty}$ is localised Reynolds number.

Thus, the reduced Sherwood and Nusselt numbers are given by

$$Sh_x = Re_x^{-1/2} Sh = -\phi'(0), \ \ Nu_x = Re_x^{-1/2} Nu = -\theta'(0). \tag{13}$$

METHODOLOGY

BVP4C of MATLAB is used as the numerical technique to get the outcomes of the present model. Bvp4c is a built-in solver of MATLAB which is a collocation formula used to solve a global system of algebraic equations imposing collocation conditions on the subintervals over the boundary. This formula implements the three stage Lobatto IIIa finite difference numerical method [33]. The equations (9) - (11) under the bcs (12) are first reduced to first order equations as shown below.

Take

$$y1 = f, y2 = f', y3 = f'', y4 = \theta, y5 = \theta', y6 = \phi, y7 = \phi'$$

Then the first order differential matrix is obtained as

$$\frac{dy}{dx} = \begin{bmatrix} y2, \\ y3 \\ \dfrac{\upsilon y5y3 - e^{\upsilon y4}(y1y3 - y2^2 + 2\beta y1y2y3 + A^2)}{1 - \beta e^{\upsilon y4} y1^2} \\ y5 \\ \epsilon y5 - Pre^{\epsilon y4}(Nby5y7 + Nty4^2 + y1y5) \\ y7 \\ Pry7(-Ley1 + Nte^{\epsilon y4}y5 + \dfrac{Nt}{Nb}\{-\epsilon y5 + Pre^{\epsilon y4}(y1y5 + Nty4^2)\} \end{bmatrix}$$

RESULTS AND DISCUSSIONS

Reduced system of governing ODEs (9) – (11) so obtained are highly coupled and nonlinear, and cannot be solved analytically. Hence the equations under the bcs (12) are first reduced to first order equations as shown in section methodology and are solved by coding to create appropriate programming in MATLAB by making use of the solver bvp4c and implementing the Lobatto IIIa finite difference numerical method. Finally, the numerical computations for nanoparticles concentration, fluid temperature and its velocity profiles for distinct values of the influencing variables are carried out and the outcomes are displayed graphically in Figures (2) - (14).

To check the effectiveness of the present numerical computations, comparisons with the prior analytical outcomes of Khan and Pop [19] under the absence of slip effects, conductivity and viscosity parameters are carried out and shown in Table 1 and Table 2. It is evident from the tables that the two results are exact upto 3 decimal places showing the relevancy of the present study.

Table 1. Comparison table taking variation in thermophoresis parameter

Nt	$-\phi'(0)$ **Previous [19]**	$-\phi'(0)$ **Present**	$-\theta'(0)$ **Previous [19]**	$-\theta'(0)$ **Present**
0.2	2.2740	2.2740	0.6932	0.6932
0.3	2.5286	2.5286	0.5201	0.5201
0.4	2.7952	2.7951	0.4026	0.4026
0.5	3.0351	3.0351	0.3211	0.3210

Table 2. Comparison table for rates of heat transfer with variation in Prandtl number

Pr	$-\theta'(0)$ **Previous [19]**	**Present results**	Pr	$-\theta'(0)$ **Previous [19]**	**Present results**
0.07	0.0663	0.0663	0.70	0.4539	0.4539
0.20	0.1691	0.1691	2.00	0.9113	0.9113

The range of controlling parameters in plotting the Figures 02 – 14 are taken as $0 \le \beta < 1, 0.1 \le Nb \le 1.2, 0.1 \le Nt \le 0.5, 0 \le \epsilon \le 2, Pr = 4, 0.01 \le \gamma \le 2.5, 0 \le \upsilon \le 1.5, 0 \le A \le 1.5, 0.1 \le S \le 3, Le = 1, 0.01 \le \lambda \le 1.5, 0.01 \le \delta \le 0.35$. The Figures 02 – 14 yield that the temperature and velocity of Maxwellian fluid and nanoparticles concentration in the nanofluid decrease monotonically across the boundary. At the surface, the value of the fluid property is maximum with a monotonic fall in the property by the end of boundary.

The effects of the variation of dimensionless viscosity parameter υ on the nanoparticles volume fraction, fluid temperature and fluid motion are depicted in Figures 2-4. From the figures, it is evident that the growth in viscosity slows down the fluidic motion. The nanoparticles concentration and the fluid temperature upsurge with the hike in viscosity parameter. Increase in viscosity parameter results in greater temperature contrast between the surface and encompassing fluid which diminishes the hydrodynamic boundary layer thereby conferring the depicted results.

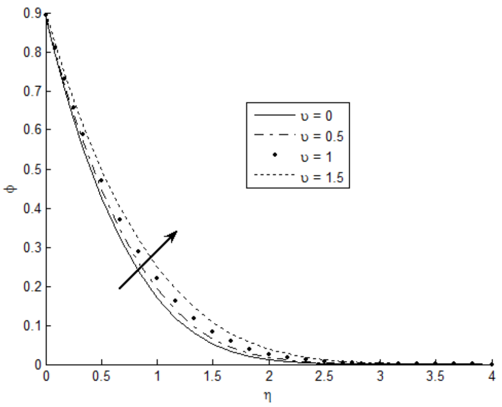

Figure 2. Changing viscosity on nanoparticles concentration

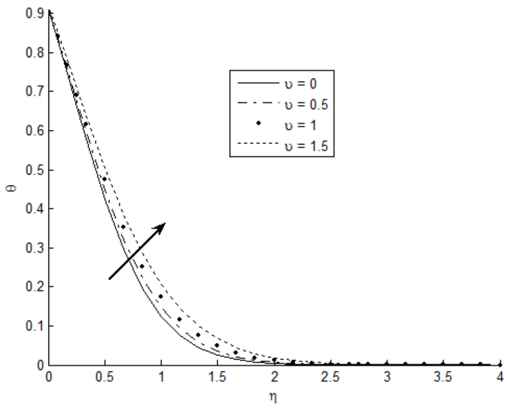

Figure 3. Changing viscosity on fluid temperature

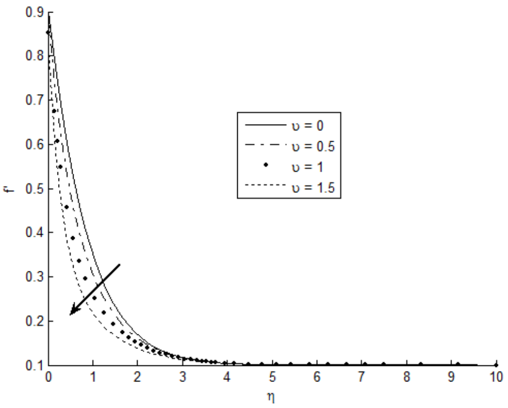

Figure 4. Changing viscosity on fluid velocity

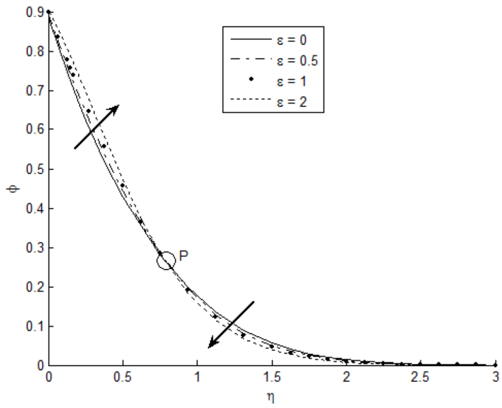

Figure 5. Changing thermal conduction on nanoparticles concentration

The effects of various values of thermal conductivity ϵ on the concentration and temperature distributions are plotted in Figures 5 and 6. The growth in thermal conductivity cools down the fluid temperature thereby thinning down the diameter of thermal boundary layer. The nanoparticles volumetric content enhances near the surface but far away from the sheet it gets reversed giving a point of inflexion P where no change in concentration is observed with the rise in thermal conductivity.

In Figures 7 and 8, the distribution of nanoparticles concentration and fluid temperature for variation in nanofluid parameters are shown. The nanofluid parameters Nb and Nt have positive effects on the fluid temperature. This is due to the generation of irregular motion known as Brownian motion by the nanoparticles and the thermophoretic force creating fast flow away from the sheet. Also, these lead to the decline in concentration of the nanoparticles by the Brownian motion parameter Nb but reverse result for thermophoresis parameter Nt.

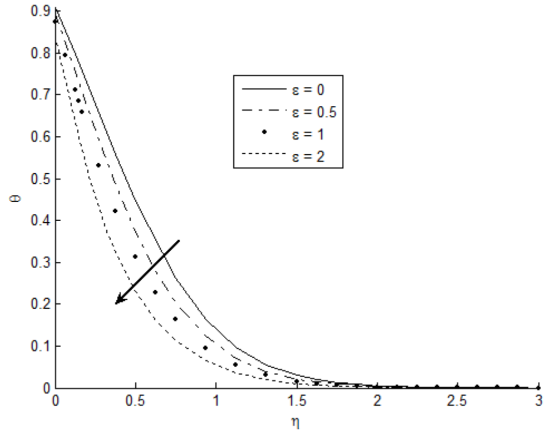

Figure 6. Changing thermal conduction on temperature

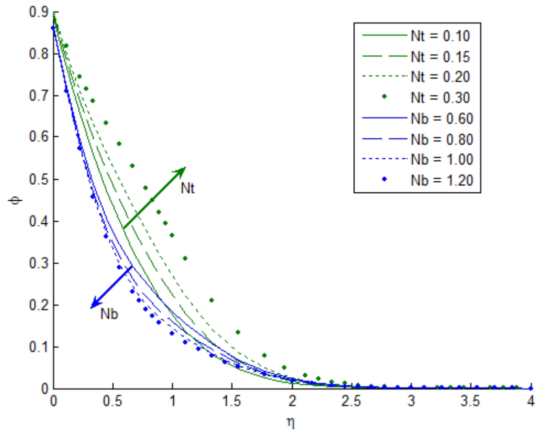

Figure 7. Changing nanofluid parameters on nanoparticles concentration

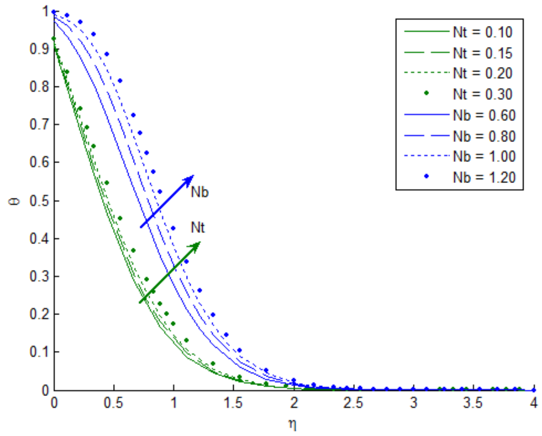

Figure 8. Changing nanofluid parameters on temperature

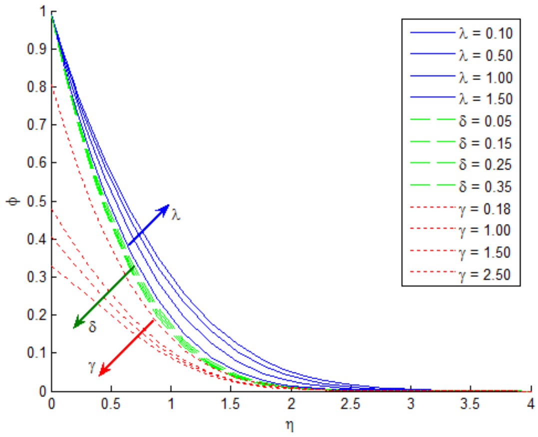

Figure 9. Changing slip parameters on nanoparticles concentration

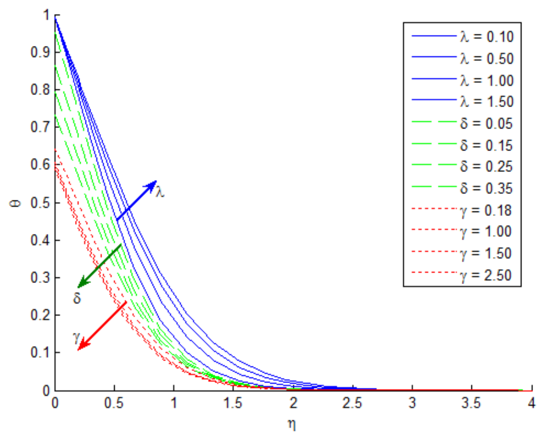

Figure 10. Changing slip parameters on temperature

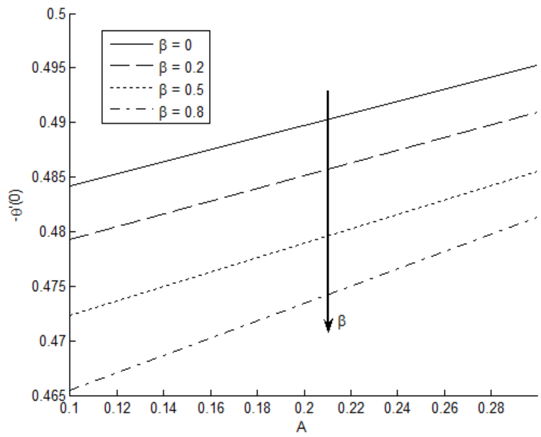

Figure 11. Nusselt number variation with Deborah number and velocity ratio

Figures 9 and 10 present the concentration and temperature distributions for distinct values of slip parameters. It's ascertained that the slip velocity λ enhances the thermal and concentration boundary layer but reverse are the results for thermal slip δ and solutal slip γ parameters. At the occurrence of slip, the velocity of fluid near the plate is not equal to the contraction rate of the plate. So as the slip velocity rises up, the fluid velocity decelerates. These further results in intensification of the fluid temperature and also movement of the nanoparticles, existing in fluid, closer towards each other thereby making the fluid more concentrated.

Finally, the mass transferring and heat transferring rates proportional to $-\phi'(0)$ and $-\theta'(0)$, respectively, are shown in the Figures 11–14 for variation in different parameters. The nanofluid parameters (Nt, Nb) boost the frequency of mass transfer but restrict the frequency of heat transfer. The increasing values of Nt and Nb result in the rising of the surface temperature thickening the thermal boundary layer and thereby reducing the Nusselt number. The elasticity and viscosity of the Maxwell fluid led to the reduction in the mass and heat transferring rates with the hike in Deborah number β while the ratio of velocities enhances these rates.

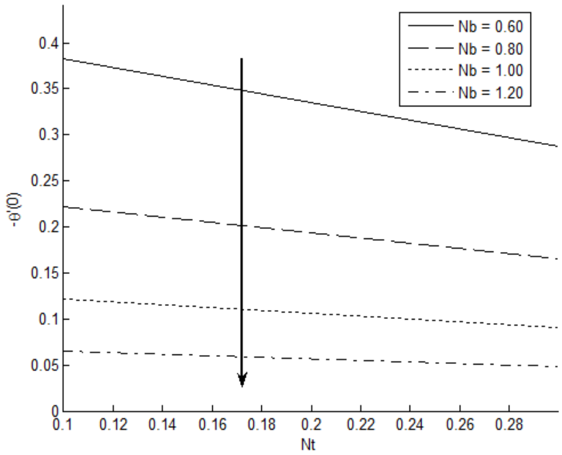

Figure 12. Nusselt number variation with nanofluid parameters

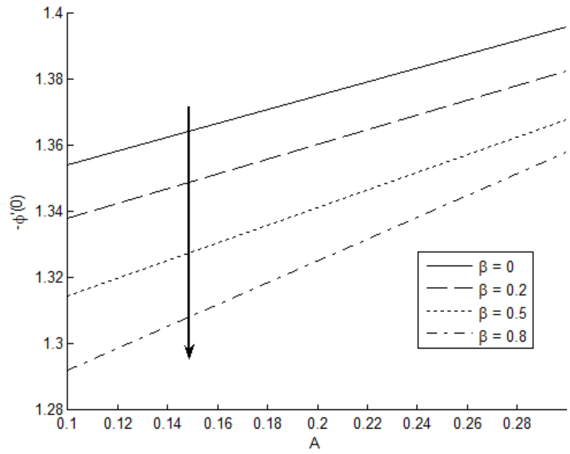

Figure 13. Sherwood number variation with velocity ratio and Deborah number

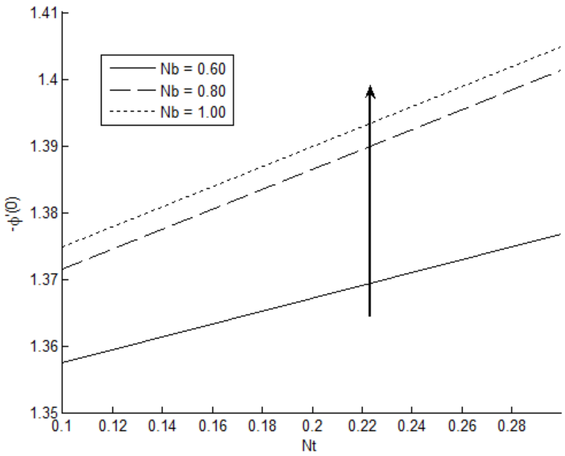

Figure 14. Sherwood number variation with nanofluid parameter

CONCLUSIONS

The cumulative impacts of changeable thermal conduction, slip effects, viscosity and suction on the movement of Maxwellian fluid past a stretchable surface are investigated from the stagnation point along with nanoparticles present in the fluid. The results are precised as follows:

- The variation in thermal conductivity results in cooling down of the fluid and makes the fluid more concentrated near the surface of the sheet but decreases the nanoparticles volumetric content distant from the sheet.
- The velocity slips, viscosity and thermophoresis parameters favored the fluid temperature and nanoparticle concentration, whereas the thermal and solutal slips reduced the diameter of thermal and concentration boundary layers.
- The presence of nanoparticles in the fluid retards the fluidic motion.
- The upsurge in Brownian motion parameter results in heating up of the fluid and thinning down the concentration boundary layer.
- The rate of heat transfer depletes on upsurge of Deborah number and nanofluid parameters but enhances for higher ratio of velocities.
- The rate of mass transfer is favored by the nanofluid parameters and velocity ratio but is opposed by the rise in Maxwell parameter.

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Declarations:

Conflict of interest. The author of the present work declares no competing financial interests.

Human and animal rights. In the present study, no human (or animal) tissue was involved.

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ВПЛИВ ТЕМПЕРАТУРНО-ЗАЛЕЖНИХ ТЕПЛОПРОВІДНОСТІ ТА В'ЯЗКОСТІ НА КОВЗАЮЧИЙ ПОТІК НАНОРІДИНИ МАКСВЕЛЛА

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Запропоновано математичну модель для перевірки впливу мінливих теплофізичних властивостей, таких як теплопровідність, ефекти ковзання та в’язкість, на нанорідину Максвелла. Теплопровідність швидко зростає через наявність у базовій рідині наночастинок, таких як метали, карбіди, оксиди тощо. Потік відбувається від застійної точки проходження розтягнутого листа з умовами ковзання. Також враховуються особливості броунівського руху, а також процеси термофорезу. За допомогою перетворень подібності ODE виводяться з рівнянь, що впливають на потік рідини. Вбудований розв’язувач MATLAB, а саме bvp4c, який є формулою спільного розташування, що реалізує чисельний метод кінцевих різниць LobattoIIIa, застосовується для чисельного розв’язання цих перетворених рівнянь. Проаналізовано графіки чисельних результатів, що представляють вплив варіацій різних параметрів на рух рідини, передачу тепла разом з масою. Це дослідження призводить до важливого аспекту, що, оскільки теплопровідність у потоці посилюється, температура рідини знижується з високою агрегацією наночастинок біля поверхні листа. Крім того, швидкість тепло- та масообміну зменшується через релаксацію рідини Максвелла. Крім того, ефективність представлених чисельних розрахунків визначається шляхом проведення порівнянь швидкостей тепло- та масопередачі з попередніми аналітичними результатами для кількох значень термофорезу та параметрів Прандтля. Ефективність його результатів може бути застосована в нанонаукових технологіях і полімерних галузях для їх розробок.

Ключові слова: *теплообмін; змінна в'язкість рідини; ефекти ковзання; змінна теплопровідність; рідина Максвелла*