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ANISOTROPIC COSMOLOGICAL MODEL IN $f(R,T)$ THEORY OF GRAVITY WITH A QUADRATIC FUNCTION OF T[†]

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In this paper, we study spatially homogeneous and anisotropic Bianchi type-I space-time filled with perfect fluid within the framework of $f(R,T)$ theory of gravity for the functional form $f(R,T) = R + 2f(T)$ with $f(T) = \alpha T + \beta T^2$, where α and β are constants. Exact solutions of the gravitational field equations are obtained by assuming the average scale factor to obey a hybrid expansion law and some cosmological parameters of the model are derived. Two special cases, leading to the power-law expansion and the exponential expansion are also considered. We investigate the physical and geometrical properties of the models by studying the evolution graphs of some relevant cosmological parameters such as the Hubble parameter (H), the deceleration parameter (q) etc.
Keywords: *Bianchi type-I universe; f(R, T) theory of gravity; Hubble parameter; Cosmological constant; Deceleration parameter*
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1. INTRODUCTION

More than two decades have passed since the first observational results from Supernovae Type Ia [1–3] with strong support from a number of astrophysical and cosmological observations such as Cosmic Microwave Background (CMB), Wilkinson Microwave Anisotropy Probe (WMAP), Large Scale Structure (LSS), Baryon Acoustic Oscillation (BAO), Galaxy redshift surveys [4–10] etc. that the universe at present is in a state of accelerated expansion. It is accepted as true that there was also a cosmic acceleration, which occurred at the very early epoch of the universe. The early time cosmic acceleration, called inflation, although there is no known direct detection for this, has theoretical explanations, but the root cause of the late time cosmic acceleration having direct detection is yet to be ascertained. Since matter contributes with force and positive pressure that decelerates the rate of cosmic expansion, therefore, as a resolution to this bizarre issue a substantial amount of energy component apart from the baryonic matter is hypothesized to be present in the universe to speed up the cosmic expansion. It is possible only when an unusual component with large negative pressure, dubbed dark energy, covering nearly 68.3% of the total energy content of the universe is present to counteract the gravitational pressure of the baryonic matter. Within the framework of General Relativity, the most efficient candidate for dark energy is the cosmological constant Λ as it works well with the observational data. But due to its problematic nature with the fine-tuning and the cosmic coincidence problems, various other dark energy models such as quintessence, k-essence, tachyon, phantom, Holographic dark energy, Chaplygin gas models etc. have been proposed in the literature.

The problem of late time cosmic acceleration has also been approached with some alternative theories of gravity, popularly known as modified theories of gravity, which are developed by modifying the geometric part of the Einstein-Hilbert action. Among the various modified theories of gravity, the simplest and the most studied one is the $f(R)$ theory of gravity, the action of which is constructed from the standard Einstein-Hilbert action simply by taking an arbitrary function $f(R)$ in place of R, where R is the Ricci scalar curvature. The other most interesting and viable alternative to General Relativity is the $f(R,T)$ theory of gravity proposed by Harko *et al.* [11] in which the gravitational Lagrangian in Einstein-Hilbert action is given by an arbitrary function $f(R,T)$ of the Ricci scalar R and the trace T of the stress-energy tensor T_{ij}. In their work, they have obtained the gravitational field equations in the $f(R,T)$ gravity in the metric formalism and presented the field equations for the three explicit forms of the functional $f(R,T)$: (i) $f(R,T) = R + 2f(T)$, (ii) $f(R,T) = f_1(R) + f_2(T)$, (iii) $f(R,T) = f_1(R) + f_2(R)f_3(T)$.

Harko *et al.* also derived the equations of motion of test particles together with the Newtonian limits in $f(R,T)$ gravity models. Further, they have investigated the constraints on the magnitude of the extra-acceleration on the precession of the perihelion of the planet Mercury. Houndjo [12] discussed transaction of matter dominated phase to an accelerated expansion phase by developing the cosmological reconstruction of $f(R,T)$ theory of gravity. Since then, many researchers have studied cosmological dynamics in $f(R,T)$ theory of gravity as it takes care of the early time inflation as well as the late time cosmic acceleration. A number of authors have also investigated Bianchi cosmological models in $f(R,T)$ theory of gravity in different contexts as Wilkinson Microwave Anisotropy Probe (WMAP) and some other experimental tests support the existence of an anisotropic phase in the early era which might have been wiped out in the course of cosmic evolution resulting in the present isotropic phase. Adhav [13] investigated LRS

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Bianchi Type I cosmological model with perfect fluid, Reddy *et al.* [14] explored Bianchi Type III and Kaluza-Klein cosmological model, Chandel and Ram [15] generated a new class of solutions of field equations from a set of known solutions for a Bianchi Type III cosmological model with perfect fluid, Chaubey and Shukla [16] studied a new class of Bianchi Type III, V, VI_0 models in presence of perfect fluid, Sahoo and Mishra [17] investigated Kaluza-Klein dark energy model in the presence of wet dark fluid, Ladke *et al.* [18] constructed higher dimensional Bianchi Type-I cosmological model, Sahoo *et al.* [19] investigated an axially symmetric space-time in presence of perfect fluid source, Agrawal and Pawar [20] investigated plane symmetric cosmological model in the presence of quark and strange quark matter, Bhoyar [21] talked about non-static plane symmetric cosmological model with magnetized anisotropic dark energy, Yadav *et al.* [22] searched the existence of bulk viscous Bianchi-I embedded cosmological model by taking into account the simplest coupling between matter and geometry, Yadav *et al.* [23] investigated a bulk viscous universe and estimated the numerical values of some cosmological parameters with observational Hubble data and SN Ia data. Singh and Beesham [24] explored a plane symmetric Bianchi Type I model by considering a specific Hubble parameter which yields a constant deceleration parameter, Chaubey *et al.* [25] considered general class of anisotropic Bianchi cosmological models in $f(R,T)$ gravity with dark energy in viscous cosmology, Bhattacharjee *et al.* [26] presented modelling of inflationary scenarios, Tiwari *et al.* [27] studied Bianchi type I cosmological model for a specific choice of the function of the trace of the energy momentum tensor. Modifications and generalisations of $f(R,T)$ theory of gravity are also considered in the literature. Singh and Bishi [28] studied Bianchi Type III cosmological model in the presence of cosmological constant Λ. Moraes *et al.* [29] investigated static wormholes in modified $f(R,T)$ gravity. Moraes and Sahoo [31] have proposed a new hybrid shape function for wormhole. Azmat *et al.* [31] studied viscous anisotropic fluid and constructed corresponding dynamical equations and modified field equations in $f(R,T)$ theory of gravity. Tretyakov [32] discussed the possibility of a further generalization of $f(R,T)$ gravity by incorporating higher derivative terms in the action and demonstrated that inflationary scenarios appear quite naturally in the theory. Recently, several authors have studied various other cosmological scenarios in the framework of $f(R,T)$ theory of gravity [33–39].

Motivated by the above-mentioned works, we focus our present work in studying spatially homogeneous and anisotropic Bianchi type-I universe with perfect fluid source in $f(R,T)$ theory of gravity for the functional form $f(R,T) = R + 2(\alpha T + \beta T^2)$, where α and β are constants. The field equations are solved by assuming the average scale factor in the form of hybrid expansion law. We organize the paper as follows: in section 2, we give a brief review of the $f(R,T)$ theory of gravity. In section 3, we derive the gravitational field equations for the Bianchi type-I metric. Exact solutions of the field equations are obtained in section 4. In section 5, some physical and kinematical properties of the model are discussed by graphically representing the evolution of graphs of some parameters of cosmological importance. Two particular scenarios are also examined when the expansion of the universe is governed by power-law expansion and exponential expansion only. We summarize the main results with some concluding remarks in section 6.

2. BRIEF REVIEW OF $f(R,T)$ GRAVITY

In $f(R,T)$ gravity proposed by Harko *et al.* (2011), the action is taken as

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci Scalar R and of the trace T of the stress-energy tensor of matter T_{ij} defined by

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}. \tag{2}$$

Here, L_m is the matter Lagrangian that generates a specific set of field equations for each choice of L_m.
By assuming the Lagrangian of matter to depend only on the metric tensor components g_{ij} and not on its derivatives, the stress-energy tensor can be obtained as

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}}. \tag{3}$$

By varying the action (1) with respect to the metric tensor components g^{ij}, the field equations of $f(R,T)$ theory of gravity in the metric formalism are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \tag{4}$$

where, $f_R(R,T)$ and $f_T(R,T)$ are the partial derivatives of $f(R,T)$ with respect to R and T respectively, ∇_i is the covariant derivative, $\Box = \nabla_k\nabla^k$ is the D'Alembert operator and

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lk}}. \tag{5}$$

The stress-energy tensor of matter T_{ij} is assumed to take the perfect fluid form so that

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}. \tag{6}$$

where ρ and p are respectively the density and pressure of the perfect fluid.
For the choice $L_m = -p$, we thus have

$$\Theta_{ij} = -2T_{ij} - pg_{ij}. \tag{7}$$

For the functional form

$$f(R,T) = R + 2f(T)\,. \tag{8}$$

where $f(T)$ is an arbitrary function of T, the gravitational field equations of $f(R,T)$ gravity are obtained from Eq. (4), as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} + f(T)g_{ij}, \tag{9}$$

where $f'(T) = \frac{d}{dT}(f(T))$. In view of eq (6), the field equations (9) become

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}. \tag{10}$$

For the choice

$$f(T) = \alpha T + \beta T^2, \tag{11}$$

where α and β are constants, the eq.(10), in presence of a time varying cosmological constant Λ, reduces to

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = [8\pi + 2\alpha + 4\beta T]T_{ij} + [2p\alpha + 4p\beta T + \alpha T + \beta T^2]g_{ij}. \tag{12}$$

3. THE METRIC AND FIELD EQUATIONS

The spatially homogeneous and anisotropic Bianchi type-I metric is given by

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2, \tag{13}$$

where the directional scale factors *A, B* and *C* are functions of the cosmic time *t* alone. In comoving coordinates, the field equations (12) take the form

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} = (8\pi + 3\alpha)\rho + 5\beta\rho^2 - 3\beta p^2 - 14\beta p\rho - \alpha p - \Lambda, \tag{14}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = -(8\pi + 3\alpha)p + 9\beta p^2 - 6\beta p\rho + \alpha\rho + \beta\rho^2 - \Lambda, \tag{15}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -(8\pi + 3\alpha)p + 9\beta p^2 - 6\beta p\rho + \alpha\rho + \beta\rho^2 - \Lambda, \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = -(8\pi + 3\alpha)p + 9\beta p^2 - 6\beta p\rho + \alpha\rho + \beta\rho^2 - \Lambda, \tag{17}$$

where an overhead dot denotes differentiation with respect to t.

4. COSMOLOGICAL SOLUTION OF THE FIELD EQUATIONS

Here, we have four field equations with six unknowns A, B, C, ρ, p and Λ. So, in order to obtain a complete solution, we have to consider two extra conditions.

Therefore, we consider the equation of state for perfect fluid as

$$p = \omega\rho, \tag{18}$$

where ω is a constant.
And, the average scale factor a defined by

$$a = (ABC)^{\frac{1}{3}}. \tag{19}$$

to obey the hybrid expansion law proposed by Akarsu *et al.* [40];

$$a = a_0\left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0} - 1\right)}, \tag{20}$$

where γ and ξ are non-negative constants and a_0 represents the present value of the scale factor and t_0 represents the present age of the universe.
From equations (15)-(17), we then obtain

$$A = a_0 l_1\left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0} - 1\right)} \exp\left[m_1\int\left\{a_0\left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0} - 1\right)}\right\}^{-3} dt\right], \tag{21}$$

$$B = a_0 l_2 \left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)} \exp\left[m_2 \int \left\{a_0 \left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)}\right\}^{-3} dt\right], \tag{22}$$

$$C = a_0 l_3 \left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)} \exp\left[m_3 \int \left\{a_0 \left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)}\right\}^{-3} dt\right]. \tag{23}$$

where l_1, l_2, l_3 and m_1, m_2, m_3 are constants of integration satisfying the relations $l_1 l_2 l_3 = 1$ and $m_1 + m_2 + m_3 = 0$.

5. PHYSICAL AND GEOMETRICAL PROPERTIES OF THE MODEL

For our model, some important cosmological parameters are:
The spatial volume

$$V = ABC = {a_0}^3 \left(\frac{t}{t_0}\right)^{3\gamma} e^{3\xi\left(\frac{t}{t_0}-1\right)}. \tag{24}$$

The mean Hubble parameter is

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{\xi}{t_0} + \frac{\gamma}{t}. \tag{25}$$

The deceleration parameter

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{\gamma {t_0}^2}{(\xi t + \gamma t_0)^2} - 1. \tag{26}$$

The expansion scalar

$$\theta = 3H = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 3\left(\frac{\xi}{t_0} + \frac{\gamma}{t}\right). \tag{27}$$

The shear scalar

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} {H_i}^2 - 3H^2\right) = \frac{{m_2}^2 + {m_3}^2 + m_2 m_3}{{a_0}^6 \left(\frac{t}{t_0}\right)^{6\gamma} e^{6\xi\left(\frac{t}{t_0}-1\right)}}. \tag{28}$$

The anisotropy parameter

$$A_m = \frac{1}{3}\sum_{i=1}^{3} \left(\frac{H_i - H}{H}\right)^2 = \frac{2({m_2}^2 + {m_3}^2 + m_2 m_3)}{9{a_0}^6 \left(\frac{\xi}{t_0} + \frac{\gamma}{t}\right)^2 \left(\frac{t}{t_0}\right)^{6\gamma} e^{6\xi\left(\frac{t}{t_0}-1\right)}}. \tag{29}$$

where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters.
The energy density (ρ), the pressure (p) and the cosmological constant (Λ) are obtained as

$$\rho = \frac{4\pi + \alpha}{4\beta(3\omega - 1)} - \frac{\sqrt{(\omega+1)\left\{(4\pi+\alpha)^2(\omega+1){a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma} e^{6\xi\left(\frac{t}{t_0}-1\right)} + 8\beta(3\omega-1)\left({m_2}^2 + {m_3}^2 + m_2 m_3 - \gamma {a_0}^6 {t_0}^{(-2)} \left(\frac{t}{t_0}\right)^{6\gamma-2} e^{6\xi\left(\frac{t}{t_0}-1\right)}\right)\right\}}}{4\beta(\omega+1)(3\omega-1){a_0}^3\left(\frac{t}{t_0}\right)^{3\gamma} e^{3\xi\left(\frac{t}{t_0}-1\right)}}, \tag{30}$$

$$p = \frac{\omega(4\pi + \alpha)}{4\beta(3\omega - 1)} - \frac{\omega\sqrt{(\omega+1)\left\{(4\pi+\alpha)^2(\omega+1){a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma} e^{6\xi\left(\frac{t}{t_0}-1\right)} + 8\beta(3\omega-1)\left({m_2}^2 + {m_3}^2 + m_2 m_3 - \gamma {a_0}^6 {t_0}^{(-2)} \left(\frac{t}{t_0}\right)^{6\gamma-2} e^{6\xi\left(\frac{t}{t_0}-1\right)}\right)\right\}}}{4\beta(\omega+1)(3\omega-1){a_0}^3\left(\frac{t}{t_0}\right)^{3\gamma} e^{3\xi\left(\frac{t}{t_0}-1\right)}}, \tag{31}$$

$$\Lambda = -\frac{({m_2}^2 + {m_3}^2 + m_2 m_3)(\omega+5)}{2(\omega+1){a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma} e^{6\xi\left(\frac{t}{t_0}-1\right)}} + \frac{\gamma\{\omega+5-6\gamma(\omega+1)\}}{2(\omega+1)t^2} - \frac{3\xi^2}{{t_0}^2} - \frac{6\gamma\xi}{t t_0} - \frac{(4\pi+\alpha)\{(\omega+5)(4\pi+\alpha)-2(8\pi+3\alpha-\alpha\omega)\}}{8\beta(3\omega-1)} +$$

$$9\left(\frac{\xi}{t_0} + \frac{\gamma}{t}\right)^2 \frac{({m_2}^2 + {m_3}^2 + m_2 m_3)}{{a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma} e^{6\xi\left(\frac{t}{t_0}-1\right)}} + \left\{\frac{\pi}{2\beta(3\omega-1)} + \right.$$

$$\left.\frac{\alpha}{8\beta(\omega+1)}\right\} \frac{\sqrt{(\omega+1)\left\{(4\pi+\alpha)^2(\omega+1){a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma} e^{6\xi\left(\frac{t}{t_0}-1\right)} + 8\beta(3\omega-1)\left({m_2}^2 + {m_3}^2 + m_2 m_3 - \gamma {a_0}^6 {t_0}^{(-2)} \left(\frac{t}{t_0}\right)^{6\gamma-2} e^{6\xi\left(\frac{t}{t_0}-1\right)}\right)\right\}}}{{a_0}^3\left(\frac{t}{t_0}\right)^{3\gamma} e^{3\xi\left(\frac{t}{t_0}-1\right)}}. \tag{32}$$

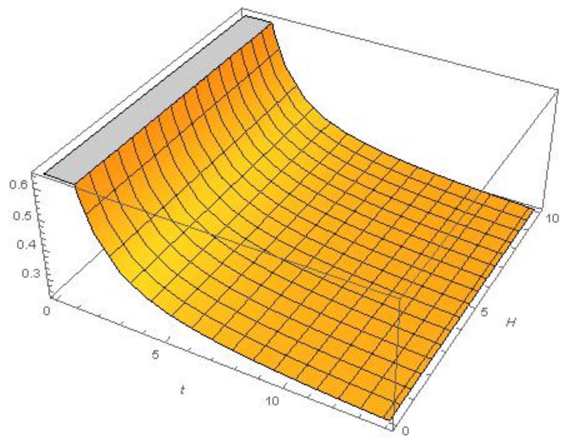

Figure 1. Variation of the Hubble parameter H v/s cosmic time t

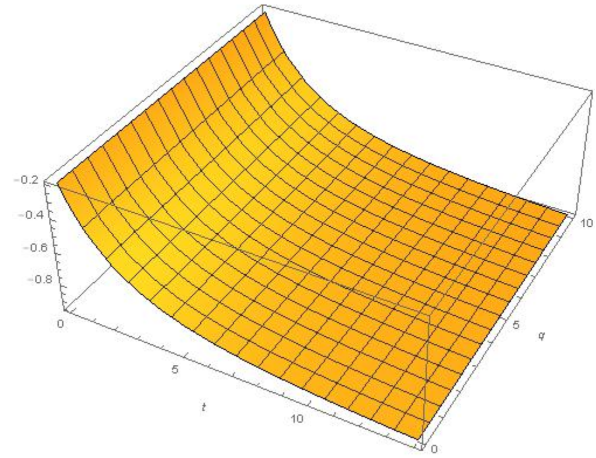

Figure 2. Variation of the deceleration parameter q v/s cosmic time t

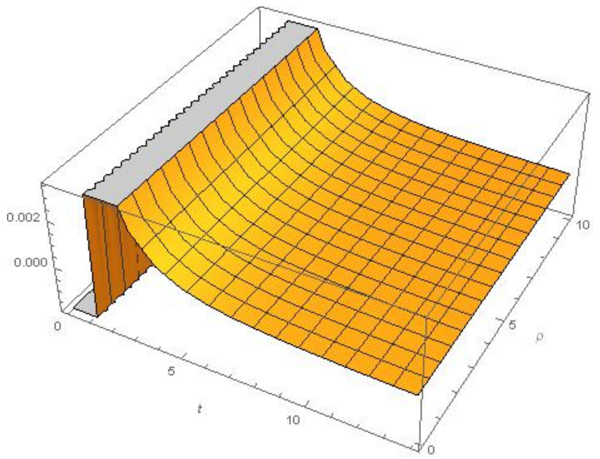

Figure 3. Variation of the energy density ρ v/s cosmic time t

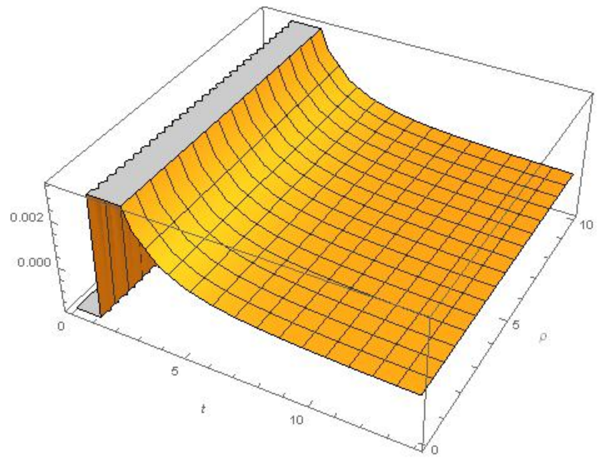

Figure 4. Variation of the pressure p v/s cosmic time t

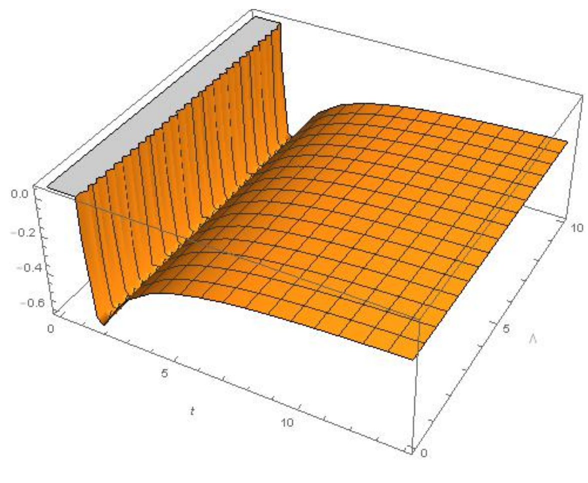

Figure 5. Variation of the cosmological constant Λ v/s cosmic time t

To explore the physical and geometrical properties of the model from the evolution graphs of the cosmological parameters, we take $\gamma = 0.6,\ \xi = 0.2,\ a_0 = 1,\ t_0 = 1,\ m_1 = 0.7,\ m_2 = 0.3,\ m_3 = -1,\ \alpha = 0.1,\ \beta = 0.1,\ \omega = 1$. From the graphs, we observe that the Hubble parameter (H) and the deceleration parameter (q) are decreasing functions of cosmic time. The energy density (ρ) and pressure (p) are also decreasing function of cosmic time. Figure 5, shows that the cosmological constant (Λ) decreases rapidly at initial stage and tend to zero in the course of evolution. The hybrid expansion law (20) is a combination of the power law expansion and the exponential expansion. It yields the power law expansion for $\xi = 0$ and the exponential expansion for $\gamma = 0$.

Case (i): When $\xi = 0$, equation (20) reduces to

$$a = a_0 \left(\frac{t}{t_0}\right)^{\gamma},$$

which is the power-law of expansion.
Then, equations (21)-(23) yield

$$A = a_0 l_1 \left(\frac{t}{t_0}\right)^{\gamma} \exp\left[m_1 a_0^{-3} t_0^{3\gamma} \frac{t^{1-3\gamma}}{1-3\gamma}\right], \tag{33}$$

$$B = a_0 l_2 \left(\frac{t}{t_0}\right)^{\gamma} \exp\left[m_2 a_0^{-3} t_0^{3\gamma} \frac{t^{1-3\gamma}}{1-3\gamma}\right], \tag{34}$$

$$C = a_0 l_3 \left(\frac{t}{t_0}\right)^{\gamma} \exp\left[m_3 a_0^{-3} t_0^{3\gamma} \frac{t^{1-3\gamma}}{1-3\gamma}\right] \tag{35}$$

Thus, when the expansion of the universe is governed by a power law expansion, then

$$V = {a_0}^3 \left(\frac{t}{t_0}\right)^{3\gamma}, \tag{36}$$

$$H = \frac{\gamma}{t}, \tag{37}$$

$$q = \frac{1}{\gamma} - 1, \tag{38}$$

$$\theta = \frac{3\gamma}{t}, \tag{39}$$

$$\sigma^2 = \frac{{m_2}^2 + {m_3}^2 + m_2 m_3}{{a_0}^6 \left(\frac{t}{t_0}\right)^{6\gamma}}, \tag{40}$$

$$A_m = \frac{2({m_2}^2 + {m_3}^2 + m_2 m_3)}{9{a_0}^6 \left(\frac{\gamma}{t}\right)^2 \left(\frac{t}{t_0}\right)^{6\gamma}}, \tag{41}$$

$$\rho = \frac{4\pi+\alpha}{4\beta(3\omega-1)} - \frac{\sqrt{(\omega+1)\left\{(4\pi+\alpha)^2(\omega+1){a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma} + 8\beta(3\omega-1)\left({m_2}^2+{m_3}^2+m_2 m_3-\gamma {a_0}^6 {t_0}^{(-2)}\left(\frac{t}{t_0}\right)^{6\gamma-2}\right)\right\}}}{4\beta(\omega+1)(3\omega-1){a_0}^3\left(\frac{t}{t_0}\right)^{3\gamma}}, \tag{42}$$

$$p = \frac{\omega(4\pi+\alpha)}{4\beta(3\omega-1)} - \frac{\omega\sqrt{(\omega+1)\left\{(4\pi+\alpha)^2(\omega+1){a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma} + 8\beta(3\omega-1)\left({m_2}^2+{m_3}^2+m_2 m_3-\gamma {a_0}^6 {t_0}^{(-2)}\left(\frac{t}{t_0}\right)^{6\gamma-2}\right)\right\}}}{4\beta(\omega+1)(3\omega-1){a_0}^3\left(\frac{t}{t_0}\right)^{3\gamma}}, \tag{43}$$

$$\Lambda = -\frac{({m_2}^2+{m_3}^2+m_2 m_3)(\omega+5)}{2(\omega+1){a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma}} + \frac{\gamma\{\omega+5-6\gamma(\omega+1)\}}{2(\omega+1)t^2} - \frac{(4\pi+\alpha)\{(\omega+5)(4\pi+\alpha)-2(8\pi+3\alpha-\alpha\omega)\}}{8\beta(3\omega-1)} + 9\left(\frac{\gamma}{t}\right)^2 \frac{({m_2}^2+{m_3}^2+m_2 m_3)}{{a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma}} + \left\{\frac{\pi}{2\beta(3\omega-1)} + \frac{\alpha}{8\beta(\omega+1)}\right\}\frac{\sqrt{(\omega+1)\left\{(4\pi+\alpha)^2(\omega+1){a_0}^6\left(\frac{t}{t_0}\right)^{6\gamma} + 8\beta(3\omega-1)\left({m_2}^2+{m_3}^2+m_2 m_3-\gamma {a_0}^6 {t_0}^{(-2)}\left(\frac{t}{t_0}\right)^{6\gamma-2}\right)\right\}}}{{a_0}^3\left(\frac{t}{t_0}\right)^{3\gamma}}. \tag{44}$$

From the Figures 6, 7, 8 and 9, we see that the Hubble parameter (*H*), energy density (ρ), pressure (p) are decreasing functions of cosmic time and the cosmological constant (Λ) decreases initially to negative value and then increases tending to zero as time evolves. The deceleration parameter (q) may be positive, negative or zero depending on the values of γ. For $\gamma > 1$, the expansion of the universe corresponding to the constructed model accelerates.
For $\gamma < 1$, the expansion decelerates and for $\gamma = 1$, the universe undergoes uniform expansion.

Case (ii): When $\gamma = 0$, equation (21) reduces to

$$a = a_0 e^{\xi\left(\frac{t}{t_0} - 1\right)},$$

which is the exponential law of expansion.

Then, equations (21)-(23) yield

$$A = a_0 l_1 e^{\xi\left(\frac{t}{t_0} - 1\right)} \exp\left[\frac{-m_1 t_0}{3\xi {a_0}^3} e^{-3\xi\left(\frac{t}{t_0} - 1\right)}\right], \tag{45}$$

$$B = a_0 l_2 e^{\xi\left(\frac{t}{t_0} - 1\right)} \exp\left[\frac{-m_2 t_0}{3\xi {a_0}^3} e^{-3\xi\left(\frac{t}{t_0} - 1\right)}\right], \tag{46}$$

$$C = a_0 l_3 e^{\xi\left(\frac{t}{t_0} - 1\right)} \exp\left[\frac{-m_3 t_0}{3\xi {a_0}^3} e^{-3\xi\left(\frac{t}{t_0} - 1\right)}\right]. \tag{47}$$

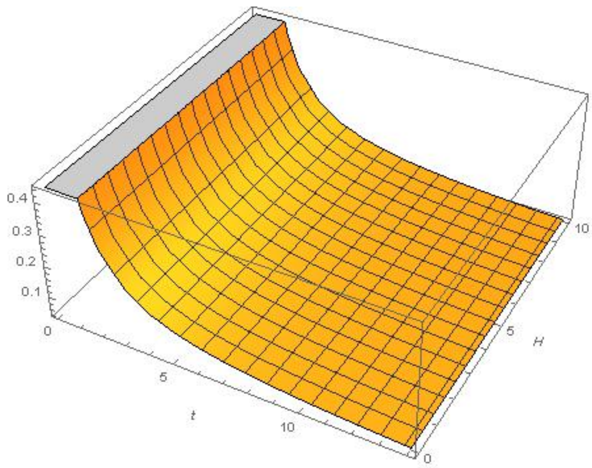

Figure 6. Variation of the Hubble parameter H v/s cosmic time t

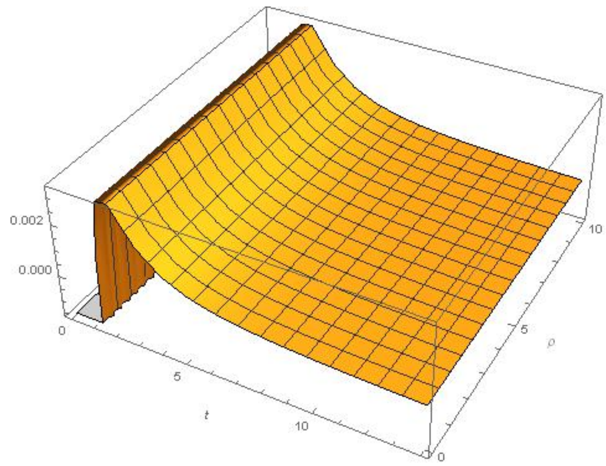

Figure 7. Variation of the energy density ρ v/s cosmic time t

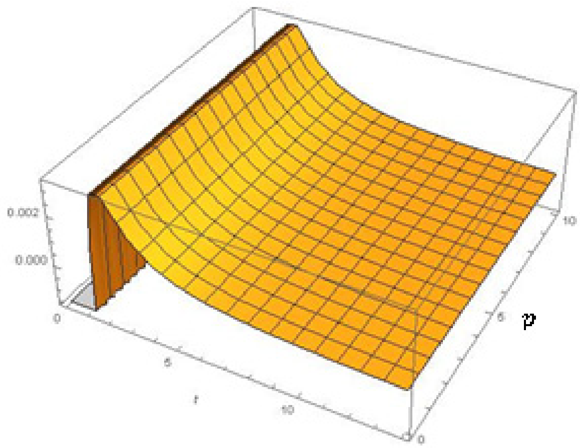

Figure 8. Variation of the pressure p v/s cosmic time t

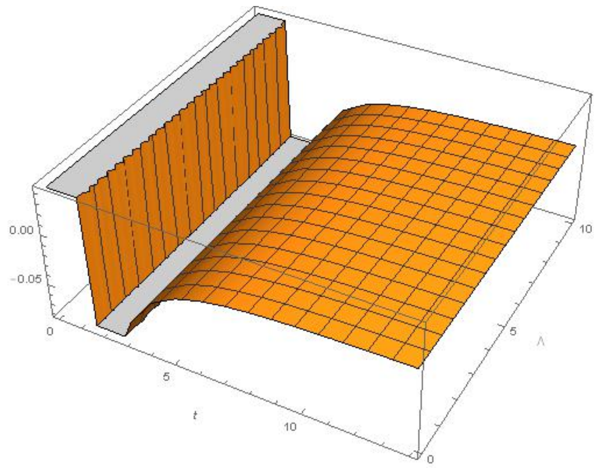

Figure 9. Variation of the cosmological constant Λ v/s cosmic time t

When the expansion of the universe is governed by the exponential law of expansion, then

$$V = {a_0}^3 e^{3\xi\left(\frac{t}{t_0} - 1\right)}, \tag{48}$$

$$H = \frac{\xi}{t_0}, \tag{49}$$

$$q = -1, \tag{50}$$

$$\theta = 3\frac{\xi}{t_0}, \tag{51}$$

$$\sigma^2 = \frac{{m_2}^2 + {m_3}^2 + m_2 m_3}{{a_0}^6 e^{6\xi\left(\frac{t}{t_0} - 1\right)}}, \tag{52}$$

$$A_m = \frac{2({m_2}^2+{m_3}^2+m_2 m_3)}{9{a_0}^6\left(\frac{\xi}{t_0}\right)^2 e^{6\xi\left(\frac{t}{t_0}-1\right)}}, \tag{53}$$

$$\rho = \frac{4\pi+\alpha}{4\beta(3\omega-1)} - \frac{\sqrt{(\omega+1)\left\{(4\pi+\alpha)^2(\omega+1){a_0}^6 e^{6\xi\left(\frac{t}{t_0}-1\right)}+8\beta(3\omega-1)({m_2}^2+{m_3}^2+m_2 m_3)\right\}}}{4\beta(\omega+1)(3\omega-1){a_0}^3 e^{3\xi\left(\frac{t}{t_0}-1\right)}}, \tag{54}$$

$$p = \frac{\omega(4\pi+\alpha)}{4\beta(3\omega-1)} - \frac{\omega\sqrt{(\omega+1)\left\{(4\pi+\alpha)^2(\omega+1){a_0}^6 e^{6\xi\left(\frac{t}{t_0}-1\right)}+8\beta(3\omega-1)({m_2}^2+{m_3}^2+m_2 m_3)\right\}}}{4\beta(\omega+1)(3\omega-1){a_0}^3 e^{3\xi\left(\frac{t}{t_0}-1\right)}}, \tag{55}$$

$$\Lambda = -\frac{({m_2}^2+{m_3}^2+m_2 m_3)(\omega+5)}{2(\omega+1){a_0}^6 e^{6\xi\left(\frac{t}{t_0}-1\right)}} - \frac{3\xi^2}{{t_0}^2} - \frac{(4\pi+\alpha)\{(\omega+5)(4\pi+\alpha)-2(8\pi+3\alpha-\alpha\omega)\}}{8\beta(3\omega-1)} + 9\left(\frac{\xi}{t_0}\right)^2\frac{({m_2}^2+{m_3}^2+m_2 m_3)}{{a_0}^6 e^{6\xi\left(\frac{t}{t_0}-1\right)}} + \left\{\frac{\pi}{2\beta(3\omega-1)} + \frac{\alpha}{8\beta(\omega+1)}\right\}\frac{\sqrt{(\omega+1)\left\{(4\pi+\alpha)^2(\omega+1){a_0}^6 e^{6\xi\left(\frac{t}{t_0}-1\right)}+8\beta(3\omega-1)({m_2}^2+{m_3}^2+m_2 m_3)\right\}}}{{a_0}^3 e^{3\xi\left(\frac{t}{t_0}-1\right)}}. \tag{56}$$

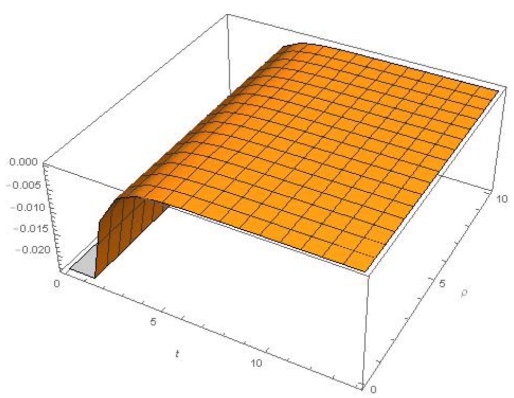

Figure 10. Variation of the energy density ρ v/s cosmic time t

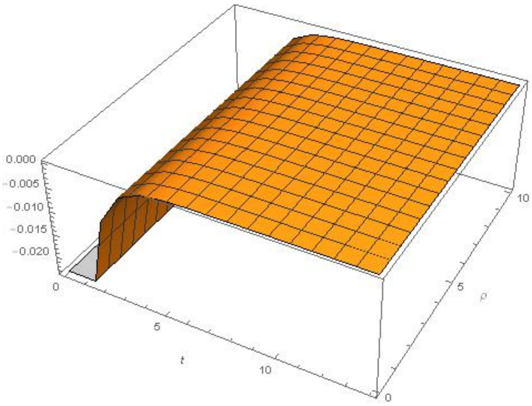

Figure 11. Variation of the pressure p v/s cosmic time t

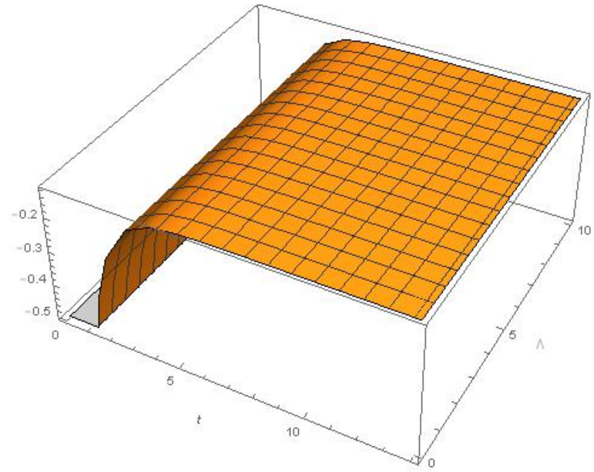

Figure 12. Variation of the cosmological constant Λ v/s cosmic time t

From the graphs, we observe that the energy density, pressure and cosmological constant initially assume negative values and then tend to zero in the course of time.

6. CONCLUDING REMARKS

In this paper, we study Bianchi type-I cosmological model within the framework of $f(R,T)$ theory of gravity considering the functional $f(R,T) = R + 2(\alpha T + \beta T^2)$, where α and β are constants. We consider the expansion of the universe to follow a hybrid expansion law and obtain exact solution of the field equations. Two particular cases are also considered when the expansion of the universe is governed by a power law and an exponential law only. We investigate the physical and kinematical properties of various cosmological parameters in all these three cases and find that

• Both the hybrid expansion law and power law of expansion induce an initial singular model of the universe as the metric coefficients A, B and C vanish at the initial moment. In case of exponential expansion law, the metric coefficients A, B and C become constants at $t = 0$.

• For hybrid law and power law of expansion, the physical parameters H, θ, σ^2, A_m assume very high value at the initial epoch and tend to zero for large t. Also, the volume of the universe is zero at the beginning and increases exponentially with time t. Hence, the universe starts with the Big Bang singularity at $t = 0$ and then expand throughout the evolution. In case of exponential law, the physical parameters H, θ become constants. Volume is initially very low and increases exponentially in the course of time while the other parameters show similar behavior as hybrid law and power law of expansion.

• In hybrid expansion law, the deceleration parameter (q) approaches -1 for large cosmic time. In case of power law of expansion, it may be positive, negative or zero showing thereby that the universe may undergo accelerating expansion, decelerating expansion or uniform expansion. The expression for the deceleration parameter q, in the case of exponential law of expansion, shows that the expansion of the universe is decelerating throughout the evolution without depending on γ.

• For hybrid law and power law of expansion, the energy density and pressure increase rapidly at the beginning but it decreases in the course of evolution and tend to 0 at late time. But in case of exponential expansion law, the energy density and pressure are negative and increase exponentially throughout the evolution of the universe and tend to 0 as time $t \to \infty$.

• The cosmological constant (Λ) decreases initially and then increases and tends to 0 at late times for hybrid law as well as power-law of expansion. In the case of exponential law, cosmological constant is negative and increases in the course of time tending to zero at late times.

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АНІЗОТРОПНА КОСМОЛОГІЧНА МОДЕЛЬ У $f(R,T)$ ТЕОРІЇ ГРАВІТАЦІЇ З КВАДРАТИЧНОЮ ФУНКЦІЄЮ ВІД Т

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У цій статті ми досліджуємо просторово-однорідний та анізотропний простір-час Біанкі типу I, заповнений ідеальною рідиною, у рамках $f(R,T)$ теорії гравітації для функціональної форми $f(R,T)=R+2f(T)$ з $f(T)=\alpha T+\beta T^2$, де α і β константи. Точні розв'язки рівнянь гравітаційного поля отримані шляхом припущення, що середній масштабний коефіцієнт підкоряється гібридному закону розширення, і виведено деякі космологічні параметри моделі. Також розглядаються два особливих випадки, що призводять до степеневого розкладу та експоненціального розкладу. Ми досліджуємо фізичні та геометричні властивості моделей, вивчаючи графіки еволюції деяких відповідних космологічних параметрів, таких як параметр Хаббла (H), параметр уповільнення (q) тощо.

Ключові слова: *Всесвіт Біанкі типу I; f(R,T) теорія гравітації; параметр Хаббла; космологічна стала; параметр уповільнення*