East European Journal of Physics. **4**. 77–86 (2023)
DOI: 10.26565/2312-4334-2023-4-08 ISSN 2312-4334

THE EFFECT OF THERMAL STRATIFICATION ON UNSTEADY PARABOLIC FLOW PAST AN INFINITE VERTICAL PLATE WITH CHEMICAL REACTION

Rupam Shankar Nath*, **Rudra Kanta Deka**, **Himangshu Kumar**
Department of Mathematics, Gauhati University, Guwahati-781014, Assam, India
**Corresponding Author e-mail: rupamnath23@gmail.com*
Received August 1, 2023; revised September 30, 2023; accepted November 3, 2023

This research paper investigates the effects of thermal stratification on unsteady parabolic flow past an infinite vertical plate with chemical reaction. Using the Laplace transform method, analytical solutions are derived to simulate the physical process of the flow. The study considers the effects of thermal stratification on the flow field, as well as the effects of chemical reaction on the velocity, and temperature field. The results of the stratification case are then compared to the case of no stratification of a similar flow field. The results of this research can be used to improve understanding of the unsteady parabolic flow in thermal stratified environments and provide valuable insight into the effects of chemical reactions on the temperature field.

Keywords: *Thermal Stratification; Chemical Reaction; Parabolic Flow; Vertical plate*

PACS: 47.55.P-, 44.25.+f, 44.05.+e, 47.11.-j

NOMENCLATURE

α	Thermal Diffusivity
β	Volumetric Coefficient of Thermal Expansion
β^*	Volumetric Coefficient of Expansion with Concentration
η	Similarity Parameter
γ	Thermal Stratification Parameter
ν	Kinematic Viscosity
τ	Non-Dimensional Skin-Friction
θ	Non-Dimensional Temperature
C	Non-Dimensional Concentration
C'	Species Concentration in the fluid
C'_∞	Concentration of the fluid far away from the Plate
C'_w	Concentration of the Plate
D	Mass Diffusion Coefficient
g	Acceleration due to gravity
Gc	Mass Grashof Number
Gr	Thermal Grashof Number
K	Non-Dimensional Chemical Reaction Parameter
K_1	Chemical Reaction Parameter
Pr	Prandtl Number
S	Non-Dimensional Thermal Stratification Parameter
Sc	Schimdt Number
t	Non-Dimensional Time
T'	Temperature of the fluid
t'	Time
T'_∞	Temperature of the fluid far away from the Plate
T'_w	Temperature of the Plate
U	Non-Dimensional Velocity
u'	Velocity of the fluid in x' direction
u_0	Velocity of the Plate
y	Non-Dimensional Coordinate Normal to the plate
y'	Coordinate Normal to the Plate

1. INTRODUCTION

The study of parabolic flow is significant because it helps to reduce energy losses in flowing fluids by decreasing viscous interactions between neighboring layers of fluid and the pipe wall. Additionally, parabolic flow is simpler to model and calculate than unstable flow, which includes the fluid's random and unpredictable

Cite as: R.S. Nath, R.K. Deka, H. Kumar, East Eur. J. Phys. 4, 77 (2023), https://doi.org/10.26565/2312-4334-2023-4-08

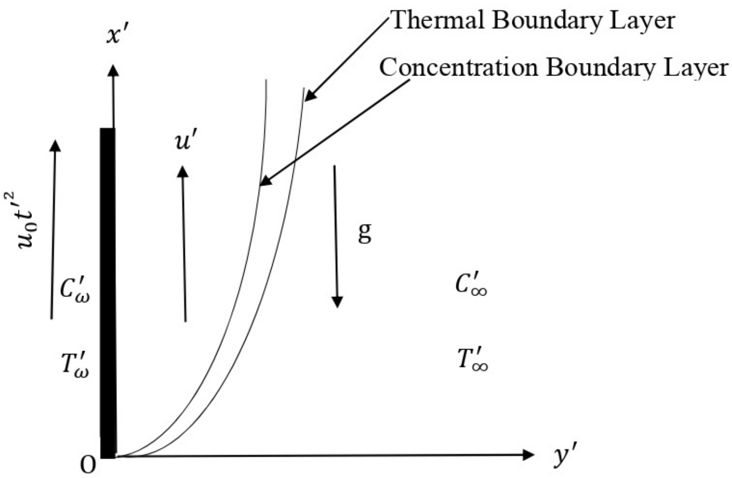

Figure 1. Physical Model and coordinate system

motion. One of the uses for parabolic flow is in mass and heat transfer procedures using an infinite vertical plate. The plate may be subjected to a variety of boundary conditions, including constant or variable temperature, constant or variable heat flow, constant or variable concentration, etc. The study of these problems has a lot of attention because they are important in engineering and industrial processes, such as cooling electronic devices, sun collectors, chemical reactors, combustion chambers, etc.

The first study on the parabolic starting flow past an infinite vertical plate was done by [1]. In addition, the MHD Parabolic flow for an infinite vertical plate was investigated by [2], while [3] studied the flow around an accelerating vertical plate. The present research is the first work to look into the combined effects of thermal stratification and chemical reaction on parabolic flow past an infinite vertical plate. [4, 5] and [6] investigated unsteady flows in a stably stratified fluid, focusing on infinite plates. Furthermore, buoyancy-driven flows in a stratified fluid were examined by [7, 8]. [9] came up with an analytical solution to describe how fluid would flow past an infinite vertical plate that had been affected chemically. In their studies, [10] and [11] examine the results of applying a chemical reaction to an infinite vertical plate under different situations. The articles [12] and [13] investigate the impacts of chemical reaction and thermal stratification on MHD flow for vertical stretching surfaces. In a similar manner, [14] investigated the effects of non-Newtonian fluid flow across a porous material on both effects. The study conducted by [15] focuses on analysing the collective influence of thermal stratification and chemical reaction on the flow of a fluid relative to an infinitely vertical plate. Furthermore, [16] conducted an investigation to examine the impact of thermal stratification on the unsteady flow of fluid past an accelerated vertical plate, while also considering the presence of a first-order chemical reaction. [17] conducted an investigation to analyse the impact of thermal stratification with velocity slip, and changing viscosity on the magnetohydrodynamic (MHD) flow of a nanofluid across a disc.

The work presents the derivation of the special solutions for the situation $Sc = 1$ and the classical solutions for the case $S = 0$, where no stratification is present. In order to compare these solutions to the original solutions, graphs are utilized to show the variations. Physical parameters that affect the concentration, temperature, and velocity profiles are discussed and illustrated graphically. These parameters include the S, K, Gr, Gc, and time t. The conclusions of this study have numerous uses across several industries and chemical plants.

2. MATHEMATICAL ANALYSIS

We examine at the unstable parabolic flow over an infinite vertical plate of a viscous, in-compressible, stratified fluid with first order chemical reaction effects. To study the flow situation, we employ a coordinate system in which the y' axis is perpendicular to the plate and the x' axis is chosen vertically upward along the plate. At first, both the plate's initial temperature T'_∞ and fluid's initial concentration C'_∞ are the same. At time $t' > 0$, the plate is moving at the velocity $u_0 t'^2$ in its own plane relative to the gravitational field. Additionally, at time $t' > 0$, the concentration level is raised to C'_w and the plate temperature is raised to T'_w. All flow variables are independent of x' and only affected by y' and t' since the plate has an infinite length. As a result, we are left with a flow that is only one dimension and has one non-zero vertical velocity component, u'. The Boussinesqs' approximation is then used to represent the equations for motion, energy, and concentration as follows:

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)+\nu\frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \alpha\frac{\partial^2 T'}{\partial y'^2}-\gamma u' \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2}-K_1 C' \tag{3}$$

with the following initial and boundary Conditions:

$$\begin{array}{llll} u'=0 & T'=T'_\infty & C'=C'_\infty & \forall y', t'\leq 0 \\ u'=u_0 t'^2 & T'=T'_w & C'=C'_w & \text{at } y'=0, t'>0 \\ u'=0 & T'\to T'_\infty & C'\to C'_\infty & \text{as } y'\to\infty, t'>0 \end{array}$$

where, $\gamma = \frac{dT'_\infty}{dx'}+\frac{g}{C_p}$ denotes the thermal stratification parameter and $\frac{dT'_\infty}{dx'}$ denotes the vertical temperature convection known as thermal stratification. In addition, $\frac{g}{C_p}$ represents the rate at which particles in a fluid do reversible work due to compression, often known as work of compression. The variable (γ) will be referred to as the thermal stratification parameter in our research because the compression work is relatively minimal. For the purpose of testing computational methods, compression work is kept as an additive to thermal stratification. And we provide non-dimensional quantities in the following:

$$U=u'\left(\frac{u_0}{\nu^2}\right)^{1/3},\quad t=t'\left(\frac{u_0^2}{\nu}\right)^{1/3},\quad y=y'\left(\frac{u_0}{\nu^2}\right)^{1/3},\quad \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},\quad C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\quad Pr=\frac{\nu}{\alpha}$$

$$Gr=\frac{g\beta(T'_w-T'_\infty)}{(\nu u_0)^{1/3}},\quad Gc=\frac{g\beta^*(C'_w-C'_\infty)}{(\nu u_0)^{1/3}},\quad Sc=\frac{\nu}{D},\quad K=K_1\left(\frac{\nu}{u_0^2}\right)^{1/3},\quad S=\frac{\gamma\nu}{u_0(T'_w-T'_\infty)}$$

The non-dimensional forms of the equations (1)-(3) are given by

$$\frac{\partial U}{\partial t} = Gr\theta+GcC+\frac{\partial^2 U}{\partial y^2} \tag{4}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}-SU \tag{5}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-KC \tag{6}$$

Non-dimensional form of initial and boundary Conditions are:

$$\begin{array}{llll} U=0 & \theta=0 & C=0 & \forall y, t\leq 0 \\ U=t^2 & \theta=1 & C=1 & \text{at } y=0, t>0 \\ U=0 & \theta\to 0 & C\to 0 & \text{as } y\to\infty, t>0 \end{array} \tag{7}$$

3. METHOD OF SOLUTION

The non-dimensional governing equations (4)- (6) with boundary conditions (7) are solved using Laplace's transform method for $Pr = 1$. Hence, the expressions for concentration, velocity and temperature with the help of [18] and [19] are given by

$$C = \frac{1}{2}\left[e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)+e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right] \tag{8}$$

$$U = \left[f_4(iA)+f_4(-iA)\right]+\frac{iA}{2S}\left\{f_1(iA)-f_1(-iA)\right\}+\frac{Gc}{2(Sc-1)}\left[C_1\left\{f_1(iA)+f_1(-iA)\right\}\right.$$

$$+(C_2 - iC_3)\{f_2(iA, B+iB_1) + f_2(-iA, B+iB_1)\} + (C_2 + iC_3)\{f_2(iA, B-iB_1)$$
$$+f_2(-iA, B-iB_1)\}] + \frac{Gc}{2iA}[(D_1 - 1)\{f_1(iA) - f_1(-iA)\} + (D_2 + iD_3)\{f_2(iA, B+iB_1)$$
$$-f_2(-iA, B+iB_1)\} + (D_2 - iD_3)\{f_2(iA, B-iB_1) - f_2(-iA, B-iB_1)\}]$$
$$-\frac{Gc}{(Sc-1)}\left[\frac{C_1}{2}\left\{e^{-2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) + e^{2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right\}\right.$$
$$+(C_2 - iC_3)\{f_3(K, B+iB_1)\} + (C_2 + iC_3)\{f_3(K, B-iB_1)\}] \quad (9)$$

$$\theta = \frac{S}{iA}[f_4(iA) - f_4(-iA)] + \frac{1}{2}\{f_1(iA) + f_1(-iA)\} + \frac{SGc}{2iA(Sc-1)}[C_1\{f_1(iA) - f_1(-iA)\}$$
$$+(C_2 - iC_3)\{f_2(iA, B+iB_1) - f_2(-iA, B+iB_1)\} + (C_2 + iC_3)\{f_2(iA, B-iB_1)$$
$$-f_2(-iA, B-iB_1)\}] + \frac{SGc}{2(Sc-1)^2}[E_1\{f_1(iA) + f_1(-iA)\} + (E_2 - iE_3)\{f_2(iA, B+iB_1)$$
$$+f_2(-iA, B+iB_1)\} + (E_2 + iE_3)\{f_2(iA, B-iB_1) + f_2(-iA, B-iB_1)\}]$$
$$-\frac{SGc}{(Sc-1)^2}\left[\frac{E_1}{2}\left\{e^{-2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) + e^{2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right\}\right.$$
$$+(E_2 - iE_3)f_3(K, B+iB_1) + (E_2 + iE_3)f_3(K, B-iB_1) \quad (10)$$

where,

$$\eta = \frac{y}{2\sqrt{t}}, \quad A = \sqrt{SGr}, \quad B = \frac{ScK}{Sc-1}, \quad B_1 = \frac{A}{Sc-1} = \frac{\sqrt{SGr}}{Sc-1}, \quad C_1 = \frac{B}{(B^2 + B_1^2)}$$
$$C_2 = \frac{-B}{2(B^2 + B_1^2)}, \quad C_3 = \frac{-B_1}{2(B^2 + B_1^2)}, \quad D_1 = \frac{B^2}{(B^2 + B_1^2)}, \quad D_2 = \frac{B_1^2}{2(B^2 + B_1^2)}$$
$$D_3 = \frac{BB_1}{2(B^2 + B_1^2)}, \quad E_1 = \frac{1}{(B^2 + B_1^2)}, \quad E_2 = \frac{-1}{2(B^2 + B_1^2)}, \quad E_3 = \frac{B}{2B_1(B^2 + B_1^2)}$$

Also, f_i's are inverse Laplace's transforms given by

$$f_1(ip) = L^{-1}\left\{\frac{e^{-y\sqrt{s+ip}}}{s}\right\}, \quad f_2(ip, q_1 + iq_2) = L^{-1}\left\{\frac{e^{-y\sqrt{s+ip}}}{s + q_1 + iq_2}\right\}$$
$$f_3(p, q_1 + iq_2) = L^{-1}\left\{\frac{e^{-y\sqrt{Sc(s+p)}}}{s + q_1 + iq_2}\right\}, \quad f_4(ip) = L^{-1}\left\{\frac{e^{-y\sqrt{s+ip}}}{s^3}\right\}$$

We separate the complex arguments of the error function contained in the previous expressions into real and imaginary parts using the formulas provided by [18].

4. SPECIAL CASE [FOR $Sc = 1$]

We came up with answers for the special case where $Sc = 1$. Hence, the solutions for the special case are as follows:

$$C^* = \frac{1}{2}\left[e^{-2\eta\sqrt{Kt}} erfc\left(\eta - \sqrt{Kt}\right) + e^{2\eta\sqrt{Kt}} erfc\left(\eta + \sqrt{Kt}\right)\right] \quad (11)$$

$$U^* = \frac{KGc}{2(K^2 + A^2)}\{f_1(iA) + f_1(-iA)\} + \frac{iA}{2}\left(\frac{1}{S} + \frac{Gc}{K^2 + A^2}\right)\{f_1(iA) - f_1(-iA)\}$$
$$-\frac{KGc}{2(K^2 + A^2)}\left[e^{-2\eta\sqrt{Kt}} erfc\left(\eta - \sqrt{Kt}\right) + e^{2\eta\sqrt{Kt}} erfc\left(\eta + \sqrt{Kt}\right)\right]$$
$$+\{f_4(iA) + f_4(-iA)\} \quad (12)$$

$$\theta^* = \frac{SKGc}{2iA(K^2 + A^2)}\{f_1(iA) - f_1(-iA)\} + \frac{1}{2}\left(1 + \frac{SGc}{K^2 + A^2}\right)\{f_1(iA) + f_1(-iA)\}$$
$$-\frac{SGc}{2(K^2 + A^2)}\left\{e^{-2\eta\sqrt{Kt}} erfc\left(\eta - \sqrt{Kt}\right) + e^{2\eta\sqrt{Kt}} erfc\left(\eta + \sqrt{Kt}\right)\right\}$$
$$+\frac{S}{iA}\{f_4(iA) - f_4(-iA)\} \quad (13)$$

5. CLASSICAL CASE ($S = 0$)

We derived solutions for the classical case of no thermal stratification ($S = 0$). We want to compare the results of the fluid with thermal stratification to the case with no stratification. Hence, the corresponding solutions for the classical case is given by :

$$
\begin{aligned}
\theta_c &= erfc(\eta) \qquad (14)\\
U_c &= \frac{Gc}{2KSc}\Big[2erfc\,(\eta) - e^{-Bt}\Big\{e^{-2\eta\sqrt{-Bt}}erfc\left(\eta - \sqrt{-Bt}\right) + e^{2\eta\sqrt{-Bt}}erfc\left(\eta + \sqrt{-Bt}\right)\Big\}\\
&-\Big\{e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) + e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\Big\}\\
&+e^{-Bt}\Big\{e^{-2\eta\sqrt{Sc(K-B)t}}erfc\left(\eta\sqrt{Sc} - \sqrt{(K-B)t}\right)\\
&+e^{2\eta\sqrt{Sc(K-B)t}}erfc\left(\eta\sqrt{Sc} + \sqrt{(K-B)t}\right)\Big\}\Big] + 2t\eta Gr\left\{\frac{e^{-\eta^2}}{\sqrt{\pi}} - \eta\ erfc\,(\eta)\right\}\\
&+\frac{t^2}{3}\left\{\left(3 + 12\eta^2 + 4\eta^4\right)erfc\,(\eta) - \frac{\eta}{\sqrt{\pi}}\left(10 + 4\eta^2\right)e^{-\eta^2}\right\} \qquad (15)
\end{aligned}
$$

5.1. Skin-Friction

The non-dimensional Skin-Friction, which is determined as shear stress on the surface, is obtained by

$$\tau = -\frac{dU}{dy}\bigg|_{y=0}$$

The solution for the Skin-Friction is calculated from the solution of Velocity profile U, represented by (9), as follows:

$$
\begin{aligned}
\tau &= t\sqrt{\frac{t}{\pi}}\cos At + t^2\sqrt{\frac{A}{2}}\,(r_1 - r_2) + \frac{t(r_1 + r_2)}{\sqrt{2A}} + \frac{(r_1 - r_2)}{4A\sqrt{2A}} - \frac{\sin At}{2A}\sqrt{\frac{t}{\pi}}\\
&+\frac{Gc}{Sc-1}\left[C_1\left\{\frac{\cos At}{\sqrt{\pi t}} + \sqrt{\frac{A}{2}}\,(r_1 - r_2) - \sqrt{ScK}\ erf(\sqrt{Kt}) - \sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}\right.\\
&+2C_2\left\{\frac{\cos At}{\sqrt{\pi t}} - \sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}\\
&+e^{-Bt}\left\{(C_2P_1 + C_3Q_1)(r_3\cos B_1t + r_4\sin B_1t) + (C_3P_1 - C_2Q_1)(r_4\cos B_1t - r_3\sin B_1t)\right\}\\
&+e^{-Bt}\left\{(C_2P_2 - C_3Q_2)(r_5\cos B_1t - r_6\sin B_1t) - (C_3P_2 + C_2Q_2)(r_6\cos B_1t + r_5\sin B_1t)\right\}\\
&\left.-2e^{-Bt}\sqrt{Sc}\left\{(C_2P_3 - C_3Q_3)(r_7\cos B_1t - r_8\sin B_1t) - (C_3P_3 + C_2Q_3)(r_8\cos B_1t + r_7\sin B_1t)\right\}\right]\\
&+\frac{A}{S}\left\{\frac{\sin At}{\sqrt{\pi t}} - \sqrt{\frac{A}{2}}\,(r_1 + r_2)\right\} + \frac{Gc}{A}\left[(D_1 - 1)\left\{\frac{-\sin At}{\sqrt{\pi t}} + \sqrt{\frac{A}{2}}\,(r_1 + r_2)\right\} - \frac{2D_2\sin At}{\sqrt{\pi t}}\right.\\
&+e^{-Bt}\left\{(D_2P_1 - D_3Q_1)(r_4\cos B_1t - r_3\sin B_1t) + (D_3P_1 + D_2Q_1)(r_3\cos B_1t + r_4\sin B_1t)\right\}\\
&\left.+e^{-Bt}\left\{(D_2P_2 + D_3Q_2)(r_6\cos B_1t + r_5\sin B_1t) - (D_3P_2 - D_2Q_2)(r_5\cos B_1t - r_6\sin B_1t)\right\}\right]
\end{aligned}
$$

The solution for the Skin-Friction for the special case is given from the expression (12), which is represented by

$$
\begin{aligned}
\tau^* &= t\sqrt{\frac{t}{\pi}}\cos At + t^2\sqrt{\frac{A}{2}}\,(r_1 - r_2) + \frac{t(r_1 + r_2)}{\sqrt{2A}} + \frac{(r_1 - r_2)}{4A\sqrt{2A}} - \frac{\sin At}{2A}\sqrt{\frac{t}{\pi}} + \frac{KGc}{K^2 + A^2}\left\{\frac{\cos At}{\sqrt{\pi t}}\right.\\
&\left.+\sqrt{\frac{A}{2}}\,(r_1 - r_2) - \sqrt{K}\ erf(\sqrt{Kt}) - \frac{e^{-Kt}}{\sqrt{\pi t}}\right\} + A\left(\frac{1}{S} + \frac{Gc}{K^2 + A^2}\right)\left\{\frac{\sin At}{\sqrt{\pi t}} - \sqrt{\frac{A}{2}}\,(r_1 + r_2)\right\}
\end{aligned}
$$

The solution for the Skin-Friction for the classical case is given from the expression (15), which is represented by

$$\tau_c = \frac{Gc}{KSc}\left[e^{-Bt}\left\{\sqrt{Sc(K-B)}\ erf(\sqrt{(K-B)t}) - \sqrt{-B}\ erf(\sqrt{-Bt})\right\} - \sqrt{ScK}\ erf(\sqrt{Kt})\right]$$

$$+\left(\frac{8t}{3}-Gr\right)\sqrt{\frac{t}{\pi}}$$

where,

$$B_2=\sqrt{B^2+(A-B_1)^2},\quad B_3=\sqrt{B^2+(A+B_1)^2},\quad B_4=\sqrt{(K-B)^2+B_1^2},\quad P_1=\sqrt{\frac{B_2-B}{2}},$$
$$Q_1=\sqrt{\frac{B_2+B}{2}},\quad P_2=\sqrt{\frac{B_3-B}{2}},\quad Q_2=\sqrt{\frac{B_3+B}{2}},\quad P_3=\sqrt{\frac{B_4-(K-B)}{2}},$$
$$Q_3=\sqrt{\frac{B_4+(K-B)}{2}},\quad \sqrt{-B+i(A-B_1)}=P_1+iQ_1,\quad \sqrt{-B+i(A+B_1)}=P_2+iQ_2,$$
$$\sqrt{K-B+iB_1}=P_3+iQ_3,\quad erf(\sqrt{iAt})=r_1+ir_2,\quad erf(P_1\sqrt{t}+iQ_1\sqrt{t})=r_3+ir_4,$$
$$erf(P_2\sqrt{t}+iQ_2\sqrt{t})=r_5+ir_6,\quad erf(P_3\sqrt{t}+iQ_3\sqrt{t})=r_7+ir_8$$

5.2. Nusselt Number

The non-dimensional Nusselt number, which is determined as the rate of heat transfer, is obtained by

$$Nu=-\left.\frac{d\theta}{dy}\right|_{y=0}$$

The solution for the Nusselt number is calculated from the solution of Temperature profile θ, represented by (10), as follows:

$$\begin{aligned}
Nu \;=\;& \frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1-r_2)-\frac{S}{A}\left[t\sqrt{\frac{t}{\pi}}\sin At-t^2\sqrt{\frac{A}{2}}(r_1+r_2)+\frac{t(r_1-r_2)}{\sqrt{2A}}-\frac{(r_1+r_2)}{4A\sqrt{2A}}\right.\\
&\left.+\frac{\cos At}{2A}\sqrt{\frac{t}{\pi}}\right]+\frac{Gc}{A(Sc-1)}\left[C_1\left\{\frac{-\sin At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1+r_2)\right\}-\frac{2C_2\sin At}{\sqrt{\pi t}}\right.\\
&+e^{-Bt}\{(C_2P_1+C_3Q_1)(r_4\cos B_1t-r_3\sin B_1t)-(C_3P_1-C_2Q_1)(r_3\cos B_1t+r_4\sin B_1t)\}\\
&\left.+e^{-Bt}\{(C_2P_2-C_3Q_2)(r_6\cos B_1t+r_5\sin B_1t)+(C_3P_2+C_2Q_2)(r_5\cos B_1t-r_6\sin B_1t)\}\right]\\
&+\frac{SGc}{(Sc-1)^2}\left[E_1\left\{\frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1-r_2)-\sqrt{ScK}\;erf(\sqrt{Kt})-\sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}\right.\\
&+2E_2\left\{\frac{cosAt}{\sqrt{\pi t}}-\sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}+e^{-Bt}\{(E_2P_1+E_3Q_1)(r_3cosB_1t+r_4\sin B_1t)\}\\
&+(E_3P_1-E_2Q_1)(r_4cosB_1t-r_3\sin B_1t)+e^{-Bt}\{(E_2P_2-E_3Q_2)(r_5cosB_1t-r_6\sin B_1t)\\
&-(E_3P_2+E_2Q_2)(r_6cosB_1t+r_5\sin B_1t)\}-2e^{-Bt}\sqrt{Sc}\{(E_2P_3-E_3Q_3)(r_7cosB_1t-r_8\sin B_1t)\\
&\left.-(E_3P_3+E_2Q_3)(r_8cosB_1t+r_7\sin B_1t)\}\right]
\end{aligned}$$

The solution for the Nusselt number for the special case is given from the expression (13), which is represented by

$$\begin{aligned}
Nu^* \;=\;& \frac{SKGc}{A(K^2+A^2)}\left\{\frac{-\sin At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1+r_2))\right\}+\left(1+\frac{SGc}{K^2+A^2}\right)\left\{\frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1-r_2)\right\}\\
&-\frac{S}{A}\left[t\sqrt{\frac{t}{\pi}}\sin At-t^2\sqrt{\frac{A}{2}}(r_1+r_2)+\frac{t(r_1-r_2)}{\sqrt{2A}}-\frac{(r_1+r_2)}{4A\sqrt{2A}}+\frac{\cos At}{2A}\sqrt{\frac{t}{\pi}}\right]\\
&-\frac{SGc}{K^2+A^2}\left\{\sqrt{K}\;erf(\sqrt{Kt})+\frac{e^{-Kt}}{\sqrt{\pi t}}\right\}
\end{aligned}$$

The solution for the Nusselt number for the classical case is given from the expression (14), which is represented by

$$Nu_c=\frac{1}{\sqrt{\pi t}}$$

5.3. Sherwood Number

The non-dimensional Sherwood number, which is determined as the rate of mass transfer, is obtained by

$$Sh = -\frac{dC}{dy}\bigg|_{y=0}$$

The solution for the Sherwood number is calculated from the solution of Concentration profile C, represented by (8), as follows:

$$Sh = \sqrt{ScK}\ erf(\sqrt{Kt}) + \sqrt{\frac{Sc}{\pi t}}e^{-Kt}$$

The solution for the Sherwood number for the special case is given from the expression (11), which is represented by

$$Sh^{*} = \sqrt{K}\ erf(\sqrt{Kt}) + \frac{1}{\sqrt{\pi t}}e^{-Kt}$$

6. RESULT AND DISCUSSIONS

We calculated the velocity, temperature, concentration, Skin friction, Nusselt number, and Sherwood number for various values of the physical parameters S, Gr, Gc, Sc, K and time t from the solutions we obtained in the previous sections. This allowed us to get a better understanding of the physical significance of the problem. Additionally, we portrayed them graphically in Figures 2- 15.

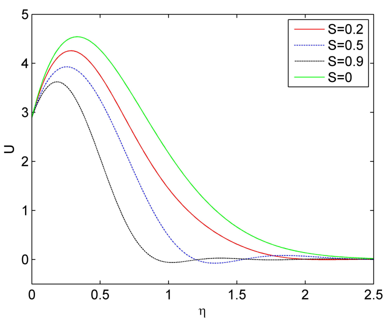

Figure 2. Effects of S on Velocity Profile for $Gr = 5, Gc = 5, t = 1.7, Sc = 0.6, K = 0.2$

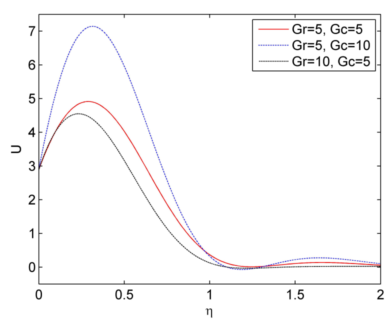

Figure 3. Effects of Gr and Gc on Velocity Profile for $S = 0.4, Sc = 0.6, t = 1.7, K = 0.2$

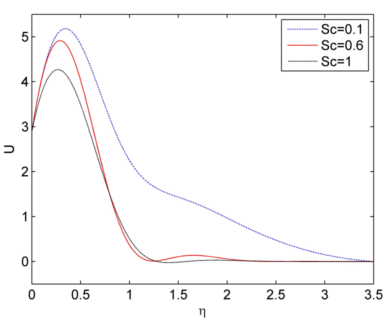

Figure 4. Effects of Sc on Velocity Profile for $Gr = 5, Gc = 5, S = 0.4, t = 1.7, K = 0.2$

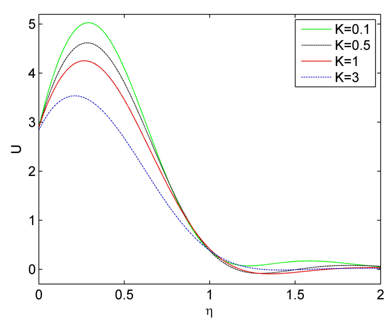

Figure 5. Effects of K on Velocity Profile for $Gr = 5, Gc = 5, S = 0.4, Sc = 0.6, t = 1.7$

The Figure 2 displays the changes in velocity profiles brought upon by thermal stratification (S). It can be seen that thermal stratification decrease the velocity. An increase in the thermal stratification parameter(S) leads to a decrease in the convective potential efficiency between the hot plate and the surrounding fluid. The reduction in the buoyancy force consequently leads to a decrease in the flow velocity. As seen in Figure 3, a rise in Gc results in a rise in velocity, but a rise in Gr results in a decline in velocity. The fluid's velocity for various values of Sc and K are represented in figures 4 and 5. As Sc and K values grow, the fluid velocity falls.

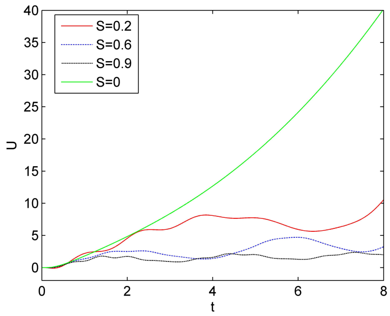

Figure 6. Effects of S on Velocity Profile against time for $Gr = 5, Gc = 5, Sc = 0.6, y = 1.6, K = 0.2$

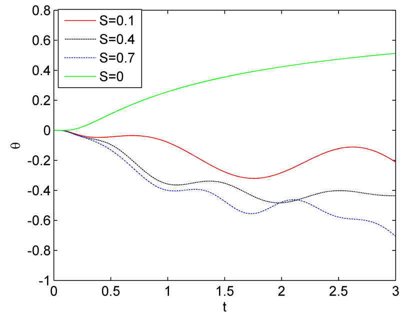

Figure 7. Effects of S on Temperature Profile against time for $Gr = 5, Gc = 5, Sc = 0.6, y = 1.6, K = 0.2$

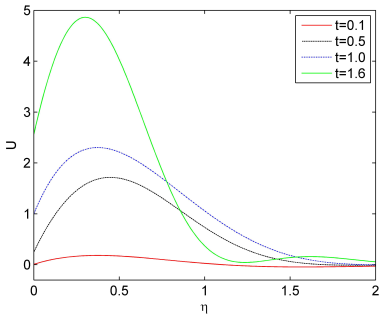

Figure 8. Velocity Profile at different time for $Gr = 5, Gc = 5, S = 0.4, Sc = 0.6, K = 0.2$

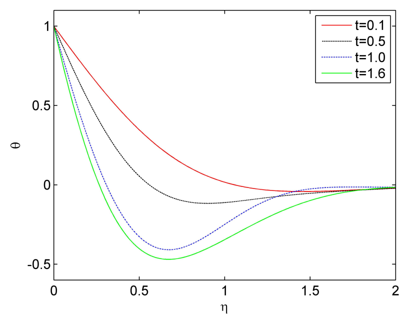

Figure 9. Temperature Profile at different time for $Gr = 5, Gc = 5, S = 0.4, Sc = 0.6, K = 0.2$

The impact of thermal stratification (S) on fluid velocity and temperature are plotted against time in Figures 6 and 7. When there is no stratification, the velocity increases indefinitely with time; but, when stratification exists, the velocity progressively approaches a stable state. The present research is more realistic than earlier ones without stratification because it applies thermal stratification, which lowers velocity and temperature in comparison to the classical scenario ($S = 0$). The figures 8 and 9 represent time-varying velocity and temperature characteristics. We have seen that the velocity rises with time and falls to zero as the distance from the plate increases.On the other hand, Temperatures decrease with time and eventually attain absolute zero as one moves away from the plate.

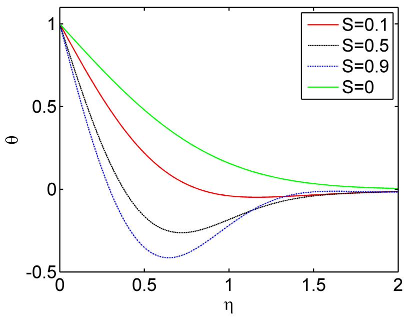

Figure 10. Effects of S on Temperature Profile for $Gr = 5, Gc = 5, , Sc = 0.6, t = 0.6, K = 0.2$

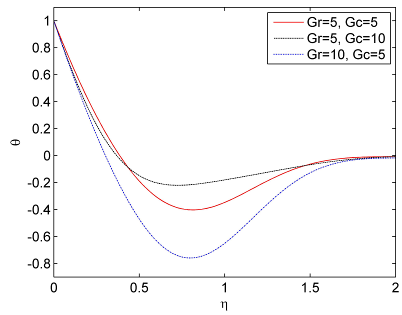

Figure 11. Effects of Gr and Gc on Temperature Profile for $S = 0.5, Sc = 0.6, t = 1.1, K = 0.2$

The impact of thermal stratification (S) on the temperature distribution is seen in Fig. 10. The presence of thermal stratification(S) will result in a decrease in the temperature gradient between the heated plate and the surrounding fluid. Therefore, the thermal boundary layer experiences an increase in thickness, resulting in a decrease in temperature. As the value of thermal stratification parameters (S) increases, it has been observed that the temperature drops. The impacts of Gr, Gc, Sc, and K are displayed in 11, 12, and 13, respectively. The temperature decreases as the value of Gr, Sc, and K is reduced, while the value of Gc is increased. The results of thermal stratification (S) on skin friction and Nusselt number are presented in Fig. 14 and 15, respectively.

They increase in presence of thermal stratification (S) compared to special case with no stratification. Thermal Stratification (S) contributes to an increase in the oscillation of the Nusselt number.

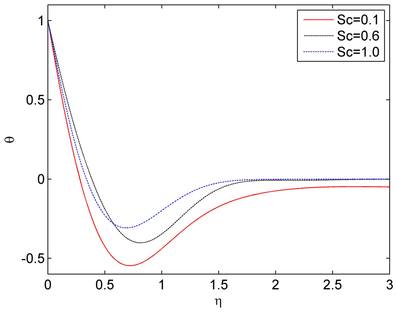

Figure 12. Effects of Sc on Concentration Profile for $Gr = 5, Gc = 5, S = 0.5, t = 1.1, K = 0.2$

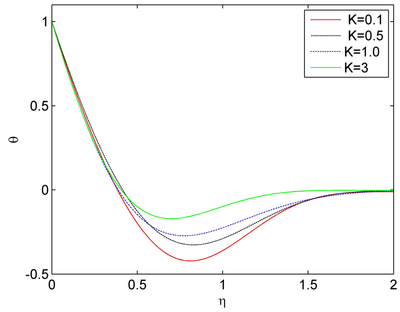

Figure 13. Effects of K on Temperature Profile for $Gr = 5, Gc = 5, S = 0.5, Sc = 0.6, t = 1.1$

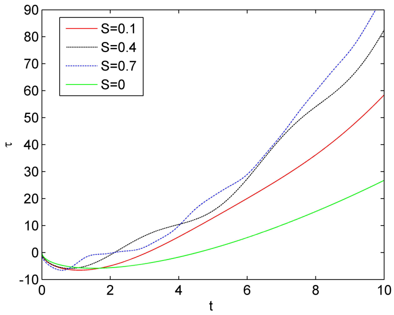

Figure 14. Effects of S on Skin friction for $Gr = 5, Gc = 5, Sc = 0.1, K = 0.2$

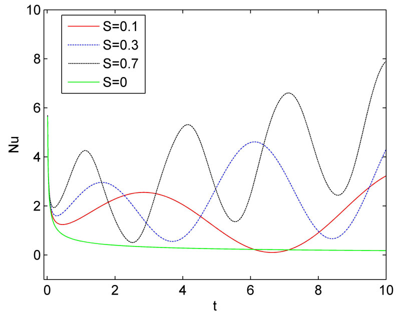

Figure 15. Effects of S on Nusselt Number for $Gr = 5, Gc = 5, Sc = 0.1, K = 0.2$

7. CONCLUSION

We looked at the effect of thermal stratification (S) on the parabolic flow through an infinite vertical plate when a chemical reaction is present. The outcomes of the current investigation have been compared with those of the classic case, which assumes that there is no stratification. The velocity of the fluid reduces as the values of S, K, and Gr grows, but it increases as the value of Gc grows. The use of thermal stratification, which reduces velocity and temperature compared to the classical situation ($S = 0$), makes this study more applicable than earlier ones. Temperature drops as K, Sc, and Gc go down; it rises when S, Gr, and time go up. The temperature is highest at the plate and then drops till it becomes zero farther away. The existence of stratification raises both the skin friction and the Nusselt number in contrast to a situation with no stratification. Furthermore, thermal Stratification (S) causes the Nusselt number to oscillate more frequently.

ORCID

Rupam Shankar Nath, https://orcid.org/0009-0002-2352-0538; **Rudra Kanta Deka**, https://orcid.org/0009-0007-1573-4890; **Himangshu Kumar**, https://orcid.org/0009-0008-9784-650X

REFERENCES

[1] R. Muthucumaraswamy, and E. Geetha, "Effects of parabolic motion on an isothermal vertical plate with constant mass flux," Ain Shams Engineering Journal, **5**(4), 1317–1323 (2014). https://doi.org/10.1016/j.asej.2014.05.011

[2] R. Muthucumaraswamy, and P. Sivakumar, "Hydro magnetic effects on parabolic flow past an infinite isothermal vertical plate with variable mass diffusion in the presence of thermal radiation and chemical reaction," International Journal of Recent Technology and Engineering, **4**(2), 5–10 (2015).

[3] A. Selvaraj, S.D. Jose, R. Muthucumaraswamy, and S. Karthikeyan, "Mhd-parabolic flow past an accelerated isothermal vertical plate with heat and mass diffusion in the presence of rotation," Materials Today: Proceedings, **46**, 3546–3549 (2021). https://doi.org/10.1016/j.matpr.2020.12.499

[4] E. Magyari, I. Pop, and B. Keller, "Unsteady free convection along an infinite vertical flat plate embedded in a stably stratified fluid-saturated porous medium," Transport in porous media, **62**, 233–249 (2006). https://doi.org/10.1007/s11242-005-1292-6

[5] A. Bhattacharya, and R.K. Deka, "Theoretical study of chemical reaction effects on vertical oscillating plate immersed in a stably stratified fluid," Research Journal of Applied Sciences, Engineering and Technology, **3**(9), 887–898 (2011). https://maxwellsci.com/print/rjaset/v3-887-898.pdf

[6] A. Shapiro, and E. Fedorovich, "Unsteady convectively driven flow along a vertical plate immersed in a stably stratified fluid," Journal of Fluid Mechanics, **498**, 333–352 (2004). https://doi.org/10.1017/S0022112003006803

[7] J.S. Park, and J.M. Hyun, "Technical note transient behavior of vertical buoyancy layer in a stratified fluid," International journal of heat and mass transfer, **41**(24), 4393–4397 (1998). https://doi.org/10.1016/S0017-9310(98)00175-6

[8] J.S. Park, "Transient buoyant flows of a stratified fluid in a vertical channel," KSME International Journal, 15, 656–664 (2001). https://doi.org/10.1007/BF03184382

[9] U.N. Das, R. Deka, and V.M. Soundalgekar, "Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction," Forschung im Ingenieurwesen, **60**(10), 284–287 (1994). https://doi.org/10.1007/BF02601318

[10] F.M.N. El-Fayez, "Effects of chemical reaction on the unsteady free convection flow past an infinite vertical permeable moving plate with variable temperature," Journal of surface engineered materials and advanced technology, **2**(2), 100-109 (2012). http://dx.doi.org/10.4236/jsemat.2012.22016

[11] B.P. Reddy, and J. Peter, "Effects of chemical reaction on mhd flow past an impulsively started infinite vertical plate with variable temperature and mass diffusion in the presence of hall current," Journal of the Serbian Society for Computational Mechanics, **13**(1), 92–108 (2019). http://doi.org/10.24874/jsscm.2019.13.01.06

[12] M.A. Mansour, N.F. El-Anssary, and A.M. Aly, "Effects of chemical reaction and thermal stratification on mhd free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering soret and dufour numbers," Chemical engineering journal, **145**(2), 340–345 (2008). https://doi.org/10.1016/j.cej.2008.08.016

[13] R. Kandasamy, K. Periasamy, and K.K.S. Prabhu, "Chemical reaction, heat and mass transfer on mhd flow over a vertical stretching surface with heat source and thermal stratification effects," International Journal of Heat and Mass Transfer, **48**(21-22), 4557–4561 (2005). https://doi.org/10.1016/j.ijheatmasstransfer.2005.05.006

[14] A.M. Megahed, and W. Abbas, "Non-newtonian cross fluid flow through a porous medium with regard to the effect of chemical reaction and thermal stratification phenomenon," Case Studies in Thermal Engineering, **29**, 101715 (2022). https://doi.org/10.1016/j.csite.2021.101715

[15] R.S. Nath, and R.K. Deka, "The effects of thermal stratification on flow past an infinite vertical plate in presence of chemical reaction," East European Journal of Physics, **3**, 23–232 (2023). https://doi.org/10.26565/2312-4334-2023-3-19

[16] N. Kalita, R.K. Deka, and R.S. Nath, "Unsteady flow past an accelerated vertical plate with variable temperature in presence of thermal stratification and chemical reaction," East European Journal of Physics, **3**, 441–450 (2023). https://doi.org/10.26565/2312-4334-2023-3-49

[17] K. Rafique, Z. Mahmood, U. Khan, S.M. Eldin, M. Oreijah, K. Guedri, and H.A. El-Wahed Khalifa, "Investigation of thermal stratification with velocity slip and variable viscosity on MHD flow of Al_2O_3-Cu-TiO_2/H_2O nanofluid over disk," Case Studies in Thermal Engineering, **49**, 103292 (2023). https://doi.org/10.1016/j.csite.2023.103292

[18] M. Abramowitz, and I.A. Stegun, editors, *Handbook of mathematical functions with Formulas, Graphs, and Mathematical Tables*, Ninth Printing, (Dover Publications Inc., NY, 1970).

[19] R.B. Hetnarski, "An algorithm for generating some inverse laplace transforms of exponential form," Zeitschrift f¨ur angewandte Mathematik und Physik ZAMP, **26**, 249–253 (1975). https://doi.org/10.1007/BF01591514

ВПЛИВ ТЕРМІЧНОЇ СТРАТИФІКАЦІЇ НА НЕСТАЦІОНАРНИЙ ПАРАБОЛІЧНИЙ ПОТІК ПОВЗ НЕСКІНЧЕННУ ВЕРТИКАЛЬНУ ПЛАСТИНУ З ХІМІЧНОЮ РЕАКЦІЄЮ

Рупам Шанкар Нат, Рудра Канта Дека, Хімангшу Кумар
Факультет математики, Університет Гаухаті, Гувахаті-781014, Ассам, Індія

У цій дослідницькій статті досліджується вплив термічної стратифікації на нестаціонарний параболічний потік повз нескінченну вертикальну пластину з хімічною реакцією. Використовуючи метод перетворення Лапласа, отримані аналітичні рішення для моделювання фізичного процесу потоку. Дослідження розглядає вплив термічної стратифікації на поле течії, а також вплив хімічної реакції на поле швидкості та температури. Результати випадку стратифікації потім порівнюються з випадком відсутності стратифікації подібного поля потоку. Результати цього дослідження можуть бути використані для покращення розуміння нестаціонарного параболічного потоку в теплових стратифікованих середовищ і надання цінного розуміння впливу хімічних реакцій на температурне поле.

Ключові слова: *термічна стратифікація; хімічна реакція; параболічний потік; вертикальна плита*