

GENERATION OF O-MODE IN THE PRESENCE OF ION-CYCLOTRON DRIFT WAVE TURBULENCE IN A NONUNIFORM PLASMA[†]

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This study aims to investigate the effect of ion-cyclotron drift wave turbulence on the generation of ordinary mode (O-mode) in the presence of density and temperature gradients. For this, a Vlasov plasma is considered where a resonant, and non-resonant modes are considered to be present in the system. Here, the non-resonant mode is a perturbation caused by O-mode in a quasi-steady state of plasma, which is characterised by the presence of low frequency ion-cyclotron resonant mode waves. The interaction between these waves is studied by the Vlasov-Maxwell set of equations and a modified Maxwellian-type distribution function for particles that includes the external force field \vec{F} and associated density and temperature gradient parameters λ_j . The study analyses the growth rate of electromagnetic O-mode at the expense of ion-cyclotron drift wave energy and the associated impact of the density and temperature gradient. This model uses the linear response theory on weakly turbulent plasma, evaluates the responses due to turbulent and perturbed fields, and obtains the nonlinear dispersion relation for O-mode.

Keywords: *Ordinary mode; Density and temperature gradients; Drift wave turbulence; Wave-particle interaction*

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INTRODUCTION

Ordinary mode (O-mode) waves are high-frequency electromagnetic plasma waves that are observed in magnetospheric plasma, as reported by satellite observations. [1] This enhanced O-mode phenomenon is a subject of interest to many plasma physicists for the explanation of many radiation phenomena in the magnetosphere and the ionosphere, including radiation phenomena in the auroral zone. O-mode instability is a purely growing mode that was first discussed by Davidson and Wu [2] in high beta plasmas. They found that the O-mode waves propagated in the perpendicular direction of the ambient magnetic field, which could become unstable in high beta plasmas and temperature anisotropic when $T_{\parallel} > T_{\perp}$ in bi-Maxwellian distribution functions.

The generation of unstable O-mode waves in the auroral region of the Earth's magnetosphere was considered by Gurnett [3] and also by Hayes & Melrose [4]. In their study, they thoroughly considered the enhanced electromagnetic radiation in the top ionospheric regions and characterised such radiation as Auroral Kilometric Radiation (AKR). Later, in the investigations done by Mellott et al. [5], it was confirmed that the AKR is composed of X and O-mode radiations. Later, it received renewed attention owing to its potential applicability to the solar wind plasma. Ibscher, Schlickeiser, and their co-authors [6-9] examined the O-mode instability and expanded it to the low-beta plasma domain by studying a counter-streaming bi-Maxwellian model. Temperature anisotropic effects on O-mode and its instability have been reviewed [10] in magnetised non-relativistic bi-Maxwellian plasma. Based on a numerical approach [11], it has been confirmed that the unstable O-mode is possible at low beta plasma in the presence of a finite counter-stream. So, further, the authors in Ref. [12] have derived an accurate analytical marginal instability condition for O-mode, where they identified that though it has large enough counter-stream parameters, the O-mode must operate for temperature anisotropy $T_{\perp}/T_{\parallel} > 1$ even larger than unity.

In our manuscript, we have considered the nonlinear wave-particle interaction process in the presence of ion-cyclotron drift wave turbulence based on the plasma turbulence theory proposed by Nambu [13] and Tystovich [14]. Based on the non-linear wave-particle interaction mechanism known as the plasma maser effect [13,15-16], it may be possible to transfer wave energy from the low-frequency mode to the high-frequency mode. The Plasma maser effect occurs in the presence of both resonant and non-resonant modes. The resonant modes are those for which the Cherenkov resonance condition $\omega - \vec{k} \cdot \vec{v} = 0$ is satisfied, while the non-resonant waves are those for which the Cherenkov condition and the nonlinear scattering conditions are not satisfied, i.e., $\Omega - \vec{k} \cdot \vec{v} \neq 0$ and $(\Omega - \omega) - (\vec{K} - \vec{k}) \cdot \vec{v} \neq 0$. Here, ω is the frequency of the resonant wave, Ω is the frequency of the non-resonant wave, and \vec{k} and \vec{K} are the corresponding wave vectors. Though most of the studies on the plasma maser effect have been carried out considering the plasma system as homogeneous [17-19], many attempts have been made to investigate the role of the density gradient parameter in the energy up-conversion process through the plasma maser effect in inhomogeneous plasma [20, 21]. Applying the plasma-maser theory, Deka and Borgohain [1] studied the amplification of O-mode through non-linear wave-particle interaction

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in the presence of drift wave turbulence in an inhomogeneous plasma. In their study, they found that the amplification process of electromagnetic radiation is possible at the expense of drift wave turbulence in space plasma. The generation of high-frequency electromagnetic O-mode waves and low-frequency ion sound waves in the upper ionosphere region has been mentioned in several studies [5, 22, 23]. Several studies have mentioned the generation of high-frequency electromagnetic O-mode waves and low-frequency ion sound waves in the upper ionosphere region. Previous works have mostly focused on investigating the impact of density gradients on wave amplification through the nonlinear wave-particle interaction associated with drift waves. However, Gogoi [24] discussed the generation of wave energy up-conversion of electromagnetic O-mode waves through plasma maser instability in inhomogeneous ionospheric plasma in the presence of electrostatic ion sound waves. In practical situations like tokamak plasmas [25-27], particle drift is often caused by temperature gradients, which may lead to the creation of drift wave turbulence [28-30].

Motley and D'Angelo first discovered electrostatic ion cyclotron waves in a Q-machine, which spread outward across the magnetic field and were stimulated by a current drawn to a small auxiliary electrode. The solar wind has been observed by spacecraft to contain ion-cyclotron waves (ICWs) at various distances (0.3 - 1 AU) from the Sun [31].

Our investigation focused on the generation of ordinary mode waves in the presence of density and temperature gradients. We studied the plasma maser effect in the presence of an ion cyclotron drift wave and analysed the energy exchange process between waves. In this case, the resonant wave is the ion cyclotron drift wave, while the non-resonant wave is the ordinary mode wave. Through resonant interaction, plasma particles can transfer their energy to non-resonant waves via a modulated field, resulting in energy up-conversion from resonant mode to non-resonant mode. In our study, we have also considered a nonlinear process that enables the exchange of wave energy among participating waves, even with significant frequency differences [15, 31, 32].

Plasma nonuniformity, associated with gradients in density and temperature, generates drift motions and supports ion cyclotron drift wave turbulence. To investigate the instability of the O-mode, we employed the Vlasov-Maxwell system of equations and a modified Maxwellian-type particle distribution function. From the nonlinear dispersion relation of the O-mode, the expression of the growth rate is obtained, and by using satellite observational data [33], we have estimated the impact of gradient parameters on the growth of the O-mode.

The paper is organised as follows: In Section 2, the geometrical and mathematical formulations are given. In Section 3, the nonlinear dispersion relation of the ordinary mode wave is discussed. In Section 4, the growth rate of the electromagnetic O-mode is presented, and in Section 5, the discussions of the work along with the conclusion are given.

MATHEMATICAL FORMULATION

We consider an inhomogeneous plasma in the presence of drift wave turbulence. In order to describe this system, we use a particle distribution function [32] involving density and temperature gradients caused by an external force in a uniform magnetic field \vec{B}_0 along the z-axis. There exists a turbulence field characterised by a wave vector $\vec{k} = (0, 0, k_{\parallel})$. We introduce an electromagnetic ordinary mode as a perturbation to the system with a propagation vector $\vec{K} = (K_{\perp}, 0, 0)$. The density gradient $\vec{\nabla}n(y)$ and temperature gradient $\vec{\nabla}T(y)$ are along the y-direction, which is also the direction of the uniform effective force field \vec{F} . This force supports $\vec{F} \times \vec{B}$ drift and is perpendicular to the embedded magnetic field \vec{B}_0 . The geometry of the model is depicted in Figure 1.

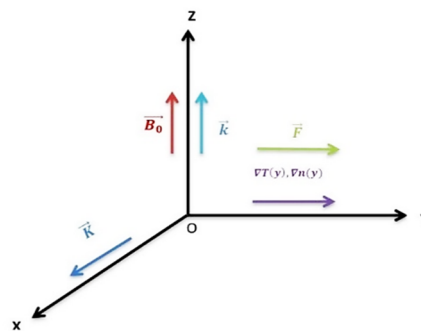


Figure 1. Geometry of the Model

In the absence of collisions, the distribution function for particles is considered Maxwellian.

$$f_j(\vec{v}) = \left(\frac{m_j}{2\pi T_j}\right)^{3/2} \exp\left(-\frac{m_j v^2}{2T_j}\right), \quad (j = e, i).$$

Here, we use the subscript $j = e$ for the electrons and $j = i$ for the ion.

In our system, field \vec{B} and \vec{F} are time invariant, so the Hamiltonian H is a constant of motion given by

$$H = \frac{1}{2} m_j \vec{v}^2 - Fy.$$

The system is translationally invariant in the x and z directions, so canonical momentum

$$p_x = m_j v_x + \frac{e_j}{c} yB,$$

$$p_z = m_j v_z.$$

where e_j is the charge of the plasma particle.

The quasi Maxwellian distribution will be

$$f_{0j}(Y, H) = \frac{N(Y)}{\left(\frac{2\pi T_j(Y)}{m_j}\right)^{3/2}} \exp\left(-\frac{H}{T_j(Y)}\right), \tag{1}$$

where $Y' = y + \frac{v_x}{\Omega}$.

For $\lambda_L/L \ll 1$, where L is the characteristic length and λ_L is the Larmor radius, we can expand (1) to the first order in the Larmor radius as

$$f_{0j}(Y, H) = f_{0j}(y, H) + \left(y + \frac{v_x}{\Omega_j}\right) \frac{\partial f_{0j}(x, H)}{\partial x},$$

where $\frac{\partial f_{0j}(x, H)}{\partial x} = \left[\frac{1}{N} \frac{dN}{dy} + \frac{dT_j}{dy} \frac{\partial}{\partial T_j} - \frac{F}{T_{0j}} \right] f_{0j}$,

and we can have a particle distribution function [32] for (1) as

$$f_{0j}(T_j, y, \vec{v}) = \left(\frac{m_j}{2\pi T_j}\right)^{3/2} \left[1 + \lambda_j \left(y + \frac{v_x}{\Omega_j}\right) \right] \exp\left[-\left(\frac{m_j \vec{v}^2}{2T_j} - \frac{Fy}{T_j}\right)\right]. \tag{2}$$

The distribution function for the guiding centre is denoted by f_{0j} . $\Omega_j = e_j B_0 / m_j c$ denotes the cyclotron frequency of the plasma particles. The parameter λ_j is associated with gradients in density and temperature. At $y = 0$, the value of λ_j can be computed using Eq. (2).

$$\lambda_j = \left[\left(\frac{\partial}{\partial T_j} \frac{dT_j}{dy} \right) + \left(\frac{1}{f_j} \frac{df_j}{dy} \right) \right]_{y=0} - \frac{F}{T_{0j}}. \tag{3}$$

$\vec{K} = (K_\perp, 0, 0)$ is the propagation vector of the ordinary mode wave; $\vec{k} = (0, 0, k_\parallel)$ is the propagation vector of the ion-cyclotron drift wave; \vec{B}_0 is the magnetic field along the positive z-axis; $\vec{\nabla}n(y)$ is the density gradient along the y-direction; $\vec{\nabla}T(y)$ is the temperature gradient along the y-direction; F is the uniform effective force field along the y-direction.

The interaction of high-frequency electromagnetic ordinary mode with low-frequency ion-cyclotron drift wave turbulence is governed by the Vlasov-Maxwell system of equations.

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} + \left\{ \frac{e_j}{m_j} \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) - \frac{\vec{F}}{m_j} \right\} \cdot \frac{\partial}{\partial \vec{v}} \right] F_{0j}(\vec{r}, \vec{v}, t) = 0, \tag{4}$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \tag{5}$$

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{J}, \tag{6}$$

$$\bar{J} = \sum_{j=e,i} e_j n_j \int \bar{v} f_{0j}(\bar{r}, \bar{v}, t) d\bar{v}, \tag{7}$$

$$\nabla \cdot \bar{E} = \sum_{j=e,i} 4\pi e_j n_j \int f_{0j}(\bar{r}, \bar{v}, t) d\bar{v}. \tag{8}$$

The unperturbed particle distribution function and fields are described by the linear response theory of a turbulent plasma [32].

$$F_{0j} = f_{0j} + \varepsilon f_{1j} + \varepsilon^2 f_{2j}, \tag{9}$$

and

$$\bar{E}_{0l} = \varepsilon \bar{E}_l + \varepsilon^2 \bar{E}_2, \tag{10}$$

where ε is a small parameter associated with the ion cyclotron drift wave turbulent field $\bar{E}_l = (0, 0, E_{ll})$, f_{0j} is the space and time averaged parts, f_{1j} and f_{2j} are fluctuating parts of the distribution function, \bar{E}_2 is the second order electric field. On putting these in Eq. (4)

$$\left[\frac{\partial}{\partial t} + \bar{v} \cdot \frac{\partial}{\partial \bar{r}} + \left\{ \frac{e_j}{m_j} \left(\varepsilon \bar{E}_l + \varepsilon^2 \bar{E}_2 + \frac{\bar{v} \times \bar{B}}{c} \right) - \frac{\bar{F}}{m_j} \right\} \frac{\partial}{\partial \bar{v}} \right] [f_{0j} + \varepsilon f_{1j} + \varepsilon^2 f_{2j}] = 0.$$

Now to the order of ε ,

$$\left[\frac{\partial}{\partial t} + \bar{v} \cdot \frac{\partial}{\partial \bar{r}} + \left\{ \frac{e_j}{m_j} \left(\varepsilon \bar{E}_l + \varepsilon^2 \bar{E}_2 + \frac{\bar{v} \times \bar{B}}{c} \right) - \frac{\bar{F}}{m_j} \right\} \frac{\partial}{\partial \bar{v}} \right] f_{1j}(\bar{r}, \bar{v}, t) = \frac{e_j}{m_j} \left(\bar{E}_l \cdot \frac{\partial}{\partial \bar{v}} f_{0j} \right). \tag{11}$$

Using the Fourier transform of the form

$$A(\bar{r}, \bar{v}, t) = \sum_{\bar{k}, \omega} A(\bar{k}, \omega, \bar{v}) \exp[i(\bar{k} \cdot \bar{r} - \omega t)], \tag{12}$$

and integrating along the unperturbed orbit, the fluctuating part of the distribution function due to the low-frequency turbulence field f_{1j} , is obtained from Eq. (11).

$$f_{1j}(\bar{k}, \omega) = \int_{-\infty}^0 \frac{e_j}{m_j} \bar{E}_l \cdot \frac{\partial}{\partial \bar{v}} f_{0j} \exp\left[i\left\{\bar{k} \cdot (\bar{r}' - \bar{r}) - \omega \tau\right\}\right] d\tau, \tag{13}$$

$$f_{1j}(\bar{k}, \omega) = \frac{ie_j}{m_j} \frac{E_{ll}}{\omega - k_{ll} v_{ll} + i \cdot 0^+} \frac{\partial}{\partial v_{ll}} f_{0j};$$

In order to attain a quasi-steady state, we introduce a perturbation to the test high-frequency ordinary mode by applying an electric field $\delta \bar{E}_{hj} = (0, 0, \delta E_{hj})$ with a propagating vector $\bar{K} = (K_{\perp}, 0, 0)$, a magnetic field $\delta \bar{B}_{hj} = (0, \delta B_{hj}, 0)$, and a frequency Ω_j . This results in a perturbed electric field, magnetic field, and particle distribution function, given by

$$\delta \bar{E} = \mu \delta \bar{E}_{hj} + \mu \varepsilon \delta \bar{E}_{lhj} + \mu \varepsilon^2 \Delta \bar{E},$$

$$\delta \bar{B} = \mu \delta \bar{B}_{hj} + \mu \varepsilon \delta \bar{B}_{lhj} + \mu \varepsilon^2 \Delta \bar{B},$$

$$\delta f = \mu \delta f_{hj} + \mu \varepsilon \delta f_{lhj} + \mu \varepsilon^2 \Delta f_j.$$

where the variables $\delta \bar{E}_{lhj}, \Delta \bar{E}, \delta \bar{B}_{lhj}, \Delta \bar{B}, \delta f_{lhj}$, and Δf_j represent modulation fields, while δf_{hj} is the fluctuating part caused by the high-frequency ordinary mode. δf_{lhj} and Δf_j refer to the particle distribution function associated with the modulation field, and μ is the smallness parameter for the perturbed field, which is much smaller than ε .

Then, putting these in Vlasov Eq. (4), resulting in an expression up to the order of $\mu, \mu \varepsilon, \mu \varepsilon^2$, we get,

$$P \delta f_{hj} = \frac{e_j}{m_j} \left(\delta \bar{E}_{hj} + \frac{\bar{v} \times \delta \bar{B}_{hj}}{c} \right) \cdot \frac{\partial f_{0j}}{\partial \bar{v}}, \tag{14}$$

$$P \delta f_{lhj} = \frac{e_j}{m_j} \delta \bar{E}_{hj} \cdot \frac{\partial f_{1j}}{\partial \bar{v}} + \frac{e_j}{m_j} \frac{\bar{v} \times \delta \bar{B}_{hj}}{c} \cdot \frac{\partial f_{1j}}{\partial \bar{v}} + \frac{e_j}{m_j} \delta \bar{E}_{lhj} \cdot \frac{\partial f_{0j}}{\partial \bar{v}} + \frac{e_j}{m_j} \bar{E}_l \cdot \frac{\partial}{\partial \bar{v}} \delta f_{hj} + \frac{e_j}{m_j} \frac{\bar{v} \times \bar{B}_l}{c} \cdot \frac{\partial}{\partial \bar{v}} \delta f_{hj} + \frac{e_j}{m_j} \frac{\bar{v} \times \delta \bar{B}_{lhj}}{c} \cdot \frac{\partial f_{0j}}{\partial \bar{v}}, \tag{15}$$

$$P\Delta f_j = \frac{e_j}{m_j} \left[\delta \bar{E}_{l_{hj}} \frac{\partial f_{1j}}{\partial \bar{v}} + \frac{\bar{v} \times \delta \bar{B}_{l_{hj}}}{c} \cdot \frac{\partial f_{1j}}{\partial \bar{v}} + \bar{E}_l \frac{\partial}{\partial \bar{v}} \delta f_{l_{hj}} + \frac{\bar{v} \times \bar{B}_l}{c} \cdot \frac{\partial}{\partial \bar{v}} \delta f_{l_{hj}} \right], \quad (16)$$

where $P = \left[\frac{\partial}{\partial t} + \bar{v} \cdot \frac{\partial}{\partial \bar{r}} - \left\{ \frac{e_j}{m_j} \left(\bar{E} + \frac{\bar{v} \times \bar{B}}{c} \right) - \frac{\bar{F}}{m_j} \right\} \frac{\partial}{\partial \bar{v}} \right]$.

We solve this differential equation for the fluctuating parts of the equation $\delta f_j, \delta f_{l_{hj}}, \Delta f_j$ over the electron trajectories using the method of characteristics, commonly known as "integration over an unperturbed orbit" [34]. Using the Fourier transform of Eq. (12) and integrating along the unperturbed orbits, we evaluate the various perturbed distribution functions from Eqs. (14) to (16) to obtain the nonlinear dielectric function of the electromagnetic ordinary mode wave of frequency Ω_j in the presence of ion-cyclotron drift wave turbulence.

Now, we calculate the modulated field $\delta E_{l_{hj}}(\bar{K} - \bar{k}, \Omega - \omega)$ using Maxwell's equation.

$$\nabla \times \delta \bar{B}_{l_{hj}} = \frac{1}{c} \frac{\partial}{\partial t} \delta \bar{E}_{l_{hj}} + \frac{4\pi i}{c} \bar{J}, \quad (17)$$

$$\bar{J} = \sum_{j=e,i} e_j n_j \int \bar{v} \delta f_{l_{hj}} d\bar{v}. \quad (18)$$

Therefore, we have

$$\nabla \times \delta \bar{B}_{l_{hj}} = \frac{1}{c} \frac{\partial}{\partial t} \delta \bar{E}_{l_{hj}} + \frac{4\pi i}{c} \sum_{j=e,i} e_j n_j \int \bar{v} \delta f_{l_{hj}} d\bar{v}, \quad (19)$$

$$\delta \bar{E}_{l_{hj}} \times \frac{c^2 |\bar{K} - \bar{k}|^2 - (\Omega - \omega)^2}{(\Omega - \omega)c} = \frac{4\pi i e_j n_j}{c} \int \bar{v} \delta f_{l_{hj}} d\bar{v}, \quad (20)$$

$$\delta E_{l_{hj}} = \frac{4\pi i e_j n_j (\Omega - \omega)}{c^2 |\bar{K} - \bar{k}|^2 - (\Omega - \omega)^2} \int v_{\parallel} \delta f_{l_{hj}} d\bar{v}, \quad (21)$$

$$\begin{aligned} \delta E_{l_{hj}} = & - \frac{i\omega_{pj}^2 (\Omega - \omega)}{R \left\{ c^2 |\bar{K} - \bar{k}|^2 - (\Omega - \omega)^2 \right\}} \left(\frac{e_j}{m_j} \right) E_l \delta E_{l_{hj}} \times \int v_{\parallel} \left[\left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega} \right) \frac{\partial}{\partial v_{\perp}} + \frac{k_{\parallel} v_{\perp}}{\omega} \frac{\partial}{\partial v_{\parallel}} \right] \times \\ & \left[\frac{K_{\perp}}{\Omega} \frac{m_j}{T_j K_{\perp}} \left\{ 1 + \left(\Omega - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j} \right) Q_{s,t} \right\} f_{0j} - \left(1 - \frac{K_{\perp} v_{\perp}}{\Omega} \cos \theta \right) \frac{\partial f_{0j}}{\partial v_{\parallel}} Q_{s,t} \right] T_{p,q} \\ & - \left[\frac{K_{\perp}}{\Omega} \frac{m_j}{T_j K_{\perp}} \left\{ 1 + \left(\Omega - \omega + k_{\parallel} v_{\parallel} - \frac{\lambda_j T_j K_{\perp}}{m \Omega_j} \right) T_{p,q} \right\} + \frac{K_{\perp} v_{\perp}}{\Omega} \cos \theta \frac{\partial}{\partial v_{\parallel}} (T_{p,q}) \right] d\bar{v}, \end{aligned} \quad (22)$$

where

$$\begin{aligned} R = & 1 + \frac{\omega_{pj}^2 (\Omega - \omega)}{\left\{ c^2 |\bar{K} - \bar{k}|^2 - (\Omega - \omega)^2 \right\}} \int v_{\parallel} \left[\left(\frac{K_{\perp}}{|\bar{K} - \bar{k}|} - \frac{|\bar{K} - \bar{k}|}{\Omega - \omega} v_{\parallel} \right) \frac{m_j}{T_j K_{\perp}} \right. \\ & \left. \left\{ 1 + \left(\Omega + k_{\parallel} v_{\parallel} - \omega - \frac{\lambda_j T_j K_{\perp}}{m \Omega_j} \right) \times T_{p,q} \right\} f_{0j} - \left(\frac{|\bar{K} - \bar{k}|}{\Omega - \omega} v_{\perp} \cos \theta + \frac{k_{\parallel}}{|\bar{K} - \bar{k}|} \right) \times \frac{\partial f_{0j}}{\partial v_{\parallel}} T_{p,q} \right] d\bar{v}, \end{aligned} \quad (23)$$

$$Q_{s,t} = \sum_{s,t} \frac{J_s(\alpha) J_t(\alpha) \exp[i(t-s)]}{s \Omega_j - \Omega + K_{\perp} v_F + \omega - k_{\parallel} v_{\parallel}}, \quad \alpha = \frac{K_{\perp} v_F}{\Omega_j},$$

and

$$T_{p,q} = \sum_{p,q} \frac{J_p(\alpha) J_q(\alpha) \exp[i(q-p)]}{p \Omega_j - \Omega + K_{\perp} v_F + \omega - k_{\parallel} v_{\parallel}}, \quad \alpha = \frac{K_{\perp} v_F}{\Omega_j}.$$

Non-linear Dispersion Relation of Ordinary Mode Wave

The nonlinear dispersion relation for electromagnetic ordinary mode waves can be obtained from Maxwell's equations.

$$\nabla \times \delta \vec{B}_{hj} = \frac{1}{c} \frac{\partial}{\partial t} \delta \vec{E}_{hj} + \frac{4\pi}{c} \vec{J}. \tag{24}$$

$$J = \sum_{j=e,i} e_j n_j \int v_{\parallel} (\delta f_{hj} + \Delta f_j) d\vec{v}. \tag{25}$$

$$\delta E_{hj} = \sum_{j=e,i} \frac{4\pi e_j n_j (\Omega - \omega)}{c^2 K_{\perp}^2 - \Omega^2} \int v_{\parallel} (\delta f_{hj} + \Delta f_j) d\vec{v}. \tag{26}$$

After simplification, we get,

$$\delta E_{hj} \epsilon_h(\vec{K}, \Omega) = 0. \tag{27}$$

Here, $\epsilon_h(\vec{K}, \Omega)$ represents the nonlinear dispersion relation of the ordinary mode wave, which is described by

$$\epsilon_h(\vec{K}, \Omega) = \epsilon_0(\vec{K}, \Omega) + \epsilon_d(\vec{K}, \Omega) + \epsilon_p(\vec{K}, \Omega). \tag{28}$$

where $\epsilon_0(\vec{K}, \Omega)$ is the linear part, $\epsilon_d(\vec{K}, \Omega)$ is the direct coupling part, and $\epsilon_p(\vec{K}, \Omega)$ is the polarisation coupling part. The expressions of these parts are given by,

$$\epsilon_0(\vec{K}, \Omega) = 1 + \frac{\omega_{pj}^2 (\Omega - \omega)}{c^2 K_{\perp}^2 - \Omega^2} \int v_{\parallel} \left[\frac{cK_{\perp}}{\Omega} \frac{m_j}{T_j K_{\perp}} \left\{ 1 + \left(\Omega - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j \Omega} \right) Q_{s,t} \right\} f_{0j} - \left(1 - \frac{K_{\perp} v_{\perp}}{\Omega_j} \cos \theta \right) \frac{\partial f_{0j}}{\partial v_{\parallel}} Q_{s,t} \right] d\vec{v}. \tag{29}$$

$$\begin{aligned} \epsilon_d(\vec{K}, \Omega) &= \frac{\omega_{pj}^2 (\Omega - \omega)}{c^2 K_{\perp}^2 - \Omega^2} \left(\frac{e_j}{m_j} \right)^2 |E_i|^2 \int v_{\parallel} \left[\left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega} \right) \frac{\partial}{\partial v_{\perp}} + \frac{k_{\parallel} v_{\perp}}{\omega} \right] Q_{s,t} \times \left[\frac{K_{\perp}}{\Omega} \frac{m_j}{T_j K_{\perp}} \left\{ 1 + \left(\Omega - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j \Omega} \right) Q_{s,t} \right\} f_{0j} - \right. \\ &\left. \left(1 - \frac{K_{\perp} v_{\perp}}{\Omega} \cos \theta \right) \frac{\partial f_{0j}}{\partial v_{\parallel}} Q_{s,t} \right] T_{p,q} - \left[\frac{K_{\perp}}{\Omega} \frac{m_j}{T_j K_{\perp}} \left\{ 1 + \left(\Omega - \omega + k_{\parallel} v_{\parallel} - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j \Omega} \right) T_{p,q} \right\} \right. \\ &\left. + \frac{K_{\perp} v_{\perp}}{\Omega} \cos \theta \frac{\partial}{\partial v_{\parallel}} (T_{p,q}) \left(\frac{1}{(k_{\parallel} v_{\parallel} - \omega + i.0)} \right) \left[\left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega} \right) \frac{\partial f_{0j}}{\partial v_{\perp}} + \frac{k_{\parallel} v_{\perp}}{\omega} \frac{\partial f_{0j}}{\partial v_{\parallel}} \right] \right] d\vec{v}. \end{aligned} \tag{30}$$

Since, the expression $\epsilon_p(\vec{K}, \Omega)$ is very lengthy, we may write it as follows:

$$\epsilon_p(\vec{K}, \Omega) = \frac{\omega_{pj}^4 (\Omega - \omega)^2}{R(c^2 K_{\perp}^2 - \Omega^2) \left\{ c^2 |\vec{K} - \vec{k}|^2 - (\Omega - \omega)^2 \right\}} \left(\frac{e_j}{m_j} \right)^2 |E_i|^2 (A + B)(C + D). \tag{31}$$

where A, B, C, and D are obtained as:

$$\begin{aligned} A &= \int v_{\parallel} \left[\left[\left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega} \right) \frac{\partial}{\partial v_{\parallel}} + \frac{k_{\parallel} v_{\perp}}{\omega} \frac{\partial}{\partial v_{\parallel}} \right] Q_{s,t} \times \left[\left(\frac{K_{\perp}}{|\vec{K} - \vec{k}|} - \frac{\vec{K} - \vec{k}}{\Omega - \omega} v_{\parallel} \right) \frac{m_j}{T_j K_{\perp}} \left\{ 1 + \left(\Omega + k_{\parallel} v_{\parallel} - \omega - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j \Omega} \right) \times T_{p,q} \right\} f_{0j} - \right. \right. \\ &\left. \left. \left(\frac{|\vec{K} - \vec{k}|}{\Omega - \omega} v_{\perp} \cos \theta + \frac{k_{\parallel}}{|\vec{K} - \vec{k}|} \right) \times \frac{\partial f_{0j}}{\partial v_{\parallel}} T_{p,q} \right] \right] d\vec{v}. \end{aligned} \tag{32}$$

$$\begin{aligned} B &= - \int v_{\parallel} \left[\left(\frac{K_{\perp}}{|\vec{K} - \vec{k}|} - \frac{|\vec{K} - \vec{k}|}{\Omega - \omega} v_{\parallel} \right) \frac{m_j}{T_j K_{\perp}} \left\{ 1 + \left(\Omega - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j \Omega} \right) Q_{s,t} \right\} \times \left[\left(\frac{1}{k_{\parallel} v_{\parallel} - \omega + i.0} \right) \times \left[\left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega} \right) \frac{\partial f_{0j}}{\partial v_{\perp}} + \frac{k_{\parallel} v_{\perp}}{\omega} \frac{\partial f_{0j}}{\partial v_{\parallel}} \right] \right. \right. \\ &\left. \left. + \left(\frac{|\vec{K} - \vec{k}|}{\Omega - \omega} v_{\perp} \cos \theta + \frac{k_{\parallel}}{|\vec{K} - \vec{k}|} \right) \times \frac{\partial}{\partial v_{\parallel}} \left[\left(\frac{1}{k_{\parallel} v_{\parallel} - \omega + i.0} \right) \times \left[\left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega} \right) \frac{\partial f_{0j}}{\partial v_{\perp}} + \frac{k_{\parallel} v_{\perp}}{\omega} \frac{\partial f_{0j}}{\partial v_{\parallel}} \right] Q_{s,t} \right] \right] d\vec{v}. \end{aligned} \tag{33}$$

$$C = \int v_{\parallel} \left[\left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega} \right) \frac{\partial}{\partial v_{\parallel}} + \frac{k_{\parallel} v_{\perp}}{\omega} \frac{\partial}{\partial v_{\perp}} \right] \times \left[\frac{Ck_{\perp}}{\Omega} \frac{m_j}{T_j K_{\perp}} \left\{ 1 + \left(\Omega - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j \Omega} \right) \times Q_{p,q} \right\} f_{0j} - \left(1 - \frac{K_{\perp} v_{\perp}}{\Omega} \cos \theta \right) \frac{\partial f_{0j}}{\partial v_{\parallel}} Q_{s,t} \right] T_{p,q} d\vec{v}. \quad (34)$$

$$D = - \int v_{\parallel} \left[\left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega} \right) \frac{\partial f_{0j}}{\partial v_{\perp}} + \frac{k_{\parallel} v_{\perp}}{\omega} \frac{\partial f_{0j}}{\partial v_{\parallel}} \right] \times \left(\frac{1}{k_{\parallel} v_{\parallel} - \omega + i0} \right) \times \left[\frac{Ck_{\perp}}{\Omega} \frac{m_j}{T_j K_{\perp}} \left\{ 1 + \left(\Omega - \omega + k_{\parallel} v_{\parallel} - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j \Omega} \right) \times T_{p,q} \right\} + \frac{K_{\perp} v_{\perp}}{\Omega} \cos \theta \frac{\partial}{\partial v_{\parallel}} T_{p,q} \right] d\vec{v}. \quad (35)$$

Instabilities

The growth rate of the ordinary mode wave is computed using the formula:

$$\frac{\gamma_h}{\Omega} = - \left[\frac{Im \epsilon_d + Im \epsilon_h + \frac{1}{2} \left(\frac{\partial^2 \epsilon_0}{\partial \Omega \partial t} \right)}{\Omega \left(\frac{\partial \epsilon_0}{\partial \Omega} \right)} \right]_{\Omega = \Omega_j}. \quad (36)$$

As a result of the reverse absorption effect, the second term of the growth rate expression in Eq. (36) reduces to zero. Now, from Eq. (29), we estimate the linear term of the dielectric function for the ordinary mode wave, considering that the term $s = t = p = q = 1$ contributes the most to Bessel's sum for the ordinary mode wave.

$$\epsilon_0(\vec{K}, \Omega) = 1 + \frac{\omega_{pj}^2 (\Omega - \omega) \int_0^{\infty} J_1^2 \left(\frac{K_{\perp} v_{\perp}}{\Omega_j} \right) 2\pi v_{\perp} dv_{\perp} f_{0j}(v_{\perp})}{(c^2 K_{\perp}^2 - \Omega^2)(\Omega_j - \Omega + K_{\perp} v_F)} - \frac{\omega_{pj}^2 (\Omega - \omega) K_{\perp} \cos \theta \int_0^{\infty} J_1^2 \left(\frac{K_{\perp} v_{\perp}}{\Omega_j} \right) 2\pi v_{\perp}^2 f_{0j}(v_{\perp}) dv_{\perp}}{\Omega (c^2 K_{\perp}^2 - \Omega^2)(\Omega_j - \Omega + K_{\perp} v_F)}. \quad (37)$$

where v_F is the particle drift velocity defined by $v_F = -\frac{F}{m_j \Omega_j}$.

From equation (37), we obtain,

$$\begin{aligned} \Omega \frac{\partial \epsilon_0}{\partial \Omega}(\vec{K}, \Omega) &= \frac{\Omega}{(c^2 K_{\perp}^2 - \Omega^2)} \left(\frac{\omega_{pj}^2 \int_0^{\infty} J_1^2 \left(\frac{K_{\perp} v_{\perp}}{\Omega_j} \right) 2\pi v_{\perp} f_{0j}(v_{\perp})}{(\Omega_j - \Omega + \omega_F)} + \frac{\omega_{pj}^2 \Omega (\Omega - \omega) \int_0^{\infty} J_1^2 \left(\frac{K_{\perp} v_{\perp}}{\Omega_j} \right) 2\pi v_{\perp} dv_{\perp} f_{0j}(v_{\perp})}{(\Omega_j - \Omega + \omega_F)^2} \right. \\ &+ \left. \frac{2\omega_{pj}^2 \Omega (\Omega - \omega) \int_0^{\infty} J_1^2 \left(\frac{K_{\perp} v_{\perp}}{\Omega_j} \right) 2\pi v_{\perp} dv_{\perp} f_{0j}(v_{\perp})}{(c^2 K_{\perp}^2 - \Omega^2)(\Omega_j - \Omega + \omega_F)} \right) + \left(1 - \frac{K_{\perp} \cos \theta}{2\Omega} \int_0^{\infty} J_1^2 \left(\frac{K_{\perp} v_{\perp}}{\Omega_j} \right) 2\pi v_{\perp} dv_{\perp} f_{0j}(v_{\perp}) \right) + \\ &\frac{\omega_{pj}^2 \int_0^{\infty} J_1^2 \left(\frac{K_{\perp} v_{\perp}}{\Omega_j} \right) 2\pi v_{\perp} dv_{\perp} f_{0j}(v_{\perp}) \int_0^{\infty} J_1^2 \left(\frac{K_{\perp} v_{\perp}}{\Omega_j} \right) 2\pi v_{\perp}^2 dv_{\perp} f_{0j}(v_{\perp}) K_{\perp} \cos \theta (\Omega - \omega)}{\Omega^2 (\Omega_j - \Omega + \omega_F)}. \end{aligned} \quad (38)$$

where $\omega_F = K_{\perp} v_F$.

The imaginary component of the dispersion relation contributes to the growth rate. Here, we consider the contribution of polarisation coupling to the growth rate, which is the dominant contribution of the plasma maser effect. Now, we compute $Im \epsilon_p(\vec{K}, \Omega)$ from Eq. (31).

$$Im \epsilon_p(\vec{K}, \Omega) = \frac{\omega_{pj}^4 (\Omega - \omega)^2}{R (c^2 K_{\perp}^2 - \Omega^2) c^2 K^2 - (\Omega - \omega)^2} \left(\frac{e_j}{m_j} \right)^2 |E_i|^2 (A \times Im D + C \times Im B). \quad (39)$$

where $K = |\vec{K} - \vec{k}|$.

Hence, the dominant portion comes from $A \times \text{Im } D$.

$$\begin{aligned}
 A = & \frac{\int_0^\infty J_1^2 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp}{v_d v_j^2 |\bar{K} - \bar{k}| (\Omega_j + K_\perp v_F - \Omega)} - \frac{\int_0^\infty J_1^4 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp (\Omega - \lambda_j v_{0j} \alpha)}{v_d v_j^2 |\bar{K} - \bar{k}| (\Omega_j - \Omega + \omega_F) (\Omega_j - \Omega - \omega + \omega_F)} + \frac{1}{v_j^2} \left(\frac{1}{v_d |\bar{K} - \bar{k}|} + \frac{|\bar{K} - \bar{k}|}{(\Omega - \omega) K_\perp} \right) \times \\
 & \left[\frac{\int_0^\infty J_1^2 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp}{(\Omega_j - \Omega + \omega_F)} + \frac{\int_0^\infty J_1^4 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp (\Omega - \lambda_j v_{0j} \alpha)}{(\Omega_j - \Omega + \omega_F)^2} \right] - \tag{40} \\
 & \frac{2|\bar{K} - \bar{k}| \cos \theta \int_0^\infty J_1^4 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^3 f_{0j}(v_\perp) dv_\perp}{v_j^2 (\Omega - \omega) (\Omega_j - \Omega + \omega_F)^2} - \frac{k_\parallel \int_0^\infty J_1^4 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp}{v_j^2 |\bar{K} - \bar{k}| (\Omega_j - \Omega + \omega_F)^2} \\
 \text{Im } \varepsilon_p(\bar{K}, \Omega) = & \left(\frac{\omega_{pj}}{\Omega_j} \right)^2 \left(\frac{\Omega_j - \omega}{c k_\parallel} \right)^2 \left(\frac{e_j}{m_j} \right)^2 |E_t|^2 \times \frac{\int_0^\infty J_1^2 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp}{v_d v_j^2 |\bar{K} - \bar{k}| \omega_F} - \frac{\int_0^\infty J_1^4 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp (\Omega - \lambda_j v_{0j} \alpha)}{v_d v_j^2 |\bar{K} - \bar{k}| \omega_F (\omega_F - \omega)} \\
 & + \frac{1}{v_j^2} \left(\frac{1}{v_d |\bar{K} - \bar{k}|} + \frac{|\bar{K} - \bar{k}|}{(\Omega_j - \omega) K_\perp} \right) \times \left(\frac{\int_0^\infty J_1^2 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp}{\omega_F} + \frac{\int_0^\infty J_1^4 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp (\Omega - \lambda_j v_{0j} \alpha)}{\omega_F^2} \right) - \\
 & \frac{2|\bar{K} - \bar{k}| \int_0^\infty J_1^4 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^3 f_{0j}(v_\perp) dv_\perp}{v_j^2 (\Omega_j - \omega) \omega_F^2} - \frac{k_\parallel \int_0^\infty J_1^4 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp}{v_j^2 |\bar{K} - \bar{k}| \omega_F^2} \times \\
 & \frac{\sqrt{\pi} K_\perp v_d c}{v_j^5 \Omega |k_\parallel|} \left[\frac{\sqrt{\pi} v_j}{4} + \frac{(\Omega - \lambda_j v_{0j} \alpha)}{\omega_F} \int_0^\infty J_1^2 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^2 f_{0j}(v_\perp) dv_\perp - \frac{4\sqrt{\pi} K_\perp v_d \int_0^\infty J_1^4 \left(\frac{K_\perp v_\perp}{\Omega_j} \right) 2\pi v_\perp^3 f_{0j}(v_\perp) dv_\perp}{v_j^2 |k_\parallel| \Omega_j \omega_F} \right] \exp \left[-\frac{v_d^2}{v_j^2} \right]. \tag{41}
 \end{aligned}$$

where $\alpha = \frac{K_\perp v_F}{\Omega_j}$.

Using Eqs. (29), (39), (40), and (41) in Eq. (36), we obtain the growth of ordinary mode waves due to the polarisation coupling term as

$$\frac{\gamma_p}{\Omega} = \pi \left(\frac{\omega_{pj}}{\Omega_j} \right)^2 \left(\frac{e_j}{m_j} \right)^2 \frac{c^2 k_\parallel^2}{c^2 K_\perp^2 - \Omega^2} \left(1 - \frac{\lambda_j T_j K_\perp}{m_j \Omega_j \Omega} \right) \frac{E_t^2}{|k_\parallel| v_j^3} \exp \left\{ -\left(\frac{v_d}{v_j} \right)^2 \right\}. \tag{42}$$

DISCUSSIONS AND CONCLUSIONS

Drift waves are the characteristic feature of inhomogeneous plasma and are observed in magnetosphere and ionospheric plasma as well as in fusion devices. Drift waves are in phase with plasma particles, and energy transfer through wave particle interaction is possible in such a case. This study on nonlinear energy exchange among drift wave and O-mode through wave particle interaction is called the plasma maser effect, which may help in understanding electromagnetic instabilities in fusion plasma and in the magnetosphere in inhomogeneous regions. Such a nonlinear process may be useful in developing methods to control and minimise the effect of gradient features on plasma confinement and stability.

Temperature gradients in ionospheric plasma are responsible for the drift motion of plasma particles and associated drift wave turbulence phenomena. We have considered drift ion-cyclotron waves in the magnetospheric region and in the upper ionospheric plasma [1]. The low-frequency ion-cyclotron wave energy is taking part in the growth of the O-mode,

as shown in Eq. (42). The external force involved in the drift motion of particles is also contributing to the growth of the O-mode. The role of gradient parameters ‘ λ_j ’ in the amplification process for O-mode may also be analysed from Eq. (42).

In earlier work on the growth of O-mode through nonlinear wave-particle interaction [1, 26], analysis of the effect of external force and gradient parameters for density and temperature was not included.

By using the following satellite observational data from magnetospheric plasma [33],

$$\Omega = 2.804 \times 10^6 \text{ Hz}, \Omega_j = 2.8 \times 10^6 \text{ Hz}, K_{\perp} = 10^{-3} \text{ m}^{-1}, \omega_{pj} = 2.84 \times 10^8 \text{ Hz}, v_j = 4.19 \times 10^5 \text{ ms}^{-1}, k_{\parallel} \sim 10^{-5} \text{ m}^{-1}, E_l \sim 10^{-4} \text{ Vm}^{-1}, E_l \sim 10^{-4} \text{ Vm}^{-1},$$

To demonstrate the influence of gradient parameters, we express the growth rate of Eq. (42) as a function of λ_j ,

$$\frac{\gamma_p}{\Omega} = \pi \left(\frac{\omega_{pj}}{\Omega_j} \right)^2 \left(\frac{e_j}{m_j} \right)^2 \frac{c^2 k_{\parallel}^2}{c^2 K_{\perp}^2 - \Omega^2} \left(1 - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j \Omega} \right) \frac{E_l^2}{|k_{\parallel}| v_j^3} \exp \left\{ - \left(\frac{v_d}{v_j} \right)^2 \right\},$$

$$\Rightarrow \frac{\gamma_p}{\Omega} = M \times R(\lambda_j),$$

where $M = \pi \left(\frac{\omega_{pj}}{\Omega_j} \right)^2 \left(\frac{e_j}{m_j} \right)^2 \frac{c^2 k_{\parallel}^2}{c^2 K_{\perp}^2 - \Omega^2} \frac{E_l^2}{|k_{\parallel}| v_j^3} \exp \left\{ - \left(\frac{v_d}{v_j} \right)^2 \right\},$

and $R(\lambda_j) = \left(1 - \frac{\lambda_j T_j K_{\perp}}{m_j \Omega_j \Omega} \right).$

Case 1: When both temperature and density gradients are not present, i.e., plasma is homogeneous, then $R(\lambda_j) = 1.$

$$\Rightarrow \frac{\gamma_p}{\Omega} = M \times 1,$$

$$\Rightarrow \frac{\gamma_p}{\Omega} = \pi \left(\frac{\omega_{pj}}{\Omega_j} \right)^2 \left(\frac{e_j}{m_j} \right)^2 \frac{c^2 k_{\parallel}^2}{c^2 K_{\perp}^2 - \Omega^2} \frac{E_l^2}{|k_{\parallel}| v_j^3} \exp \left\{ - \left(\frac{v_d}{v_j} \right)^2 \right\},$$

$$\Rightarrow \frac{\gamma_p}{\Omega} = 10^{-2},$$

$$\Rightarrow \gamma_p = 10^{-2} \Omega.$$

Case 2: When the temperature gradient is ignored,

$$R(\lambda_j) = \left(1 - \frac{\lambda_{2j} T_j K_{\perp}}{m_j \Omega_j \Omega} \right),$$

where $\lambda_{2j} = \frac{1}{f_j} \frac{df_j}{dy} - \frac{F}{T_{0j}},$

$$\Rightarrow \frac{\gamma_p}{\Omega} = M \times R(\lambda_{2j}),$$

$$\Rightarrow \frac{\gamma_p}{\Omega} = 10^{-3} \lambda_{2j},$$

$$\Rightarrow \gamma_p = \Omega 10^{-3} \lambda_{2j}.$$

Case 3: When the density gradient is ignored,

$$R(\lambda_j) = \left(1 - \frac{\lambda_{1j} T_j K_{\perp}}{m_j \Omega_j \Omega} \right),$$

where $\lambda_{1j} = \frac{\partial}{\partial T_j} \frac{dT_j}{dy} - \frac{F}{T_{0j}},$

$$\Rightarrow \frac{\gamma_p}{\Omega} = M \times R(\lambda_{1j}),$$

$$\Rightarrow \frac{\gamma_p}{\Omega} = 10^{-3} \lambda_{1j},$$

$$\Rightarrow \gamma_p = \Omega 10^{-3} \lambda_{1j}.$$

Case 4: When all the gradients are present,

$$\begin{aligned}\frac{\gamma_p}{\Omega} &= M \times R(\lambda_j), \\ \Rightarrow \frac{\gamma_p}{\Omega} &= 10^{-5}, \\ \Rightarrow \gamma_p &= 10^{-5} \Omega.\end{aligned}$$

As a result, we have observed that the gradients in density and temperature, along with the frequency related to particle pressure, may influence the amplification process of electromagnetic O-mode instability through electrostatic ion cyclotron drift wave turbulence.

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Data Availability Statement

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

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ГЕНЕРАЦІЯ О-МОДИ ЗА НАЯВНОСТІ ІОННО-ЦИКЛОТРОННОЇ ТУРБУЛЕНТНОСТІ ДРЕЙФОВОЇ ХВИЛІ В НЕОДНОРІДНІЙ ПЛАЗМІ

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Це дослідження має на меті дослідити вплив турбулентності іонно-циклотронної дрейфової хвилі на генерацію звичайної моди (О-моди) за наявності градієнтів густини та температури. Для цього розглядається плазма Власова, де в системі присутні як резонансні, так і нерезонансні моди. Тут нерезонансний режим – це збурення, викликане О-модю в квазістаціонарному стані плазми, яке характеризується наявністю низькочастотних хвиль іонно-циклотронного резонансного режиму. Взаємодія між цими хвилями вивчається системою рівнянь Власова-Максвелла та модифікованою функцією розподілу максвеллівського типу для частинок, яка включає зовнішнє силове поле та пов'язані параметри градієнта густини та температури. У дослідженні аналізується швидкість зростання електромагнітної О-моди за рахунок енергії іонно-циклотронного дрейфу хвилі та пов'язаного з цим впливу градієнта густини та температури. Ця модель використовує теорію лінійного відгуку на слабко турбулентну плазму, оцінює відгуки через турбулентні та збурені поля та отримує нелінійне співвідношення дисперсії для О-моди.

Ключові слова: звичайний режим; градієнти густини і температури; турбулентність дрейфової хвилі; взаємодія хвиля-частинка