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A STUDY OF EVOLUTION OF COSMOLOGICAL PARAMETERS BASED ON A DARK ENERGY MODEL IN THE FRAMEWORK OF BRANS-DICKE GRAVITY[†]

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The objective of the present study is to find the characteristics of evolution of a homogeneous and isotropic universe in the framework of Brans-Dicke (BD) theory of gravity. FLRW space-time, with zero spatial curvature, has been used to obtain BD field equations. Scale factor and Hubble parameter have been obtained from an ansatz for the deceleration parameter, assumed on the basis of its property of signature flip indicating a change of phase from deceleration to acceleration. Validation of the model has been achieved by a suitable parametrization of that ansatz. Expressions for energy density, pressure, equation of state (EoS) parameter, cosmological constant, gravitational constant have been derived and depicted graphically. The gravitational constant is found to decrease with time at a gradually decreasing rate. The Hubble parameter, deceleration parameter and energy density decrease with time, which is in agreement with many other studies. The value of the EoS parameter at the present epoch is negative, and it becomes more negative with time. The cosmological constant increases very rapidly in the early universe from negative to smaller negative values, becoming positive finally, with a much slower change thereafter. A cosmographic and a geometrical analysis have been carried out. It is observed that a gradual transition takes place from a regime of quintessence to phantom dark energy. An important finding of this study is that the signature flip of the deceleration parameter takes place almost simultaneously with the signature flip of the cosmological constant, implying a connection between accelerated expansion and dark energy, which is represented here by the cosmological constant. Unlike the common practice of using arbitrary units, proper SI units for all measurable quantities have been used. This theoretical investigation provides the reader with a simple method to formulate models in the framework of BD theory.
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1. INTRODUCTION

Based on the observations regarding the Type 1a supernova, it was established that the universe is expanding with acceleration and the present phase of acceleration was preceded by a phase of deceleration [1-5]. It has been a great challenge since then to explain the cosmic acceleration. According to the general theory of relativity (GTR), there is a strange form of energy, named Dark Energy (DE), which causes this acceleration. One of the parameters representing DE is the cosmological constant (Λ) in ΛCDM model which is known to be consistent with several important astrophysical observations related with supernova, baryon acoustic oscillation and cosmic microwave background. But it cannot account for the structure formation at small scales. DE models with scalar fields, such as phantom, quintessence and tachyon models were formulated to account for the phenomenon of late-time acceleration by Copeland et al. [6].

Several models were formulated to explain the dynamical behavior of DE [7-9]. One of the most significant among the theories which are modified versions of GTR is the one formulated by Brans and Dicke, which supports Mach's principle and weak equivalence principle [10]. The theory of $f(R,T)$ gravity is another such important theory of modified gravity [11, 12]. Brans and Dicke made a pioneering contribution to the theoretical exploration of the scalar tensor theories, and the elegant theoretical framework built up by them is known as Brans-Dicke theory of gravity. The scalar field (ϕ) in this theory evolves with time and we have $\phi = 1/G$ where G stands for the gravitational constant. A dimensionless constant ω in BD theory couples the scalar field with gravity. BD theory can generate the results of GTR if ϕ is constant and ω is infinitely large [13]. An important role is played by the scalar field ϕ in explaining the characteristics of the inflationary universe [10, 14, 15]. Based on BD theory, various cosmological models have been formulated by several researchers to account for the observed features [16-20]. The dynamics of Bianchi Type-V universe have been studied by Prasad et al. under the framework of BD theory [21]. DE models in BD theory have also been constructed where the dimensionless parameter ω is a function of the scalar field (ϕ) [22].

The main objective of the present study is to construct a cosmological model, in the framework of Brans-Dicke theory of gravity, to find and analyze the features of time evolution of different cosmological quantities. For this purpose, we have built a model starting from an empirical expression for the deceleration parameter (q) which is based upon the fact that q undergoes a signature flip as time goes on. Expressions for Hubble parameter (H) and scale factor (a) have

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been derived from that ansatz for q which clearly demonstrates (as a function of time) the change of the mode of expansion of the universe from deceleration to acceleration. The values of the arbitrary constants involved in this model have been calculated by using the presently accepted values of some cosmological quantities obtained from observational data. Considering the homogeneity and isotropy of the universe at large scale, we have used FLRW metric to obtain the field equations. To extract information from these equations, a power-law relation between the scalar field (ϕ) and the scale factor (a) has been used in the present study. Employing these equations, we have derived expressions for energy density, cosmic pressure, cosmological constant, gravitational constant, equation of state (EoS) parameter and shown their time variation graphically. For a cosmographic analysis, we have shown the variation of *jerk*, *snap* and *lerk* parameters in terms of redshift (z) graphically. To analyze the DE model characteristics, we have employed the theoretical tools named Statefinder diagnostic and *Om* diagnostic. It is observed that there is a gradual transition from quintessence dark energy regime to a phantom dark energy regime in the universe. A novel finding is that, the signature flip of the cosmological constant is almost simultaneous with the signature flip of the deceleration parameter, pointing towards a role of dark energy (represented by the cosmological constant) in causing cosmic acceleration.

2. FIELD EQUATIONS

The action for the Brans-Dicke theory of gravity is expressed as,

$$S = \int_{\mathcal{R}} \left\{ \phi(R - 2\Lambda) + \omega\phi^{-1}\phi^{\mu}\phi_{\mu} + \frac{16\pi}{c^4} L_m \right\} \sqrt{-g}\, d^4x, \tag{1}$$

where, ϕ represents the BD scalar field, which is reciprocal of the gravitational constant ($G = 1/\phi$). R stands for the Ricci scalar. The symbol Λ represents the cosmological constant. ω is called the BD parameter which represents a dimensionless coupling constant. The symbol ϕ_μ denotes the ordinary derivative of ϕ with respect to x^μ. The matter Lagrangian density is denoted by the symbol L_m.

The field equation, which is given below, is obtained by the variation of action (i.e., S in eqn. 1) through infinitesimal changes in the metric tensor $g^{\mu\nu}$.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi}{\phi c^4} T_{\mu\nu} - \frac{\omega}{\phi^2}\left(\phi_\mu\phi_\nu - \frac{1}{2} g_{\mu\nu}\phi^\gamma\phi_\gamma\right) - \frac{1}{\phi}\left(\phi_{\mu;\nu} - g_{\mu\nu}\Box\phi\right) - \Lambda g_{\mu\nu} \tag{2}$$

In the above equation, $T_{\mu\nu}$ denotes the energy-momentum tensor, $g_{\mu\nu}$ is the metric tensor and $R_{\mu\nu}$ stands for the Ricci tensor. The semicolon (;) in this equation stands for the covariant derivative and the symbol $\Box$ represents the d'Alembert operator.

Through a variation of the scalar field (ϕ) in BD action (i.e., S in eqn. 1) we get the following equation.

$$\Box\phi = \frac{1}{3+2\omega}\left(\frac{8\pi T}{c^4} + 2\Lambda\phi\right) \tag{3}$$

where, $T = g^{\mu\nu}T_{\mu\nu}$ is the trace of energy-momentum tensor $T_{\mu\nu}$.

The matter content of the universe is considered to be that of a perfect fluid distribution, which is given by,

$$T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu} \tag{4}$$

where, ρ and p are cosmic fluid's energy density and pressure respectively. $h_{\mu\nu} = u_\mu u_\nu - g_{\mu\nu}$ where u_μ represents the four-velocity vector of cosmic fluid with $g_{\mu\nu}u^\mu u^\nu = 1$.

It has been concluded from recent cosmological findings that the observable universe is homogeneous and isotropic at large scales. To take into account this aspect of the cosmos, we have chosen FLRW metric to represent the space-time geometry of the universe. This metric is given by,

$$ds^2 = c^2dt^2 - a^2\left(\frac{1}{1+kr^2}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2\right) \tag{5}$$

In equation (5), the symbol a denotes the scale factor. The coordinates (co-moving) of the spherical polar system are (r, θ, ϕ). The symbol k is regarded as the curvature parameter which has three values $1, 0, -1$, denoting respectively three characteristics of the expanding universe, namely *open*, *flat* and *closed*.

In co-moving coordinate system, Brans-Dicke field equations (eqns. 2, 3) and the energy-momentum tensor (eqn. 4) for a FLRW metric (eqn. 5) with zero curvature (i.e., $k = 0$) lead to the set of differential equations given below (eqns. 6-8).

$$H^2 + H\frac{\dot{\phi}}{\phi} - \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \frac{8\pi\rho}{3\phi c^2} + \frac{\Lambda c^2}{3} \tag{6}$$

$$2\dot{H}+3H^2+2H\frac{\dot{\phi}}{\phi}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\ddot{\phi}}{\phi}=-\frac{8\pi p}{\phi c^2}+\Lambda c^2 \tag{7}$$

$$(3+2\omega)\left(3H\frac{\dot{\phi}}{\phi}+\frac{\ddot{\phi}}{\phi}\right)=\frac{8\pi(\rho-3p)}{\phi c^2}+2\Lambda c^2 \tag{8}$$

In equations (6), (7) and (8), H represents the Hubble parameter (given by, $H=\frac{\dot{a}}{a}$). A dot over any parameter represents the conventional derivative of that parameter with respect to time (t).

Based on the three above equations, we have obtained the following expressions for the parameters Λ, ρ and p, represented as functions of a, ϕ and their time derivatives.

$$\Lambda(t)=\frac{1}{2c^2}\left[6\left(\frac{\dot{a}}{a}\right)^2+\omega\left(\frac{\dot{\phi}}{\phi}\right)^2+6\frac{\ddot{a}}{a}-2\omega\frac{\ddot{\phi}}{\phi}-6\omega\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}\right] \tag{9}$$

$$\rho(t)=\frac{\phi c^2}{8\pi}\left[3\left(\frac{\dot{a}}{a}\right)^2+3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2-\Lambda c^2\right] \tag{10}$$

$$p(t)=\frac{\phi c^2}{8\pi}\left[\Lambda c^2-2\frac{\ddot{a}}{a}-\left(\frac{\dot{a}}{a}\right)^2-\frac{\ddot{\phi}}{\phi}-2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2\right] \tag{11}$$

3. SOLUTION OF FIELD EQUATIONS

Considering the change of phase of the cosmic expansion from deceleration to acceleration [1-5], we assume an ansatz regarding the time evolution of the deceleration parameter (q). It is given by,

$$q(t)=At^{-n}-B \tag{12}$$

where $A, B, n > 0$. This function of time, $q(t)$, undergoes a change of sign (with time) from positive to negative.

Since $q(t)=-1-\frac{\dot{H}}{H^2}$, where H is the Hubble parameter, we may write equation (12) as,

$$\frac{\dot{H}}{H^2}=B-1-At^{-n} \tag{13}$$

Integrating equation (13), we get the following expression for the Hubble parameter.

$$H=\left[(1-B)t+\frac{At^{1-n}}{1-n}+C\right]^{-1} \tag{14}$$

where C is a constant of integration.
Substituting $H=\frac{\dot{a}}{a}$ in equation (14) we get the following differential equation.

$$\frac{\dot{a}}{a}=\left[(1-B)t+\frac{At^{1-n}}{1-n}+C\right]^{-1} \tag{15}$$

Equation (15) can be solved analytically for $C=0$, which corresponds to its simplest solution. That solution leads to the following expression for the scale factor (a).

$$a(t)=b[(B-1)(n-1)t^n+A]^{\frac{1}{n(1-B)}} \tag{16}$$

where b is a constant of integration.

The expressions for a, H and q (in terms of time) are dependent upon the parameters A, B, b and n. The inter-dependence of these parameters can be determined by using the values of the cosmological quantities (H_0, q_0, ρ_0) obtainable from observational data. The symbols, H_0, q_0, ρ_0 denote respectively the values of H, q, ρ at the present time (i.e., $t=t_0$) where t_0 is the present age of the universe.

Using the fact that $q=q_0$ at $t=t_0$ in equation (12), we get,

$$A=(q_0+B){t_0}^n \tag{17}$$

Similarly, using the fact that $H=H_0$ at $t=t_0$ in equation (14), we get,

$$B = 1 + \frac{A{t_0}^{-n}}{1-n} - \frac{1}{H_0 t_0} \tag{18}$$

Solving equations (17) and (18) for A and B one gets,

$$A = \left[q_0 + \frac{(1-n)(1-H_0 t_0) - q_0 H_0 t_0}{n H_0 t_0}\right] {t_0}^n \tag{19}$$

$$B = \frac{(1-n)(1-H_0 t_0) - q_0 H_0 t_0}{n H_0 t_0} \tag{20}$$

Thus, A and B are found to be dependent upon the parameter n.
Using the fact that $a = a_0$ at $t = t_0$, in equation (16), we get,

$$a_0 = b[(B-1)(n-1){t_0}^n + A]^{\frac{1}{n(1-B)}} \tag{21}$$

where, a_0 is the value of the scale factor at the present time. Using equation (21), we obtain b in terms of n and a_0, as given below.

$$b = a_0[(B-1)(n-1){t_0}^n + A]^{-\frac{1}{n(1-B)}} \tag{22}$$

Substituting the equations (19) and (20) in equation (22) we get,

$$b = a_0\left[\left(\frac{(1-n)(1-H_0 t_0) - q_0 H_0 t_0}{n H_0 t_0} - 1\right)(n-1){t_0}^n + \left[q_0 + \frac{(1-n)(1-H_0 t_0) - q_0 H_0 t_0}{n H_0 t_0}\right]{t_0}^n\right]^{-\frac{1}{n\left[1 - \frac{(1-n)(1-H_0 t_0) - q_0 H_0 t_0}{n H_0 t_0}\right]}} \tag{22A}$$

Thus, among the four arbitrary parameters of this model (A, B, b and n), it is observed from equations (19), (20) and (22A) that A, B, b can be expressed as functions of n. We have chosen $a_0 = 1$ in the present study. Hence q, H and a are dependent on n only. Taking $H_0 t_0 \approx 1$ [18], we obtain $B \approx -\frac{q_0}{n}$ from equation (20), satisfying the condition $B > 0$, since $q_0 < 0$ for an accelerating universe and $n > 0$ by definition. For A to be positive, we must have $(q_0 + B){t_0}^n > 0$ (according to eqn. 17). It means $q_0 - \frac{q_0}{n} > 0$, leading to $n < 1$ (using $B = -\frac{q_0}{n}$ and $q_0 < 0$). Thus, the range of values for n is obtained as, $0 < n < 1$.

An ansatz for the scalar field parameter (ϕ), where it has a power-law relation with the scale factor (a), has been used in the present formulation, based on some recent studies [17-20]. This ansatz is,

$$\phi = \phi_0 \left(\frac{a}{a_0}\right)^m \tag{23}$$

where, m is an arbitrary constant and $\phi_0 = \frac{1}{G_0}$, where G_0 is the present value of the gravitational constant G. In BD theory, the gravitational term $G(t) = 1/\phi(t)$. Based on equation (23), the first and the second order time-derivatives of ϕ are given by the following two equations.

$$\dot{\phi} = mH\phi \tag{24}$$

$$\ddot{\phi} = mH^2\phi\{(m-1) - q\} \tag{25}$$

Equation (24) and (25) have been obtained by using the relations: $\frac{\dot{a}}{a} = H$ and $\frac{\ddot{a}}{a} = -qH^2$. Using equations (24) and (25) in equations (9), (10) and (11), we obtain the following expressions for cosmological term (Λ), energy density (ρ) and pressure (p).

$$\Lambda(t) = \frac{H^2}{2c^2}\left[(6 - \omega m^2 - 4\omega m) + 2q(\omega m - 3)\right] \tag{26}$$

$$\rho(t) = \frac{\phi c^2 H^2}{8\pi}\left[(3m + 2\omega m) + q(3 - \omega m)\right] \tag{27}$$

$$p(t) = \frac{\phi c^2 H^2}{8\pi}\left[(2 - \omega m^2 - 2\omega m - m^2 - m) + q(\omega m + m - 1)\right] \tag{28}$$

Putting $\rho = \rho_0$, $\phi = \phi_0$, $H = H_0$, $q = q_0$ in equation (27), we get the following expression for m in terms of ω.

$$m = \frac{8\pi\rho_0 - 3q_0(\phi_0 c^2 H_0{}^2)}{(\phi_0 c^2 H_0{}^2)(3+2\omega-q_0\omega)} \tag{29}$$

Hence, along with n, ω is also a free parameter in this model. Λ, ρ and p are dependent on both n and ω, while q, H and a are dependent on n only.

The equation of state (EoS) parameter (η) is defined as $\eta = \frac{p}{\rho}$. Using equations (27) and (28), η can be expressed as,

$$\eta = \frac{(2-\omega m^2 - 2\omega m - m^2 - m) + q(\omega m + m - 1)}{(3m+2\omega m) + q(3-\omega m)} \tag{30}$$

Using equation (12) in (26), (27), (28) and (30), the expressions for Λ, ρ, p and η take new forms represented by equations (31), (32), (33) and (34), respectively, as given below.

$$\Lambda(t) = \frac{1}{2c^2}\left((1-B)t + \frac{At^{1-n}}{1-n}\right)^{-2} \begin{bmatrix} (6-\omega m^2 - 4\omega m) + \\ 2(At^{-n} - B)(\omega m - 3) \end{bmatrix} \tag{31}$$

$$\rho(t) = \frac{\phi_0 c^2}{8\pi}\left(\frac{a}{a_0}\right)^m \left((1-B)t + \frac{At^{1-n}}{1-n}\right)^{-2} \begin{bmatrix} (3m+2\omega m) + \\ (At^{-n} - B)(3-\omega m) \end{bmatrix} \tag{32}$$

$$p(t) = \frac{\phi_0 c^2}{8\pi}\left(\frac{a}{a_0}\right)^m \left((1-B)t + \frac{At^{1-n}}{1-n}\right)^{-2} \begin{bmatrix} (2-\omega m^2 - 2\omega m - m^2 - m) + \\ (At^{-n} - B)(\omega m + m - 1) \end{bmatrix} \tag{33}$$

$$\eta(t) = \frac{p(t)}{\rho(t)} = \frac{(2-\omega m^2 - 2\omega m - m^2 - m) + (At^{-n} - B)(\omega m + m - 1)}{(3m+2\omega m) + (At^{-n} - B)(3-\omega m)} \tag{34}$$

4. COSMOGRAPHIC ANALYSIS

For a cosmographic analysis, we have determined the time dependence of jerk(j), snap(s) and lerk(l) parameters, which are defined as, $j(t) = a^2 \frac{d^3a}{dt^3}\left(\frac{da}{dt}\right)^{-3}$, $s(t) = a^3 \frac{d^4a}{dt^4}\left(\frac{da}{dt}\right)^{-4}$ and $l(t) = a^4 \frac{d^5a}{dt^5}\left(\frac{da}{dt}\right)^{-5}$ respectively [23, 24]. These are dimensionless quantities which depend upon the scale factor (a) and its third, fourth and fifth order time derivatives. They allow us to determine the rate of cosmic expansion more accurately by a model independent analysis of the evolution of the universe. Using equation (16) we get the following expressions (eqns. 35, 36 and 37) for *j*, *s* and *l* respectively.

$$j(t) = \frac{C_1 t^{2n} + C_2 t^n + C_3}{(n-1)t^{2n}} \tag{35}$$

where $C_1 = 2B^2n - Bn - 2B^2 + B$,

$C_2 = -ABn^2 + An^2 - 3ABn + 4AB - A$,

and $C_3 = A^2n - 2A^2$.

$$s(t) = \frac{C_4 t^{3n} + C_5 t^{2n} + C_6 t^n + C_7}{(n-1)^2 t^{3n}} \tag{36}$$

where $C_4 = 6B^3n^2 - 7B^2n^2 + 2Bn^2 - 12B^3n + 14B^2n - 4Bn + 6B^3 - 7B^2 + 2B$,

$C_5 = -AB^2n^4 + 2ABn^4 - An^4 - 5AB^2n^3 + 6ABn^3 - An^3 - 5AB^2n^2 - 4ABn^2 + 3An^2 + 29AB^2n - 18ABn + An - 18AB^2 + 14AB - 2A$,

$C_6 = 4A^2Bn^3 - 4A^2n^3 + 7A^2n^2 - 22A^2Bn + 4A^2n + 18A^2Bt^n - 7A^2$,

and $C_7 = -A^3n^2 + 5A^3n - 6A^3$.

$$l(t) = \frac{C_8 t^{4n} + C_9 t^{3n} + C_{10} t^{2n} + C_{11} t^n + C_{12}}{(n-1)^3 t^{4n}} \tag{37}$$

where $C_8 = 24B^4n^3 - 46B^3n^3 + 29B^2n^3 - 6Bn^3 - 72B^4n^2 + 138B^3n^2 - 87B^2n^2 + 18Bn^2 + 72B^4nt^{4n} - 138B^3n + 87B^2n - 18Bn - 24B^4 + 46B^3 - 29B^2 + 6B$,

$C_9 = -AB^3n^6 + 3AB^2n^6 - 3ABn^6 + An^6 - 8AB^3n^5 + 19AB^2n^5 - 14ABn^5 + 3An^5 - 16AB^3n^4 + 18AB^2n^4 + 2ABn^4 - 4An^4 + 10AB^3n^3 - 60AB^2n^3 + 50ABn^3 - 10An^3 + 161AB^3n^2 - 163AB^2n^2 + 23ABn^2 + 9An^2 - 242AB^3n + 321AB^2n - 116ABn + 7An + 96AB^3 - 138AB^2 + 58AB - 6A,$

$C_{10} = 11A^2B^2n^5 - 22A^2Bn^5 + 11A^2n^5 + +19A^2B^2n^4 - 8A^2Bn^4 - 11A^2n^4 - 65A^2B^2n^3 + 130A^2Bn^3 - 40A^2n^3 - 115A^2B^2n^2 - 10A^2Bn^2 + 40A^2n^2 + 294A^2B^2n - 228A^2Bn + 29A^2n - 144A^2B^2 + 138A^2B - 29A^2,$

$C_{11} = -11A^3Bn^4 + 11A^3n^4 + 30A^3Bn^3 - 45A^3n^3 + 35A^3Bn^2 + 35A^3n^2 - 150A^3Bn + 45A^3n + 96A^3B - 46A^3,$

and $C_{12} = A^4n^3 - 9A^4n^2 + 26A^4n - 24A^4$.

5. STATEFINDER AND OM DIAGNOSTICS

Sahni et al. introduced two parameters r and s which are defined as [25],

$$r = a^2 \frac{d^3a}{dt^3}\left(\frac{da}{dt}\right)^{-3} \tag{38}$$

$$s = \frac{2(1-r)}{3(1-2q)} \tag{39}$$

The expression for r is the same as the expression for the jerk parameter $j(t)$.

Like H and q, these dimensionless parameters are functions of the scale factor (a) and its higher order derivatives. These parameters help us to differentiate between any DE model and the ΛCDM model. As per statefinder diagnostic, (s, r) and (q, r) trajectories are plotted in $s - r$ and $q - r$ planes, respectively, to analyze the evolution of the universe under the frameworks of different models of dark energy. Since q, r and s involve only the scale factor (a) and its time derivatives of higher orders, this method is independent of the framework of gravity. Thus, this diagnostic is model independent.

Using equations (12) and (16) in equations (38) and (39), r and s are expressed as,

$$r = \frac{C_1t^{2n}+C_2t^n+C_3}{(n-1)t^{2n}} \tag{40}$$

$$s = \frac{2\left\{1-\frac{C_1t^{2n}+C_2t^n+C_3}{(n-1)t^{2n}}\right\}}{3\{1-2(At^{-n}-B)\}} \tag{41}$$

where, $C_1 = 2B^2n - Bn - 2B^2 + B$, $C_2 = -ABn^2 + An^2 - 3ABn + 4AB - A$ and $C_3 = A^2n - 2A^2$.

ΛCDM model is represented by the point $(s, r) = (0, 1)$ in $s - r$ plane. Standard cold dark matter (SCDM) model is represented by the point $(s, r) = (1, 1)$ in FLRW background. The point $(q, r) = (-1, 1)$ stands for steady state (SS) model and $(q, r) = (0.5, 1)$ stands for SCDM model in $q - r$ plane.

Om diagnostic has been used in recent cosmological studies to distinguish between the standard ΛCDM model and various other DE models [26]. In this theory, a parameter called $Om(z)$ is defined as

$$Om(z) = \frac{[E(z)]^2-1}{z^3+3z^2+3z} \tag{42}$$

where, $E(z) = \frac{H(z)}{H_0}$ and $z(\equiv \frac{a_0}{a} - 1)$ is the redshift parameter. The positive curvatures of $Om(z)$ trajectories imply phantom behaviour while negative curvatures of $Om(z)$ trajectories indicate quintessence behaviour of dark energy. Constant value of $Om(z)$ for a model indicates that its behaviour is the same as that of the ΛCDM model.

6. RESULTS AND DISCUSSION

To plot some cosmological quantities graphically with respect to redshift (z), we have derived the following relation (redshift versus time) based on equation (16).

$$z = \frac{a_0}{a} - 1 = \frac{a_0}{b}\left[(B-1)(n-1)t^n + A\right]^{-\frac{1}{n(1-B)}} - 1 \tag{43}$$

Since the behaviour of the deceleration parameter (q) determines how the phase of decelerated expansion of the universe changes into the phase of accelerated expansion, we have derived the following expression for Λ as a function of q, using equations (12), (14) and (26).

$$\Lambda(q)=\frac{[(6-\omega m^2-4\omega m)+2q(\omega m-3)]}{2c^2}\left[(1-B)\left\{\left(\frac{q+B}{A}\right)^{-\frac{1}{n}}\right\}+\frac{A\left\{\left(\frac{q+B}{A}\right)^{-\frac{1}{n}}\right\}^{1-n}}{1-n}+C\right]^{-2} \quad (44)$$

To validate the present model, the arbitrary constants associated with the formulation have been so adjusted that the values of H_0, q_0, ρ_0, G_0, t_0 are obtained correctly from the model, as discussed in Section-3 of this article. For this purpose, we have used the following values of these parameters [18].

$H_0 = 2.34\times10^{-18}\ sec^{-1}$, $q_0 = -0.55$, $\rho_0 = 8.91\times10^{-10}\ J/m^3$, $G_0 = 6.67\times10^{-11} N\ m^2\ Kg^{-2}$, $t_0 = 4.13\times10^{17}\ sec.$

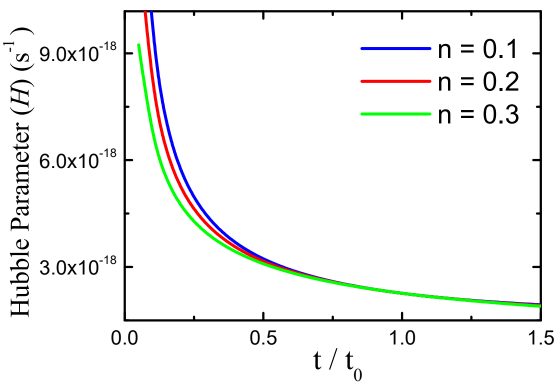

Figure 1. Plots of Hubble parameter (H) versus time

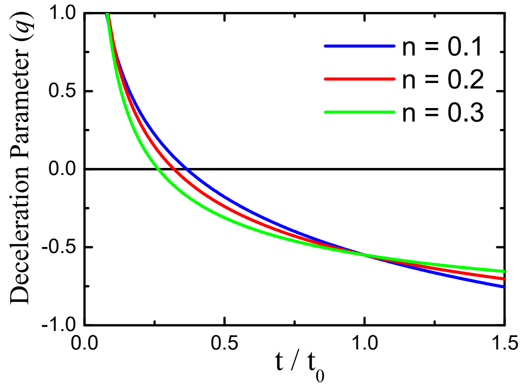

Figure 2. Plots of deceleration parameter (q) versus time

Figures 1 and 2 show the time evolution of Hubble parameter (H) and deceleration parameter (q) respectively, for three values of the parameter n. It is observed that H decreases with time, which is consistent with recent studies based on various models [7, 17-19]. The deceleration parameter shows a signature flip indicating a change from decelerated expansion to accelerated expansion, in accordance with the observed features [7, 17-19].

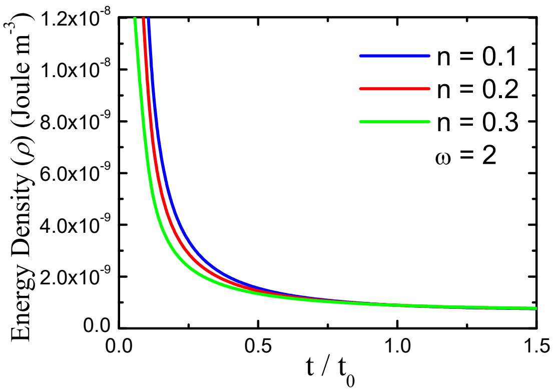

Figure 3. Plots of energy density (ρ) versus time

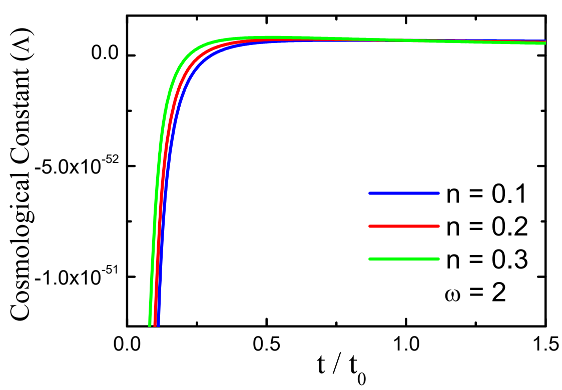

Figure 4. Plots of cosmological Constant (Λ) versus time

Figures 3, 4, 5 and 6 show respectively the behavior of energy density (ρ), cosmological constant (Λ), Gravitational constant (G) and EoS parameter (η) with respect to time, for three values of the parameter n. These four cosmological quantities depend also on the Brans-Dicke parameter ω. It is found from equation (29) that $m > 0$ for $\omega > -1.11$. Using equation (32) we have found that, for some values of ω, in the range of $\omega < -1.11$, ρ becomes negative. Due to this discrepancy regarding the sign of ρ values, we have chosen to use ω values belonging to the range of $\omega > -1.11$. For all calculations we have used $\omega = 2$, leading to $m = 0.61$, which are consistent with a recent study in the framework of BD theory [19]. Another study by Goswami et al, in BD framework, also used positive values of ω [18]. As per equation (23), ϕ increases with time if $m > 0$, implying that $G\ (\equiv 1/\phi)$ decreases with time, as shown by Figure 5, which is in agreement with some recent studies based on different theoretical models and experimental observations [18, 27, 28]. Figure 3 shows that ρ decreases with time, as obtained from many other studies [9, 17, 19, 29, 30]. Figure 4 shows that Λ rises very steeply in the early universe, becoming positive from negative and then changes slowly. This behavior is consistent with the findings of various other studies [31-34]. As per Figure 6, η is negative and decreases gradually with time, with $\eta(t = t_0) = -0.8$, which is consistent with values obtained from observational data [35, 36]. According to the plots of Figure 6, the universe presently has a quintessence dark energy regime $(\eta > -1)$ and it is making a gradual transition towards a phantom dark energy regime $(\eta < -1)$.

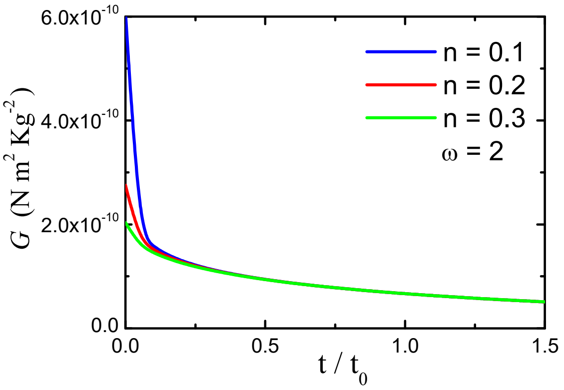

Figure 5. Plots of gravitational constant (G) versus time

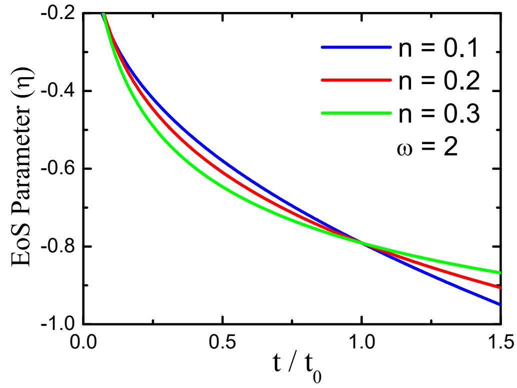

Figure 6. Plots of EoS Parameter (η) versus time

Figure 7 depicts the variation of cosmic pressure (p) with respect to time for three values of n. It is negative and it becomes less negative with time. Negative pressure is associated with DE, causing the accelerated expansion of the universe. This behavior is consistent with the findings of several studies [37, 38].

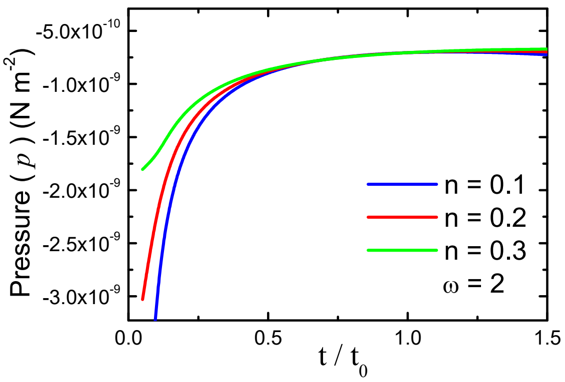

Figure 7. Plots of pressure (p) versus time

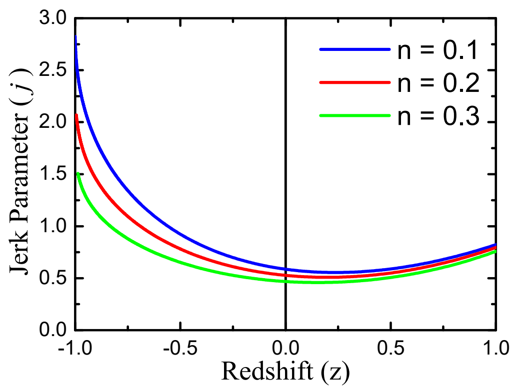

Figure 8. Plots of jerk parameter (j) versus redshift (z)

Figures 8, 9 and 10 represent respectively the plots of jerk (j), snap (s) and lerk (l) parameters against redshift (z). Their behaviours are similar to the results of a recent study under BD framework [19].

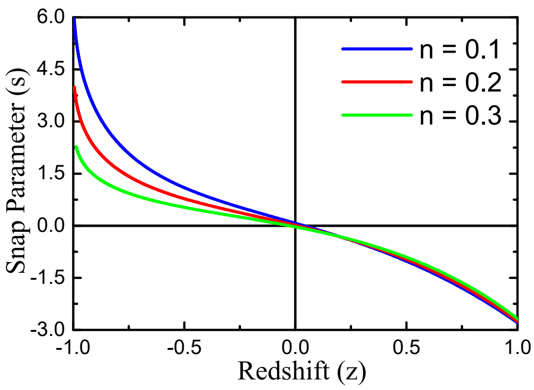

Figure 9. Plots of snap parameter (s) versus redshift (z)

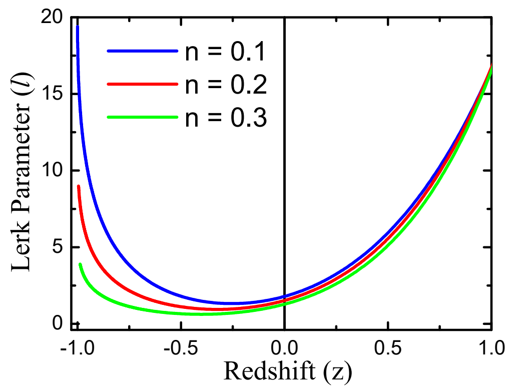

Figure 10. Plots of lerk parameter (l) versus redshift (z)

Positive values for j and l and negative values for s represent accelerated expansion [39]. Figures 8 and 10 show j and l to have positive values for all values of n. As per Figure 9, the values of the snap parameter (s) undergo a transition with time from negative to positive (based on the fact that z decreases as t increases). From equation (36) it is found that the value of the snap parameter at the present time (i.e., at $t = t_0$) is negative for $n > 0.227$, which is consistent with the plot for $n = 0.3$ in Figure 9 where s is negative at $t = t_0$ (i.e., $z = 0$). It provides a clear guideline for choosing the values of n for an accelerating universe. It has been shown in Section-3 of this article that $0 < n < 1$, based on the requirements for a proper parameterization of the ansatz that we have chosen for the deceleration parameter (represented by eqn. 12). Figure 8 shows that, the rate of increase of j with time (i.e., as z decreases) is larger for smaller values of the parameter n. We observe almost the same behaviour for the plots of s in Figure 9. In Figure 10, the values of l initially decrease with time and, at some point of time in future (i.e., $z < 0$) the values increase with time, having the largest rate of rise for the lowest value of the parameter n. For plotting the graphs in Figures 8-10, we did not have to express jerk (j), snap (s) and lerk (l) parameters as functions of redshift (z). Expressions for these parameters, in terms of z, would have been extremely complicated and difficult to handle. We generated datasets for j, s, l and z as functions of time (using eqns. 35-37, 43 respectively) for three different values of the parameter n, using Microsoft Excel. Based on these datasets, we have plotted j, s, l as functions of z.

Figures 11 and 12 show the plots of (s, r) and (q, r) trajectories. Their natures are found to be close to those obtained from a different model in the BD framework [19]. In Figure 11, trajectories begin in the Chaplygin gas (CG) region ($s < 0, r > 1$), and enter the quintessence region ($s > 0, r < 1$). Then they merge together and reach the Chaplygin gas region again after passing through the point (0, 1) which stands for the ΛCDM model, for all values of n. In Figure 12, the line $r = 1$ represents the evolution of ΛCDM model. The trajectories, starting from the region of decelerated expansion ($q > 0$), are found to reach and cross the $r = 1$ line for all three values of n. Here, $q = 0.5$ line represents matter-dominated era. These two figures show that the constructed model is presently behaving as a quintessence dark energy model. Its predictions for the future evolution of the universe will be like the Chaplygin gas model, after passing through an intermediate stage having the behavior which is consistent with that of the ΛCDM model.

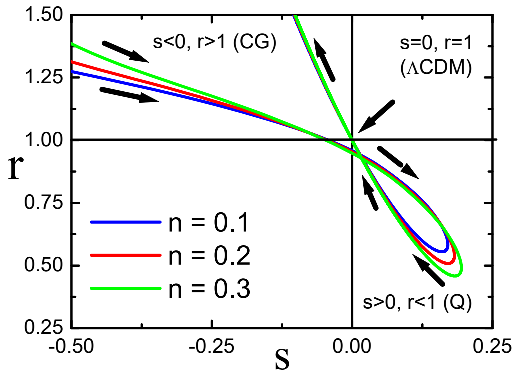

Figure 11. Plots of (s, r) trajectories for statefinder diagnostic

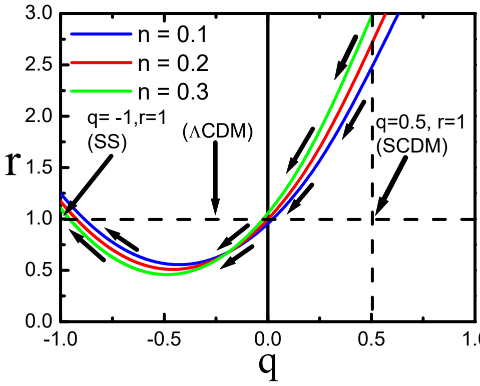

Figure 12. Plots of (q, r) trajectories for statefinder diagnostic

Figure 13 depicts the variation of $Om(z)$ as a function of z for different values of n. It is known that, if the curvature of $Om(z)$ is positive with respect to z, the model is a phantom dark energy model ($\eta < -1$) and, if the curvature is negative, it is a quintessence dark energy model ($\eta > -1$). For zero curvature, it represents the ΛCDM model [40-42]. It is observed that, $Om(z)$ rises steeply as z increases and it decreases slowly beyond $z = -0.75$ (approximately). Its decreasing behavior at $z = 0$ (i.e., the present time) indicates that our model has the characteristics of a quintessence DE model at the present time. Since z decreases with time, Figure 13 shows a transition of the model characteristics from those of a quintessence DE model to those of a phantom DE model, which is consistent with the inferences drawn from Figure 6. This transition takes more time to occur for greater values of the parameter n.

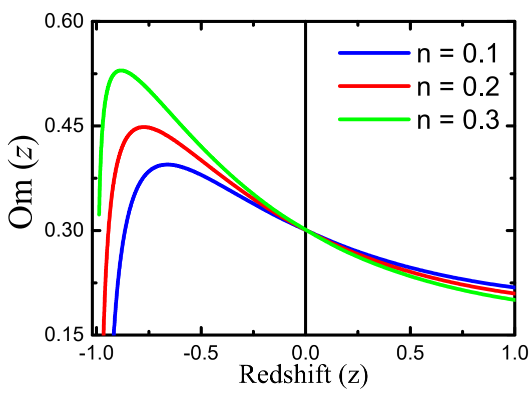

Figure 13. Plots of Om (z) versus redshift (z)

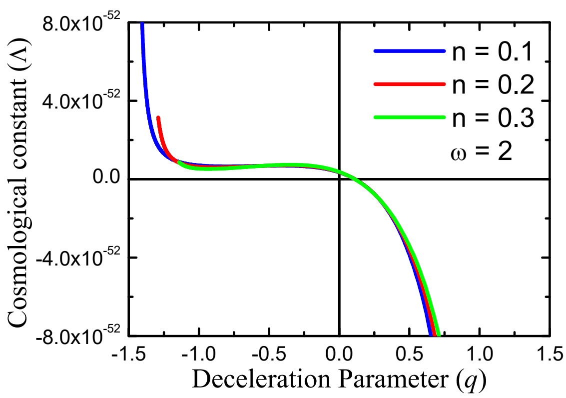

Figure 14. Plots of cosmological constant (Λ) versus deceleration parameter (q)

Figure 14 shows the variation of the cosmological constant (Λ) as a function of the deceleration parameter (q). These plots are based on equation (44). It is observed that, as Λ changes its sign from negative to positive, q undergoes a signature flip from positive to negative, indicating the phase transition (i.e., deceleration to acceleration) to be associated with some phenomena involving dark energy which is represented by Λ.

7. CONCLUSION

In the present article, we have constructed a cosmological model using FLRW metric for zero spatial curvature, in the Brans-Dicke framework with cosmological constant (Λ). Solutions of the field equations have been obtained from a proper parameterization of the deceleration parameter $q(t) = At^{-n} - B$ with $A, B, n > 0$. The choice of this expression is based on the phenomenon of signature flip of the deceleration parameter as obtained from astrophysical observations. The characteristics of the physical and geometrical parameters have been depicted graphically. We have a detailed interpretation of these graphs in Section-6 of this article. A significant finding of the present study is that the time at which the deceleration parameter changes its sign (from positive to negative) is almost the same as the time at which the cosmological parameter changes its sign (from negative to positive), indicating clearly that the change of phase from decelerated expansion to accelerated expansion is governed by some dark energy dynamics which is generally regarded as being represented by the characteristics of the cosmological parameter (Λ), in calculations under different gravitational frameworks. The plots based on the statefinder parameters show that, for all values of the parameter n, the (s, r) trajectories enter the quintessence region from Chaplygin gas region, ending up finally in the Chaplygin gas region after passing through the point representing ΛCDM. Thus, the future characteristics of the universe, based on this model, is like those obtained from Chaplygin gas model. The (q, r) trajectories start evolving from a region close to SCDM and move ahead crossing the line representing ΛCDM. The findings of the present study are sufficiently consistent with the findings of models constructed under various other theoretical frameworks, and they are also in reasonable agreement with observational data. As a future extension of the present work, we have plans to use some new ansatzes representing deceleration parameter to determine the time evolution of various cosmological quantities and find their average behaviour under different theoretical frameworks of gravitation. The construction of the present model might be helpful to the researchers in studying the evolution of the universe under the Brans-Dicke framework by formulating more such models in future.

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ДОСЛІДЖЕННЯ ЕВОЛЮЦІЇ КОСМОЛОГІЧНИХ ПАРАМЕТРІВ НА ОСНОВІ МОДЕЛІ ТЕМНОЇ ЕНЕРГІЇ В РАМКАХ ГРАВІТАЦІЇ БРАНСА-ДІКЕ

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Мета цього дослідження полягає в тому, щоб знайти характеристики еволюції однорідного та ізотропного Всесвіту в рамках теорії гравітації Бренса-Дікке (BD). Для отримання рівнянь поля BD використовувався простір-час FLRW з нульовою просторовою кривизною. Масштабний коефіцієнт і параметр Хаббла були отримані з підходу для параметра уповільнення, прийнятого на основі його властивості перевороту знаку, що вказує на зміну фази від уповільнення до прискорення. Перевірка моделі була досягнута відповідною параметризацією цього підходу. Виведено та зображено графічно вирази для густини енергії, тиску, параметра рівняння стану (EoS), космологічної постійної, гравітаційної постійної. Встановлено, що гравітаційна стала зменшується з часом зі швидкістю, що поступово зменшується. Параметр Хаббла, параметр уповільнення та щільність енергії зменшуються з часом, що узгоджується з багатьма іншими дослідженнями. Значення параметра EoS в сучасну епоху від'ємне, а з часом воно стає ще від'ємнішим. Космологічна стала дуже швидко зростає в ранньому Всесвіті від негативних до менших негативних значень, стаючи врешті позитивною, з набагато повільнішими змінами після цього. Проведено космографічний та геометричний аналіз. Спостерігається поступовий перехід від режиму «квінтесенції» до фантомної темної енергії. Важливим висновком цього дослідження є те, що характерна зміна параметра уповільнення відбувається майже одночасно з характерною зміною космологічної постійної, що означає зв'язок між прискореним розширенням і темною енергією, яка тут представлена космологічною сталою. На відміну від загальноприйнятої практики використання довільних одиниць, для всіх вимірних величин використовуються правильні одиниці СІ. Це теоретичне дослідження надає читачеві простий метод формулювання моделей у рамках теорії BD.

Ключові слова: *гравітація Бранса-Дікке; темна енергія; гравітаційна стала; космологічна стала; космографічний аналіз; Ом діагностика; діагностика стану*