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FIVE - DIMENSIONAL PLANE SYMMETRIC COSMOLOGICAL MODEL WITH QUADRATIC EQUATION OF STATE IN $f(R,T)$ THEORY OF GRAVITY[†]

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In this paper, we analysed the five-dimensional plane symmetric cosmological model containing perfect fluid in the context of $f(R,T)$ gravity. Field equations have solved for two class of $f(R,T)$ gravity i.e., $f(R,T) = R + f(T)$ and $f(R,T) = f_1(R)f_2(T)$ with the inclusion of cosmological constant Λ and quadratic equation of state parameters in the form $p = \alpha\rho^2 - \rho$, where α is a constant and strictly $\alpha \neq 0$. In order to derive the exact solutions, we utilize volumetric power law and exponential law of expansion. The physical and geometrical aspects of model have discussed.

Keywords: *Quadratic equation of state; $f(R,T)$ gravity; cosmological constant; five-dimensional plane symmetric cosmological model*

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1. INTRODUCTION

Over a significant period of time, scientific understanding suggested that the expansion of the universe was decelerating. However, recent astrophysical observations have indicated that our universe is actually undergoing an accelerated expansion [1, 2, 3, 4, 5]. This phenomenon is attributed to the presence of a mysterious form of energy, known as dark energy, which possesses a negative pressure. Dark energy comprises approximately 69% of the total energy content of the universe, while dark matter constitutes about 26%, and baryonic matter (ordinary matter) makes up the remaining 5%. The precise nature of dark energy and dark matter remains largely unknown. Scientists have proposed various theoretical explanations to account for the accelerated expansion of the universe. One approach involves the consideration of dark energy candidates such as quintessence [6], phantom models [7, 8], polytropic gas models [9], k-essence [10], tachyons [11], chaplygin gas [12, 13], and the cosmological constant Λ. The cosmological constant Λ represents a straightforward and natural candidate for explaining the expansion of the universe. It is essentially a modification to Einstein's field equations, serving as a classical correction factor. Incorporating the cosmological constant into the field equations is an effective means of generating accelerated expansion. However, this approach faces significant challenges, including the fine-tuning problem and the cosmic coincidence problem in cosmology [14, 15]. An alternative avenue involves modifying the geometric component of Einstein's Hilbert action, leading to the formulation of modified theories of gravity, such as $f(R)$ [16], $f(T)$ [17], $f(G)$ [18], $f(R,T)$ [19] theories of gravity. These modified gravity theories play a crucial role in successfully explaining the motion of galaxy clusters and the rotation curves of galaxies within the universe. By altering the underlying gravitational framework, these theories provide alternative explanations for the observed accelerated expansion while addressing certain shortcomings associated with the cosmological constant approach.

Harko et al. [19] have developed a novel modified theory of gravity called $f(R,T)$ gravity, which extends the concept of $f(R)$ gravity. This theory introduces an arbitrary function within the gravitational Lagrangian, involving both the Ricci scalar R and the trace T of the energy-momentum tensor. By employing metric formalization, the researchers derived the dynamic field equations for various choices of the Lagrangian. Several investigations have focused on studying plane symmetric cosmological models within the framework of $f(R,T)$gravity. Chirde and Shekh [20] have explored plane symmetric models of dark energy represented as a wet dark fluid in the context of $f\ (R,T)$ gravity. Pawar and Agrawal [21] examined plane symmetric cosmological models incorporating quark and strange quark matter within the framework of $f(R,T)$ gravity. Shamir [22] have investigated exact static solutions for plane symmetric systems in $f(R,T)$gravity. Shaikh and Bhoyar [23] discussed a deterministic solution of field equation for plane symmetric cosmological model with Λ in modified theory of gravity. Mollah et al. [24] have explored Bianchi type-III universe with quadratic equation of state in

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Lyra geometry and they found the shear free dark energy cosmological model universe for large values of cosmic time t. Katore et al. [25] explored plane symmetric cosmological models incorporating perfect fluid and dark energy within the framework of general relativity (GR). In summary, Harko et al. proposed the $f(R,T)$gravity an extension of $f(R)$ gravity and various researchers have since investigated different aspects of plane symmetric cosmological models within this modified theory. These studies have examined dark energy models, quark matter, static solutions and perfect fluid dynamics within the context of $f(R,T)$ gravity, thereby contributing to our understanding of the universe's behaviour.

In recent times, several researchers have focused on investigating various cosmological models within the framework of modified $f\left(R,T\right)$ gravity, aiming to elucidate the evolution of the universe during both early and late times. Adhav [26] examined a locally-rotationally-symmetric (LRS) Bianchi-I spacetime model assuming a constant expansion rate in the $f\left(R,T\right)=R+2\lambda T$ gravity theory and obtained a solution and they have studied physical behaviour of the universe. However, the solutions presented by the author were found to be mathematically and physically invalid due to an incorrect field equation. Singh and Beesham [27] on the other hand considered the correct field equations and extended the solutions to incorporate a scalar field model (quintessence or phantom). They thoroughly explored the geometrical and physical properties associated with the solutions. Nagpal and her co-authors [28, 29, 30] have also investigated various aspects of $f\left(R,T\right)$ gravity. Katore and Hatkar [31] studied Kantowski-Sachs and Bianchi type III models incorporating a domain wall within the $f\left(R,T\right)$ theory. Singh and Singh [32] examined the anisotropic LRS Bianchi type-I metric with dark energy within the framework of the modified $f\left(R,T\right)$ theory. Aditya et al. [33] analysed a plane-symmetric dark energy model incorporating a massive scalar field. Singh et al. [34] have studied a spherically symmetric spacetime in a 5D setting was explored within the framework of $f\left(R,T\right)$ gravity, where the $f\left(R,T\right)$ gravity theory itself behaves as a dark energy model. Biswal et al. [35] presented a five-dimensional Kaluza-Klein cosmological model within the $f\left(R,T\right)$ theory of gravity, considering the presence of domain walls and obtained the solutions using Berman's proposed special law of variation parameter leading to a constant deceleration parameter. Dasunaidu et al. [36] investigated non-static five-dimensional spherically symmetric cosmological models in the presence of massive strings within the framework of $f\left(R,T\right)$ gravity. Pawar et al. [37] have discussed Bianchi type-V model in presence of perfect fluid with heat conduction using modified theory of gravity.

In the realm of relativity and cosmology, the equation of state plays a significant role as it defines the relationship between combined matter, temperature, pressure, and energy density within any region of space. The quadratic equation of state holds particular importance in Brane world models and the study of dark energy and general relativistic dynamics in different models [38, 39]. Furthermore, the quadratic equation of state governs the evolution of the universe from the Planck epoch to the de-Sitter epoch, making it an increasingly relevant topic. Thus, exploring the quadratic equation of state becomes crucial. The general form of the quadratic equation of state can be expressed as

$$p=\ p_0+\alpha\rho\ +\ B\rho^2 \tag{1}$$

Where, $\alpha,\ \ \beta,\ \ p_0$ are parameters. Equation (1) indicates the first terms of the Taylor expansion of any equation of state parameter of the form $p=p(\rho)$ about $\rho=0$.

Ananda and Bruni [40] examined the Robertson-Walker cosmological model with a non-linear quadratic equation of state. Ananda and Bruni [41] also investigated the impact of the quadratic equation of state, described by the equation,

$$p\ =\ \alpha\rho+\frac{\rho^2}{\rho_c} \tag{2}$$

on anisotropic homogeneous and inhomogeneous cosmological models in general relativity, aiming to achieve isotropization of the universe as the initial singularity is approached.

Nojiri and Odintsov [42] have discussed the modifications to the general equation of state, including inhomogeneous and Hubble parameter-dependent terms, in the late-time universe. Capozziello et al. [43] presented observational constraints on dark energy models with a quadratic equation of state. Nojiri and Odintsov [42] and Capozziello et al. [43] demonstrated that the quadratic equation of state can describe dark energy or unified dark matter. Mahanta et al. [44] have explored Bianchi type-V universe in the context of $f\left(R,T\right)$ gravity for time varying cosmological constant and quadratic equation of state. Aygün et al. [45] have studied Mader space- time in presence of perfect fluid for different quadratic equation of state models in modified $f\left(R,T\right)$ gravity. Rahman [46] discussed an electromagnetic mass model with a quadratic equation of state in the context of general relativity. Chavanis [47] proposed a cosmological model based on a quadratic equation of state, unifying vacuum, radiation, and dark energy. Additionally, Chavanis [48] investigated a cosmological model that describes early inflation, intermediate decelerating expansion, and late accelerating expansion using a quadratic equation of state. Feroze and Siddiqui [49] explored charged anisotropic matter models with a quadratic equation of state in general relativity. Malaver [50] studied strange quark star models with a quadratic equation of state, obtaining a class of models characterized by anisotropic compact spheres, where the gravitational potential Z

depends on an adjustable parameter n. Bhar et al. [51] investigated compact stellar models obeying a quadratic equation of state. Maharaj and Takisa [52] derived new exact solutions of the Einstein-Maxwell field equations by considering a static and spherically symmetric spacetime with charged anisotropic matter distribution and a quadratic equation of state. Sharma and Ratanpal [53] obtained a class of solutions describing the interior of a static spherically symmetric compact anisotropic star, demonstrating that the model admits a quadratic equation of state. Singh and Bishi [54] discussed the Bianchi type-I cosmological model containing perfect fluid with a quadratic equation of state and cosmological constant within the framework of $f(R,T)$ gravity. Singh and Bishi [55] analysed solutions with a quadratic equation of state in the context of $f\ (R,T)$ gravity, including a cosmological constant Λ, for the Bianchi I transit universe, as expressed by equation

$$p = \alpha\rho^2 - \rho \tag{3}$$

where $\alpha \neq 0$ is a constant.

In summary, various researchers have investigated different aspects of the quadratic equation of state in cosmological and stellar models considering its implications for the behaviour of the universe, dark energy, anisotropy, compact objects, and other phenomena in the framework of general relativity and modified gravity theories. Motivated by the previous studies in cosmology, this paper focuses on investigating a higher-dimensional plane symmetric cosmological model that involves a perfect fluid with a quadratic equation of state within the framework of $f(R,T)$ theory. The structure of the paper is outlined as follows:

Section 2 provides a concise overview of the gravitational field equations derived from the modified $f(R,T)$ gravity theory. In Section 3, the metric and field equations for the specific case of $f(R,T) = R + 2f(T)$ are discussed. Section 4 presents the solutions obtained for the metric and field equations using the volumetric power law and exponential expansion law. Section 5 explores the field equations for the case of $f(R,T) = f_1(R) + f_2(T)$, along with their corresponding solutions using power law and exponential expansion laws. Finally, in Section 6, concluding remarks are provided to summarize the findings and implications of the study. In essence, this paper delves into the analysis of a higher-dimensional plane symmetric cosmological model within the $f(R,T)$ theory, specifically focusing on the presence of a perfect fluid with a quadratic equation of state. The paper follows a structured format, presenting the theoretical background, field equations, solutions, and concluding remarks in a coherent manner.

2. GRAVITATIONAL FIELD EQUATIONS OF $f(R,T)$ GRAVITY

The $f(R,T)$ theory of gravity is the modification or generalization of general relativity which is proposed by Harko et al. (2011). The action principle is,

$$s = \frac{1}{16\pi G}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \tag{4}$$

Where, $f(R,T)$ is an arbitrary function of Ricci scalar R and trace T of the energy momentum tensor of matter T_{ij} . L_m is the matter Lagrangian density. The energy momentum tensor of matter is defined as,

$$T_{\alpha\beta} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{\alpha\beta}} \tag{5}$$

on varying the action with respect to metric tensor $g_{\alpha\beta}$, the field equations of $f(R,T)$ gravity are obtained as $f_R(R,T)$ gravity are obtained as

$$f_R(R,T)R_{\alpha\beta} - \frac{1}{2}f(R,T)g_{\alpha\beta} - f_R(R,T)\left(\nabla_\alpha\nabla_\beta - g_{\alpha\beta}\Box\right) = 8\pi T_{\alpha\beta} - f_T(R,T)\left(T_{\alpha\beta} + \Theta_{\alpha\beta}\right) \tag{6}$$

where,

$$\Theta_{\alpha\beta} = -2T_{\alpha\beta} + g_{\alpha\beta}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g_{\alpha\beta}\partial g^{lk}} \tag{7}$$

Here, $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}, \Box = \nabla^\alpha\nabla_\alpha$ where ∇_α is the covariant derivative
Now contraction of equation (6) gives

$$f_R(R,T)R + 3f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)(T + \Theta) \tag{8}$$

where $\Theta = g^{\alpha\beta}\Theta_{\alpha\beta}$ eqn. (8) gives relation between Ricci scalar R and the trace T of energy momentum tensor. In the present study, we assume that the stress energy tensor of matter is given by,

$$T_{\alpha\beta} = (\rho + p)\ u_\alpha u_\beta - pg_{\alpha\beta} \tag{9}$$

Where, p and ρ indicates pressure and density of fluid. Here, $u^{\alpha} = (0,0,0,0,1)$ is the five-velocity vector in co-moving co-ordinate system and satisfies the conditions, $u_{\alpha}u^{\alpha} = 1$ and $u^{\alpha}\nabla_{\beta}\, u_{\alpha} = 0$. We choose matter Lagrangian as $L_m = -p$,which yields the

$$\theta_{\alpha\beta} \; = \; -pg_{\alpha\beta} - 2T_{\alpha\beta} \tag{10}$$

It may be mentioned that these field equations depend on physical nature of matter field. As $f(R,\, T)$ gravity depends on matter field, many theoretical models corresponding to different matter could be derived. The Three classes of three of these models are given as follow

$$f\left(R,T\right) \; = \left\{ \begin{array}{c} R + 2f\left(T\right) \\ f_1\left(R\right) + f_2\left(T\right) \\ f_1\left(R\right) + f_2\left(R\right) f_3\left(T\right) \end{array} \right.$$

In this present work, we have focused on two classes of $f\left(R,T\right)$. i.e. first class $f\left(R,T\right) = \; R + 2f\left(T\right)$ and second class $f\left(R,T\right) = f_1\left(R\right) + f_2\left(T\right)$. for the choice of $f\left(R,T\right) = R + 2f\left(T\right)$ gravitational field equation of $f\left(R,T\right)$ modified gravity with the help of eqn. (8) and (9), eqn. (6) becomes

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R \; = 8\pi T_{\alpha\beta} + 2f^{'}\left(T\right)T_{\alpha\beta} + \left[f\left(T\right) + 2Pf^{'(T)}\right]g_{\alpha\beta} \tag{11}$$

where, an overhead prime denotes differentiation with respect to the argument T.
for the choice of $f\left(R,T\right) = f_1\left(R\right) + f_2\left(T\right)$ gravitational field equation of $f\left(R,T\right)$ gravity with the help of eqn. (8) and (9), eqn. (6) becomes

$$f_1^{'}\left(R\right)R_{\alpha\beta} - \frac{1}{2}f_1\left(R\right)g_{\alpha\beta} + \left(g_{\alpha\beta\Box} - \nabla_{\alpha}\nabla_{\beta}\right)f_1^{'}\left(R\right) = \left(8\pi + f_2'\left(T\right)\right)T_{\alpha\beta} + \left(f_2'\left(T\right)p + \frac{1}{2}f_2\left(T\right)\right)g_{\alpha\beta} \tag{12}$$

3. METRIC AND FIELD EQUATIONS FOR $f(R,T) = R + 2f(T)$

Higher dimensional plane symmetric cosmological model given by

$$ds^2 = dt^2 - {R_1}^2\left(dx^2 + dy^2\right) - {R_2}^2dz^2 - {R_3}^2d\omega^2 \tag{13}$$

here $R_1,\; R_2,\; \; R_3$ are matric potentials which are functions of cosmic time t.
Now using a co-moving coordinate system, the field equations (11) with the help of equation (9) for the metric (13) can be explicitly written as

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} \; = \; \left(8\pi + 4\lambda\right)p \; - \lambda\rho - \Lambda \tag{14}$$

$$2\frac{\ddot{R}_1}{R_1} + 2\frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \left(\frac{\dot{R}_1}{R_1}\right)^2 + \frac{\ddot{R}_3}{R_3} \; = \; \left(8\pi + 4\lambda\right)p \; - \lambda\rho - \Lambda \tag{15}$$

$$2\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \left(\frac{\dot{R}_1}{R_1}\right)^2 + 2\frac{\dot{R}_1\dot{R}_2}{R_1R_2} \; = \left(8\pi + 4\lambda\right)p \; - \lambda\rho - \Lambda \tag{16}$$

$$2\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + 2\frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} + \left(\frac{\dot{R}_1}{R_1}\right)^2 \; = \; -\left(8\pi + 3\lambda\right)\rho \; + 2p\lambda - \Lambda \tag{17}$$

Here overhead dot represents derivative with respect to t.
Dynamical parameters for five-dimensional plane symmetric cosmological model are defined as follows:
The spatial volume $V = a^4\left(t\right) = {R_1}^2R_2R_3$
The directional Hubble parameters

$$H_x \; = H_y \; = \; \frac{\dot{R}_1}{R_1}\; , \; H_z = \frac{\dot{R}_2}{R_2}, H_{\omega} = \frac{\dot{R}_3}{R_3}$$

The generalized mean Hubble's parameter H is given as

$$H = \; \frac{1}{4}\; \left(2\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) \tag{18}$$

The expansion Scalar θ is given by,

$$\theta \ = \ 4H \ = 2\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \tag{19}$$

The Shear Scalar and the mean anisotropic parameter are defined as

$$\sigma^2 = \frac{4}{2}\Delta H^2 \tag{20}$$

and

$$A_m \ = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \tag{21}$$

4. SOLUTIONS OF FIELD EQUATIONS

After solving eqns. (14) – (17), we get

$$\frac{R_1}{R_2} = k_1 exp\left[x_1 \int \frac{dt}{V}\right] \tag{22}$$

$$\frac{R_2}{R_3} = k_2 exp\left[x_2 \int \frac{dt}{V}\right] \tag{23}$$

$$\frac{R_3}{R4} = k_3 exp\left[x_3 \int \frac{dt}{V}\right] \tag{24}$$

where k_1, k_2, k_3 *and* $x_{1,} x_2, x_3$ are constant of integration which satisfies the relation

$$k_3 = k_1 k_2 \ and \ x_3 = x_2 + x_1$$

Using above Eqns. (22) (23) and (24), we can write the metric functions A, B and C explicitly as

$$R_1 = K_1 V^{\frac{1}{4}} \ \ exp\left[X_1 \int \frac{dt}{V}\right] \tag{25}$$

$$R_2 = K_2 V^{\frac{1}{4}} exp\left[X_2 \int \frac{dt}{V}\right] \tag{26}$$

$$R_3 \ = \ K_3 V^{\frac{1}{4}} \ exp\left[X_3 \int \frac{dt}{V}\right] \tag{27}$$

where $K_1, \ K_2, \ K_3$ *and* $X_{1,} \ X_2, \ X_3$ are constant of integration which satisfies the relation
${K_1}^2 K_2 K_3 = 1$ and $2X_1 + X_2 + X_3 \ = \ 0$
Now, using Eq. (14) and (17), we obtain

$$\rho^2 \ = \frac{1}{\alpha\,(8\pi + 2\lambda)} \ \left[\frac{\ddot{R}_1}{R_1} \ + \frac{\ddot{R}_2}{R_2} \ + \frac{\ddot{R}_3}{R_3} - \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} - \left(\frac{\dot{R}_1}{R_1}\right)^2 - \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2}\right] \tag{28}$$

To solve the Einstein's modified field equations for the system having four equations and six unknowns (R_1, R_2, R_3, p, ρ, Λ). To obtain the complete solution, we need two more physically plausible relations.
1. Quadratic Equation of State
2. Expansion Law
Power Law

$$V = V_1 t^n \tag{29}$$

Exponential law

$$V = V_1 e^{4\beta t} \tag{30}$$

4.1. Power Law Model

$$V = V_1 t^n$$

where, V_1 and n are constant.
Then metric potential become

$$R_1 = K_1 {V_1}^{\frac{1}{4}} t^{\frac{n}{4}} exp\left[\frac{X_1}{V_1}\frac{t^{1-n}}{1-n}\right] \tag{31}$$

$$R_2 = K_2 {V_1}^{\frac{1}{4}}\ t^{\frac{n}{4}}\ exp\left[\frac{X_2}{V_1}\frac{t^{1-n}}{1-n}\right] \tag{32}$$

$$R_3 = K_3 {V_1}^{\frac{1}{4}}\ t^{\frac{n}{4}}\ exp\left[\frac{X_3}{V_1}\frac{t^{1-n}}{1-n}\right] \tag{33}$$

As the time t approaching zero, the analysis indicates that all the metric potentials become zero. As a result, the model exhibits an initial singularity.
The directional Hubble parameter $H_x = H_y, H_z, H_\omega$ are given as

$$H_x = H_y = \frac{n}{4t} + \frac{X_1}{V_1 t^n} \tag{34}$$

$$H_z = \frac{n}{4t} + \frac{X_2}{V_1 t^n} \tag{35}$$

$$H_\omega = \frac{n}{4t} + \frac{X_3}{V_1 t^n} \tag{36}$$

Mean Hubble parameter H is given by

$$H = \frac{n}{4t} \tag{37}$$

Anisotropy parameter of the expansion is

$$\Delta = \frac{4X^2}{n^2 {V_1}^2 t^{2(n-1)}} \tag{38}$$

where $2{X_1}^2 + {X_2}^2 + {X_3}^2 = X^2$
Dynamical scalar is given by

$$\theta = 4H = \frac{n}{t} \tag{39}$$

Deceleration parameter q is given by,

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{4}{n} - 1 \tag{40}$$

Shear scalar

$$\sigma^2 = \frac{4}{2}\Delta H^2 = \frac{X^2}{2{V_1}^2 t^{2n}} \tag{41}$$

Using equation (28)

$$\rho = \sqrt{\frac{1}{\alpha\,(8\pi + 2\lambda)}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1 t^n)^2} - \frac{3n}{t^2}\right]} \tag{42}$$

Using Eqn. (42) in Eqn. (3) pressure is obtained as

$$p = \frac{1}{8\pi + 2\lambda}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1 t^n)^2} - \frac{3n}{t^2}\right] - \sqrt{\frac{1}{\alpha\,(8\pi + 2\lambda)}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1 t^n)^2} - \frac{3n}{t^2}\right]} \tag{43}$$

With the help of Eqn. (15)

$$\Lambda = \frac{(4\pi + 2\lambda)}{(4\pi + \lambda)}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1 t^n)^2} - \frac{3n}{t^2}\right] - (8\pi + 5\lambda)\sqrt{\frac{1}{\alpha\,(8\pi + 2\lambda)}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1 t^n)^2} - \frac{3n}{t^2}\right]}$$

$$-6\left(\frac{n}{4t}\right)^2 - \frac{\left[3{X_1}^2 + {X_3}^2 + 2X_1X_3\right]}{(V_1t^n)^2} + \frac{2n}{t^2} \tag{44}$$

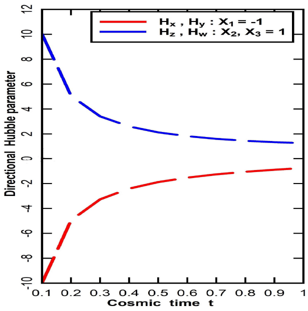

Figure 1.
Directional Hubble Parameter Vs Cosmic time t for $n, V_1, n, X_2, X_3 = 1, X_1 = -1$.

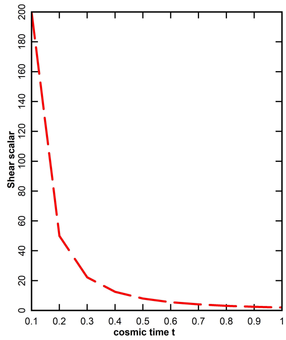

Figure 2.
Shear scalar Vs Cosmic time t for $n, V_1, X_2, X_3 = 1, X_1 = -1$.

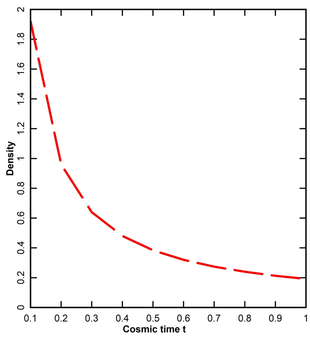

Figure 3.
Density Vs Cosmic time t for α, $V_1, X_2, X_3, \lambda, n = 1, X_1 = -1$.

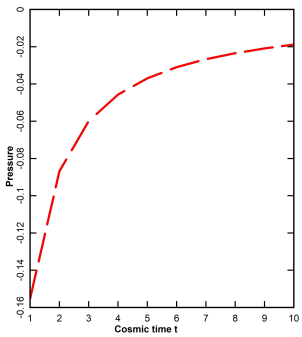

Figure 4.
Pressure Vs Cosmic time t for α, $V_1, X_2, X_3, \lambda, n = 1, X_1 = -1$.

4.2. Exponential Law Model

We consider model for exponential expansion.

$$V = V_1 e^{4\beta t}$$

Then the scale factor can be obtained by using

$$R_1 = K_1 {V_1}^{\frac{1}{4}} e^{\beta t} \exp\left(\frac{-X_1}{4V_1\beta} e^{-4\beta t}\right) \tag{45}$$

$$R_2 = K_2 {V_1}^{\frac{1}{4}} e^{\beta t} \left(\frac{-X_2}{4V_1\beta} e^{-4\beta t}\right) \tag{46}$$

$$R_3 = \; K_3 {V_1}^{\frac{1}{4}} e^{\beta t} exp\left(\frac{-X_3}{4V_1\beta} e^{-4\beta t}\right) \tag{47}$$

Evidently, the metric potentials maintain constant values during the initial period and subsequently undergo time-dependent evolution without encountering any form of singularity. Eventually, they diverge towards infinity. This aligns with the concept of the big bang scenario, reminiscent of the findings presented in the Shaikh and Bhoyar [23].
The directional Hubble parameter are given as

$$H_x = \left(\frac{X_1}{V_1 e^{4\beta t}} + \; \beta\right) \tag{48}$$

$$H_Z = \left(\frac{X_2}{V_1 e^{4\beta t}} + \ \beta \right) \tag{49}$$

$$H_\omega = \left(\frac{X_3}{V_1 e^{4\beta t}} + \beta \right) \tag{50}$$

Mean Hubble parameter

$$H = \beta \tag{51}$$

At the time t equals zero, the directional Hubble parameters possess finite values. These parameters deviate from the average Hubble parameter due to the influence of factor β.
Anisotropy parameter

$$\Delta = \frac{X^2}{4\beta^2 (V_1 e^{4\beta t})^2} \tag{52}$$

where $2{X_1}^2 + {X_2}^2 + {X_3}^2 = X^2$
Dynamical scalar is given by

$$\sigma^2 = \frac{X^2}{2(V_1 e^{\beta t})^2} \tag{53}$$

Deceleration parameter is given by

$$q = -1 \tag{54}$$

With increasing time t, the expansion's anisotropy diminishes exponentially until it reaches null. As a result, the space converges towards isotropy in accordance with this model.
Using the values of metric potentials $R_1, R_2, R_3$and substituting the quadratic equation of state in the form $p \ = \ \alpha\rho^2 - \rho$ we obtained the energy density ρ and Λ.

$$\rho \ = \sqrt{\frac{1}{\alpha\,(8\pi + 2\lambda)} \ \left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3\,)}{(V_1 e^{4\beta t})^2} \right]} \tag{55}$$

$$p = \ \frac{1}{(8\pi + 2\lambda)} \left[\frac{{X_2}^2 + {X_3}^2 - \ \ X_1\,(X_2 + X_3)}{\left(V_1 e^{4\beta t}\right)^2} \right] - \sqrt{\frac{1}{\alpha\,(8\pi + 2\lambda)} \left[\frac{{X_2}^2 + {X_3}^2 - X_1\,(X_2 + X_3)}{(V_1 e^{4\beta t})^2} \right]} \tag{56}$$

$$\Lambda = \frac{(4\pi + 2\lambda)}{(4\pi + \lambda)} \left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3\,)}{(V_1 e^{4\beta t})^2} \right] - \frac{(8\pi + 5\lambda)}{V_1 e^{4\beta t}} \sqrt{\frac{1}{\alpha\,(8\pi + 2\lambda)} \ \left[{X_2}^2 + {X_3}^2 - X_1\,(X_2 + X_3\,) \right]}$$
$$-6\beta^2 - \frac{\left[3{X_1}^2 + {X_3}^2 + 2X_1X_3 \right]}{(V_1 e^{4\beta t})^2} \tag{57}$$

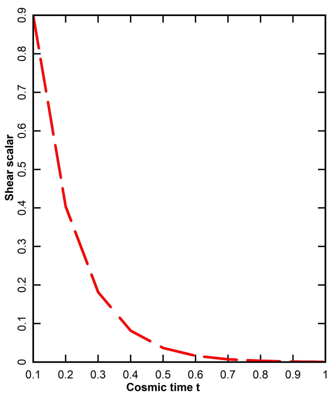

Figure 5.
Shear Scalar vs Cosmic time t for β, $V_1, X_2, X_3 = 1, X_1 = -1$.

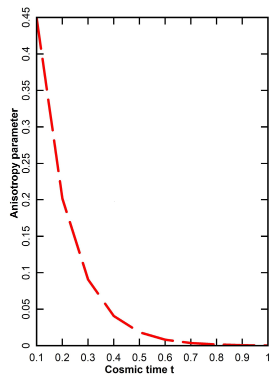

Figure 6.
Anisotropy Parameter vs Cosmic time t for β, $V_1, X_2, X_3 = 1, X_1 = -1$.

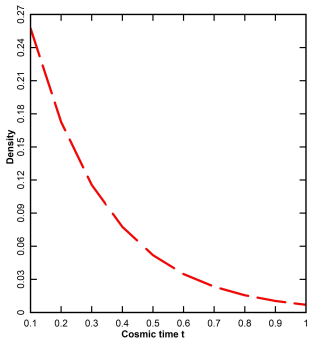

Figure 7.
Density vs Cosmic time t for β, $V_1, X_2, X_3, \lambda, \alpha = 1, \pi = 3.14, X_1 = -1$.

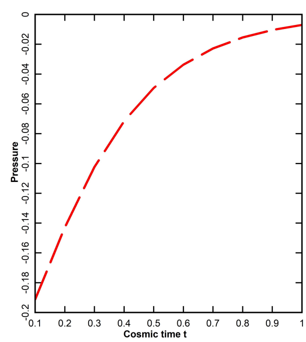

Figure 8.
Pressure vs Cosmic time t for β, $V_1, X_2, X_3, \lambda = 1, \pi = 3.14, X_1 = -1$.

5. FIELD EQUATIONS FOR $f(R,T) = f_1(R) + f_2(R)$

In $f(R,T)$,with the choice of $f_1(R) = \lambda R$, $f_2(T) = \lambda T$ gravitational field equation (12) along with cosmological constant Λ appears as follows:

$$G_{ij} = \left(\frac{8\pi + \lambda}{\lambda}\right) T_{ij} + \left(\frac{\rho - p + 2\Lambda}{2}\right) g_{ij}$$

In this case, field equations are given by

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} = \frac{(8\pi + 2\lambda)}{\lambda}p - \frac{\rho}{2} - \Lambda \tag{58}$$

$$2\frac{\ddot{R}_1}{R_1} + 2\frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \left(\frac{\dot{R}_1}{R_1}\right)^2 + \frac{\ddot{R}_3}{R_3} = \frac{(8\pi + 2\lambda)}{\lambda}p - \frac{\rho}{2} - \Lambda \tag{59}$$

$$2\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \left(\frac{\dot{R}_1}{R_1}\right)^2 + 2\frac{\dot{R}_1\dot{R}_2}{R_1R_2} = \frac{(8\pi + 2\lambda)}{\lambda}p - \frac{\rho}{2} - \Lambda \tag{60}$$

$$2\frac{\dot{R}_1\dot{R}_3}{R_1R_3} + 2\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} + \left(\frac{\dot{R}_1}{R_1}\right)^2 = -\frac{(16\pi + 3\lambda)}{2\lambda}\rho + p - \Lambda \tag{61}$$

5.1. Power law Model

By adopting the same procedure as in subsection 4.1, we have obtained the same metric potential as in equation (31) (32) and (33). Using the values of metric potential R_1, R_2, R_3 and substituting the quadratic equation of state in the form $p = \alpha\rho^2 - \rho$,we obtained the energy density ρ, pressure p and cosmological constant Λ as follows:

$$\rho = \sqrt{\frac{\lambda}{\alpha(8\pi + \lambda)}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1t^n)^2} - \frac{3n}{t^2}\right]} \tag{62}$$

$$p = \frac{\lambda}{8\pi + \lambda}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1t^n)^2} - \frac{3n}{t^2}\right] - \sqrt{\frac{\lambda}{\alpha(8\pi + \lambda)}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1t^n)^2} - \frac{3n}{t^2}\right]} \tag{63}$$

$$\Lambda = \frac{(8\pi + 2\lambda)\ \lambda}{(8\pi + \lambda)}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1t^n)^2} - \frac{3n}{t^2}\right] - \left(\frac{16\pi + 5\lambda}{2\lambda}\right)\sqrt{\frac{\lambda}{\alpha(8\pi + \lambda)}\left[\frac{{X_2}^2 + {X_3}^2 - X_1(X_2 + X_3)}{(V_1t^n)^2} - \frac{3n}{t^2}\right]}$$

$$-6\left(\frac{n}{4t}\right)^2 - \frac{[3{X_1}^2 + {X_3}^2 + 2X_1X_3]}{(V_1t^n)^2} + \frac{2n}{t^2} \tag{64}$$

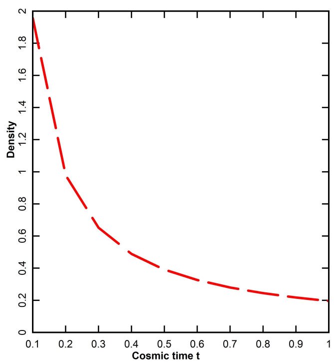

Figure 9.
Density vs Cosmic time t for β,
$V_1, X_2, X_3, \lambda, \alpha = 1, \pi = 3.14, X_1 = -1$.

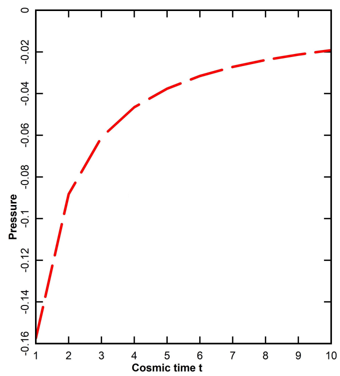

Figure 10.
Pressure vs Cosmic time t for β,
$V_1, X_2, X_3, \lambda, \alpha = 1, \pi = 3.14, , X_1 = -1$.

In this context, we have observed a comparable outcome similar to findings presented in section 4.1

5.2. Exponential Law Model

By adopting the same procedure as in subsection 4.2, we have obtained the same metric potential as in equation (45) (46) and (47). Using the values of metric potential R_1, R_2, R_3 and substituting the quadratic equation of state in the form $p = \alpha\rho^2 - \rho$, we obtained the energy density ρ, pressure p and cosmological constant Λ as follows:

$$\rho = \sqrt{\frac{\lambda}{\alpha\,(8\pi+\lambda)}\left[\frac{{X_2}^2+{X_3}^2-X_1(X_2+X_3)}{(V_1e^{4\beta t})^2}\right]} \tag{65}$$

$$p = \frac{\lambda}{(8\pi+\lambda)}\left[\frac{{X_2}^2+{X_3}^2-X_1\,(X_2+X_3)}{(V_1e^{4\beta t})^2}\right] - \sqrt{\frac{\lambda}{\alpha\,(8\pi+\lambda)}\left[\frac{{X_2}^2+{X_3}^2-X_1(X_2+X_3)}{(V_1e^{4\beta t})^2}\right]} \tag{66}$$

$$\Lambda = \frac{(8\pi+2\lambda)}{(8\pi+\lambda)}\left[\frac{{X_2}^2+{X_3}^2-X_1(X_2+X_3)}{(V_1e^{4\beta t})^2}\right] - \frac{(16\pi+5\lambda)}{2\lambda}\sqrt{\frac{\lambda\,\left[{X_2}^2+{X_3}^2-X_1\,(X_2+X_3)\right]}{\alpha\,(8\pi+\lambda)\,(V_1e^{4\beta t})^2}}$$

$$-6\beta^2 - \frac{\left[3{X_1}^2+{X_3}^2+2X_1X_3\right]}{(V_1e^{4\beta t})^3} \tag{67}$$

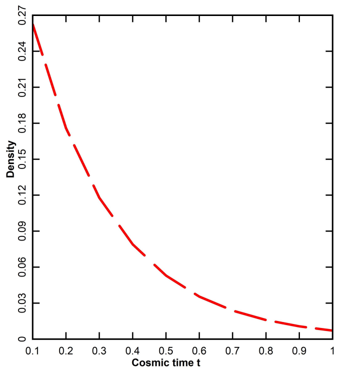

Figure 11.
Density vs Cosmic time t for β,
$V_1, X_2, X_3, \lambda, \alpha = 1, \pi = 3.14, X_1 = -1$.

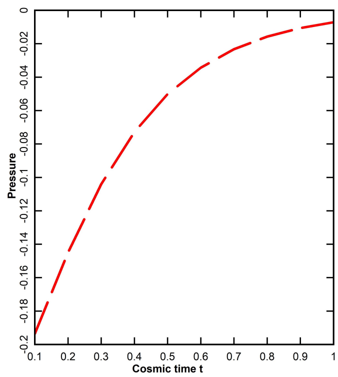

Figure 12.
Pressure vs Cosmic time t for β,
$V_1, X_2, X_3, \lambda, \alpha = 1, \pi = 3.14, , X_1 = -1$.

In this context, we have observed a comparable outcome similar to findings presented in section 4.2

6. OBSERVATION AND DISCUSSION

To decipher the exact solution of five- dimensional plane symmetric cosmological model we assumed power law expansion and exponential expansion law, in the previous section. We have found that

- In section 4.1, for Power Law Model.
  - Figure 1 represent that he directional Hubble parameter H_x and H_y in the direction x and y are increasing function of cosmic time t whereas directional Hubble parameter H_z , H_w in the direction z, w are decreasing function of cosmic time t.
  - Now from Figure 2 and equation (41), it is clear that the shear scalar is decreasing function of cosmic time t. At initial epoch, when time $t = 0$ shear scalar start with infinite value and vanishes as $t \to \infty$.
  - Figure 3 of equation (42), illustrate the variation of energy density ρ against time t . Here it is observed that energy density is positive function that decreases over time.
  - By analysing Figure 4 of equation (43), it is observed that pressure is increasing function of time. Initially, it begins from large negative value and gradually approaches to small negative value close to zero.
- In section 4.2, for exponential expansion model
  - From Figure 5 and 6, it is clear that shear scalar and anisotropy parameter are decreasing function of time t whereas for large time t universe approaches to isotropy.
  - The graph of energy density and pressure are shown in Figure 7 and Figure 8 respectively. Figure 7 shows that energy density is positive decreasing function over time t.
  - Figure 8 reveals that the pressure starts from significantly large negative value and approaches to small negative value close to zero. This result consistent with prior studies referenced in reference [23]
- In section 5
  - In this context, we have observed a comparable outcome similar to findings presented in section 4.1 and 4.2. for power law model we found that pressure Figure 10 and density Figure 9 exhibited identical graphs to pressure Figure 4 and density Figure 3
  - Similarly, for exponential expansion model, we found that density Figure 11 and pressure Figure 12 exhibited identical graphs to density Figure 7 and pressure Figure 8

7. CONCLUSION

We investigated the intricate details of a five-dimensional plane symmetric cosmological model governed by $f(R, T)$ gravity, taking into consideration the influence of a cosmological constant and employing a quadratic equation of state. The motivation for this inquiry originates from the desire to investigate modified gravity theories that go beyond the traditional general relativity framework, allowing for a more thorough understanding of the universe's behaviour and evolution. We studied two unique classes of functionals within the $f(R, T)$ gravity framework. The first is represented by $f(R, T) = R + f(T)$, and the second by $f(R, T) = f1(R)\ f2(T)$. To examine the model's behaviour, we used a quadratic equation of state and an expansion rule to find an exact solution to the field equations, yielding vital insights into the evolution of the cosmos within the considered framework. From both the models of $f(R, T)$ we have following findings.

In power law model, several intriguing findings emerge from our investigation. Because of the positive average Hubble parameter ($H > 0$), we see an expanding cosmos. However, as time passes towards infinity, the expansion slows and eventually approaches zero. This behaviour is consistent with our assumptions and lends support to the idea of a universe approaching zero expansion asymptotically. Interestingly, we discover that the pace of expansion of the universe is extremely fast during the early stages of cosmic time ($0 < t < 1$), followed by a slowing of the growth for $t > 1$. This behaviour implies a transition from a quick early phase to a more steady expansion over time. These findings are consistent with prior studies referenced in references [56, 57], which validates our methodology and increases the credibility of our findings. Furthermore, we have detected anisotropy and shearing in the cosmos throughout its whole existence. This implies that the universe has directional deviations and a lack of perfect symmetry, which could have serious ramifications for the distribution and evolution of matter on cosmic scales.

Additionally, our analysis indicates that the universe undergoes accelerated expansion for values of the parameter $n > 4$, as deduced from equation (40). This finding aligns with current observations [58], providing further support for the validity of the model and its ability to reproduce essential features of the real universe.

Examining the evolution of physical quantities, we find that the density of the cosmic fluid decreases as cosmic time progresses, as depicted in Figure 3. This decrease in density suggests that the universe becomes less dense as it expands, a result that is in line with our expectations and consistent with our understanding of cosmological expansion. Furthermore, our investigation reveals the presence of negative pressure, as illustrated in Figure 4. This negative pressure is a characteristic feature of dark energy. Its presence within the model lends support to the notion that dark energy plays a significant role in shaping the dynamics of our cosmos.

In exponential expansion model, we have obtained significant findings regarding this particular model including the deceleration parameter (q) and the rate of change of Hubble parameter $\left(\frac{dH}{dt}\right)$.These findings are indicative of the model's ability to provide highly accurate values for the Hubble parameter, while effectively representing both the inflationary era during the early stages of the universe and the late-time evolution. In this model, the directional Hubble parameters have finite values at both the beginning $t \to 0$ and the far future $t \to \infty$ while the mean Hubble parameter remains constant. The expansion scalar, which measures the rate of expansion of the universe remains constant throughout its evolution, indicating uniform exponential expansion.Anisotropic expansion of universe measures constant value at initial time t while it decreases as time progresses and finally tends to zero at infinite time.From Figure 5 the shear scalar is finite at initial epoch while the Shear Scalar approaches zero as time approaches infinity i.e. $\sigma \to 0$ as $t \to \infty$. This indicates that the anisotropy of the universe diminishes over time, eventually tending towards isotropy. The sign of the deceleration parameter q determines whether the universe is accelerating or decelerating. A positive value of q corresponds to the standard decelerating model while a negative value indicates acceleration. Current cosmological observations support the notion that the expansion of the universe is accelerating at present, whereas it was decelerating in the past. From the equation (54), we find that the deceleration parameter is negative aligning with the current observations of Type Ia supernovae and the cosmic microwave background (CMB). This result resembles with the Singh and Bishi [54].The expansion scalar, which measures the rate at which the universe expands remains constant throughout the entire evolution, thereby indicating a uniform exponential expansion.At the initial time the anisotropy parameter remains constant and diminishes as time advances.This implies that the universe was initially anisotropic but steadily moves towards isotropy as time elapses. This behavior is visually depicted in Figure5.

Overall, our study provides a comprehensive analysis of a five-dimensional plane symmetric cosmological model within the framework of $f(R,T)$ gravity. By incorporating a quadratic equation of state and exact solutions to the field equations, we have gained valuable insights into the nature of the universe, its expansion, anisotropy, and the presence of dark energy. Our findings not only contribute to the advancement of theoretical cosmology but also align with current observations, consolidating our understanding of the universe's behavior at both early and late cosmic timescales.

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П'ЯТИВИМІРНА ПЛОСКА СИМЕТРИЧНА КОСМОЛОГІЧНА МОДЕЛЬ З КВАДРАТНИМ РІВНЯННЯМ СТАНУ В $f(R,T)$ ТЕОРІЇ ГРАВІТАЦІЇ

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У цій статті ми проаналізували п'ятивимірну плоску симетричну космологічну модель, що містить ідеальну рідину, у контексті $f(R,T)$ гравітації. Рівняння поля розв'язані для двох класів $f(R,T)$ гравітації, тобто $f(R,T) = R + f(T)$ і $f(R,T) = f_1(R)f_2(T)$ із включенням космологічної сталої Λ і квадратного рівняння параметрів стану у вигляді $p = \alpha\rho^2 - \rho$, де α — константа і строго $\alpha \neq 0$. Щоб отримати точні рішення, ми використовуємо об'ємний степеневий закон і експоненціальний закон розширення. Розглянуто фізичні та геометричні аспекти моделі.

Ключові слова: *Квадратне рівняння стану; $f(R,T)$ гравітація; космологічна стала; п'ятивимірна плоска симетрична космологічна модель*