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RENYI HOLOGRAPHIC DARK ENERGY MODEL IN *f(R)* GRAVITY WITH HUBBLE'S IR CUT-OFF[†]

Kishor S. Wankhade[a#], Alfred Y. Shaikh[b$], Siraj N. Khan[c*]
[a]*Department of Mathematics, Yashwantrao Chavan Arts and Science Mahavidyalaya, Mangrulpir Dist. Washim - 444403, India*
[b]*Department of Mathematics, Indira Gandhi Mahavidyalaya, Ralegaon - 445402, India*
[c]*Prof. Ram Meghe College of Engineering and Management, Badnera, Amravati - 444701, India*
[#]*E-mail: wankhade.kishor@rediffmail.com;* [$]*E-mail: shaikh_2324ay@yahoo.com*
**Corresponding Author e-mail: sirajnk2311@gmail.com*
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In the present study, a homogeneous and anisotropic LRS Bianchi type-I universe model is considered with an interacting dark matter and Renyi holographic dark energy model (RHDE) in $f(R)$ gravity. The deceleration parameter (DP) shows a signature flipping for a universe which was decelerating in past and accelerating at present epoch. Therefore, the DP is a most physically justified parameter to analyze the solution of cosmological model. In order to find an exact solution of the field equations of the model, the shear scalar is considered to be proportional to the expansion scalar. We have considered $f(R)=bR^n$, the depiction model of $f(R)$ which is the function of Ricci scalar R. The physical and geometrical characteristics of the universe model have been studied.
Keywords: *f*(*R*) *Gravity; RHDE; dark matter; Cosmology; Bianchi type-I space-time*
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INTRODUCTION

Observational cosmic data show that our Universe is currently expanding at a faster rate [1–5]. Dark energy (DE), which has negative pressure and accounts for 70% of the exotic component, is what propels the universe's cosmic expansion [6–9]. To investigate the universe and its accelerated expansion, modified theories of gravity provide an alternative approach. Some appropriate characteristics of modified theories of gravity are found in [10]. In the literature, several modified theories, including $f(R)$ gravity [11–15], $f(T)$ gravity [16–20], and $f(G)$ gravity [21–23] have been proposed with the changes of the Einstein–Hilbert action. Many researchers have worked on modified theories of gravity in recent past on different aspects of Cosmology [24-34]. In fact, the Ricci scalar $f(R)$ theory uses a conventional Einstein-Hilbert action that contains an arbitrary function R. The authors of Nojiri et al. [35] provided a comprehensive overview of modified theories of gravitation. Theoretical models of workable dark energy are described in [36]. The Noether symmetry technique is used to show spherically symmetric solutions in [37]. The exact solutions of static spherically symmetric space-times in $f(R)$ gravity coupled to nonlinear electrodynamics have been studied by Hollenstein and Lobo [38]. $f(R)$ gravity has been studied by a number of researchers in various cosmological contexts [39–53].

Holographic dark energy (HDE) has a variety of characteristics that have been studied in [54–58]. In [59–61], the holographic concept serves as the foundation for the potential of HDE. The HDE theory is also a helpful approach for addressing the DE conundrum in [62]. It was put forth based on the quantum characteristics of black holes (BH), which have been thoroughly studied in the literature to research quantum gravity. Studying the cosmic ramifications of holographic dark energy is more natural because Newton's gravitational constant is made dynamical in the Scalar Tensor Theory. According to [63], the holographic principle refers to a system's entropy, which is determined by its surrounding surface area rather than its volume. If we assume that the infrared (IR) cutoff is equal to the size of the universe, then the holographic energy density is rather near to the dark energy density. We can discover the cosmological characteristics of the vacuum energy with the aid of the HDE theory. The decreased Planck mass $M_p^2=8\pi G$ and the numerical constant d are used to calculate the HDE energy density $\rho_{de}=3d^2M_p^2L^{-2}$. Numerous investigations have examined the interaction of holographic dark energy with matter using various IR cutoffs, including particle horizons, future horizons, and Hubble horizons. The authors of [64] suggested an IR cut-off made up of local Hubble scale values and temporal derivative Hubble scales. Sheykhi et al. in [65] explore the astrophysical implications of New Holographic DE (NHDE) by using the Hubble radius $L=H^{-1}$ as the system's IR cutoff. Many extended entropy formalisms have been used to investigate cosmological and gravitational events, but Tsallis and Renyi entropies offer the most accurate universe model. Sharma-Mittal HDE is compatible with the expansion of the universe and it is stable whenever it dominates the cosmos. The horizon is assigned to the Tsallis and Renyi entropies to investigate the cosmic ramifications. The generalized entropies

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have been used to develop three HDE models: the Tsallis HDE (THDE) [66], the Renyi HDE (RHDE) [67] and the Sharma-Mittal HDE (SMHDE) [68]. Jawad and colleagues studied the THDE, RHDE and SMHDE models [69]. Maity and Debnath have examined all the aforementioned HDE models as well as New Agegraphic DE (NADE) in the context of flat D-dimensional fractal Universe in loop quantum cosmology [70]. The same authors analysis of THDE, RHDE, and SMHDE in [71] used the Nojiri- Odinstov (NO) cut-off as the IR cut-off. In [72], Sharma and Dubey looked at the RHDE in relation to Brans-Dicke cosmology. An interacting model of the Renyi holographic dark energy in the Brans-Dicke theory of gravity was built by the authors in [73]. Numerous relativists have recently worked on RHDE in various cosmological contexts, as seen in [74-83].

The main goal for this work is to illuminate the cosmic expansion for a homogeneous and anisotropic LRS Bianchi type I universe model with an interacting dark matter and RHDE within the framework of $f(R)$ gravity by taking the Hubble horizon as a candidate for the IR-cutoff i.e., $L = 1/H$. The deceleration parameter (DP) shows a signature flipping for a universe which was decelerating in past and accelerating at present epoch. Therefore, the DP is a most physically justified parameter to analyze the solution of cosmological model. In order to find an exact solution of the field equations of the model, the shear scalar is considered to be proportional to the expansion scalar. We have considered $f(R) = bR^n$, the depiction model of $f(R)$ which is the function of Ricci scalar R.

$f(R)$ GRAVITY FORMALISM

The $f(R)$ gravity is one of the generalizations of the general theory of relativity. The three primary approaches to $f(R)$ gravity are "affine gravity", "Palatini formalism" and "metric approach". The action is varied with regard to the metric tensor and the connection used in the metric approach is a Levi-Civita connection. While in Palatini formalism, the metric and the connection are independent of one another and the two aforementioned parameters can vary on their own. In metric-affine $f(R)$ gravity, the metric tensor and connection both operate independently, with the assumption that the matter action also depends on the connection. Action in this theory is provided by

$$S = \int \sqrt{-g}\left(f(R) + L_m\right) d^4x \,. \tag{1}$$

Here, g the metric determinant, L_m the matter Lagrangian and a general function $f(R)$ of the Ricci scalar are present. It should be noted that this action can be obtained simply by replacing R by $f(R)$ in the default Einstein-Hilbert action. From this action, the associated field equations are discovered as

$$F(R)R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \Box F(R) = -T_{\mu\nu}\,, \tag{2}$$

where $\Box \equiv \nabla^\mu \nabla_\mu$, $F(R) \equiv df(R)/dR$, $T_{\mu\nu}$ the standard matter energy momentum tensor is obtained from the Lagrangian L_m, and ∇_μ is the covariant derivative.

METRIC AND FIELD EQUATION COMPONENTS

Cosmologies with anisotropic models are being investigated in recent times. The spatially homogeneous and anisotropic Bianchi type models are more interesting because of their ability to explain the cosmic evolution in early phase of universe. More ever, Bianchi type models have simple mathematical forms. In this work, we are interested to explore $f(R)$ gravity using Locally Rotationally Symmetric (LRS) Bianchi type-I space-time. The line element is of the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)\left[dy^2 + dz^2\right], \tag{3}$$

where A and B are functions of cosmic time t only.
Consider the stress energy momentum tensor for interacting two fluid as

$$T_{\mu\nu} = \overline{T}_{\mu\nu} + \hat{T}_{\mu\nu} \tag{4}$$

where, $\overline{T}_{\mu\nu} = \rho_m u_\mu u_\nu$ and $\hat{T}_{\mu\nu} = (\rho_r + p_r)u_\mu u_\nu + p_r g_{\mu\nu}$, with comoving coordinates $u^\mu = (0,0,0,1)$ and $u^\mu u_\nu = -1$, where u^μ is the four velocity vector of the fluid, p_r is pressure of RHDE, ρ_m and ρ_r are energy densities of dark matter and RHDE respectively. Now we define some kinematical quantities of the space-time such as average scale factor and volume respectively as

$$a^3 = V = AB^2 \,. \tag{5}$$

The mean Hubble parameter, which expresses the volumetric expansion rate of the universe is

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{6}$$

where H_1, H_2 and H_3 are the directional Hubble parameters in the directions of *x*, *y* and *z* axes respectively. Anisotropy parameter, for discussing whether universe approach isotropy or not, is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{7}$$

The expansion scalar and shear scalar are respectively defined as

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \tag{8}$$

$$\sigma^2 = \frac{3}{2}H^2 A_m. \tag{9}$$

Here dot indicates a derivative with regard to time.
Using equations (2), (3), and (4), the field equations can be expressed as follows

$$\left(\frac{\ddot{A}}{A} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right)F(R) + \frac{1}{2}f(R) + 2\frac{\dot{B}}{B}\dot{F} + \ddot{F} = p_r, \tag{10}$$

$$\left(\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right)F(R) + \frac{1}{2}f(R) + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{F} + \ddot{F} = p_r, \tag{11}$$

$$\left(\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B}\right)F(R) + \frac{1}{2}f(R) + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{F} = -\left(\rho_m + \rho_r\right). \tag{12}$$

SOLUTION OF THE FIELD EQUATIONS

The deceleration parameter is defined in terms of the scale factor and scale factor is a function of time. So, it always motives the researchers to investigate the time dependent deceleration parameter rather than the constant deceleration parameter. Thus, we have interested to investigate the time dependent deceleration parameter. The deceleration parameter is defined as $q = -a\ddot{a}/\dot{a}^2$, where a is the average scale factor. We have considered the special form of time varying deceleration parameter $q = -1 + \beta/1 + a^{\beta}$, where $\beta > 0$ is a constant. Consequently, the Hubble's parameter is

$$H = 1 + a^{-\beta}. \tag{13}$$

Again, integrating the above equation, we have

$$a = \left(e^{\beta t + c} - 1\right)^{1/\beta}. \tag{14}$$

Using equation (5), we get $AB^2 = \left(e^{\beta t + c} - 1\right)^{3/\beta}$. Now, in order to solve the field equations, we take into account that expansion scalar is proportional to shear scalar [84-85] which results as $A = B^m$ where $m > 1$ is an arbitrary constant. Hence, we get

$$A = \left(e^{\beta t + c} - 1\right)^{\frac{3m}{\beta(m+2)}}, \tag{15}$$

$$B = \left(e^{\beta t + c} - 1\right)^{\frac{3}{\beta(m+2)}}, \tag{16}$$

Using equations (15) and (16), the metric in (3) filled with the fluid in (4) in the framework of $f(R)$ gravity becomes

$$ds^2 = -dt^2 + \left(e^{\beta t + c} - 1\right)^{\frac{6m}{\beta(m+2)}} dx^2 + \left(e^{\beta t + c} - 1\right)^{\frac{6}{\beta(m+2)}}\left[dy^2 + dz^2\right]. \tag{17}$$

TWO FLUIDS INTERACTING MODEL

When dark matter and holographic Renyi dark energy interact, the overall energy density fulfils the continuity equation as $(\dot{\rho}_m)+(\dot{\rho}_r)+3H(\rho_m+\rho_r+p_r)=0$. However, when their individual energy densities do not conserve, the equation for the continuity of matter becomes [86],

$$(\dot{\rho}_m)+3H(\rho_m)=Q, \tag{18}$$

$$(\dot{\rho}_r)+3H(\rho_r+p_r)=-Q. \tag{19}$$

The quantity $Q\geq 0$, expresses the interaction (for $Q>0$) and non-interaction ($Q=0$) term between RHDE and dark matter components. Since we are interested in investigating the interaction between RHDE and dark matter, it should be noted that an ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction Q. In our work, we consider the interacting term as $Q=3\kappa H\rho_m$, where κ is the coupling constant to determine its appropriateness.

DYNAMICAL PROPERTIES OF THE MODEL

The average scale factor and spatial volume are respectively obtained as

$$a=\left(e^{\beta t+c}-1\right)^{1/\beta}, \tag{20}$$

$$V=a^3=\left(e^{\beta t+c}-1\right)^{3/\beta}. \tag{21}$$

The mean Hubble parameter and expansion scalar yield as

$$H=\frac{1}{\left(1-e^{-(\beta t+c)}\right)}, \tag{22}$$

$$\theta=\frac{3}{\left(1-e^{-(\beta t+c)}\right)}. \tag{23}$$

The anisotropy parameter is

$$A_m=\frac{2(m-1)^2}{(m+2)^2}. \tag{24}$$

The shear scalar is determined as

$$\sigma^2=\frac{3(m-1)^2}{(m+2)^2}\frac{1}{\left(1-e^{-(\beta t+c)}\right)^2}. \tag{25}$$

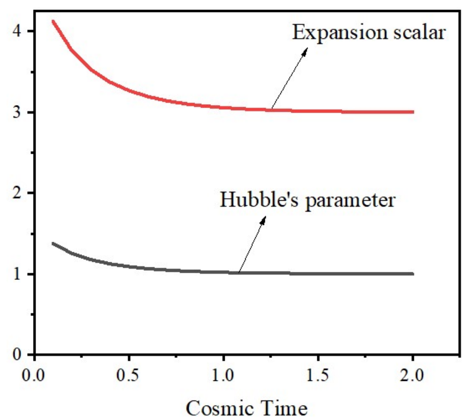

Figure 1. Hubble parameter and Expansion scalar versus cosmic time for $c=1$, $m=2$ and $\beta=3$

It can be shown that the universe is expanding as cosmic time advances because both the average scale factor and spatial volume are rising. It is depicted from Figure 1 that the mean Hubble parameter is acting as a constant throughout the universe's expansion and that the expansion scalar gets less as cosmic time grows. Expansion scalar is constant because of $t\to\infty$. It implies that the universe's expansion is initially more rapid and then it slows down over time. Anisotropy parameter is observed to be constant during the expansion of the universe and does not depend on cosmic time t in contrast to shear scalar, which is observed to depend on cosmic time and change over the course of the universe's evolution.

The deceleration parameter is obtained as

$$q=-1+\beta e^{-(\beta t+c)} \tag{26}$$

The DP shows a transition of universe for β > 1 and again lies at an accelerating phase for β ≤ 1. Summing up the results, it can be concluded that, the deceleration parameter plays a vital role in account of accelerated expansion of the universe. The model with time varying deceleration parameter represents an expanding universe in accelerated phase. From Figure 2, it is noted that the deceleration parameter, which reflects the typical accelerating expansion, $q\to -1$ is negative throughout the expansion of the universe. As cosmic time lengthens, recent theoretical observations are consistent with this scenario.

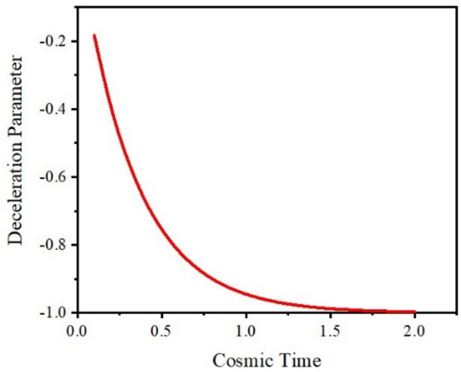

Figure 2. Deceleration parameter versus cosmic time for $c=1$ and $\beta=3$.

The Ricci scalar R is obtained as

$$R=\frac{(-6)}{M_1\left(1-e^{-(\beta t+c)}\right)}\left(\beta M_1+\frac{M_2}{\left(1-e^{-(\beta t+c)}\right)}\right), \tag{27}$$

where $M_1=(m+2)^2$ and $M_2=3\left(m^2+2m+3\right)-\beta(m+2)^2$.

The depiction model of $f(R)$ is

$$f(R)=b\left\{\frac{(-6)}{M_1\left(1-e^{-(\beta t+c)}\right)}\left(\beta M_1+\frac{M_2}{\left(1-e^{-(\beta t+c)}\right)}\right)\right\}^n. \tag{28}$$

Also,

$$F(R)=bn\left\{\frac{(-6)}{M_1\left(1-e^{-(\beta t+c)}\right)}\left(\beta M_1+\frac{M_2}{\left(1-e^{-(\beta t+c)}\right)}\right)\right\}^{n-1}. \tag{29}$$

Energy density for RHDE has the form

$$\rho_r=\frac{3d^2}{8\pi L^2}\left(1+\pi\delta L^2\right)^{-1},$$

with d and δ being constants [67], [69]. Here, by considering the Hubble horizon as a candidate for the IR-cutoff, i.e., $L=1/H$, the energy density for RHDE from above equation is obtained as

$$\rho_r=\frac{3d^2}{8\pi}\frac{1}{\tau^2\left(1+\pi\delta\tau^2\right)}, \tag{30}$$

where, $\tau=1-e^{-(\beta t+c)}$.

The isotropic pressure is

$$p_r = \frac{3}{(m+2)\tau}\left\{\left[\beta m + \frac{m(3m-\beta(m+2)+6)}{(m+2)\tau}\right]F + 2\dot{F}\right\} + \frac{1}{2}f + \ddot{F}\,, \tag{31}$$

where

$$\dot{F} = -bn(n-1)\beta\left(\frac{-6}{M_1}\right)^{n-1}\left(\frac{1-\tau}{\tau}\right)\left(\beta M_1 + \frac{2M_2}{\tau}\right)\left(\beta M_1 + \frac{M_2}{\tau}\right)^{n-2},$$

and

$$\ddot{F} = \left\{\begin{array}{l} bn(n-1)\left(\frac{-6}{M_1}\right)^{n-1}\left(\frac{1-\tau}{\tau^n}\right)\left(\beta M_1 + \frac{M_2}{\tau}\right)^{n-3} \\ \left[\left\{\left(\beta M_1 + \frac{M_2}{\tau}\right)\left[2M_2\left(\frac{1-\tau}{\tau^2}\right) + \left(\beta M_1 + \frac{2M_2}{\tau}\right)\left(1 + 2\frac{(1-\tau)}{\tau}\right)\right] + (n-2)\left(\frac{1-\tau}{\tau}\right)\left(\beta M_1 + \frac{2M_2}{\tau}\right)^2\right\}\right] \end{array}\right\}$$

The RHDE equation of state parameter is found to be

$$\omega_r = \frac{8\pi}{3d^2}\tau^2\left(1+\pi\delta\tau^2\right)\left\{\frac{3}{(m+2)\tau}\left\{\left[\beta m + \frac{m(3m-\beta(m+2)+6)}{(m+2)\tau}\right]F + 2\dot{F}\right\} + \frac{1}{2}f + \ddot{F}\right\}. \tag{32}$$

The RHDE energy density is always positive and decreases with cosmic time t as depicted in Figure 3. As the cosmos expands, its energy density drops until it reaches zero at a massive expansion. In Figure 4, RHDE pressure versus cosmic time t is depicted graphically.

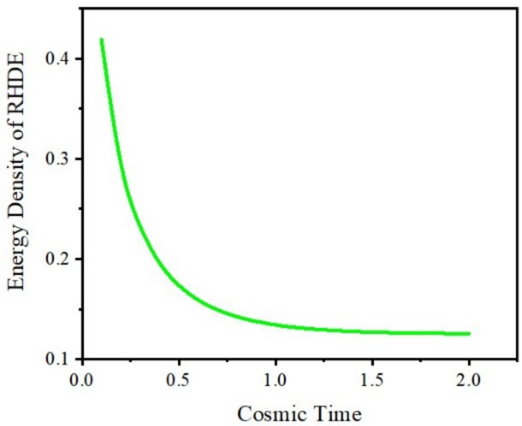

Figure 3. Energy density of RHDE versus cosmic time for $\beta = 3$, $c = 1$, $d = 1.2$ and $\delta = 4$

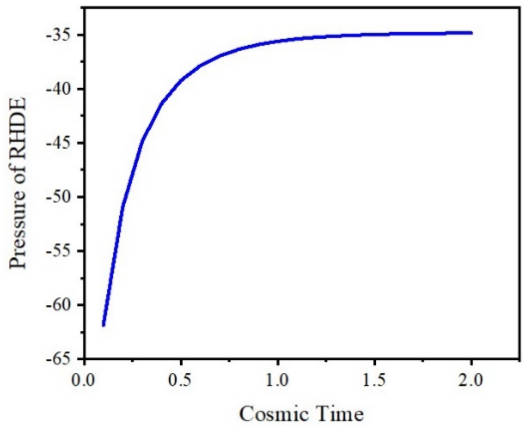

Figure 4. Pressure of RHDE versus cosmic time for $\beta = 3$, $c = 1$, $m = 2$, $n = 2$ and $b = 1$.

It is evident that RHDE exerts a negative pressure throughout the universe's expansion. The year 2009 saw a result that limits the dark energy equation of state parameter to $-1.44 < \omega_r < -0.92$ by combining cosmic data sets from galaxy clustering, CMBR anisotropy, and luminosity distances of high redshift SNe-Ia.

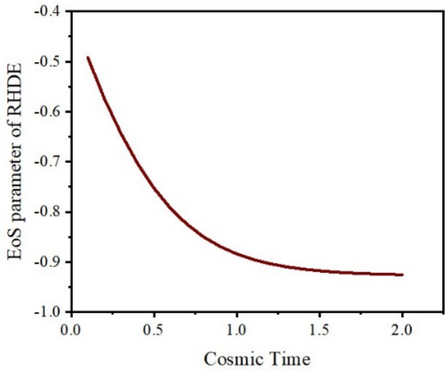

Figure 5. EoS parameter of RHDE versus cosmic time for $\beta = 3$, $c = 1$, $d = 1.2$, $\delta = 4$, $m = 2$, $n = 2$ and $b = 1$.

In Figure 5, it is shown that the equation of state parameter initially reflects the quintessence region and that as time goes on, it evolves around the ΛCDM model. The equation of state parameter behaves like a cosmological constant $\omega_r = -1$ as the universe expands, which is consistent with recent theoretical results.

Stability factor is

$$\vartheta_r^2 = -\frac{8\pi}{3d^2\beta}\left(\frac{\tau^2}{1-\tau}\right)\frac{\left(1+\pi\delta\tau^2\right)}{\left[\frac{\pi\delta}{\left(1+\pi\delta\tau^2\right)}+\frac{2}{\tau}\right]}\left\{\frac{1}{2}f+\dddot{F}+\frac{3}{(m+2)\tau}\left\{\begin{array}{l}-\beta\left(\frac{1-\tau}{\tau}\right)\left\{\left[\beta m+\frac{2m\left(3m-\beta(m+2)+6\right)}{(m+2)\tau}\right]F+2\dot{F}\right\}\\ +\left[\beta m+\frac{m\left(3m-\beta(m+2)+6\right)}{(m+2)\tau}\right]\dot{F}+2\ddot{F}\end{array}\right\}\right\} \tag{33}$$

where

$$\dddot{F}=\left\{\begin{array}{l}-bn(n-1)\beta\left(\frac{-6}{M_1}\right)^{n-1}\left(\tau_1\right)^{n-3}\left(\frac{1-\tau}{\tau^n}\right)\\ \left\{\left(\frac{1-\tau}{\tau^2}\right)\left\{2\tau_2\tau_3+\frac{M_2}{\tau}\left[2M_2\left(\frac{3-2\tau}{\tau}\right)+\beta M_1(2-\tau)\right]+(n-2)\tau_1\tau_3\right\}\right.\\ \left.+\left\{\tau_1\left[2M_2\left(\frac{1-\tau}{\tau^2}\right)+\tau_2\left(\frac{2-\tau}{\tau}\right)\right]+(n-2)\left(\frac{1-\tau}{\tau}\right)\tau_2^2\right\}\left\{(n-3)M_2\left(\frac{1-\tau}{\tau^2\tau_1}\right)+\left(1+n\left(\frac{1-\tau}{\tau}\right)\right)\right\}\right\}\end{array}\right\},$$

and

$$\tau_1=\left(\beta M_1+\frac{M_2}{\tau}\right),\ \tau_2=\left(\beta M_1+\frac{2M_2}{\tau}\right),\ \tau_3=\left[2M_2\left(\frac{3-\tau}{\tau}\right)+\beta M_1\right].$$

From Figure 6, it is clear that the universe is unstable because RHDE has a negative stability factor during the universe's expansion.

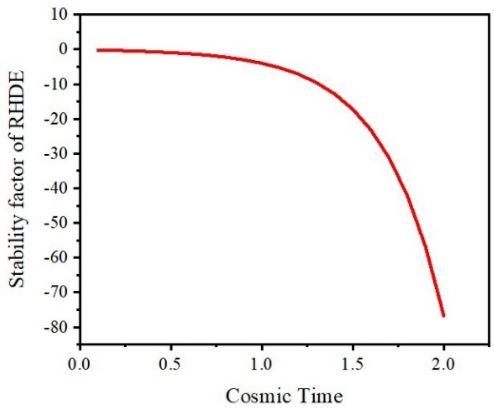

Figure 6. Stability factor of RHDE versus cosmic time for $\beta=3$, $c=1$, $d=1.2$, $\delta=4$, $m=2$, $n=2$ and $b=1$.

CONCLUSION

In the present study, a homogeneous and anisotropic LRS Bianchi type I universe model is considered with an interacting dark matter and RHDE in $f(R)$ gravity.

- It is found that the energy density of RHDE is always positive and decreases as a function of cosmic time t.
- As time goes on, the equation of state parameter evolves around the ΛCDM model, which is familiar from the universe's current accelerated expansion. The equation of state parameter initially represents the quintessence area [87-88].
- The average scale factor and the spatial volume increase with increasing cosmic time.
- Expansion scalar is constant because of $t \to \infty$. It implies that the universe expands more quickly initially and then less quickly as time goes on.
- The deceleration parameter is negative during the entire universe's expansion, which is consistent with the usual accelerating expansion [89-90].
- The stability factor has been negative during the universe's expansion, indicating that the cosmos is unstable.
- As cosmic time lengthens, recent theoretical observations are consistent with this scenario.

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ORCID
Alfred Y. Shaikh, https://orcid.org/0000-0001-5315-559X; **Siraj N. Khan,** https://orcid.org/0000-0003-2246-7050

REFERENCES

[1] A.G. Riess, *et al*., (Supernova Search Team): Astron. J. **116**, 1009 (1998). https://doi.org/10.1086/300499
[2] A.G. Riess *et al*., (Supernova Search Team): Astron. J. **607**, 665 (2004). https://doi.org/10.1086/383612
[3] D.N. Spergel *et al*., Astrophys. J. Suppl. Ser. **148**, 175 (2003). https://doi.org/10.1086/377226
[4] D.N. Spergel *et al*., Astrophys. J. Suppl. Ser. **170**, 377 (2007). https://doi.org/10.1086/513700
[5] M. Tegmark *et al*., Phys. Rev. D, **69**, 103501 (2004). https://doi.org/10.1103/PhysRevD.69.103501
[6] P. Astier, and R. Pain, Comptes Rendus Physique, **13**(6), 521 (2012). https://doi.org/10.1016/j.crhy.2012.04.009
[7] E.J. Copeland, M. Sami, and S. Tsujikawa, Int. J. of Mod. Phys. D, **15**, 1753 (2006). http://dx.doi.org/10.1142/S021827180600942X
[8] R.R. Caldwell, and M. Kamionkowski, Ann. Rev. Nucl. Part. Sci. **59**, 397 (2009). https://doi.org/10.1146/annurev-nucl-010709-151330
[9] A. Silvestri, and M. Trodden, Rep. Prog. Phys. **72**, 096901 (2009). 10.1088/0034-4885/72/9/096901
[10] S. Nojiri, and S.D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **115**, 4 (2007). https://doi.org/10.1142/S0219887807001928
[11] S. Capozziello, Int. J. Mod. Phys. D, **11**, 483 (2002). https://doi.org/10.1142/S0218271802002025
[12] S. Nojiri, and S.D. Odintsov, Phys. Rev. D, **74**(8), 086005 (2006). https://doi.org/10.1103/PhysRevD.74.086005
[13] S. Nojiri, and S.D. Odintsov, Phys. Lett. B, **657**(4), 238 (2007). https://doi.org/10.1016/j.physletb.2007.10.027
[14] S. Nojiri, and S.D. Odintsov, Phys. Rept., **505**, 59 (2011). https://doi.org/10.1016/j.physrep.2011.04.001
[15] S.K. Tripathy, and B. Mishra, The Eur. Phys. J. Plus, **131**, 273 (2016). https://doi.org/10.1140/epjp/i2016-16273-5
[16] E.V. Linder, Phys. Rev. D, **81**, 127301 (2010). https://doi.org/10.1103/PhysRevD.81.127301
[17] R. Myrzakulov, The Eur. Phys. J. C, **71**(9), 1752 (2011). https://doi.org/10.1140/epjc/s10052-011-1752-9
[18] S.H. Chen, J.B. Dent, S. Dutta, and E.N. Saridakis, Phys. Rev. D, **83**(2), 023508 (2011). https://doi.org/10.1103/PhysRevD.83.023508
[19] J.B. Dent, S. Dutta, and E.N. Saridakis, J. of Cosm. and Astroparticle Phys., **2011**, 009 (2011). https://doi.org/10.1088/1475-7516/2011/01/009
[20] T. Harko, F.S. Lobo, G. Otalora, and E.N. Saridakis, Phys. Rev. D, **89**(12), 124036 (2014). https://doi.org/10.1103/PhysRevD.89.124036
[21] S. Nojiri, and S.D. Odintsov, Phys. Lett. B, **631**(1), 1 (2005). https://doi.org/10.1016/j.physletb.2005.10.010
[22] B. Li, J.D. Barrow, and D.F. Mota, Phys. Lett. B, **76**(4), 044027 (2007). https://doi.org/10.1103/PhysRevD.76.044027
[23] G. Konas, and E.N. Saridakis, Phys. Rev. D, **90**, 084044 (2014). https://doi.org/10.1103/PhysRevD.90.084044
[24] V.J. Dagwal, Can. J. Phys., **98**, 636 (2020). https://doi.org/10.1139/cjp-2019-0226
[25] V.J. Dagwal, D.D. Pawar, Ind. J. of Phys., **95**, 1923 (2021). https://doi.org/10.1007/s12648-020-01830-3
[26] A.Y. Shaikh, A.S. Shaikh, and K.S. Wankhade, (2020). https://arxiv.org/abs/2006.12300
[27] V.J. Dagwal, and D.D. Pawar, Ind. J. of Phys., **95**, 177 (2021). https://doi.org/10.1007/s12648-020-01691-w
[28] V.J. Dagwal, and D.D. Pawar, Mod. Phys. Lett. A, **35**, 1950357 (2020). https://doi.org/10.1142/S0217732319503577
[29] D.D. Pawar, R.V. Mapari, and J.L. Pawade, Pramana – J. of Phys., **95**, 1 (2021). https://doi.org/10.1007/s12043-020-02058-w
[30] A.Y. Shaikh, and K.S. Wankhade, Theoretical Physics, **2**(1), 34-43 (2017). https://dx.doi.org/10.22606/tp.2017.21006
[31] V.J. Dagwal, Ind. J. of Phys. **96**, 3361 (2022). https://doi.org/10.1007/s12648-021-02256-1
[32] V.J. Dagwal, D.D. Pawar, and Y.S. Solanke, Mod. Phys. Lett. A, **35**, 2050316 (2020). https://doi.org/10.1142/S0217732320503162
[33] D.D. Pawar, and R.V. Mapari, J. of Dyn. Sys. and Geom. Theories, **20**, 115 (2022). https://doi.org/10.1080/1726037X.2022.2079268
[34] D.D. Pawar, R.V. Mapari, and P.K. Agrawal, J. Astrophys. Astron. **40**, 1 (2019). https://doi.org/10.1007/s12036-019-9582-5
[35] S. Nojiri, S.D. Odintsov, and V.K. Oikonomou, Phys. Rep. **692**, 1 (2017). https://doi.org/10.1016/j.physrep.2017.06.001
[36] S. Nojiri, and S.D. Odintsov, (2008). https://arxiv.org/abs/0807.0685v1
[37] S. Capozziello, A. Stabile, and A. Troisi, Class. Quantum Grav. **24**, 2153 (2007). https://doi.org/10.1088/0264-9381/24/8/013
[38] L. Hollenstein, and F.S.N. Lobo, Phys. Rev. D, **78**, 124007 (2008). https://doi.org/10.1103/PhysRevD.78.124007
[39] M.J. Reboucas, and J. Santos, Phys. Rev. D, **80**, 063009 (2009). https://doi.org/10.1103/PhysRevD.80.063009
[40] M. Sharif, and M.F. Shamir, Class. Quantum Grav. **26**, 235020 (2009). https://doi.org/10.1088/0264-9381/26/23/235020
[41] M. Sharif, and M.F. Shamir, Gen. Relativ. Gravit. **42**, 2643 (2010). https://doi.org/10.1007/s10714-010-1005-5
[42] S. Capozziello, A. Stabile, and A. Troisi, Int. J. Theor. Phys. **49**, 1251 (2010). https://doi.org/10.1007/s10773-010-0307-4
[43] K. Bamba, S. Nojiri, and S.D. Odintsov, Phys. Lett. B, **698**, 451 (2011). https://doi.org/10.1016/j.physletb.2011.03.038
[44] Y.B. Wu, Y.Y. Zhao, R.G. Cai, J.B. Lu, J.W. Lu, and X.J. Gao, Phys. Lett. B, **717**, 323 (2012). https://doi.org/10.1016/j.physletb.2012.10.008
[45] S. Capozziello, and M. De Laurentis, Ann. Phys. **524**, 545 (2012). https://doi.org/10.1002/andp.201200109
[46] M. Sharif, and Z. Yousaf, Mon. Not. R. Astron. Soc. **440**, 3479 (2014). https://doi.org/10.1093/mnras/stu533
[47] S.D. Odintsov, V.K. Oikonomou, Phys. Rev. D, **92**, 124024 (2015). https://doi.org/10.1103/PhysRevD.92.124024
[48] S. Capozziello, and M. De Laurentis, Phys. Rep., **509**, 167 (2017). https://doi.org/10.1016/j.physrep.2011.09.003
[49] M.Z. Bhatti, Z. Yousaf, and M. Ilyas, J. Astrophys. Astr. **39**, 69 (2018). https://doi.org/10.1007/s12036-018-9559-9
[50] Z. Yousaf, K. Bamba, M.Z. Bhatti, and U. Ghafoor, Phys. Rev. D, **100**, 024062 (2019). https://doi.org/10.1103/PhysRevD.100.024062
[51] Z. Yousaf, A. Ikram, and M.Z. Bhatti, Can. J. Phys., **98**, 474 (2020). https://doi.org/10.1139/cjp-2019-0360
[52] A.S. Agrawal, S.K. Tripathy, and B. Mishra, Chin. J. of Phys. **71**, 333 (2021). https://doi.org/10.1016/j.cjph.2021.03.004
[53] K.S. Wankhade, A.Y. Shaikh, and S.N. Khan, Prespacetime Journal, **13**(3), 365, 379 (2022). https://prespacetime.com/index.php/pst/article/download/1852/1734
[54] A.G. Cohen, D.B. Kaplan, and A.E. Nelson, Phys. Rev. Lett. **82**, 4971 (1999). https://doi.org/10.1103/PhysRevLett.82.4971
[55] S.D.H. Hsu, Phys. Lett. B, **594**(1-2), 13 (2004). https://doi.org/10.1016/j.physletb.2004.05.020
[56] M. Li, Phys. Lett. B, **603**(1-2), 1 (2004). https://doi.org/10.1016/j.physletb.2004.10.014
[57] S.H. Shekh, Physics of the Dark Universe, **33**, 100850 (2021). https://doi.org/10.1016/j.dark.2021.100850

[58] M.R. Setare, and E.N. Saridakis, J. Cosm. Astropart. Phys., **2009**, 002 (2009). https://doi.org/10.1088/1475-7516/2009/03/002
[59] L. Susskind, J. Math. Phys. **36**, 6377 (1995). https://doi.org/10.1063/1.531249
[60] P. Horova, and D. Minic, Phys. Rev. Lett., **85**, 1610 (2000). https://doi.org/10.1103/PhysRevLett.85.1610
[61] M.R. Setare, Phys. Lett. B, **644**(2-3), 99 (2007). https://doi.org/10.1016/j.physletb.2006.11.033
[62] S. Ghaffari, M.H. Dehghani, and A. Sheykhi, Phys. Rev. D, **89**, 123009 (2014). https://doi.org/10.1103/PhysRevD.89.123009
[63] S. Nojiri, and S.D. Odintsov, Gen. Relativ. Gravit. **38**, 1285 (2006). https://doi.org/10.1007/s10714-006-0301-6
[64] L.N. Granda, and A. Oliveros, Phys. Lett. B, **669**, 275 (2008). https://doi.org/10.1016/j.physletb.2008.10.017
[65] A. Sheykhi, M.H. Dehghani, and S. Ghaffari, Int. J. Mod. Phys. D, **25**, 1650018 (2016). https://doi.org/10.1142/S0218271816500188
[66] M. Tavayef, A. Sheykhi, K. Bamba, and H. Moradpour, Phys. Lett. B, **781**, 195 (2018). https://doi.org/10.1016/j.physletb.2018.04.001
[67] H. Moradpour, S.A. Moosavi, I.P. Lobo, J.P.M. Graca, A. Jawad, and I.G. Salako, Eur. Phys. J. C, **78**, 829 (2018). https://doi.org/10.1140/epjc/s10052-018-6309-8
[68] A.S. Jahromi, S.A. Moosavi, H. Moradpour, J.P.M. Grac, I.P. Lobo, I.G. Salako, and A. Jawad, Phys. Lett. B, **780**, 21 (2018). https://doi.org/10.1016/j.physletb.2018.02.052
[69] A. Jawad, K. Bamba, M. Younas, S. Qummer, and S. Rani, Symmetry, **10**, 635 (2018). https://doi.org/10.3390/sym10110635
[70] S. Maity, and U. Debnath, Eur. Phys. J. Plus, **134**, 514 (2019). https://doi.org/10.1140/epjp/i2019-12884-6
[71] S. Maity, and U. Debnath, Int. J. of Geom. Methods in Mod. Phys. **17**(11), 2050170 (2020). https://doi.org/10.1142/S0219887820501704
[72] U.K. Sharma, and V.C. Dubey, Modern Physics Letters A, **35**(34), 2050281 (2020). https://doi.org/10.1142/S0217732320502818
[73] V.C. Dubey, U.K. Sharma, and A. Al Mamon, Adv. in High Energy Phys. **2021**, 6658862 (2021). https://doi.org/10.1155/2021/6658862
[74] S. Nojiri, and S.D. Odintsov, Eur. Phys. J. C, **77**, 528 (2017). https://doi.org/10.1140/epjc/s10052-017-5097-x
[75] A. Iqbal, and A. Jawad, Physics of the Dark Universe, **26**, 100349 (2019). https://doi.org/10.1016/j.dark.2019.100349
[76] S. Ghaffari, A.H. Ziaie, V.B. Bezerra, and H. Moradpour, Modern Phys. Lett. A, **35**, 1, 1950341 (2019). https://doi.org/10.1142/S0217732319503413
[77] U.K. Sharma, and V.C. Dubey, New Astronomy, **80**, 101419 (2020). https://doi.org/10.1016/j.newast.2020.101419
[78] A. Dixit, V.K. Bhardwaj, and A. Pradhan, (2020). https://arxiv.org/abs/2010.10847v1
[79] S. Chunlen, and P. Rangdee, (2020). https://arxiv.org/abs/2008.13730v2
[80] S. Ali, S. Khan, S. Sattar, and A. Abebe, (2020). https://arxiv.org/abs/2011.10046v2
[81] V.C. Dubey, A.K. Mishra, and U.K. Sharma, Astrophys. Space Sci. **365**, 129 (2020). https://doi.org/10.1007/s10509-020-03846-x
[82] A. Saha, S. Ghose, A. Chanda, and B.C. Paul, (2021). https://arxiv.org/abs/2101.04060
[83] A.Y. Shaikh, (2021). https://arxiv.org/abs/2105.04411v1
[84] M.F. Shamir, J. Exp. Theor. Phys. **123**, 607 (2016). https://doi.org/10.1134/S1063776116110182
[85] C.B. Collins, J. Math. Phys. **18**, 2116 (1977). https://doi.org/10.1063/1.523191
[86] H. Wei, and R.G. Cai, Eur. Phys. J. C, **59**, 99 (2009). https://doi.org/10.1140/epjc/s10052-008-0799-8
[87] A.Y. Shaikh, and B. Mishra, Int. J. of Geom. Methods in Mod. **17**(11), 2050158 (2020). https://doi.org/10.1142/S0219887820501583
[88] A.Y. Shaikh, S.V. Gore, and S.D. Katore, New Astronomy, **80**, 101420 (2020). https://doi.org/10.1016/j.newast.2020.101420
[89] A.Y. Shaikh, A.S. Shaikh, and K.S. Wankhade, Pramana – J. Phys., **95**, 19 (2021). https://doi.org/10.1007/s12043-020-02047-z
[90] A.Y. Shaikh, and B. Mishra, Commun. Theor. Phys. **73** ,025401 (2021). https://doi.org/10.1088/1572-9494/abcfb2

ГОЛОГРАФІЧНА МОДЕЛЬ ТЕМНОЇ ЕНЕРГІЇ РЕНЬЇ У f(R) ГРАВІТАЦІЇ З ІЧ-ОБРІЗАННЯМ ХАББЛА

Кішор С. Ванкхаде[a], Альфред Ю. Шейх[b], Сірадж Н. Хан[c]
[a]*Департамент математики, Махавідьялая, округ Мангрулпір. Васим - 444403, Індія*
[b]*Департамент математики, Індіра Ганді Махавідьялая, Ралегаон - 445402, Індія*
[c]*Коледж інженерії та менеджменту професора Рама Меге, Баднера, Амраваті - 444701, Індія*

У цьому дослідженні розглядається однорідна та анізотропна модель Всесвіту LRS Bianchi типу I із взаємодіючою темною матерією та голографічною моделлю темної енергії Реньї (RHDE) у $f(R)$ гравітації. Параметр уповільнення (DP) показує характерне перевертання для Всесвіту, який уповільнювався в минулому та прискорювався в нинішню епоху. Таким чином, DP є найбільш фізично виправданим параметром для аналізу рішення космологічної моделі. Щоб знайти точний розв'язок польових рівнянь моделі, скаляр зсуву вважається пропорційним скаляру розширення. Ми розглянули $f(R) = bR^n$, моделлю зображення $f(R)$ якої є функція скаляра Річчі R. Досліджено фізико-геометричні характеристики моделі Всесвіту.

Ключові слова: $f(R)$ *гравітація; RHDE; темна матерія; космологія; простір-час Бьянкі типу I*