East European Journal of Physics. **3**. 441–450 (2023)
DOI: 10.26565/2312-4334-2023-3-49 ISSN 2312-4334

UNSTEADY FLOW PAST AN ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE IN PRESENCE OF THERMAL STRATIFICATION AND CHEMICAL REACTION[†]

Nitul Kalita[*], **Rudra Kanta Deka**, **Rupam Shankar Nath**
Department of Mathematics, Gauhati University, Guwahati-781014, Assam, India
[*]*Corresponding Author e-mail: nitulkalita9602@gmail.com*

Received July 11, 2023; revised July 24, 2023; accepted July 26, 2023

This work aims to investigate the effect of thermal stratification on fluid flow past an accelerated vertical plate in the presence of first order chemical reaction. The dimensionless unsteady coupled linear governing equations are solved by Laplace transform technique for the case when the Prandtl number is unity. The important conclusions made in this study the effect of thermal stratification is compared with the scenario in which there was no stratification. The results of numerical computations for different sets of physical parameters, such as velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number are displayed graphically. It is shown that the steady state is attained more quickly when the flow is stratified.

Keywords: *Thermal Stratification, Chemical Reaction, Heat and Mass Transfer, Vertical Plate, Accelerated*

PACS: 47.55.P-, 44.25.+f, 44.05.+e, 47.11.-j

1. INTRODUCTION

Thermal stratification is a natural phenomenon that may be seen in many natural systems, such as lakes and seas. The presence of chemical reactions might further complicate the flow's dynamics. In this paper, we investigate how flow dynamics and interactions with chemical processes are impacted by thermal stratification.The applications of this study are wide. It may be used to build more efficient chemical reactors and heat exchangers. It may also be used to look at how the performance of cooling systems in electrical equipment is affected by thermal stratification.

[1] investigated the influence of a chemical reaction on the behavior of an unsteady flow through an accelerating vertical plate, where the mass transfer was variable and without considering stratification. The purpose of this research is to determine how fluid flow past an accelerated vertical plate impacts the interaction between thermal stratification and chemical reaction. [2] and [3] investigated the unsteady flow of a thermally stratified fluid past a vertically accelerated plate under a variety of conditions. Researchers [4], [5], and [6] have investigated steady flows in a stable stratified fluid with a focus on infinite vertical plates. [7] and [8] both investigated buoyancy-driven flows in a stratified fluid. The interaction between thermal stratification and chemical reaction to change MHD flow for vertical stretching surfaces has been studied by researchers [9] and [10]. These two phenomena were also investigated by [11], who investigated the impact of non-Newtonian fluid flow in a porous medium. The unsteady MHD flow past an accelerating vertical plate with a constant heat flux and ramped plate temperature respectively was researched by [12] and [13].

In this paper, we derived the special solutions for $Sc = 1$ and classical solutions for the case $S = 0$ (without stratification). These solutions are compared with the primary solutions, and graphs are used to demonstrate the differences. The impacts of physical parameters on velocity, temperature, and concentration profiles, including the stratification parameter (S), thermal Grashof number (Gr), mass Grashof number (Gc), Schimdt number (Sc) and Chemical Reaction Parameter (K), are explored and presented in graphs. The results of this research have a wide range of applications in a variety of industries and chemical factories.

2. MATHEMATICAL ANALYSIS

We consider a fluid that is stratified, viscous, and in-compressible, traveling along an accelerating vertical plate with first-order chemical reaction present. As can be seen in fig. 1, we use a coordinate system in which the y' axis is perpendicular to the plate and the x' axis is taken vertically upward along the plate to study the flow situation. The starting temperature T'_∞ and initial fluid concentration C'_∞ of the plate and fluid are the same. At time $t' > 0$, the plate is subjected to an impulsive constant acceleration u_0, and the concentration and temperature of the plate are increased to C'_w and T'_w, respectively. All flow variables are independent of x'

[†]***Cite as:*** N. Kalita, R.K. Deka, R.S. Nath, East Eur. J. Phys. 3, 441 (2023), https://doi.org/10.26565/2312-4334-2023-3-49

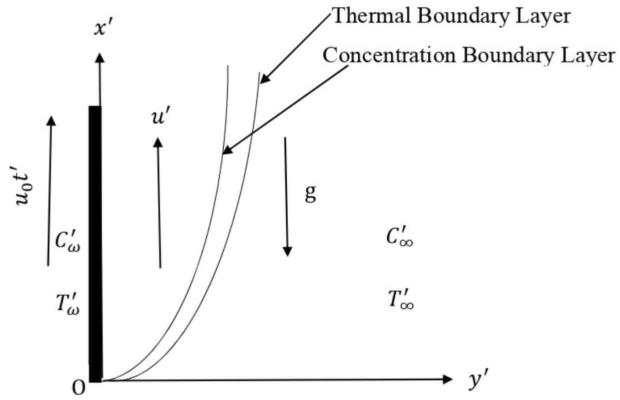

Figure 1. Physical Model and coordinate system

and only affected by y' and t' since the plate has an infinite length. As a result, we are left with a flow that is only one dimension and has one non-zero vertical velocity component, u'. The Boussinesqs' approximation is then used to represent the equations for motion, energy, and concentration as follows:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \alpha\frac{\partial^2 T'}{\partial y'^2} - \gamma u' \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_1 C' \tag{3}$$

with the following initial and boundary Conditions:

$$\begin{array}{llll} u' = 0 & T' = T'_\infty & C' = C'_\infty & \forall y', t' \leq 0 \\ u' = u_0 t' & T' = T'_\infty + (T'_w - T'_\infty)At' & C' = C'_w & \text{at } y' = 0, t' > 0 \\ u' = 0 & T' \to T'_\infty & C' \to C'_\infty & \text{as } y' \to \infty, t' > 0 \end{array}$$

where, α is the thermal diffusivity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with concentration, η is the similarity parameter, ν is the kinematic viscosity, g is the acceleration due to gravity, D is the mass diffusion coefficient. Also, $\gamma = \frac{dT'_\infty}{dx'} + \frac{g}{C_p}$ denotes the thermal stratification parameter and $\frac{dT'_\infty}{dx'}$ denotes the vertical temperature convection known as thermal stratification. In addition, $\frac{g}{C_p}$ represents the rate of reversible work done on fluid particles by compression, often known as work of compression. The variable (γ) will be referred to as the thermal stratification parameter in our research because the compression work is relatively minimal. For the purpose of testing computational methods, compression work is kept as an additive to thermal stratification.

and we provide non-dimensional quantities in the following:

$$U = \frac{u'}{(u_0\nu)^{1/3}}, \quad t = \frac{t'u_0^{2/3}}{\nu^{1/3}}, \quad y = \frac{y'u_0^{1/3}}{\nu^{2/3}}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gr = \frac{g\beta(T'_w - T'_\infty)}{u_0}$$

$$Gc = \frac{g\beta^*(C'_w - C'_\infty)}{u_0}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D}, \quad K = \frac{K_1\nu^{1/3}}{u_0^{2/3}}, \quad S = \frac{\gamma\nu^{2/3}}{u_0^{1/3}(T'_w - T'_\infty)}$$

where, $A = \left(\frac{u_0^2}{\nu}\right)^{1/3}$ is the constant.

The non-dimensional forms of the equations (1)-(3) are given by

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial y^2} \tag{4}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - SU \tag{5}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - KC \tag{6}$$

Non-dimensional form of initial and boundary Conditions are:

$$\begin{array}{llll} U=0 & \theta=0 & C=0 & \forall y, t\leq 0 \\ U=t & \theta=t & C=1 & \text{at } y=0, t>0 \\ U=0 & \theta\to 0 & C\to 0 & \text{as } y\to\infty, t>0 \end{array} \tag{7}$$

3. METHOD OF SOLUTION

The non-dimensional governing equations (4)- (6) with boundary conditions (7) are solved using Laplace's transform method for $Pr = 1$. Hence, the expressions for concentration, velocity and temperature with the help of [14] and [15] are given by

$$C = \frac{1}{2}\left[e^{-2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) + e^{2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right] \tag{8}$$

$$\begin{aligned} U = {} & \frac{1}{2}\left\{f_4(iA) + f_4(-iA)\right\} + \frac{iA}{2S}\left\{f_4(iA) - f_4(-iA)\right\} + \frac{Gc}{2(Sc-1)}\left[C_1\left\{f_1(iA) + f_1(-iA)\right\}\right. \\ & +(C_2 - iC_3)\left\{f_2(iA, B+iB_1) + f_2(-iA, B+iB_1)\right\} + (C_2 + iC_3)\left\{f_2(iA, B-iB_1)\right. \\ & \left.\left.+f_2(-iA, B-iB_1)\right\}\right] + \frac{Gc}{2iA}\left[(D_1 - 1)\left\{f_1(iA) - f_1(-iA)\right\} + (D_2 + iD_3)\left\{f_2(iA, B+iB_1)\right.\right. \\ & \left.-f_2(-iA, B+iB_1)\right\} + (D_2 - iD_3)\left\{f_2(iA, B-iB_1) - f_2(-iA, B-iB_1)\right\}\Big] \\ & -\frac{Gc}{(Sc-1)}\left[\frac{C_1}{2}\left\{e^{-2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) + e^{2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right\}\right. \\ & \left.+(C_2 - iC_3)\left\{f_3(K, B+iB_1)\right\} + (C_2 + iC_3)\left\{f_3(K, B-iB_1)\right\}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} \theta = {} & \frac{S}{2iA}\left\{f_4(iA) - f_4(-iA)\right\} + \frac{1}{2}\left\{f_4(iA) + f_4(-iA)\right\} + \frac{SGc}{2iA(Sc-1)}\left[C_1\left\{f_1(iA) - f_1(-iA)\right\}\right. \\ & +(C_2 - iC_3)\left\{f_2(iA, B+iB_1) - f_2(-iA, B+iB_1)\right\} + (C_2 + iC_3)\left\{f_2(iA, B-iB_1)\right. \\ & \left.\left.-f_2(-iA, B-iB_1)\right\}\right] + \frac{SGc}{2(Sc-1)^2}\left[E_1\left\{f_1(iA) + f_1(-iA)\right\} + (E_2 - iE_3)\left\{f_2(iA, B+iB_1)\right.\right. \\ & \left.\left.+f_2(-iA, B+iB_1)\right\} + (E_2 + iE_3)\left\{f_2(iA, B-iB_1) + f_2(-iA, B-iB_1)\right\}\right] \\ & -\frac{SGc}{(Sc-1)^2}\left[\frac{E_1}{2}\left\{e^{-2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) + e^{2\eta\sqrt{ScKt}} erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right\}\right. \\ & \left.+(E_2 - iE_3) f_3(K, B+iB_1) + (E_2 + iE_3) f_3(K, B-iB_1)\right] \end{aligned} \tag{10}$$

where,

$$\eta = \frac{y}{2\sqrt{t}}, \quad A = \sqrt{SGr}, \quad B = \frac{ScK}{Sc-1}, \quad B_1 = \frac{A}{Sc-1} = \frac{\sqrt{SGr}}{Sc-1}, \quad C_1 = \frac{B}{(B^2 + B_1^2)}$$

$$C_2 = \frac{-B}{2(B^2 + B_1^2)}, \quad C_3 = \frac{-B_1}{2(B^2 + B_1^2)}, \quad D_1 = \frac{B^2}{(B^2 + B_1^2)}, \quad D_2 = \frac{B_1^2}{2(B^2 + B_1^2)}$$

$$D_3 = \frac{BB_1}{2(B^2 + B_1^2)}, \quad E_1 = \frac{1}{(B^2 + B_1^2)}, \quad E_2 = \frac{-1}{2(B^2 + B_1^2)}, \quad E_3 = \frac{B}{2B_1(B^2 + B_1^2)}$$

Also, f_i's are inverse Laplace's transforms given by

$$f_1(ip) = L^{-1}\left\{\frac{e^{-y\sqrt{s+ip}}}{s}\right\}, \quad f_2(ip, q_1 + iq_2) = L^{-1}\left\{\frac{e^{-y\sqrt{s+ip}}}{s + q_1 + iq_2}\right\}$$

$$f_3(p, q_1 + iq_2) = L^{-1}\left\{\frac{e^{-y\sqrt{Sc(s+p)}}}{s + q_1 + iq_2}\right\}, \quad f_4(ip) = L^{-1}\left\{\frac{e^{-y\sqrt{s+ip}}}{s^2}\right\}$$

We separate the complex arguments of the error function contained in the previous expressions into real and imaginary parts using the formulas provided by [15].

4. SPECIAL CASE [FOR SC=1]

We came up with answers for the special case where $Sc = 1$. Hence, the solutions for the special case are as follows:

$$C^* = \frac{1}{2}\left[e^{-2\eta\sqrt{Kt}}erfc\left(\eta - \sqrt{Kt}\right) + e^{2\eta\sqrt{Kt}}erfc\left(\eta + \sqrt{Kt}\right)\right] \quad (11)$$

$$\begin{aligned} U^* &= \frac{1}{2}\{f_4(iA) + f_4(-iA)\} + \frac{KGc}{2(K^2+A^2)}\{f_1(iA) + f_1(-iA)\} \\ &+\frac{iAGc}{2(K^2+A^2)}\{f_1(iA) - f_1(-iA)\} + \frac{iA}{2S}\{f_4(iA) - f_4(-iA)\} \\ &-\frac{KGc}{2(K^2+A^2)}\left[e^{-2\eta\sqrt{Kt}}erfc\left(\eta - \sqrt{Kt}\right) + e^{2\eta\sqrt{Kt}}erfc\left(\eta + \sqrt{Kt}\right)\right] \end{aligned} \quad (12)$$

$$\begin{aligned} \theta^* &= \frac{1}{2}\{f_4(iA) + f_4(-iA)\} + \frac{SKGc}{2iA(K^2+A^2)}\{f_1(iA) - f_1(-iA)\} \\ &+\frac{SGc}{2(K^2+A^2)}\{f_1(iA) + f_1(-iA)\} + \frac{S}{2iA}\{f_4(iA) - f_4(-iA)\} \\ &-\frac{SGc}{2(K^2+A^2)}\left\{e^{-2\eta\sqrt{Kt}}erfc\left(\eta - \sqrt{Kt}\right) + e^{2\eta\sqrt{Kt}}erfc\left(\eta + \sqrt{Kt}\right)\right\} \end{aligned} \quad (13)$$

5. CLASSICAL CASE (S=0)

We derived solutions for the classical case of no thermal stratification ($S = 0$). We want to compare the results of the fluid with thermal stratification to the case with no stratification. Hence, the corresponding solutions for the classical case is given by :

$$\theta_c = = t\left\{\left(1+2\eta^2\right)erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}e^{-\eta^2}\right\} \quad (14)$$

$$\begin{aligned} U_c &= \frac{Gc}{2KSc}\left[2erfc(\eta) - e^{-Bt}\left\{e^{-2\eta\sqrt{-Bt}}erfc\left(\eta - \sqrt{-Bt}\right) + e^{2\eta\sqrt{-Bt}}erfc\left(\eta + \sqrt{-Bt}\right)\right\}\right. \\ &-\left\{e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) + e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right\} \\ &\left.+e^{-Bt}\left\{e^{-2\eta\sqrt{Sc(K-B)t}}erfc\left(\eta\sqrt{Sc} - \sqrt{(K-B)t}\right) + e^{2\eta\sqrt{Sc(K-B)t}}erfc\left(\eta\sqrt{Sc} + \sqrt{(K-B)t}\right)\right\}\right] \\ &+\frac{\eta Grt^2}{3}\left\{\frac{4}{\sqrt{\pi}}(1+\eta^2)e^{-\eta^2} - \eta(6+4\eta^2)\ erfc(\eta)\right\} + t\left\{\left(1+2\eta^2\right)erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}e^{-\eta^2}\right\} \end{aligned} \quad (15)$$

5.1. Skin-Friction

The non-dimensional Skin-Friction, which is determined as shear stress on the surface, is obtained by

$$\tau = -\left.\frac{dU}{dy}\right|_{y=0}$$

The solution for the Skin-Friction is calculated from the solution of Velocity profile U, represented by (9), as follows:

$$\begin{aligned} \tau &= \sqrt{\frac{t}{\pi}}\cos At + t\sqrt{\frac{A}{2}}(r_1 - r_2) + \frac{(r_1+r_2)}{2\sqrt{2A}} + \frac{A}{S}\left[\sqrt{\frac{t}{\pi}}\sin At - t\sqrt{\frac{A}{2}}(r_1+r_2) + \frac{(r_1-r_2)}{2\sqrt{2A}}\right] \\ &+\frac{Gc}{Sc-1}\left[C_1\left\{\frac{\cos At}{\sqrt{\pi t}} + \sqrt{\frac{A}{2}}(r_1-r_2) - \sqrt{ScK}\ erf(\sqrt{Kt}) - \sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}\right. \\ &+2C_2\left\{\frac{\cos At}{\sqrt{\pi t}} - \sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\} + e^{-Bt}\{(C_2P_1 + C_3Q_1)(r_3\cos B_1t + r_4\sin B_1t) \\ &+(C_3P_1 - C_2Q_1)(r_4\cos B_1t - r_3\sin B_1t)\} + e^{-Bt}\{(C_2P_2 - C_3Q_2)(r_5\cos B_1t - r_6\sin B_1t) \\ &-(C_3P_2 + C_2Q_2)(r_6\cos B_1t + r_5\sin B_1t)\} - 2e^{-Bt}\sqrt{Sc}\{(C_2P_3 - C_3Q_3) \\ &\left.(r_7\cos B_1t - r_8\sin B_1t) - (C_3P_3 + C_2Q_3)(r_8\cos B_1t + r_7\sin B_1t)\}\right] \end{aligned}$$

$$+\frac{Gc}{A}\left[(D_1-1)\left\{\frac{-\sin At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}\,(r_1+r_2)\right\}-\frac{2D_2\sin At}{\sqrt{\pi t}}\right.$$
$$+e^{-Bt}\{(D_2P_1-D_3Q_1)(r_4\cos B_1t-r_3\sin B_1t)+(D_3P_1+D_2Q_1)(r_3\cos B_1t+r_4\sin B_1t)\}$$
$$\left.+e^{-Bt}\{(D_2P_2+D_3Q_2)(r_6\cos B_1t+r_5\sin B_1t)-(D_3P_2-D_2Q_2)(r_5\cos B_1t-r_6\sin B_1t)\}\right]$$

The solution for the Skin-Friction for the special case is given from the expression (12), which is represented by

$$\tau^* = \sqrt{\frac{t}{\pi}}\cos At+t\sqrt{\frac{A}{2}}\,(r_1-r_2)+\frac{(r_1+r_2)}{2\sqrt{2A}}+\frac{KGc}{K^2+A^2}\left[\frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}\,(r_1-r_2)\right.$$
$$\left.-\sqrt{K}\ erf(\sqrt{Kt})-\frac{e^{-Kt}}{\sqrt{\pi t}}\right]+\frac{A}{S}\left[\sqrt{\frac{t}{\pi}}\sin At-\ t\sqrt{\frac{A}{2}}\,(r_1+r_2)+\frac{(r_1-r_2)}{2\sqrt{2A}}\right]$$
$$+\frac{AGc}{K^2+A^2}\left\{\frac{\sin At}{\sqrt{\pi t}}-\sqrt{\frac{A}{2}}\,(r_1+r_2)\right\}$$

The solution for the Skin-Friction for the classical case is given from the expression (15), which is represented by

$$\tau_c = \frac{Gc}{KSc}\left[e^{-Bt}\left\{\sqrt{Sc(K-B)}\ erf(\sqrt{(K-B)t})-\sqrt{-B}\ erf(\sqrt{-Bt})\right\}-\sqrt{ScK}\ erf(\sqrt{Kt})\right]$$
$$+2\sqrt{\frac{t}{\pi}}\left(1-\frac{tGr}{3}\right)$$

5.2. Nusselt Number

The non-dimensional Nusselt number, which is determined as the rate of heat transfer, is obtained by

$$Nu=-\left.\frac{d\theta}{dy}\right|_{y=0}$$

The solution for the Nusselt number is calculated from the solution of Temperature profile θ, represented by (10), as follows:

$$Nu = \sqrt{\frac{t}{\pi}}\cos At+t\sqrt{\frac{A}{2}}\,(r_1-r_2)+\frac{(r_1+r_2)}{2\sqrt{2A}}-\frac{S}{A}\left[\sqrt{\frac{t}{\pi}}\sin At-t\sqrt{\frac{A}{2}}\,(r_1+r_2)+\frac{(r_1-r_2)}{2\sqrt{2A}}\right]$$
$$+\frac{Gc}{A(Sc-1)}\left[C_1\left\{\frac{-\sin At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}\,(r_1+r_2)\right\}-\frac{2C_2\sin At}{\sqrt{\pi t}}\right.$$
$$+e^{-Bt}\{(C_2P_1+C_3Q_1)(r_4\cos B_1t-r_3\sin B_1t)-(C_3P_1-C_2Q_1)(r_3\cos B_1t+r_4\sin B_1t)\}$$
$$\left.+e^{-Bt}\{(C_2P_2-C_3Q_2)(r_6\cos B_1t+r_5\sin B_1t)+(C_3P_2+C_2Q_2)(r_5\cos B_1t-r_6\sin B_1t)\}\right]$$
$$+\frac{SGc}{(Sc-1)^2}\left[E_1\left\{\frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}\,(r_1-r_2)-\sqrt{ScK}\ erf(\sqrt{Kt})-\sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}\right.$$
$$+2E_2\left\{\frac{cosAt}{\sqrt{\pi t}}-\sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}+e^{-Bt}\{(E_2P_1+E_3Q_1)(r_3cosB_1t+r_4\sin B_1t)\}$$
$$+(E_3P_1-E_2Q_1)(r_4cosB_1t-r_3\sin B_1t)+e^{-Bt}\{(E_2P_2-E_3Q_2)(r_5cosB_1t-r_6\sin B_1t)$$
$$-(E_3P_2+E_2Q_2)(r_6cosB_1t+r_5\sin B_1t)\}-2e^{-Bt}\sqrt{Sc}\{(E_2P_3-E_3Q_3)(r_7cosB_1t-r_8\sin B_1t)$$
$$\left.-(E_3P_3+E_2Q_3)(r_8cosB_1t+r_7\sin B_1t)\}\right]$$

The solution for the Nusselt number for the special case is given from the expression (13), which is represented by

$$Nu^* = \frac{SKGc}{A(K^2+A^2)}\left[\frac{-\sin At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}\,(r_1+r_2))\right]+\left(1+\frac{SGc}{K^2+A^2}\right)\left[\frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}\,(r_1-r_2)\right]$$
$$-\frac{S}{A}\left[\sqrt{\frac{t}{\pi}}\sin At-t\sqrt{\frac{A}{2}}\,(r_1+r_2)+\frac{(r_1-r_2)}{2\sqrt{2A}}\right]-\frac{SGc}{K^2+A^2}\left\{\sqrt{K}\ erf(\sqrt{Kt})+\frac{e^{-Kt}}{\sqrt{\pi t}}\right\}$$

$$+\sqrt{\frac{t}{\pi}}\cos At+t\sqrt{\frac{A}{2}}\left(r_1-r_2\right)+\frac{(r_1+r_2)}{2\sqrt{2A}}$$

The solution for the Nusselt number for the classical case is given from the expression (14), which is represented by

$$Nu_c=\frac{1}{\sqrt{\pi t}}$$

5.3. Sherwood Number

The non-dimensional Sherwood number, which is determined as the rate of mass transfer, is obtained by

$$Sh=-\frac{dC}{dy}\bigg|_{y=0}$$

The solution for the Sherwood number is calculated from the solution of Concentration profile C, represented by (8), as follows:

$$Sh \quad = \quad \sqrt{ScK}\ erf(\sqrt{Kt})+\sqrt{\frac{Sc}{\pi t}}e^{-Kt}$$

The solution for the Sherwood number for the special case is given from the expression (11), which is represented by

$$Sh^{*} \quad = \quad \sqrt{K}\ erf(\sqrt{Kt})+\frac{1}{\sqrt{\pi t}}e^{-Kt}$$

where,

$$B_2=\sqrt{B^2+(A-B_1)^2},\quad B_3=\sqrt{B^2+(A+B_1)^2},\quad B_4=\sqrt{(K-B)^2+B_1^2},\quad P_1=\sqrt{\frac{B_2-B}{2}},$$

$$Q_1=\sqrt{\frac{B_2+B}{2}},\quad P_2=\sqrt{\frac{B_3-B}{2}},\quad Q_2=\sqrt{\frac{B_3+B}{2}},\quad P_3=\sqrt{\frac{B_4-(K-B)}{2}}$$

$$Q_3=\sqrt{\frac{B_4+(K-B)}{2}},\quad \sqrt{-B+i(A-B_1)}=P_1+iQ_1,\quad \sqrt{-B+i(A+B_1)}=P_2+iQ_2,$$

$$\sqrt{K-B+iB_1}=P_3+iQ_3,\quad erf(\sqrt{iAt})=r_1+ir_2,\quad erf(P_1\sqrt{t}+iQ_1\sqrt{t})=r_3+ir_4,$$

$$erf(P_2\sqrt{t}+iQ_2\sqrt{t})=r_5+ir_6,\quad erf(P_3\sqrt{t}+iQ_3\sqrt{t})=r_7+ir_8$$

6. RESULT AND DISCUSSIONS

In order to better understand the physical significance of the problem, we calculated the velocity, temperature, concentration, Skin friction, Nusselt number, and Sherwood number using the solutions we found in the previous sections, for different values of the physical parameters S, Gr, Gc, Sc, K and time t. Additionally, we represented them graphically in Figures 2 to 13.

The effect of thermal stratification (S) on the velocity profiles is seen in Figure 2. It can be seen that there is a decrease in velocity as a result of thermal stratification. An increase in the values of Gr and Gc leads to a rise in the value of the velocity, as seen in Figure 3. Figures 4 and 5 depicted the fluid's velocity at various values of Sc and K. The fluid velocity decreases as the values of Sc and K increase.

Figures 6 and 7 portray the effect of thermal stratification on fluid velocity and temperature against time. Without stratification, the velocity and temperature increase over time in an exponential manner; but, when stratification takes place, they finally stabilize. Due to the application of thermal stratification, which reduces velocity and temperature in comparison with the standard case ($S = 0$). Hence, this research with stratification is more realistic than prior ones without stratification.

The combined effect of thermal stratification and chemical reaction on temperature can be seen in Figure 8. The temperature seems to decrease when the thermal stratification parameter is increased, yet chemical reactions enhance the temperature. The effects of Gr, Gc and Sc on the temperature profile are shown in Figures 9 and 10, respectively. For higher Gr, Sc, and lower Gc values, the temperature decreases.

Figure 11 illustrates how the parameters Sc and K influence the concentration of the fluid. The concentration decreases when the Sc and K parameters are increased. Figures 12 and 13 illustrate the skin friction and Nusselt number variations produced by thermal Stratification. They considerably rise in the presence of stratification compared to the absence of stratification. Additionally, stratification increases the frequency of oscillations for both skin friction and the Nusselt number.

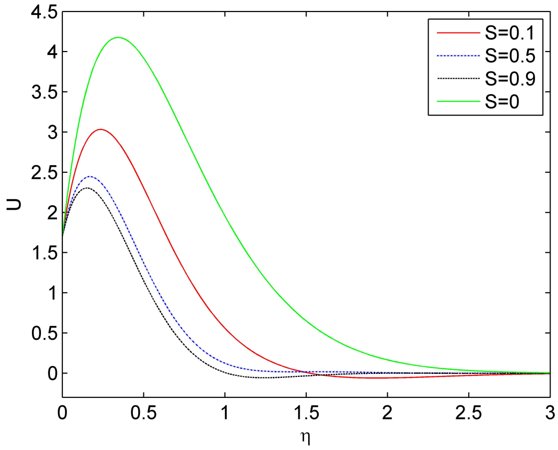

Figure 2. Effects of S on Velocity Profile for $Gr = 5, Gc = 5, t = 1.7, Sc = 0.5, K = 0.2$

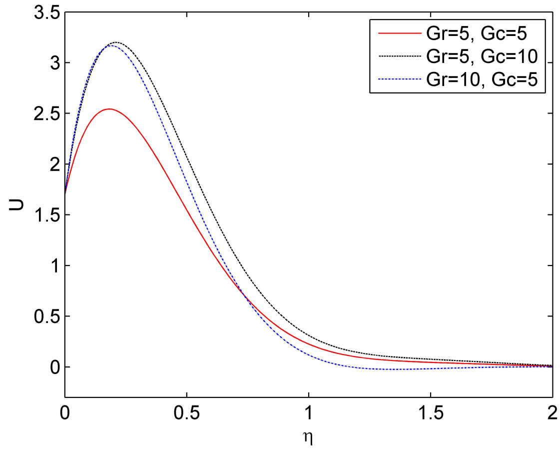

Figure 3. Effects of Gr and Gc on Velocity Profile for $S = 0.4, Sc = 0.5, t = 1.7, K = 0.2$

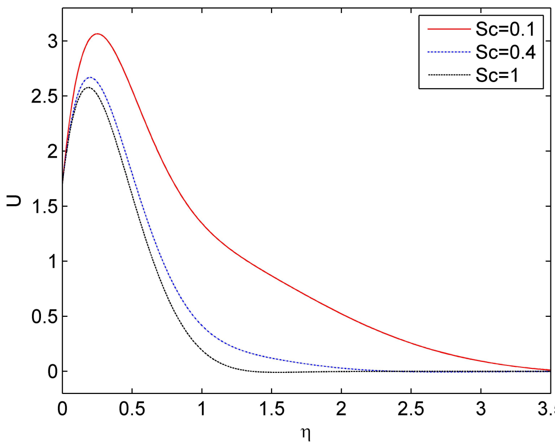

Figure 4. Effects of Sc on Velocity Profile for $Gr = 5, Gc = 5, S = 0.4, t = 1.7, K = 0.2$

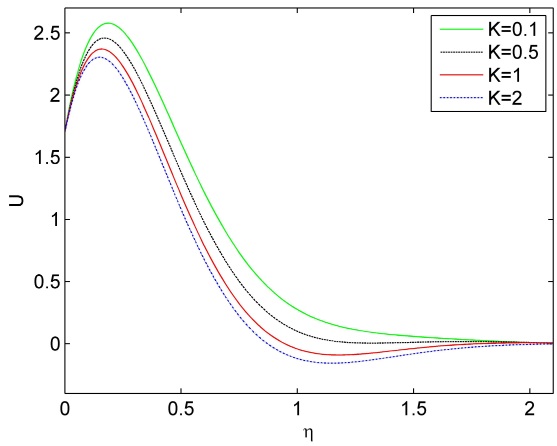

Figure 5. Effects of K on Velocity Profile for $Gr = 5, Gc = 5, S = 0.4, Sc = 0.5, t = 1.7$

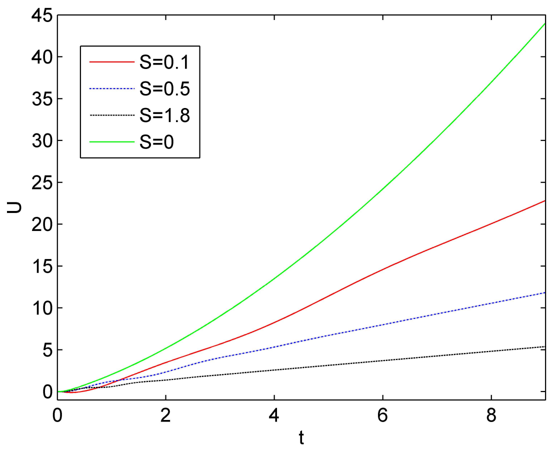

Figure 6. Effects of S on Velocity Profile against time for $Gr = 5, Gc = 5, Sc = 0.5, y = 1, K = 0.2$

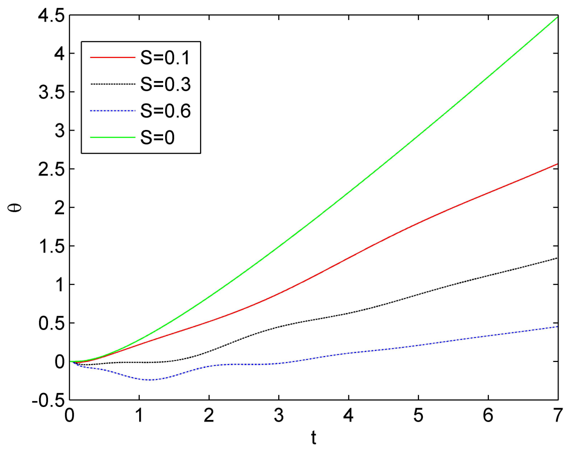

Figure 7. Effects of S on Temperature Profile against time for $Gr = 5, Gc = 5, Sc = 0.5, y = 1, K = 0.2$

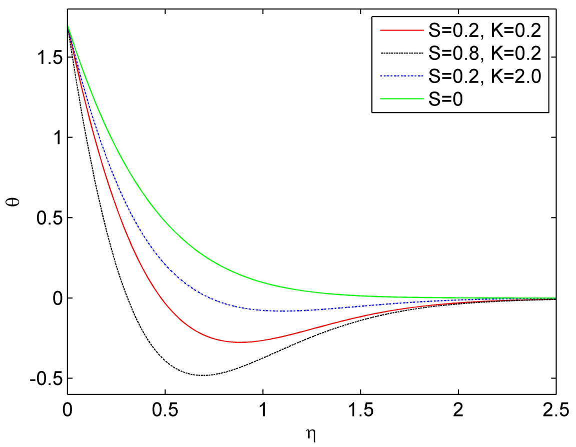

Figure 8. Effects of S and K on Temperature Profile for $Gr = 5, Gc = 5, Sc = 0.5, t = 1.7$

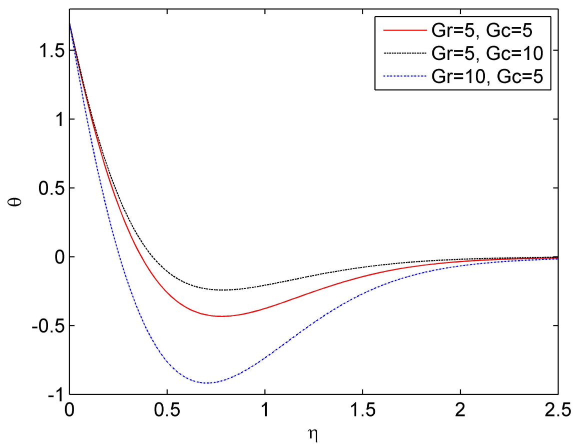

Figure 9. Effects of Gr and Gc on Temperature Profile for $S = 0.4, Sc = 0.5, t = ‘1.7, K = 0.2$

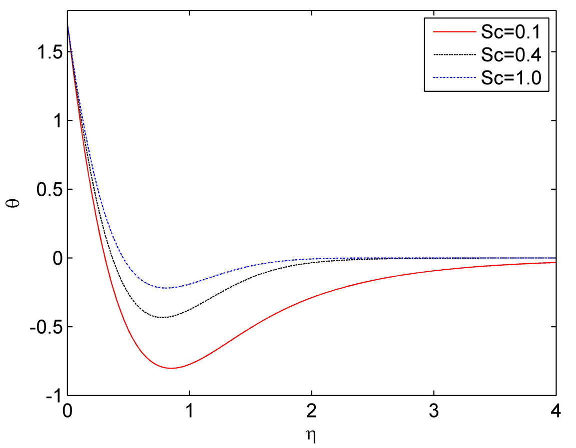

Figure 10. Effects of Sc on Temperature Profile for $Gr = 5, Gc = 5, S = 0.4, t = 1.7, K = 0.2$

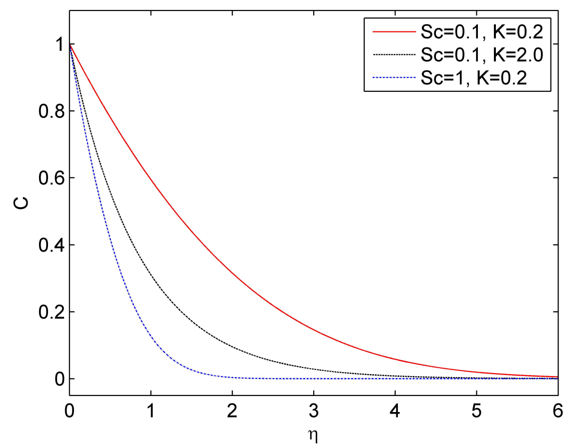

Figure 11. Effects of Sc and K on concentration Profile

7. CONCLUSION

We explored how chemical reactions impact the flow through an accelerated vertical plate in the presence of thermal stratification. The outcomes of the current study are compared with those of the classical situation

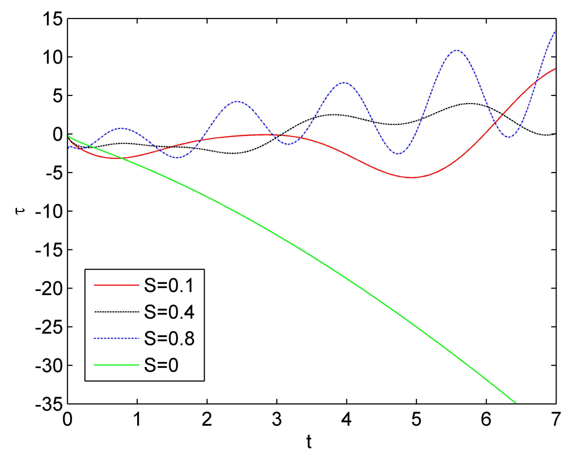

Figure 12. Effects of S on Skin friction for $Gr = 5, Gc = 5, Sc = 0.5, K = 0.2$

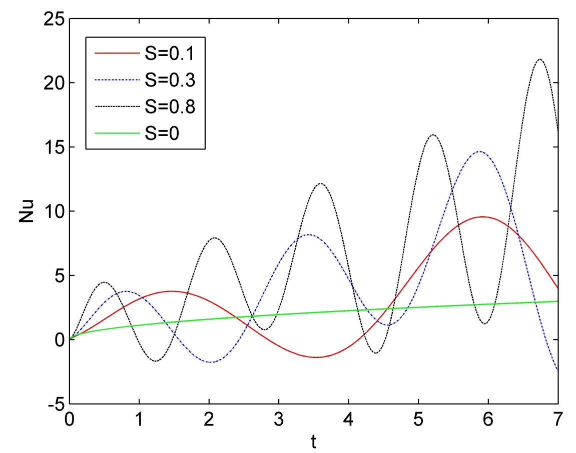

Figure 13. Effects of S on Nusselt Number for $Gr = 5, Gc = 5, Sc = 0.5, K = 0.2$

in which stratification does not take place. As S, Sc and K grow, the fluid's velocity decreases, whereas an increase in Gr, Gc increases it. This research is more practical than earlier ones because it applies thermal stratification, which lowers velocity and temperature in comparison to the classical scenario ($S = 0$). The temperature decreases when K and Gc decreases, and it increases when S, Gr increases. Thermal stratification increases the recurrence of oscillations in the skin friction and Nusselt number.

Acknowledgments

One of the author (R.S. Nath) would like to thank University Grant Commission (UGC), New Delhi, India for their financial help in the form of Junior Research Fellowship (JRF) .

ORCID

Nitul Kalita, https://orcid.org/0000-0001-8348-2225; **Rudra Kanta Deka**, https://orcid.org/0009-0007-1573-4890; **Rupam Shankar Nath**, https://orcid.org/0009-0002-2352-0538

REFERENCES

[1] R. Muthucumaraswamy, N. Dhanasekar and G. E. Prasad, "Rotation effects on unsteady flow past an accelerated isothermal vertical plate with variable mass transfer in the presence of chemical reaction of first order," Journal of Applied Fluid Mechanics, **6**(4) , 485-490 (2013) https://doi.org/10.36884/jafm.6.04.19561

[2] R. K.Deka and B. C. Neog,"Unsteady natural convection flow past an accelerated vertical plate in a thermally stratified fluid," Theoretical and Applied Mechanics **36**(4) 261-274 (2009) https://doi.org/10.2298/TAM090426

[3] B. S. Goud, P. Srilatha, K. R. Babu and L. Indira, "Finite element approach on MHD flow through porous media past an accelerated vertical plate in a thermally stratified fluid," Journal of Critical Reviews, **7**(16), 69-74 (2020)

[4] A. Bhattacharya, and R. K. Deka. "Theoretical Study of Chemical Reaction Effects on Vertical Oscillating Plate Immersed in a Stably Stratified Fluid," Research Journal of Applied Sciences, Engineering and Technology, **3**(9), 887-898 (2011). https://maxwellsci.com/print/rjaset/v3-887-898.pdf

[5] E. Magyari, I. Pop, and B. Keller, "Unsteady Free Convection along an Infinite Vertical Flat Plate Embedded in a Stably Stratified Fluid-Saturated Porous Medium," Transp. Porous. Med. **62**, 233–249 (2006). https://doi.org/10.1007/s11242-005-1292-6

[6] A. Shapiro, and E. Fedorovich, "Unsteady convectively driven flow along a vertical plate immersed in a stably stratified fluid," Journal of Fluid Mechanics, **498**, 333-352 (2004). https://doi.org/10.1017/S0022112003006803

[7] J.S. Park, and J.M. Hyun, "Technical Note Transient behavior of vertical buoyancy layer in a stratified fluid," International Journal of Heat and Mass Transfer, **41**(24), 4393-4397 (1998), https://doi.org/10.1016/S0017-9310(98)00175-6

[8] J.S. Park, "Transient buoyant flows of a stratified fluid in a vertical channel," KSME International Journal, **15**, 656–664 (2001). https://doi.org/10.1007/BF03184382

[9] R. Kandasamy, K. Periasamy, and K.K.S. Prabhu, "Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects," International Journal of Heat and Mass Transfer, **48**(21-22), 4557-4561 (2005). https://doi.org/10.1016/j.ijheatmasstransfer.2005.05.006

[10] M.A. Mansour, N.F. El-Anssary, and A.M. Aly, "Effects of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret and Dufour numbers," Chemical Engineering Journal, **145**(2), 340-345 (2008). https://doi.org/10.1016/j.cej.2008.08.016

[11] A.M. Megahed, and W. Abbas, "Non-Newtonian Cross fluid flow through a porous medium with regard to the effect of chemical reaction and thermal stratification phenomenon," Case Studies in Thermal Engineering, **29**, 101715 (2022). https://doi.org/10.1016/j.csite.2021.101715

[12] M. Narahari and L. Debnath, "Unsteady magnetohydrodynamic free convection flow past an accelerated vertical plate with constant heat flux and heat generation or absorption," ZAMM- Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, **93**(1), 38–49 (2013) https://doi.org/10.1002/zamm.201200008

[13] Y. D. Reddy, B. Shankar Goud abd M. Anil Kumar,"Radiation and heat absorption effects on an unsteady MHD boundary layer flow along an accelerated infinite vertical plate with ramped plate temperature in the existence of slip condition," Partial Differential Equations in Applied Mathematics, **4**, 100166 (2021), https://doi.org/10.1016/j.padiff.2021.100166

[14] R.B. Hetnarski, "An algorithm for generating some inverse Laplace transforms of exponential form," Journal of Applied Mathematics and Physics (ZAMP), **26**, 249–253 (1975). https://doi.org/10.1007/BF01591514.

[15] Abramowitz, Milton, I.A. Stegun, and R.H. Romer. "Handbook of mathematical functions with formulas, graphs, and mathematical tables." American Journal of Physics, **56**(10), 958 (1988). https://doi.org/10.1119/1.15378

НЕСТІЙКИЙ ПОТІК ПОВЗ ПРИСКОРЕНУ ВЕРТИКАЛЬНУ ПЛАСТИНУ ЗІ ЗМІННОЮ ТЕМПЕРАТУРОЮ ЗА НАЯВНОСТІ ТЕРМОСТРАТИФІКАЦІЇ ТА ХІМІЧНОЇ РЕАКЦІЇ

Нітул Каліта, **Рудра Канта Дека**, **Рупам Шанкар Натх**

Факультет математики, Університет Гаухаті, Гувахаті-781014, Ассам, Індія

Ця робота спрямована на дослідження впливу термічної стратифікації на потік рідини повз прискорену вертикальну пластину за наявності хімічної реакції першого порядку. Безрозмірні нестаціонарні пов'язані лінійні керуючі рівняння розв'язуються методом перетворення Лапласа для випадку, коли число Прандтля дорівнює одиниці. Важливі висновки, зроблені в цьому дослідженні, вплив термічної стратифікації порівнюють зі сценарієм, в якому стратифікації не було. Результати чисельних обчислень для різних наборів фізичних параметрів, таких як швидкість, температура, концентрація, тертя, число Нуссельта та число Шервуда, відображаються графічно. Показано, що стаціонарний стан досягається швидше, коли потік стратифікований.

Ключові слова: *термічна стратифікація; хімічна реакція; тепло- та масообмін; вертикальна пластина; прискорення*