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NUCLEON-NUCLEON ELASTIC SCATTERING FOR MOTION IN THE SHIFTED DENG-FAN POTENTIAL[†]

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The scattering theory's main objective is to comprehend an object by hurling something at it. One can learn details about an object by observing how it bounces off other objects. The potential that exists between the two particles is the thing that one seeks to comprehend. In a time-independent approach to scattering, one assumes that the incident beam has been activated for a very long time and that the entire system is in a stationary state. For short-range local potentials, the variable phase methodology is highly useful in solving quantum mechanical scattering problems. Variable phase methodology/phase-function technique has been explicitly utilized for non-relativistic nucleon-nucleon scattering phenomenon with the fundamental central local potential term and without spin-orbit force. Working under this methodology, scattering phase shifts, total scattering cross section, and Differential cross section have been investigated for a new nuclear potential model "Shifted Deng-Fan potential". Real nucleon-nucleon scattering systems (n-p) and (p-p) have been treated for this purpose with partial waves up to $\ell = 2$ in the low and moderate energy region. For $\ell > 0$ waves, interacting repulsive barrier potential has been incorporated with the existing central part. Our results for the considered potential model show a close contest with that of the experimental data.
Keywords: *Shifted Deng-Fan Potential; Phase function method; Scattering Phase shifts; Scattering cross sections; (n-p) and (p-p) systems*
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INTRODUCTION

It is a well-known fact that the exact solution of the Schrödinger equation is significant in quantum mechanics as they enclose all necessary information regarding the quantum system under consideration. Most of the quantum systems can only be treated by approximation methodologies [1,2,3], as exact analytic solutions are feasible only for a few simple cases such as the hydrogen atom, the harmonic oscillator and others [4,5,6] in all partial waves and all energies. In a quest to find a suitable potential for diatomic interaction to describe the vibrational spectrum, Deng and Fan, in 1957, proposed a new molecular potential model [7] that is exponential in nature and was called Generalized Morse potential [8]. This potential is a modification of the Morse potential also known as Deng-Fan molecular potential (DF). Numerous studies were performed for this potential by researchers in various applications [9]. This potential has been adequately utilized in describing the nucleons' mobility in the mean field produced from the interactions of the nuclei [10]. Dong treated the Deng-fan potential as a pertinent alternative to the Morse potential for vibrational spectrum and electromagnetic transitions [11,12] of the diatomic molecules. Mesa [13] applied this potential for energy spectra studies of the diatomic molecules. Oyewumi [14] utilized the Nikiforov–Uvarov method to obtain bound state solutions of the Deng–Fan molecular potential for several diatomic molecules like HCl, LiH, H2 and so on. Many of the other works have been accomplished with this potential via different quantum mechanical wave equations [15-22] by utilizing several standard approximation prescriptions to the solution in both relativistic and non-relativistic domains.

A modified form of the DF called Shifted Deng-Fan potential (SDF)was proposed by Hamzavi et al. in 2012[23] for the calculation of ro-vibrational energy levels for few of the diatomic molecules. In the modified form, the DF potential is shifted by the dissociation energy (D). Ref. [23] also demonstrated that DF and Morse potential are qualitatively similar but SDF and Morse potentials are very much similar for large values of r i.e in the regions $r \approx r_e$ and $r > r_e$, however, they differ at $r \approx 0$. Here, r_e is the equilibrium diatomic separation. Louis [24] solved the Dirac equation for the Manning-Rosen plus shifted Deng-Fan (MRSDF) potential in the presence of spin and pseudospin (pspin) symmetries and by including a Coulomb-like tensor potential. All of above works [7-24] pertains to molecular spectroscopy and molecular dynamics. Within the framework of the shifted Deng Fan potential (SDF), Sajedi [25] studied the cluster structure of astrophysically important ^{19}Ne nucleus. In recent past, working with the exponential class of potentials, our group obtained exact analytical solution of the elastic Deng Fan potential [26] scattering of a particle in S-wave and obtained the phase parameters using the Jost function methodology for the systems under consideration in the nuclear realm.

In this article, we present the study of the non-relativistic nuclear scattering treatment of the SDF potential in terms of the fundamental nucleon-nucleon scattering both charged and uncharged. In support of our justification, we present phase shift observables, total scattering cross section for proton-neutron (p-n) and proton-proton (p-p) scattering & differential cross section studies for proton-proton (p-p) scattering. A comparison is drawn for the obtained data against

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the well-established experimental data. This is indirect way of knowing how much the SDF potential model treatment is justified in nuclear realm. We make use of the standard Phase function approach (PFM) [27] for numerical solution of the Schrödinger wave equation for motion of a particle in the SDF potential. Our study on the selected systems reproduces well consistent data in phase parameters, elastic scattering cross section and differential cross section for partial wave calculations up to $\ell = 2$.

2. METHODOLOGY

Computing scattering phase shifts $\delta_\ell(k)$ as a function of centre of mass energy $\left(E_{c.m} = k^2 > 0\right.$ in the theoretical limit of $\left.\hbar^2/2m = 1\right)$ is one among the core problems of quantum scattering theory. Phase-function method (PFM) [27] is an alternative to the traditional Schrödinger equation approach. This technique is powerful for its capability in straightforward physical interpretation of its equations and basic quantities. Moreover, it introduces the same type of approach in the bound state problem. This methodology [27] arises from the fact in the theory that certain second order linear homogeneous differential equations can be reduced to their first order non-linear equations of Riccati type called Riccati equations. The Riccati equations are satisfied by the phase functions $\delta_\ell(k,r)$ that is having the meaning of the phase shifts at each point of the wave function for scattering by the potential $V(r)$ cut-off at that point. Similar investigations with PFM have been made by authors treating the local [28-34] and nonlocal [35-37] potentials. With the Shifted Deng Fan (SDF) [23-25] potential in all partial wave treatment, we incorporate a screened centrifugal barrier which is repulsive term. Thus, with a modified form of SDF potential [26], the effective potential becomes

$$V_{sDF}(r) = V_N(r) = v_0 + v_1 \frac{e^{-\alpha r}}{\left(1-e^{-\alpha r}\right)} + v_2 \frac{e^{-2\alpha r}}{\left(1-e^{-\alpha r}\right)^2} + \frac{\ell(\ell+1)}{r^2} \tag{1}$$

Where v_0 , v_1 and v_2 are the strength parameters with dimension of fm^{-2} and α is the inverse range parameter with dimension of fm^{-1} and r is the inter nucleon distance between the particles.

For a local potential [1-3,6-33], $\delta_\ell(k,r)$ satisfies a first-order non-linear differential equation [27] written as

$$\delta_\ell'(k,r) = -k^{-1}V(r)\left[\cos\delta_\ell(k,r)\hat{j}_\ell(kr) - \sin\delta_\ell(k,r)\hat{\eta}_\ell(kr)\right]^2, \tag{2}$$

where $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ are the Riccati–Bessel and Riccati Neuman functions [38].

The resulting Phase equations for S, P and D waves and corresponding $\ell = 0,1\,\&\,2$ values

$$\delta_0'(k,r) = -k^{-1}V(r)\left[\sin\left(\delta_0(k,r)+kr\right)\right]^2, \tag{3}$$

$$\delta_1'(k,r) = -\frac{V(r)}{k^3r^2}\left[\sin\left(\delta_1(k,r)+kr\right) - kr\cos\left(\delta_1(k,r)+kr\right)\right]^2 \tag{4}$$

and

$$\delta_2'(k,r) = -k^{-1}V(r)\left[\left(\frac{3}{k^2r^2}-1\right)\sin\left(\delta_2(k,r)+kr\right) - \frac{3}{kr}\cos\left(\delta_2(k,r)+kr\right)\right]^2. \tag{5}$$

The quantity k represents the momentum of the scattering particle (center of mass momentum) and is related to the centre of mass energy by the relation $k = \sqrt{2\mathrm{mE}}/\hbar$. Thus, k^2 is the energy of the scattering particle in the limit of $\hbar = 1$ and $2m = 1$ where m is the mass of the particle (reduced mass of the two particle system) scattering off the considered potential. Phase equation is solved initializing from the origin up to the asymptotic region, given the initial condition $\delta_\ell(k,0) = 0$. In the course of solving the phase equation, the phase $\delta_\ell(k,r)$ is built up by the potential in additive manner as one moves away from the origin to its asymptotic value which implies $\delta_\ell(k) = \lim_{r\to\infty}\delta_\ell(k,r)$. One can also calculate the amplitude function $A_\ell(k,r)$ by utilizing the phase function $\delta_\ell(k,r)$,

3. RESULTS AND DISCUSSIONS

Nuclear shifted Deng Fan potential (SDF) (Eq. (1)) is parameterized for the standard phase shifts [39, 40] of different states of the (n-p) and (p-p) systems by solving the differential equations (3)-(5) numerically. Proper optimization to the step size of the 'r' value is significant in the phase accumulation calculation within the range of the interaction. Thus, one has to judiciously optimize the step size in order to have proper phase parameters. The parameters for different states of the (n-p) system are given in Table 1. For stated states of the (p-p) system, we have utilized the same corresponding (n-p) states' parameters as it is an established fact that for nuclear force, (n-p) and (p-p) interactions are equivalent. But with (p-p), a Coulombic repulsion force is associated. To take care of this Coulomb force in (p-p) interaction, a Coulombic potential term is added to the existing nuclear SDF (Eq.1).

Table 1. List of parameters for states of (n-p) scattering system

System	States	α(fm^{-1})	v1(fm^{-2})	v2(fm^{-2})	v0(fm^{-2})
n-p/p-p	$^1S_0$	0.868	-0.7174	-0.050	0.014
	$^3S_1$	0.874	-2.000	1.760	-0.235
	$^1P_1$	0.756	-2.165	2.980	-0.006
	$^3P_0$	0.874	-3.430	2.500	-0.045
	$^3P_1$	0.756	-2.250	3.550	-0.010
	$^3P_2$	0.756	-2.410	1.300	-0.010
	$^1D_2$	0.350	-1.700	0.005	-0.060
	$^3D_1$	0.017	-0.550	5.800	-0.0102
	$^3D_2$	0.400	-1.682	0.050	-0.011
	$^3D_3$	0.350	-2.865	0.003	-0.008

Solving the equations (3)-(5), with the substitution of the values of the parameters from Table 1, we calculate the scattering phase shifts for neutron-proton (n-p) and (p-p) systems up to partial waves $\ell = 2$. We have used the calculated reduced mass to be $m_{np} = 0.5039\,amu$ for the n-p system, and the value of $\hbar^2/2m = 41.47\,MeV\,fm^2$. In our numerical routine, the parameters are given free running to fit the desired phase shifts for the various states of the concerned systems. The (n-p) scattering phase shifts are presented in the Figures **1-3**.

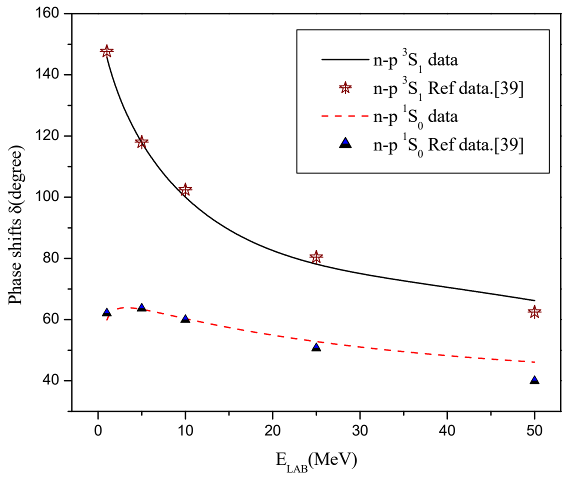

Figure 1. (n-p) S-wave scattering phase shifts as a function of laboratory energy

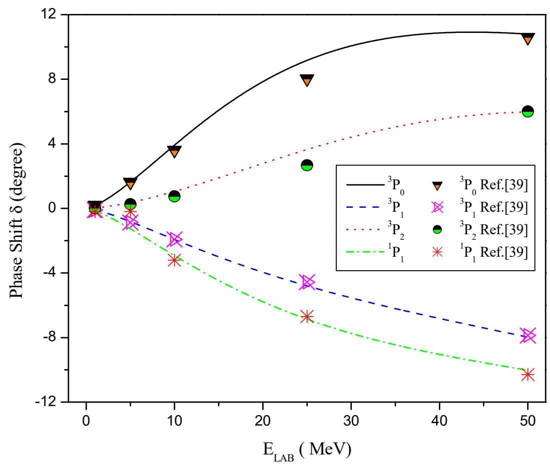

Figure 2. (n-p) P-wave scattering phase shifts as a function of laboratory energy

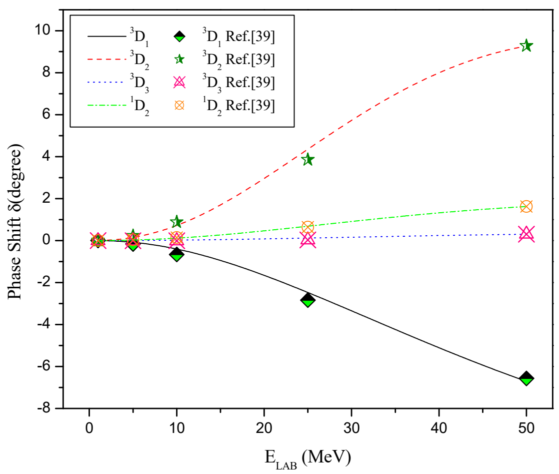

Figure 3. (n-p) D-wave scattering phase shifts as a function of laboratory energy

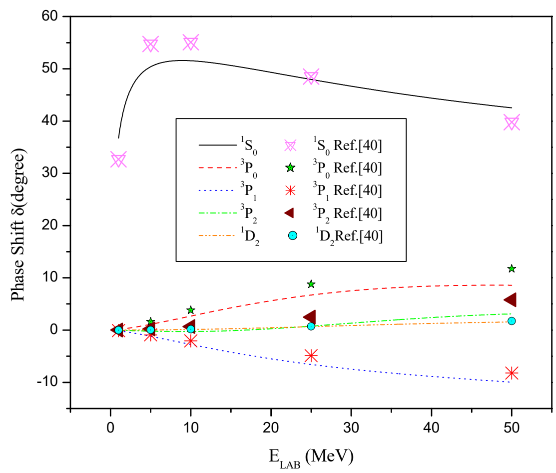

Figure 4. (p-p) S-P- and D-wave scattering phase shifts as a function of laboratory energy

And the (p-p) scattering phase shifts are presented in the Figure **4**. From the figures of obtained results, one can see that our obtained phase shifts are in close agreement with the experimental results of Gross and Stadler [39] and Wiringa [40] data. Fig. **1** shows that the parameters for the $^3S_1$ and $^1S_0$ states from Table **1** for (n-p) system reproduces the close experimental phase parameter results of Ref. [39] up to laboratory energy 50 MeV. Similarly, Figs. **2** & **3** depict the phase shift values for the P- and D- wave states and show close agreement with the results of Ref. [39]. However, the P-wave states $^3P_0$ and $^3P_1$ show some deviation in phase shift around the laboratory energy of 25 MeV although the trend is exactly matching. Beyond 50 MeV, the phase shifts start differing significantly with the energy for the reason that with increasing energy, the reaction channels come into effect dominantly over the elastic channel. Also, for (p-p) scattering in Fig. 4, shows correct trend for different states with some deviation in the phase shift results with increasing laboratory energy. The difference in their numerical values is for the reason that nuclear potentials are highly state dependent and cannot be generated properly from any known interaction unlike atomic cases. And in our case, the potential is only the spherical central term without spin-dependence and tensor potential.

For the Shifted Deng Fan Potential (SDF) model scattering of the (n-p) and (p-p) systems, the interacting potential forms for all the partial wave states have been presented in Figs. 5- 8. It is an well-established fact that nuclear potentials are highly state dependent and therefore potentials for each different states of S-, P- and D- waves are shown in Figs. 5-8, against the variable ‘r’ for the (n-p) and (p-p) systems. From these figures, one can notice that the potentials are fully consistent with the phase shifts produced.

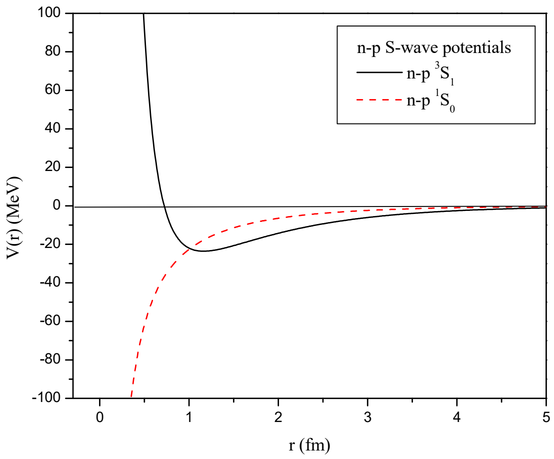

Figure 5. n-p S-wave potentials as a function of r

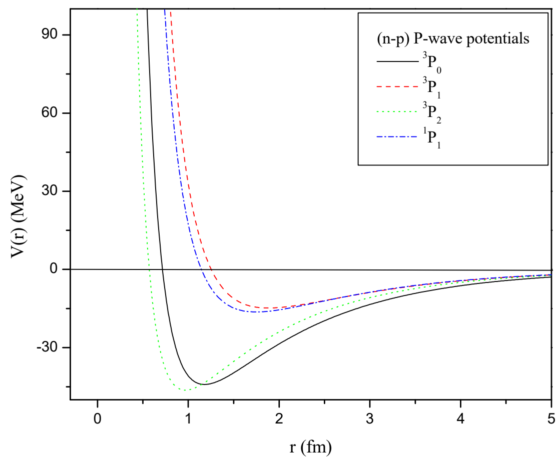

Figure 6. (n-p) P- wave potentials as a function of r

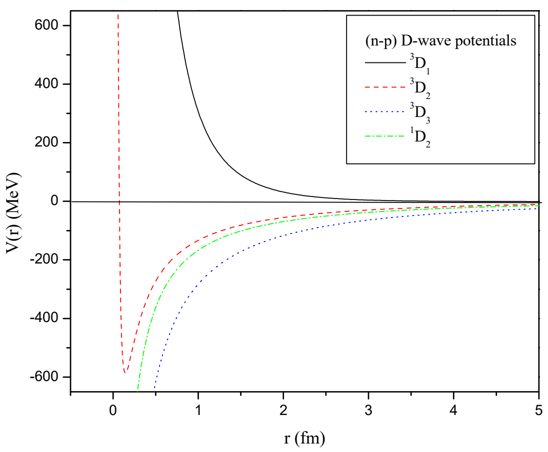

Figure 7. D- wave (n-p) potentials as a function of r

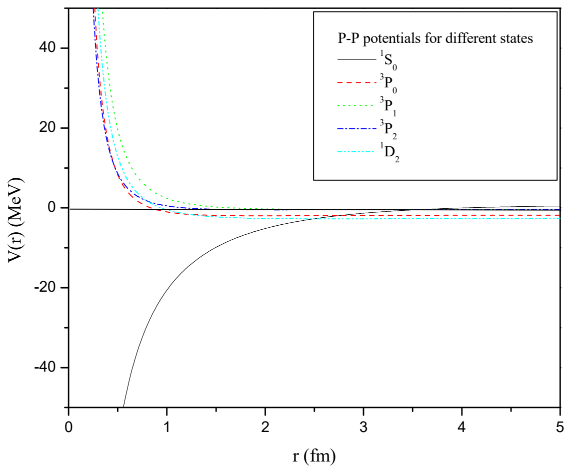

Figure 8. (p-p) potentials for different states as a function of r

4. SCATTERING CROSS SECTION

In general, two particle interactions, a beam of particles is directed at a layer of matter. The effect of this layer is composed additively of the effects of the individual units and the individual nuclei act as independent scattering centers. Upon scattering, scattered current is uniformly distributed over a sphere of radius r. The cross section of a scattering is then defined as the ratio of number of events of a given type per unit time per nucleus to the number of incident particles per unit area per unit time [41,42]. The concept of cross section cannot be used if many scattering centers are taken to act coherently with incident ones. Scattering cross section in core of its idea is an effective area proportional to the intrinsic rate at which a given radiation-target interaction occurs. Dimensionally cross section is an area with the base unit of barn (10^{-28} meter). We desire to investigate to what extent our SDF model calculations will be able to reproduce realistic cross section data in view of small discrepancies between the results of our phase shift analysis and of other calculations.

For combined coulomb and nuclear potential scattering as for charged particle interaction, the differential scattering amplitude is expressed as

$$f_{nC}(\theta) = f_C(\theta) + f_n(\theta) \,, \tag{6}$$

where

$$f_C(\theta) = -\left\{ \frac{\eta}{2\chi \sin^2 \theta/2} \right\} \exp\left[-i\eta \ln \sin^2(\theta/2) + 2i\sigma_0(\eta) \right], \tag{7}$$

and

$$f_n(\theta) = \frac{1}{2i\chi} \sum_{\ell=0}^{\infty} (2\ell+1) \exp(2i\sigma_\ell(\eta)) P_\ell(\cos\theta) (\exp(2i\delta_\ell^n) - 1) \,. \tag{8}$$

The quantity δ_ℓ^n is the Coulomb-distorted nuclear phase shift. The negative sign in front of Eq. (7) originates from the fact that the Coulomb force between two protons is repulsive. The Coulomb-distorted nuclear cross section $\sigma_{nC}(\theta)$ is given by

$$\sigma_{nC}(\theta) = |f_C(\theta) + f_n(\theta)|^2 = |f_{nC}(\theta)|^2 \,. \tag{10}$$

For identical particle, like (p-p), scattering

$$\sigma(\theta) = |f(\theta) + f(\pi - \theta)|^2 \,. \tag{11}$$

One may calculate the total scattering cross section by integrating the differential cross section $\sigma(\theta)$ over the entire solid angle and the angle integrated cross section is

$$\sigma_S = \frac{4\pi}{k^2} \sum_{L=0}^{\infty} (2L+1) \sin^2 \delta_\ell \,, \tag{12}$$

where δ_ℓ is the total scattering phase shift.

Note that this integrated cross section is sometimes called the total cross section because it is the total after integration over all angles. The elastic scattering of neutrons by proton and proton by proton have been investigated by a number of researchers [43-48]. In the present text we calculate differential and total scattering cross sections for the (p-p) & (n-p) systems and compare them with the data [47- 48] available in the literature by exploiting Eqs. (7)- (12). The (p-p) differential cross sections are portrayed in Figs. 9 & 10 together with the Ref. [47] over the whole angular range. However, the experimental results [47] are available only up to angle 50°. We have obtained satisfying data from our calculation for two different laboratory energies of 6.141MeV and 9.918 MeV as shown in Figs. 9 &10 consecutively.

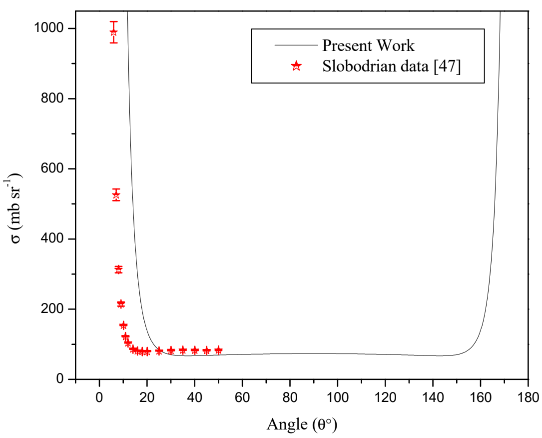

Figure 9. Differential p-p scattering cross section as a function of θ at E_{Lab}=6.141 MeV

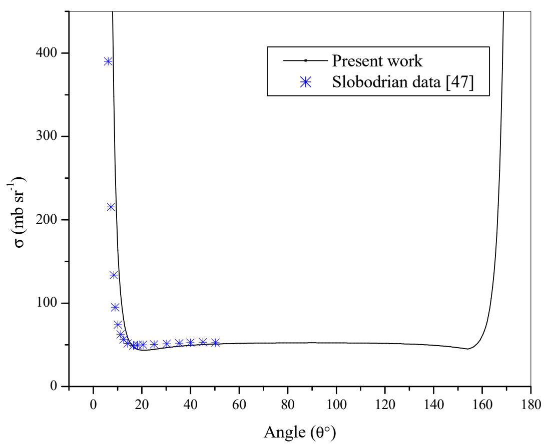

Figure 10. Differential p-p scattering cross section as a function of θ at E_{Lab}=9.918 MeV

The angular distributions for 6.141 MeV differ quantitatively in a narrow margin with those of Slobodrian et al. [47] up to angle 20^0 while for larger energy 9.918 MeV our results are in good conformity with Ref. [47]. Total cross section results for both the systems (n-p) and (p-p) have been obtained up to laboratory energies of 50 MeV (Figs. 11 & 12) and compared with the experimental data [48].

The total cross section calculations are performed including the contributions of S, P and D waves for neutron-proton (n-p) and proton-proton (p-p) scattering systems. Our (n-p) cross sectional data are in excellent agreement with the Arndt data [48] while for (p-p) system it shows qualitative agreement but with a slight quantitative disagreement in the energy range 2- 20 MeV.

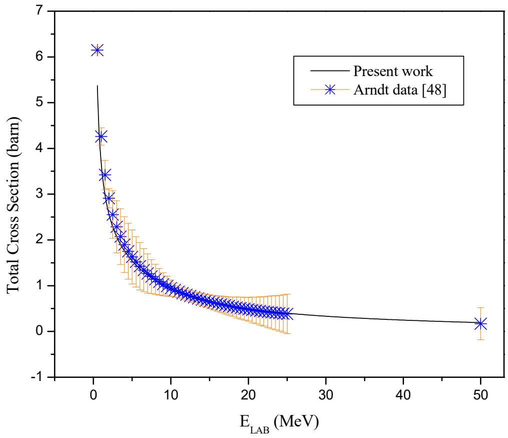

Figure 11. (n-p) total scattering cross section as a function of laboratory energy

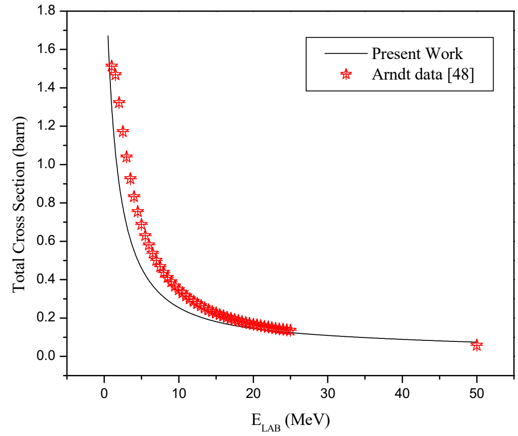

Figure 12. Total p-p scattering data with laboratory energy

5. CONCLUSIONS

We have parameterized the Shifted Deng Fan potential [23-25] for the phase shift parameters of the (n-p) and (p-p), spin independent, non-relativistic quantum nuclear scattering. Thus, obtained phase shifts along with the parameters are used to obtain total scattering cross section and differential scattering cross section values. Having obtained close agreements for scattering phase parameters with standard data of Gross and Stadler [38] & Wiringa [39], differential cross section data with Slobodrian [47], total cross section results with Arndt et al. [48], it is vivid that under standard PFM [27], Shifted Deng Fan potential (SDF) model scattering has the capability of producing the correct nature of phase shifts of respective states. And this simple minded, only three parameter attractive potential suffices to reproduce the most of the low energy nuclear interaction environment. In future, our group is aiming to explore this potential with other standard methodologies and several newer real scattering systems. We are hopeful that the present representation of the SDF potential in nuclear domain in the context of non-relativistic quantum scattering physics is expected to explore new possibilities.

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НУКЛОН-НУКЛОННЕ ПРУЖНЕ РОЗСІЯННЯ ПРИ РУСІ У ЗМІЩЕНОМУ ПОТЕНЦІАЛІ ДЕНГА-ФАНА

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Основна мета теорії розсіювання полягає в тому, щоб зрозуміти об'єкт, якщо щось в нього кинути. Можна дізнатися подробиці про об'єкт, спостерігаючи, як він відскакує від інших об'єктів. Потенціал, який існує між двома частинками, - це те, що ми прагнемо зрозуміти. У незалежному від часу підході до розсіювання передбачається, що падаючий промінь був активований протягом дуже тривалого часу і що вся система перебуває в стаціонарному стані. Для короткодіючих локальних потенціалів методологія змінної фази дуже корисна при розв'язанні задач квантово-механічного розсіювання. Методологія змінної фази/техніка фазової функції була явно використана для нерелятивістського явища нуклон-нуклонного розсіювання з фундаментальним центральним локальним потенційним членом і без спін-орбітальної сили. Працюючи за цією методологією, фазові зсуви розсіювання, загальний переріз розсіювання та диференціальний переріз були досліджені для нової моделі ядерного потенціалу «зміщений потенціал Денга-Фана». Реальні нуклон-нуклонні системи розсіювання (n-p) і (p-p) були оброблені для цієї мети парціальними хвилями $\ell = 2$ до в області низьких і помірних енергій. Для хвиль $\ell > 0$ взаємодіючий бар'єрний потенціал відштовхування було включено в існуючу центральну частину. Наші результати для розглянутої потенційної моделі показують близьку конкуренцію з результатами експериментальних даних.

Ключові слова: *зміщений потенціал Денга-Фана; метод фазової функції; фазовий зсув розсіювання; перерізи розсіювання; (n-p) і (p-p) системи*