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A STUDY OF EVOLUTION OF COSMOLOGICAL PARAMETERS BASED ON DARK ENERGY MODELS IN KALUZA-KLEIN FRAMEWORK[†]

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The purpose of the present study is to determine the characteristics of time evolution of various cosmological quantities, based on four models constructed for a universe undergoing accelerated expansion. This formulation is done in the framework of Kaluza-Klein space-time, for zero spatial curvature. To solve the field equations, an ansatz is chosen for each model in such a way that it leads to a signature flip of the deceleration parameter, to ensure its consistency with recent astrophysical observations indicating a change from a decelerated expansion to an accelerated expansion of the universe. Based on these four models, time evolutions of several cosmological parameters are obtained and their variations are shown graphically against time. The arbitrary constants, associated with each model, are so tuned that the model correctly predicts the values of the Hubble parameter, deceleration parameter, energy density and gravitational constant at the present time. The findings from these models are consistent with each other, and they are in agreement with the observed features. The gravitational constant (G) shows a rapid fall in the early universe, followed by an extremely slow rise which continues at the present time. Taking (G) as a constant in two of the four models, the cosmological constant is found to be independent of time. A significant finding is that the signature flip of the deceleration parameter almost coincides with the signature flip of the cosmological constant (Λ), pointing towards a relation between the accelerated expansion and the dark energy which is represented by Λ. Other plots with respect to Λ also depict dark energy's role in governing cosmic evolution. Considering its dynamical nature, Λ is referred to as cosmological term (instead of cosmological constant) in the text. Contrary to the common trend of using arbitrary units, the SI units for all measurable quantities are used.
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1. INTRODUCTION

Worldwide research in recent years, based on observational data, has proved beyond doubt that the universe is expanding with acceleration. It has caused a remarkable shift of the focus of research interests from a mere expansion to the mysteries of acceleration in the expansion process. Had gravitation been the only controlling force, which is attractive, the expansion would surely have been a decelerated one. An energy of an exotic form, designated as Dark Energy (DE), generating a negative pressure, is considered to be responsible for the cosmic acceleration, based on the experimental observations from supernova 1a [1, 2]. Extensive research is going on to find the characteristics of DE which remains a cosmological mystery. Based on recent research using supernova data, it has been found that the universe had changed its mode of expansion in the past, from deceleration to acceleration, causing a signature flip of the deceleration parameter from positive to negative [3-5]. One finds mainly two approaches of studying this field in the scientific literature to unravel the mysteries of the accelerated expansion of the universe. One of these approaches is to formulate models for dark energy and study their dynamics. Another approach is to construct cosmological models under the framework of modified theories of gravity (modified version of Einstein's theory) and study their dynamics. The cosmological constant (Λ) is regarded as the simplest candidate representing dark energy. There are other models like quintessence, phantom, k-essence and quintom which are known to account for dark energy [6-9]. Although Λ was used in the field equations by Einstein as a constant [10], it is presently regarded as a dynamical quantity due to certain limitations pertaining to Cosmological Problem and Coincidence Problem [11]. Einstein's theory of gravitation has undergone several modifications resulting in the birth new theories like f(R) and f(R,T) theories of gravity [12-14], and scalar tensor theories of gravitations such as Brans-Dicke (BD) and Saez-Ballester (SB) [15, 16]. Investigations of various types have been carried out in recent times by constructing DE cosmological models based on the above-mentioned theories [17-20].

Kaluza and Klein made an attempt to unify gravitational force with electro-magnetic force in the third decade of the twentieth century resulting into the formulation of Kaluza-Klein (KK) theory [21, 22]. There is an extra dimension, namely the fifth dimension, in the KK theory which was required to unify the two forces mentioned earlier. In the five dimensional model constructed by Chodos and Detweiler, it was shown that there is a contraction of the extra dimension due to cosmic evolution [23]. The present four-dimensional form of the universe is considered theoretically to be preceded by an era that required a multidimensional description. As time goes on, the extra dimensions shrink in

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such a way that they no longer remain detectable by the experimental facilities at our disposal. This phenomenon has motivated many researchers to work in the field of cosmological models in higher dimensions. KK theory is essentially the theory of general relativity in five dimensions. Some of the authors who have carried out significant studies in five-dimensional space-time are Chodos & Detweller [23], Witten [24], Appelquist et al. [25], Appelquist & Chodos [26] and Marchiano [27]. One of the articles that attracted our attention to the formulation of DE models, based on Kaluza-Klein theory, is an investigation by Mukhopadhyay *et al*. [28].

To ensure the authenticity of predictions based on a theoretical formulation, one must construct more than one model (under the same framework) and validate them using some standard results obtained from observational data. An objective of the present work is to carry out a theoretical study, to determine the time dependence of some cosmological quantities, by constructing models with sufficient inter-model consistency. Another objective is to determine the dependence of cosmic evolution upon dark energy, represented here by the cosmological term (Λ). In order to fulfill the second objective, we aim to formulate expressions relating Λ with some parameters and also to examine graphically the variation of some parameters as functions of Λ.

For the present study, we have chosen the framework of the Kaluza-Klein space-time (with zero spatial curvature). The main theoretical formulation (described in Section 2) of our study is divided into two parts (A & B). The third part (Part-C) uses the results of the preceding parts to derive expressions involving Λ. Part-A has two models based on the assumption that the equation of state (EoS) parameter has a constant value (as in ref. no. 28). Part-B consists of two models formulated on an assumption that the gravitational term (G) is a constant quantity. Using these four models, we have determined the nature of time dependence of several cosmological quantities such as, scale factor (a), Hubble parameter (H), deceleration parameter (q), energy density (ρ), EoS parameter (ω), cosmological term (Λ), gravitational term (G) and $\dot{G}/G$. Part-A is based on two empirical expressions for Λ (for Models 1 & 2) and Part-B is based on two empirical expressions for H (for Models 3 & 4). Results of the studies, using these four models, are mostly in agreement with astrophysical observations regarding a universe expanding with acceleration. To ensure the authenticity or theoretical validity of each model, we have tuned the constant parameters (associated with each model) in a way such that the values of H_0, q_0, ρ_0 and G_0 obtained from the model are consistent with their presently accepted values. The symbols H_0, q_0, ρ_0 and G_0 denote respectively the values of H, q, ρ and G at the present time, i.e. $t = t_0$, where t_0 is the present age of the universe. Time dependence of several cosmological quantities has been shown graphically and they are consistent with the findings of other recent investigations, some of whom are based on theoretical frameworks or models different from ours.

An important feature of this study is that, unlike many other contemporary studies, we have used proper units (SI) for all measurable quantities (H, ρ, G), instead of plotting them in arbitrary units. To depict the time dependence of a parameter pictorially, we have plotted it graphically against t/t_0 (where t_0 is the present age of the universe, about 13.7×10^9 years). This way of graphical depiction is expected to give the readers a concrete picture of how a cosmological parameter evolves with time. A significant and unique finding of our study is that the change of sign of the deceleration parameter (indicating transition from deceleration to acceleration) occurs almost simultaneously with the signature flip of the cosmological term Λ (which represents dark energy) from negative to positive.

2. FIELD EQUATIONS AND MODELS

The metric of Kaluza-Klein space-time is given by,

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 + (1-kr^2)d\psi^2\right] \tag{1}$$

Here, $d\Omega^2 = d\theta^2 + sin^2\theta\, d\phi^2$ and $a(t)$ stands for the scale factor. The symbol k stands for the curvature parameter where we have $k = -1, 0, 1$ respectively for spatially open, flat and closed universe.

For a universe, which is constituted by a perfect fluid, the energy-momentum tensor is expressed as,

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - p g_{\mu\nu} \tag{2}$$

Here, $\mu, \nu = 0,1,2,3,4$ and u_μ (five-velocity) satisfies the relation $u^\nu u_\nu = 1$. The symbols p and ρ denote, respectively, the pressure of the cosmic fluid and the energy density of the universe.
Einstein's field equations are expressed as,

$$R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \tag{3}$$

In equation (3), $R_{\mu\nu}$ and R denote the Ricci curvature tensor and Ricci scalar respectively. The symbol Λ denotes what was initially introduced by Einstein as cosmological constant. The symbol $g_{\mu\nu}$ denotes the metric tensor for general relativity and G stands for the gravitational constant. Considering their dynamical nature in Models 1 & 2, Λ and G have been referred to as the cosmological term and the gravitational term respectively in the present article.

Based on equations (1), (2) and (3), we obtain the following two equations from which the time dependence of the scale factor (a) can be determined.

$$8\pi G\rho + \Lambda = 6\left(\frac{\dot{a^2}}{a^2} + \frac{k}{a^2}\right) \tag{4}$$

$$8\pi Gp - \Lambda = -3\left(\frac{\ddot{a}}{a} + \frac{\dot{a^2}}{a^2} + \frac{k}{a^2}\right) \tag{5}$$

For a universe with zero spatial curvature (i.e., $k = 0$), equations (4) and (5) can be expressed as the following two equations respectively.

$$8\pi G\rho + \Lambda = 6\frac{\dot{a^2}}{a^2} = 6H^2 \tag{6}$$

$$8\pi Gp - \Lambda = -3\dot{H} - 6H^2 \tag{7}$$

We have used the expression for the Hubble parameter (H), i.e., $H = \frac{\dot{a}}{a}$, to obtain equations (6) and (7) from equations (4) and (5) respectively.

The continuity equation is given by,

$$\dot{\rho} + 4H(p + \rho) = 0 \tag{8}$$

For a barotropic equation of state, we have the following relation between pressure and density.

$$p = \omega\rho \tag{9}$$

Here ω is called the equation of state (EoS) parameter.
Using equation (9) in equation (7) we obtain,

$$8\pi G\omega\rho - \Lambda = -3\dot{H} - 6H^2 \tag{10}$$

Using equation (9) in equation (8) we obtain,

$$\dot{\rho} + 4H\rho(\omega + 1) = 0 \tag{11}$$

Combining equation (6) with equation (10) we obtain,

$$\dot{H} = \frac{(\omega+1)(\Lambda - 6H^2)}{3} \tag{12}$$

2.1. PART-A

In this part of our study, we assume the EoS parameter (ω) to be independent of time, as has been done in many recent studies, such as the one by Mukhopadhyay *et al*. [28]. Here we formulate two models (Models 1 & 2) based on two different empirical expressions for the cosmological parameter (Λ).

Solving equation (11) by integration we obtain the following expression for energy density (ρ).

$$\rho = C_1 a^{-4(\omega+1)} \tag{13}$$

where C_1 is the constant of integration. Its value can be determined by using the values of energy density (ρ) and the scale factor (a) at present time ($t = t_0$). Choosing a scale factor with $a(t_0) = 1$ we get $C_1 = \rho_0$ where ρ_0 denotes the value of ρ at $t = t_0$.

2.1.1. Model-1

Here we use the following ansatz for the cosmological term (Λ).

$$\Lambda = 6H^2 - \beta t^m \tag{14}$$

where β and m are constants.
Substituting equation (14) into equation (12) and integrating we obtain,

$$H = \frac{\dot{a}}{a} = -\frac{(1+\omega)\beta}{3}\frac{t^{m+1}}{m+1} \tag{15}$$

Integrating equation (15), the scale factor (a) is obtained as.

$$a = C_2\, exp\left[-\frac{\beta\,(1+\omega)}{3\,(m+1)\,(m+2)}t^{m+2}\right] \tag{16}$$

where C_2 is the constant of integration.
Using equation (16), the expression for the deceleration parameter is obtained as,

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{3(m+1)^2}{\beta(1+\omega)} t^{-(m+2)} \tag{17}$$

Substituting equation (16) into equation (13) we obtain the following expression for the energy density (ρ).

$$\rho = D_1 \, exp\left[\frac{4\,\beta\,(1+\omega)^2}{3\,(m+1)\,(m+2)}\; t^{(m+2)}\right] \tag{18}$$

where $D_1 = C_1 C_2^{-4(\omega+1)}$.
Substituting equation (15) into equation (14), we get the following expression for the cosmological term (Λ).

$$\Lambda = 6\left[\frac{\beta\,(1+\omega)}{3\,(m+1)}\right]^2 t^{2\,(m+1)} - \beta t^m \tag{19}$$

Using equations (15), (18) and (19) in equation (6), we get the following expression for the gravitational term (G).

$$G = \frac{\beta}{8\pi D_1} t^m exp\left[-\frac{4\beta(1+\omega)^2}{3(m+1)(m+2)} t^{m+2}\right] \tag{20}$$

Using equation (20) we get the following expression for $\dot{G}/G$,

$$\frac{\dot{G}}{G} = mt^{-1} - \frac{4\beta(1+\omega)^2}{3(m+1)} t^{m+1} \tag{21}$$

2.1.2. Model-2

Here we use the following ansatz for the cosmological term (Λ).

$$\Lambda = 6H^2(1 + \alpha t^n) \tag{22}$$

where α and n are constants.
Substituting equation (22) in equation (12) and integrating we get,

$$H = \frac{\dot{a}}{a} = -\frac{(n+1)}{2\alpha(1+\omega)} t^{-(n+1)} \tag{23}$$

Integrating equation (23), the expression of the scale factor (a) is obtained as,

$$a = C_3 \, exp\left[\frac{n+1}{2\,\alpha\,n\,(1+\omega)} t^{-n}\right] \tag{24}$$

where the constant of integration is C_3.
Using equation (24), the deceleration parameter (q) is obtained as,

$$q = -1 - 2\alpha(1+\omega)t^n \tag{25}$$

Substituting equation (24) into equation (13), we get the following expression for the energy density (ρ).

$$\rho = D_2 \, exp\left[-\frac{2(n+1)}{\alpha\,n} t^{-n}\right] \tag{26}$$

Here $D_2 = C_1 C_3^{-4(\omega+1)}$.
Substituting equation (23) into equation (22), the cosmological constant (Λ) is obtained as,

$$\Lambda = 6\left[\frac{(n+1)}{2\alpha(1+\omega)}\right]^2 t^{-2(n+1)}(1 + \alpha t^n) \tag{27}$$

Using equations (23), (26) and (27), in equation (6), the following expression is obtained for the gravitational term (G).

$$G = -\frac{3(n+1)^2}{16\pi D_2(1+\omega)^2\alpha}\; t^{-(n+2)} exp\left[\frac{2(n+1)}{\alpha n} t^{-n}\right] \tag{28}$$

Using equation (28), we get the following expression for $\dot{G}/G$.

$$\frac{\dot{G}}{G} = -(n+2)t^{-1} - \frac{2(n+1)}{\alpha} t^{-(n+1)} \tag{29}$$

2.2. PART-B

In this part of our study, we assume G to be independent of time, having a value of $G = 6.67 \times 10^{-11}\, Nm^2 Kg^{-2}$ (i.e., the value of the gravitational constant). This assumption is logically valid because the value of $\dot{G}/G$ is very small ($\approx 10^{-11}\; Yr^{-1}$ at $t = t_0$, as obtained from Models 1 & 2). Here we formulate two models based on two different empirical expressions for the Hubble parameter (H).

Using equations (6) and (10) we get,

$$\omega = -1 - \frac{3}{8\pi G\rho}\dot{H} \tag{30}$$

Substituting equation (30) into equation (11) we get,

$$\dot{\rho} = \frac{3H}{2\pi G}\dot{H} \tag{31}$$

Solving equation (31) for ρ by integration we obtain,

$$\rho = \rho_0 + \frac{3}{4\pi G}(H^2 - H_0^2) \tag{32}$$

In obtaining equation (32) from the solution of equation (31), we have used the fact that $H = H_0$ and $\rho = \rho_0$ at $t = t_0$ (the present time).
Substituting equation (32) into equation (6) we obtain the following expression for Λ.

$$\Lambda = 6H_0^2 - 8\pi G\rho_0 \tag{33}$$

Equation (33) shows that Λ is time independent.

Using the values of the parameters G, H_0 and ρ_0 in equation (33), we get $\Lambda = 1.768 \times 10^{-35}$. The values of different cosmological parameters (at $t = t_0$) used in this article are given in Section-3.

2.2.1. Model-3

In this model, we use the following ansatz for the Hubble parameter (H).

$$H = H_0\left(\frac{t}{t_0}\right)^k \tag{34}$$

Here, k is a constant.
At $t = t_0$, the ansatz yields $H = H_0$.
Using equation (34), we get the following expression for the deceleration parameter.

$$q = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{k}{H_0 t_0}\left(\frac{t}{t_0}\right)^{-(k+1)} \tag{35}$$

Using the fact that $q = q_0$ at $t = t_0$, we get,

$$k = -H_0 t_0(1 + q_0) \tag{36}$$

Substituting equation (34) into equation (32) we get the following expression for the energy density.

$$\rho = \rho_0 + \frac{3H_0^2}{4\pi G}\left[\left(\frac{t}{t_0}\right)^{2k} - 1\right] \tag{37}$$

Using equations (34) and (37) in equation (30), we get the EoS parameter (ω) as,

$$\omega = -1 - k\,\frac{3H_0}{8\pi G}\left(\frac{t^{k-1}}{t_0^k}\right)\left[\frac{1}{\rho_0 + \frac{3H_0^2}{4\pi G}\left[\left(\frac{t}{t_0}\right)^{2k} - 1\right]}\right] \tag{38}$$

Using the relation $H = \frac{\dot{a}}{a}$, equation (34) can be rewritten as,

$$\frac{\dot{a}}{a} = H_0\left(\frac{t}{t_0}\right)^k \tag{39}$$

Solving the above equation by integration and taking $a = 1$ at $t = t_0$, we get,

$$a = Exp\left[\frac{H_0 t_0}{(k+1)}\left(\left(\frac{t}{t_0}\right)^{k+1} - 1\right)\right] \tag{40}$$

2.2.2 Model-4

In this model, we choose an ansatz for the Hubble parameter (H), which is given by,

$$H = \frac{\mu}{t} + \gamma \tag{41}$$

where γ and μ are arbitrary constants.
Using equation (41) we obtain the following expression for the deceleration parameter.

$$q = -1 + \frac{\mu}{(\mu+\gamma t)^2} \tag{42}$$

Using the fact that, $H = H_0$, $q = q_0$ at $t = t_0$, we obtain the following values of the parameters μ and γ (based on eqns. 41 and 42).

$$\mu = H_0^2\, t_0^2\, (1 + q_0) \tag{43}$$

$$\gamma = H_0 - \frac{H_0^2\, t_0^2\, (1+q_0)}{t_0} \tag{44}$$

Substituting equation (41) into equation (32) we obtain the following expression for the energy density (ρ).

$$\rho = \rho_0 + \frac{3}{4\pi G}\left[\left(\gamma + \frac{\mu}{t}\right)^2 - H_0^2\right] \tag{45}$$

Substituting equation (41) and (45) into equation (30), we obtain the following expression for the EoS parameter (ω).

$$\omega = -1 + \frac{3\mu}{8\pi G t^2}\left[\frac{1}{\rho_0 + \frac{3}{4\pi G}\left[\left(\gamma+\frac{\mu}{t}\right)^2 - H_0^2\right]}\right] \tag{46}$$

Using the relation $H = \frac{\dot{a}}{a}$, equation (41) can be written in the following way,

$$\frac{\dot{a}}{a} = \frac{\mu}{t} + \gamma \tag{47}$$

Solving equation (47) by integration and using $a = 1$ for $t = t_0$, we get,

$$a = e^{\gamma(t-t_0)}\left(\frac{t}{t_0}\right)^{\mu} \tag{48}$$

2.3 PART-C
Expressions relating Λ with q and ρ based on Models 1 & 2

We know that Λ represents dark energy and its value is related with dark energy density [46]. In this part of the study we find relations connecting Λ to q and ρ, based on Part-A.

Combining equation (19) with equations (17) and (18), we get the following two equations relating Λ with q and ρ respectively.

$$\Lambda = 6\left[\frac{\beta\,(1+\omega)}{3\,(m+1)}\right]^2 \left(\left[(q+1)\,\frac{\beta(1+\omega)}{3(m+1)^2}\right]^{\frac{-1}{m+2}}\right)^{2\,(m+1)} - \beta\left(\left[(q+1)\,\frac{\beta(1+\omega)}{3(m+1)^2}\right]^{\frac{-1}{m+2}}\right)^{m} \tag{49}$$

$$\Lambda = 6\left[\frac{\beta\,(1+\omega)}{3\,(m+1)}\right]^2 \left(\left[\left(ln\left(\frac{\rho}{D_1}\right)\right)\left(\frac{3\,(m+1)\,(m+2)}{4\,\beta\,(1+\omega)^2}\right)\right]^{\frac{1}{m+2}}\right)^{2\,(m+1)} - \beta\left(\left[\left(ln\left(\frac{\rho}{D_1}\right)\right)\left(\frac{3\,(m+1)\,(m+2)}{4\,\beta\,(1+\omega)^2}\right)\right]^{\frac{1}{m+2}}\right)^{m} \tag{50}$$

Combining equation (27) with equations (25) and (26), we get the following two equations relating Λ with q and ρ respectively.

$$\Lambda = 6\left[\frac{(n+1)}{2\alpha(1+\omega)}\right]^2 \left(\left(\frac{q+1}{-2\alpha(1+\omega)}\right)^{\frac{1}{n}}\right)^{-2(n+1)} \left(1 + \alpha\left(\left(\frac{q+1}{-2\alpha(1+\omega)}\right)^{\frac{1}{n}}\right)^{n}\right) \tag{51}$$

$$\Lambda = 6\left[\frac{(n+1)}{2\alpha(1+\omega)}\right]^2 \left(\left[\left(ln\left(\frac{\rho}{D_2}\right)\right)\left(\frac{\alpha\, n}{-2(n+1)}\right)\right]^{-\frac{1}{n}}\right)^{-2(n+1)} \left(1 + \alpha\left(\left[\left(ln\left(\frac{\rho}{D_2}\right)\right)\left(\frac{\alpha\, n}{-2(n+1)}\right)\right]^{-\frac{1}{n}}\right)^{n}\right) \tag{52}$$

Equations (49) and (51) are expressions for the cosmological term (Λ) in terms of deceleration parameter (q), based on Models 1 and 2 respectively. Equations (50) and (52) are expressions for the cosmological term (Λ) in terms of energy density (ρ), based on Models 1 and 2 respectively.

3. RESULTS AND DISCUSSION

To find the time dependence of various cosmological quantities (a, H, q, ρ, G, Λ, ω, $\dot{G}/G$), one needs to know the values (or the permissible ranges of values) of the constant parameters associated with each of the four models constructed here. These values can be estimated from the considerations of the following requirements.

1) $a > 0$ by definition. Here we have chosen $a = 1$ at $t = t_0$,
2) $H > 0$ since the universe is expanding,
3) $\rho > 0$ by definition,

4) the values of H_0, q_0, ρ_0, G_0, which are obtained from the models, should be consistent with their currently accepted values obtained from observational data,

5) the time-variation of q, as obtained from these models, must be such that it undergoes a change of sign from positive to negative, to be consistent with the fact that the present accelerated expansion of the universe was preceded by a phase of deceleration [3-5].

The currently accepted values (in SI units) of some cosmological parameters, used in the present study are given below [29-31].

$H_0 = 2.39 \times 10^{-18}\, sec^{-1}$, $q_0 = -0.55$, $\rho_0 = 9.90 \times 10^{-27} Kg\, m^{-3}$, $G_0 = 6.67 \times 10^{-11} N\, m^2\, Kg^{-2}$, $t_0 = 4.34 \times 10^{17}\, sec$.

Based on the five requirements, mentioned above, we have obtained the following set of values for the constant parameters associated with Models 1 & 2.

$\alpha = -1.230 \times 10^{9}$, $\beta = 1.230 \times 10^{-9}$, $m = -1.467$, $n = -0.533$, $\omega = -0.535$, $C_1 = 9.9 \times 10^{-27}$, $C_2 = C_3 = 0.143$.

Following values have been obtained for the constant parameters associated with the Models 3 & 4, using equations (36), (43) and (44).

$$k = -0.467,\ \mu = 0.484,\ \gamma = 1.274 \times 10^{-18},\ \Lambda = 1.768 \times 10^{-35}$$

Figure 1 shows the time dependence of the scale factor (a), as obtained from Models 1, 2, 3, 4 respectively. These graphs manifest almost the same behavior for a, indicating clearly that it increases monotonically with time, which is appropriate for an expanding universe and it is found to be consistent with the findings of a recent study based on a different model in Kaluza-Klein framework [28].

Figure 2 shows the time dependence of the Hubble parameter (H), as obtained from Models 1, 2, 3, 4. These graphs are almost in agreement with each other, indicating a fall in the value of H with time, at a gradually decreasing rate. It is quite similar to the nature of time dependence of H as obtained from some other recent studies [28, 30, 31].

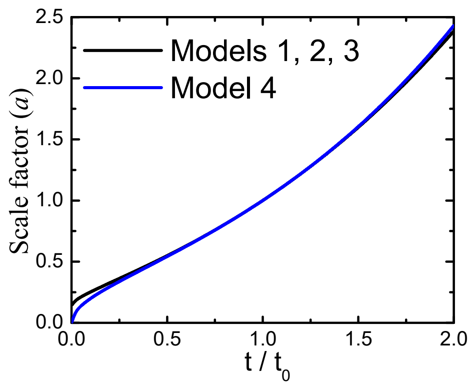

Figure 1. Variation of scale factor (a) as a function of time for Models 1-4

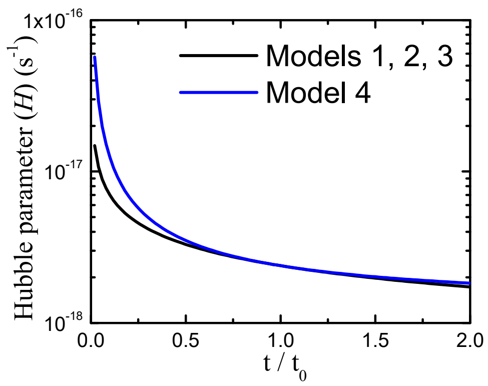

Figure 2. Variation of Hubble parameter (H) as a function of time for Models 1-4

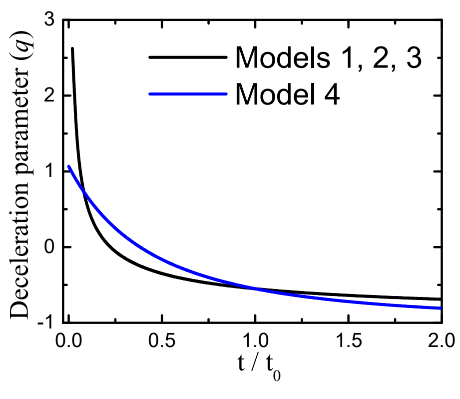

Figure 3. Variation of deceleration parameter (q) as a function of time for Models 1-4

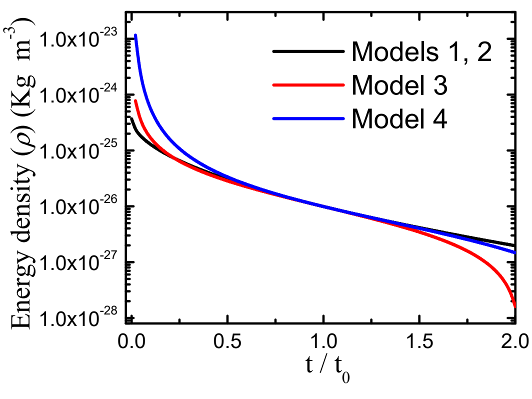

Figure 4. Variation of energy density (ρ) as a function of time for Models 1-4

Figure 3 shows the time variation of the deceleration parameter (q) for Models 1, 2, 3 and 4. Each of these graphs shows that q undergoes a signature flip from positive to negative, implying a change from a decelerated expansion to an accelerated expansion, in absolute agreement with the results based on observational data [3-5]. It is observed that q is a monotonically decreasing function of time, with a gradually decreasing rate of change. These characteristics of q are similar to the results of some recent investigations based on various theoretical formulations [28, 30-32].

Figure 4 depicts the time evolution of the energy density (ρ) as obtained from Models 1, 2, 3 and 4. For a greater visual clarity of data, ρ has been plotted in the logarithmic scale. These plots show that it decreases with time with a decreasing rate of change. This behavior of ρ is in agreement with the results of some recent studies carried out by methods different from ours [28, 31, 32].

Figure 5 shows the time variation of the cosmological term (Λ), based on Models 1 and 2. It is observed that Λ is negative in the early universe. It increases very steeply with time initially, becoming asymptotic to a positive value which is nearly 1.59×10^{-35}. Here, the rate of increase of Λ is found to be very high in the early universe but it gradually decreases to a much smaller value. This observation is consistent with the results of some recent studies carried out by various methods [33-37].

According to the Models 3 & 4, Λ is independent of time (eqn. 33), with a value of 1.77×10^{-35}. It is important to note that the value of Λ at present time (i.e., $t = t_0$), obtained from Models 1 & 2, is 1.77×10^{-35}, which is the same as the value obtained from Models 3 & 4.

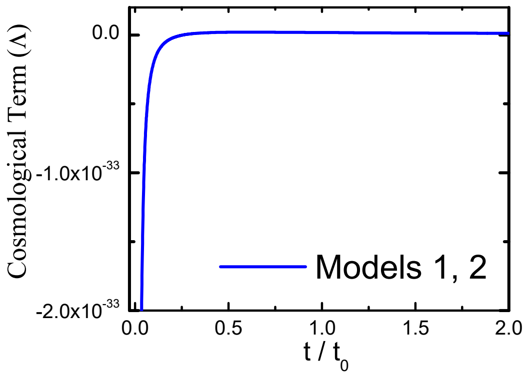

Figure 5. Variation of cosmological term (Λ) as a function of time for Models 1 and 2

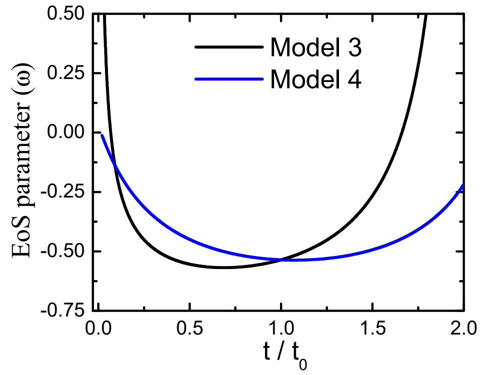

Figure 6. Variation of EoS parameter (ω) as a function of time for Models 3 and 4

Figure 6 shows the time variation of the EoS parameter (ω), as obtained from Models 3 and 4. It is observed that ω initially decreases and later increases with time for both models. This nature of time evolution is found to be close to the findings of some recent investigations based on models of various types [38-40]. These figures show that its values are in the quintessence ($\omega > -1$) region over almost the entire span of time shown in these plots. According to these plots we have, $\omega_0 = -0.535$. It is observed that this value is the same as the one that we must set for ω, for Models 1 & 2 (where ω is a constant quantity), to ensure that the predictions from these models are in agreement with the observed features of the universe.

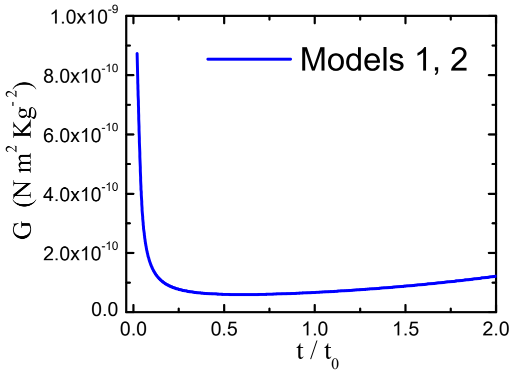

Figure 7. Variation of gravitational term (G) as a function of time for Models 1 and 2

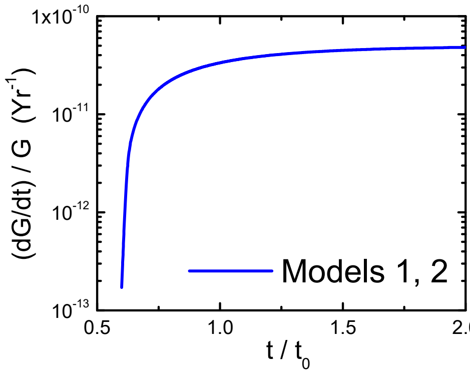

Figure 8. Variation of $\dot{G}/G$ as a function of time for Models 1 and 2

Figure 7 depicts the time variation of the gravitational term (G) based on Models 1 & 2. Each of these plots shows a steep fall in G in the early universe, followed by a slow rise. G is found to be increasing with time for almost the entire span of time shown in these plots. It is observed that G has a rising trend at the present time ($t = t_0$). There are several studies, conducted by different methods, where the gravitational term is found to increase with time [28, 41-44].

Figure 8 shows the time dependence of $\dot{G}/G$ as obtained from Models 1 and 2. In both plots, $\dot{G}/G$ is found to be increasing very steeply with time in the early universe, from negative to smaller negative values, at a gradually decreasing rate. Nearly at $t = 0.59t_0$, its sign changes from negative to positive. The value of $(\dot{G}/G)_{t=t_0}$ is $3.35 \times 10^{-11}\ Yr^{-1}$ for both models. These values are consistent with those obtained from recent observations [45]. The positive sign of $(\dot{G}/G)_{t=t_0}$ corresponds to the fact that G is presently increasing with time, which is shown by Figure 7. It is evident from the values of $\dot{G}/G$ that we have an extremely slow change of G with time, where dG/dt is of the order of 10^{-21} (in $N\,m^2\,Kg^{-2}\,Yr^{-1}$) at the present time (i.e., $t = t_0$). Its smallness implies that the change of G is extremely slow. This observation justifies our decision to consider G to be independent of time for the calculations in Part-B of the present article.

The equation of state parameter (ω) is a very important tool to find the characteristics of the accelerated expansion of the universe. The values of ω, used in many studies, are 0, 1/3, 1 and -1, for respectively the pressure-less dust, radiation, stiff-fluid and vacuum-fluid dominated universe [46]. The ranges of values of ω, obtained from galaxy clustering statistics [47] and the observational results from SN 1a data [48], are $-1.33 < \omega < -0.79$ and $-1.67 < \omega < -0.62$ respectively. However, one does not necessarily have to regard ω to have a constant value. It can be regarded as a function of time [49, 50]. On account of the unavailability of observational results, one often uses a constant value of ω for calculations [51, 52], as we have done in Models 1 & 2 of this article.

Figure 9 depicts the variation of deceleration parameter (q) as a function of cosmological term (Λ). Here, a significant observation is that the time at which q changes its sign from positive to negative is almost the same as the time at which Λ changes its sign from negative to positive. This plot is based on the data generated by equations (49) and (51). This observation indicates that the transition from the phase of deceleration to the phase of acceleration is somehow connected to some phenomena involving dark energy which is regarded here as being represented by Λ.

Figure 10 shows the variation of energy density (ρ) with respect to the cosmological term (Λ). It is based on the data generated by equations (50) and (52). Here, we find that ρ decreases very rapidly after Λ changes its sign from negative to positive. It means that the rate of decrease of ρ with time is much greater than the rate of change of Λ at that stage of cosmic evolution, indicating a considerably faster expansion of the universe compared to the era of negative Λ. Like Figure 9, this figure also indicates a relation between the accelerated expansion and dark energy whose density is estimated in terms of the value of Λ [46].

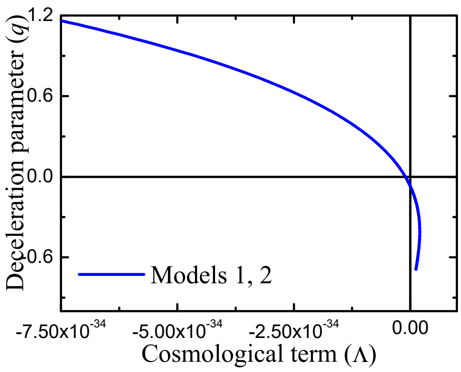

Figure 9. Variation of deceleration parameter (q) as a function of cosmological term (Λ) for Models 1, 2

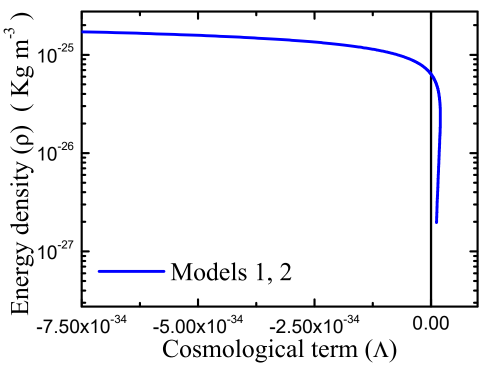

Figure 10. Variation of energy density (ρ) as a function of cosmological term (Λ) for Models 1, 2

Figures 11 and 12 show, respectively, the variations of Hubble parameter (H) and the cosmic expansion rate ($da/dt \equiv aH$) with respect to the cosmological term (Λ). In each of these two figures, it is observed that, the evolution of the dependent variable becomes much faster as the cosmological term (Λ) undergoes a signature flip from negative to positive. This behaviour may be regarded as a manifestation of acceleration in the expansion process controlled by dark energy (represented here by Λ). Quite significantly, the signature flip of Λ is found to be almost coincident with the signature flip of the deceleration parameter, as depicted by Figure 9.

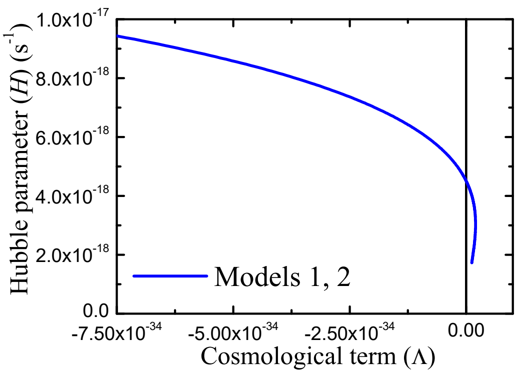

Figure 11. Variation of Hubble parameter (H) as a function of cosmological term (Λ) for Models 1, 2

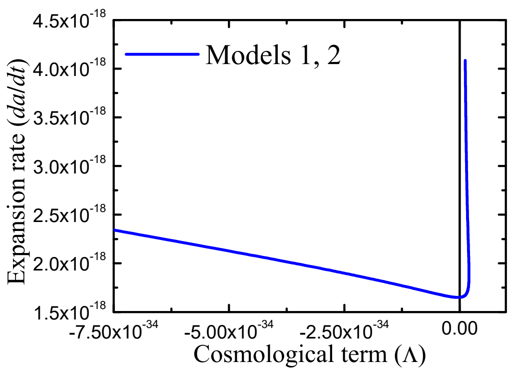

Figure 12. Variation of cosmic expansion rate ($\dot{a} \equiv aH$) as a function of cosmological term (Λ) for Models 1, 2

4. CONCLUSIONS

In the framework of Kaluza-Klein space-time, with zero spatial curvature, we have constructed four theoretical models based on four different ansatzes, and the results of this investigation have been depicted graphically. According to each of these four models, the deceleration parameter is a function of time and it changes sign from positive to negative, implying a change from a phase of decelerated expansion to a phase of acceleration, which is consistent with the conclusions drawn from recent astrophysical observations [3-5]. These four models are consistent with each other in terms of the predictions made by them regarding the characteristics of time evolution of different cosmological quantities. The results of the present investigation are also consistent with the findings of several other studies, carried out under theoretical frameworks different from ours, as discussed in the previous section. In Models 1 & 2, the EoS parameter (ω) is regarded as a constant and its value is determined from the requirement that the models should account for the observed features of the universe with sufficient accuracy. These two models enabled us to determine the time dependence of the cosmological term (Λ) and the gravitational term (G) among other parameters. It is interesting to note that, Λ has come out to be independent of time in Models 3 & 4, where we have regarded G as a constant quantity. The value of Λ obtained here is the same as the value of Λ_0 (i.e., the value of the dynamical Λ term at $t = t_0$) obtained from the Models 1 & 2. Time dependence of ω has been obtained from Models 3 & 4. The value of ω_0 obtained here is the same as the value of constant ω estimated by Models 1 & 2. All these aspects show an inter-model consistency, confirming the authenticity of predictions based on these models. A notable finding is that the signature flip of the deceleration parameter happens simultaneously with the signature flip of the dynamical Λ term. Through the plots in Figures 9-12, we have made an attempt to demonstrate, in a simple way, the role played by dark energy (represented here by Λ) in controlling the evolution of various parameters that characterize an expanding universe. A unique feature of the present article is that we have shown graphically the time evolution of $\dot{G}/G$, whose value at $t = t_0$ has been obtained from observations in several ways, as discussed in details in an article by Ray et. al. [45]. The present work can be regarded as a mathematical exercise aimed at determining the characteristics of time dependence of various cosmological quantities, under the framework of Kaluza-Klein space-time and our findings are in accordance with the observed features of the expanding universe. These findings are based on four models which are mathematically simpler than many other studies we have come across. Unlike a common convention, our graphical depiction of time evolution of every parameter is with respect to t/t_0 (instead of just t in arbitrary units) which enables the reader to get the exact time at which the parameter attains a certain value. A limitation of the present work is that we have not been able assume any ansatz using which the time evolution of both ω and G can be determined from the same model. As a future extension of this study, we have plans to work with some new ansatzes in this regard and compare the results of investigations with those obtained observationally and also with those obtained here, making thereby an attempt to achieve an improvement over the present work. The present work might be helpful to other researchers for studying cosmic evolution under various other theoretical frameworks.

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ДОСЛІДЖЕННЯ ЕВОЛЮЦІЇ КОСМОЛОГІЧНИХ ПАРАМЕТРІВ НА ОСНОВІ МОДЕЛЕЙ ТЕМНОЇ ЕНЕРГІЇ В ТЕОРІЇ КАЛУЦИ-КЛЯЙНА

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Метою цього дослідження є визначення характеристик часової еволюції різних космологічних величин на основі чотирьох моделей, побудованих для Всесвіту, що зазнає прискореного розширення. Це формулювання виконано в рамках простору-часу Калуци-Клейна для нульової просторової кривизни. Щоб розв'язати рівняння поля, для кожної моделі вибирається підхід таким чином, щоб це призвело до характерного перевороту параметра уповільнення, щоб забезпечити його узгодженість з нещодавніми астрофізичними спостереженнями, які вказують на зміну від уповільненого розширення до прискореного розширення Всесвіту. На основі цих чотирьох моделей отримані часові еволюції кількох космологічних параметрів і їх варіації показані графічно в залежності від часу. Довільні константи, пов'язані з кожною моделлю, налаштовані так, що модель правильно прогнозує значення параметра Хаббла, параметра уповільнення, щільності енергії та гравітаційної постійної в даний час. Висновки цих моделей узгоджуються один з одним, і вони узгоджуються з спостережуваними особливостями. Гравітаційна постійна (*G*) показує швидке падіння в ранньому Всесвіті, а потім надзвичайно повільне зростання, яке триває в даний час. Взявши *G* за константу в двох із чотирьох моделей, космологічна стала не залежить від часу. Важливим відкриттям є те, що характерний переворот параметра уповільнення майже збігається з характерним переворотом космологічної постійної (Λ), що вказує на зв'язок між прискореним розширенням і темною енергією, яка представлена Λ. Інші сюжети щодо Λ також зображують роль темної енергії в управлінні космічною еволюцією. Враховуючи його динамічну природу, Λ у тексті згадується як космологічний термін (замість космологічної постійної). Всупереч поширеній тенденції використання довільних одиниць, для всіх вимірних величин використовуються одиниці СІ.

Ключові слова: *космологія Калуци-Клейна; темна енергія; космологічна стала; гравітаційна стала; космічне прискорення*