

## MEAN LIFETIMES OF ns, np, nd, & nf LEVELS OF N V<sup>†</sup>

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Nitrogen is one of the key elements in the evolution and formation of stellar objects. Earth's atmosphere contains 21% oxygen and 78% nitrogen; these two gases give rise to aurora when ions of the solar wind in the ionosphere collide with them. Some aerosols made of nitrogen and oxygen are also found in the atmosphere. Nitrogen, hydrogen, carbon, and oxygen are the main contributors to the origin of life on Earth. The spectrum of nitrogen ion (N V) has been studied using Quantum defect theory (QDT) and Numerical Coulombic approximation (NCA). N V has two electrons in the core, with the nucleus, and one electron outside the core. It makes it hydrogen or lithium-like. In the first part, the energies of the ns, np, nd, and nf up to  $n < 30$  were calculated with the help of QDT. In the second part, the wavelengths were calculated using the energies and line strength parameters using NCA. Very little experimental data on lifetime and transition probability are available; however, Biemont et al. have calculated the lifetime of the 48 levels of N V using coulomb approximation. In this study, we calculated the lifetime of 196 multiplets of N V. The results are compared with the available experimental and theoretical lifetimes; an excellent agreement was found between known lifetimes and calculated in this work. The lifetimes of 100 multiplets are presented for the first time. The lifetimes of each of the Rydberg series of N V were fitted, and a third-degree polynomial represents the lifetimes of each series.

**Keywords:** Nitrogen ion (N V); Lifetimes; Quantum defect theory; Numerical coulomb approximation; Lithium-like ions

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### INTRODUCTION

In the evolution of chemically peculiar stars, nitrogen plays an important role. For example, Pluto is believed to have a rocky core and layers of various ices in the surface crust; most of the ice is nitrogen. It indicates a high abundance of nitrogen on Pluto. Improved spectroscopic data is essential in determining reliable values of stellar abundances. Kancercivius calculated transition probabilities and Oscillator Strengths using analytic wave functions of helium and lithium-like atoms from np to ms ( $m < 3$ ,  $n < 6$ ). They used three different expressions for the dipole moments [1]. Fischer et al. calculated transition data, including lifetimes using the multiconfiguration Breit-Pauli method of the Li sequence. They calculated the lifetimes of all configuration levels of principal quantum number  $< 4$ , including the configuration  $1s^2 4s$  [2]. Mohammed El-Mekki used a semi-empirical Coulomb Approximation to calculate 2p levels of isoelectronic Li-sequence lifetimes. With the help of Hartree Slater's core, he calculated lifetimes for  $Z = 3 - 92$  [3]. Chen and Crasemann used the Multiconfiguration Dirac-Fock approach to calculate the transition data of the highest spin metastable  $^4F_{9/2}$  of  $1s 2p 3d$  configuration of Li-like ions for  $Z = 6$  to 42. They calculated Auger energies, Auger rate, radiative energies, and radiative rate for the transition  $2s - 2p$ ,  $3p - 3d$ ,  $1s - 2p$ , and  $2s-3d$  [4]. Blanke et al. studied decay curves of the quartet system of Li-like ions for  $n = 6 - 9$ . The decay curve measured the lifetimes of  $n = 3$  states of the configuration  $1s 2p 3d$  [5]. Tunnell and Bhalla estimated the lifetimes of the  $^4P$  state of the configuration  $1s 2p^2$  of the lithium-like ions ( $Z = 7, 8, 10$ , and 12) [6]. Kernahan et al. measured the lifetimes of 11 levels belonging to N II – N V. They considered the transition below  $500\text{\AA}$  [7]. Baudinet-Robinet and P. D. Dumont used the beam foil technique to measure the lifetimes of N V levels in the ultraviolet spectrum range. Using Coulomb approximation, they found branching fractions and combined them with the lifetime to measure the transition probabilities [8]. Kernahan et al. also used the beam-foil technique to measure the lifetimes of 41 states of N I – N V. They studied transitions in the range  $374$  to  $2064\text{\AA}$ . They compared the results with existing known lifetimes and found that they agree well for levels near the ionization stage [9]. Dumont also used the beam-foil technique to measure the lifetime of nitrogen in the UV spectrum range. They studied nitrogen spectra and found lifetimes of 24 levels in the range  $650 - 2000\text{\AA}$  [10]. Desesquelles used the technique of beam of ions by a thin Carbon foil to prepare a nitrogen beam. This technique can make an ions beam between  $100$  keV and  $16$  MeV. Most of the lines of the spectra of neutral nitrogen to N VII can be observed. The lifetimes of upper levels and transition probabilities were found by the spatial decay of the light emitted by fast-moving atoms and ions [11]. Lewis et al. also used the foil excitation technique at high energy of  $2$  MeV of nitrogen through carbon foil and measured the lifetimes of upper levels of the transition in N II to N V. The transition lines they observed were in the range of  $2000$  to  $5000\text{\AA}$ . The measured lifetime by the spatial decay of the excited levels was  $0.4$  to  $10$  ns [12]. Heroux also used the same technique Desesquelles [10] and Lewis [12] used to measure the lifetime of multiplets in N II to N V. The beam energy of nitrogen ions was the same as used by Lewis [12], but the range of wavelengths was from  $1085$  to  $162\text{\AA}$  [13]. Berry et al. studied nitrogen transition in N I-VI using the beam foil technique; they used an energy range of  $0.25-2.0$  MeV in wavelength range  $1050-3000\text{\AA}$  and  $5.5$  MeV between  $2000-5000\text{\AA}$  [14].

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### THEORY

Quantum mechanical systems are complex systems; the exact analytical solutions of the systems are often unavailable or impossible to find. Numerical approaches are one of the best alternatives to reach an approximate solution. Schrödinger equation can be solved analytically for hydrogen atoms, with the approximation that variables are separable, but this equation is difficult to solve for helium and other elements. If the system is approximated as hydrogen, the solution is achievable and close to the exact solution in many cases, such as Li and lithium-like ions. Nitrogen ion (N V) is a lithium-like ion; its transition data can be obtained by numerically solving the Schrödinger equation.

$$\left(\frac{d^2}{dr^2} - 2V(r) - \frac{l^*(l^*+1)}{r^2} + 2E\right)P(r) = 0. \quad (1)$$

The equation (1) can be written as

$$\frac{d^2R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + 2\left(E - \frac{A}{r_i} - \frac{B}{r_i^2} - \frac{l(l+1)}{r_i^2}\right) = 0. \quad (2)$$

Here  $R(r) = \frac{P(r)}{r}$ ,  $A/r_i - B/r_i^2 = V(r_i)$  is the potential observed by the electron outside the core formed by the nucleus and other electrons in the atom,  $A = -z^*$  is the effective nuclear charge where  $r_i$  is the distance between electron  $i$  and the nucleus.

The solution of equation (2) gives the radial wavefunction R as:

$$R = C \exp\left(-\frac{Z^*r}{n^*}\right) r^{l^*} L_{n^*-l^*-1}^{2l^*+1}\left(\frac{2Z^*r}{n^*}\right). \quad (3)$$

Where  $C = \left(\frac{2Z^*}{n^*}\right)^{l^*+\frac{3}{2}} \sqrt{\frac{(n^*-l^*-1)!}{2n^*\Gamma(n^*+l^*+1)}}$ ,  $n^*$  is the effective principal quantum number. The lifetime can be calculated using equation (4)

$$\tau_i = \left(\frac{\lambda(\text{\AA})}{1265.38}\right)^3 \frac{g_i}{S_{if}}, \quad (4)$$

where the line strength factor  $S_{if}$  is given by,

$$S_{if} = \sum_{n_i, n_f} |\langle f|r|i \rangle|^2. \quad (5)$$

The matrix element values in equation (5) can be obtained using the wavefunctions in equation (3).

### RESULT AND DISCUSSION

Quantum defect theory (QDT) and Numerical Coulombic approximation (NCA) were used to study the lifetimes of the nitrogen ion (N V). The energies and quantum defects were calculated with the help of QDT. These energies and quantum defects were used to calculate the lifetimes of the levels of N V. The lifetimes of any branch of the transition depend on the transition wavelength and line strength parameter. The wavelengths were calculated by the difference of energies of upper and lower levels calculated by QDT. The wavefunctions of upper and lower levels are required to calculate line strength parameters. Since nitrogen ion (N V) may be considered hydrogen-like, hydrogen wavefunction can be used to calculate the matrix element needed for the line strength parameters. The matrix elements are calculated using NCA.

The lifetimes of the Rydberg Series  $1s^2 ns$ ,  $1s^2 np$ ,  $1s^2 nd$ , and  $1s^2 nf$  levels of N V have been calculated and compared with the available experimental and theoretical values. Generally, a good agreement is found between calculated and available lifetime data. It was found that the available experimental data on lifetime is very little. The most detailed theoretical work on the lifetime of N V is by Biemont et al. However, they reported lifetimes of forty-eight levels of N V. In this work, the lifetimes of 196 multiplets are presented. Table 1 gives the lifetimes of N V levels; the first column gives the configuration, the second column gives the angular momentum third columns give the lifetimes calculated in this work, and the fourth and last column gives the corresponding lifetimes available in the literature. The lifetime calculated for  $1s^2 2p$  is slightly different from the experimental value. The reason could be its closeness to the core, which may affect the effective charge compared to the other levels that lie relatively far from the core. It was observed in the calculations of transition probability and lifetimes of the same configuration in Li I [16], Be II [17], and C IV [18] that the calculated and experimental values are slightly different. For the configuration  $1s^2 2p$  in Li-like atoms and ions, one should expect a slight difference in theoretical and experimental lifetimes. A study is needed to see if the problem is resolved by changing  $Z$  by its effective value.

**Table 1.** The lifetimes of multiplets of N V calculated in this work and compared with available data

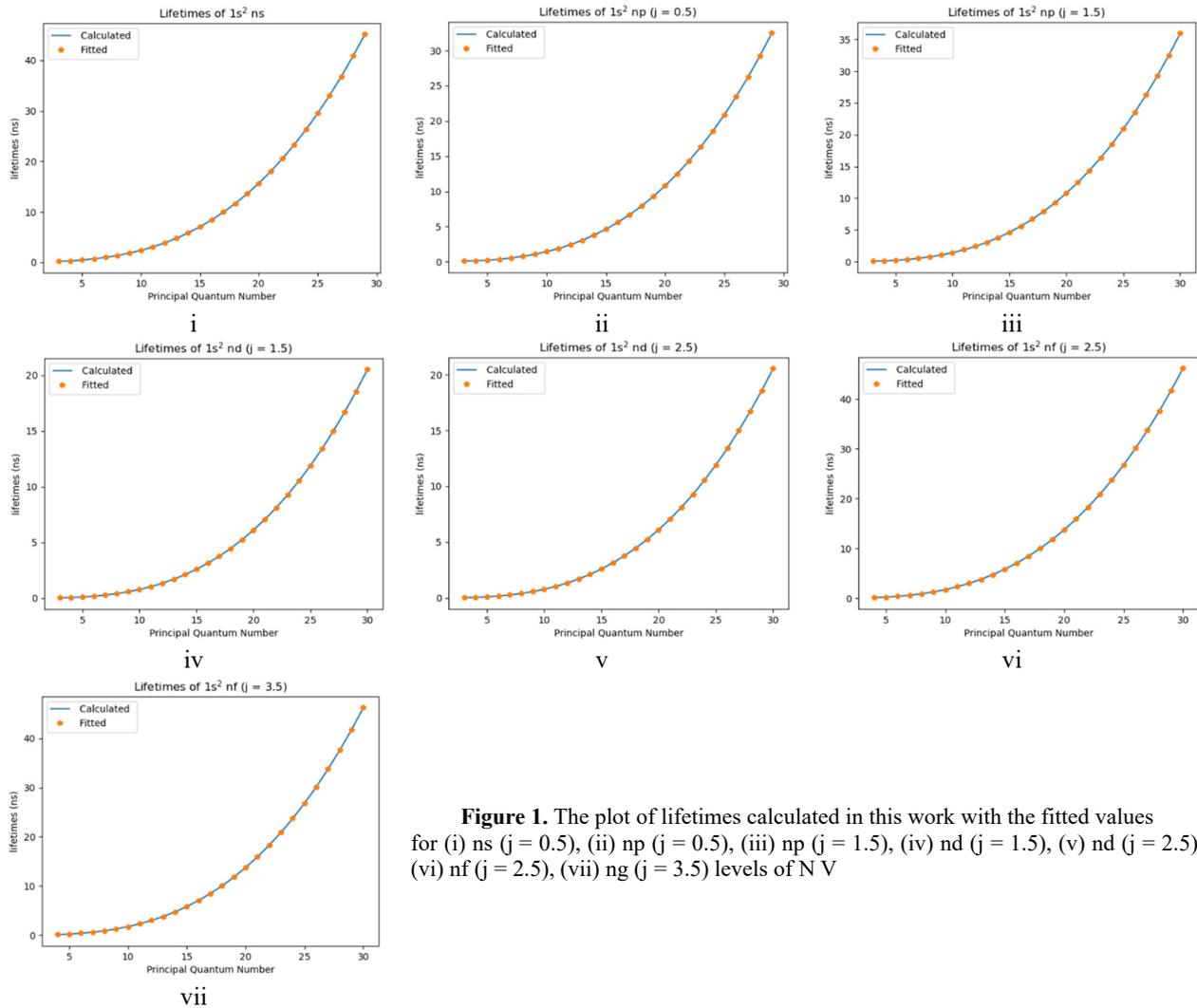
| Term<br>1s <sup>2</sup> .nl | J   | This<br>work | Other works                            | Term<br>1s <sup>2</sup> .nl | J   | This<br>work | Other works                           |
|-----------------------------|-----|--------------|--|-----------------------------|-----|--------------|---------------------------------------|
| 3d                          | 3/2 | 0.0228       | 0.02 <sup>g</sup>                      | 29d                         | 3/2 | 18.541       |                                       |
| 3d                          | 5/2 | 0.0228       |  | 29d                         | 5/2 | 18.5608      |                                       |
| 4d                          | 3/2 | 0.0527       | 0.065 <sup>a</sup> , 0.05 <sup>g</sup> | 30d                         | 3/2 | 20.5184      |                                       |
| 4d                          | 5/2 | 0.0527       | 0.95 <sup>d</sup>                      | 30d                         | 5/2 | 20.538       |                                       |
| 5d                          | 3/2 | 0.1012       | 0.1 <sup>g</sup>                       | 10f                         | 5/2 | 1.7395       |                                       |
| 5d                          | 5/2 | 0.1012       |  | 11f                         | 5/2 | 2.3098       |                                       |
| 6d                          | 3/2 | 0.1726       | 0.18 <sup>g</sup>                      | 12f                         | 5/2 | 2.9928       |                                       |
| 6d                          | 5/2 | 0.1728       |  | 13f                         | 5/2 | 3.7989       |                                       |
| 7d                          | 3/2 | 0.2715       | 0.28 <sup>g</sup>                      | 14f                         | 5/2 | 4.738        |                                       |
| 7d                          | 5/2 | 0.2718       |  | 15f                         | 5/2 | 5.8205       |                                       |
| 8d                          | 3/2 | 0.4024       | 0.41 <sup>g</sup>                      | 16f                         | 5/2 | 7.0566       |                                       |
| 8d                          | 5/2 | 0.4028       |  | 17f                         | 5/2 | 8.4564       |                                       |
| 9d                          | 3/2 | 0.5698       | 0.59 <sup>g</sup>                      | 18f                         | 5/2 | 10.0302      |                                       |
| 9d                          | 5/2 | 0.5703       |  | 19f                         | 5/2 | 11.7882      |                                       |
| 10d                         | 3/2 | 0.7781       | 0.8 <sup>g</sup>                       | 20f                         | 5/2 | 13.7405      |                                       |
| 10d                         | 5/2 | 0.7789       |  | 21f                         | 5/2 | 15.8974      |                                       |
| 11d                         | 3/2 | 1.032        | 1.05 <sup>g</sup>                      | 22f                         | 5/2 | 18.2692      |                                       |
| 11d                         | 5/2 | 1.033        |  | 23f                         | 5/2 | 20.8659      |                                       |
| 12d                         | 3/2 | 1.3359       | 1.37 <sup>g</sup>                      | 24f                         | 5/2 | 23.6978      |                                       |
| 12d                         | 5/2 | 1.3372       |  | 25f                         | 5/2 | 26.7752      |                                       |
| 13d                         | 3/2 | 1.6943       | 1.77 <sup>g</sup>                      | 26f                         | 5/2 | 30.1082      |                                       |
| 13d                         | 5/2 | 1.6959       |  | 27f                         | 5/2 | 33.7071      |                                       |
| 14d                         | 3/2 | 2.1118       | 2.12 <sup>g</sup>                      | 28f                         | 5/2 | 37.5821      |                                       |
| 14d                         | 5/2 | 2.1139       |  | 29f                         | 5/2 | 41.7421      |                                       |
| 15d                         | 3/2 | 2.5929       | 2.53 <sup>g</sup>                      | 30f                         | 5/2 | 46.1474      |                                       |
| 15d                         | 5/2 | 2.5955       |  | 4f                          | 5/2 | 0.1153       | 0.12 <sup>g</sup>                     |
| 16d                         | 3/2 | 3.1421       | 3.19 <sup>g</sup>                      | 4f                          | 7/2 | 0.1153       |                                       |
| 16d                         | 5/2 | 3.1453       |  | 5f                          | 5/2 | 0.223        | 0.22 <sup>g</sup>                     |
| 17d                         | 3/2 | 3.7639       | 3.73 <sup>g</sup>                      | 5f                          | 7/2 | 0.2231       |                                       |
| 17d                         | 5/2 | 3.7678       |  | 6f                          | 5/2 | 0.3823       | 0.61 <sup>d</sup> , 0.38 <sup>g</sup> |
| 18d                         | 3/2 | 4.463        | 4.75 <sup>g</sup>                      | 6f                          | 7/2 | 0.3824       |                                       |
| 18d                         | 5/2 | 4.4675       |  | 7f                          | 5/2 | 0.6034       | 0.6 <sup>g</sup>                      |
| 19d                         | 3/2 | 5.2437       |  | 7f                          | 7/2 | 0.6036       |                                       |
| 19d                         | 5/2 | 5.2491       |  | 8f                          | 5/2 | 0.8965       | 0.89 <sup>g</sup>                     |
| 20d                         | 3/2 | 6.1106       |  | 8f                          | 7/2 | 0.8968       |                                       |
| 20d                         | 5/2 | 6.117        |  | 9f                          | 5/2 | 1.2718       | 1.26 <sup>g</sup>                     |
| 21d                         | 3/2 | 7.0684       |  | 9f                          | 7/2 | 1.2723       |                                       |
| 21d                         | 5/2 | 7.0758       |  | 10f                         | 7/2 | 1.7402       | 1.72 <sup>g</sup>                     |
| 22d                         | 3/2 | 8.1214       |  | 11f                         | 7/2 | 2.3107       |                                       |
| 22d                         | 5/2 | 8.1299       |  | 12f                         | 7/2 | 2.994        |                                       |
| 23d                         | 3/2 | 9.2742       |  | 13f                         | 7/2 | 3.8004       |                                       |
| 23d                         | 5/2 | 9.284        |  | 14f                         | 7/2 | 4.7399       |                                       |
| 24d                         | 3/2 | 10.5314      |  | 15f                         | 7/2 | 5.8229       |                                       |
| 24d                         | 5/2 | 10.5425      |  | 16f                         | 7/2 | 7.0596       |                                       |
| 25d                         | 3/2 | 11.8974      |  | 17f                         | 7/2 | 8.46         |                                       |
| 25d                         | 5/2 | 11.91        |  | 18f                         | 7/2 | 10.0345      |                                       |
| 26d                         | 3/2 | 13.3769      |  | 19f                         | 7/2 | 11.7932      |                                       |
| 26d                         | 5/2 | 13.3911      |  | 20f                         | 7/2 | 13.7465      |                                       |
| 27d                         | 3/2 | 14.9743      |  | 21f                         | 7/2 | 15.9044      |                                       |
| 27d                         | 5/2 | 14.9903      |  | 22f                         | 7/2 | 18.2771      |                                       |
| 28d                         | 3/2 | 16.6942      |  | 23f                         | 7/2 | 20.8751      |                                       |
| 28d                         | 5/2 | 16.7118      |  | 24f                         | 7/2 | 23.7083      |                                       |

| Term<br>1s <sup>2</sup> .nl | J   | This<br>work | Other works   |
|-----------------------------|-----|--------------|---|
| 25f                         | 7/2 | 26.787       |   |
| 26f                         | 7/2 | 30.1216      |   |
| 27f                         | 7/2 | 33.7221      |   |
| 28f                         | 7/2 | 37.5987      |   |
| 29f                         | 7/2 | 41.762       |   |
| 30f                         | 7/2 | 46.2199      |   |
| 2p                          | 3/2 | 2.8423       | 2.9741 <sup>a</sup> , 3.12 <sup>b</sup> , 3.2 <sup>d</sup> , 3.3 <sup>f</sup> , 2.95 <sup>g</sup> |
| 2p                          | 1/2 | 2.871        | 2.9442 <sup>a</sup> , 3.08 <sup>b</sup> , 3.3 <sup>d</sup> , 3.3 <sup>f</sup>                     |
| 3p                          | 1/2 | 0.0821       | 0.081328 <sup>a</sup> , 0.084 <sup>c</sup> , 0.51 <sup>f</sup> 0.08 <sup>g</sup>                  |
| 3p                          | 3/2 | 0.0822       | 0.082058 <sup>a</sup>   |
| 4p                          | 1/2 | 0.1333       | 0.14 <sup>g</sup> , 0.17 <sup>c</sup>   |
| 4p                          | 3/2 | 0.1335       |   |
| 5p                          | 1/2 | 0.2224       | 0.22 <sup>g</sup>   |
| 5p                          | 3/2 | 0.2227       |   |
| 6p                          | 1/2 | 0.353        | 0.51 <sup>f</sup> , 0.36 <sup>g</sup>   |
| 6p                          | 3/2 | 0.3534       |   |
| 7p                          | 1/2 | 0.5321       | 0.54 <sup>g</sup>   |
| 7p                          | 3/2 | 0.5327       |   |
| 8p                          | 1/2 | 0.7672       | 0.77 <sup>g</sup>   |
| 8p                          | 3/2 | 0.7679       |   |
| 9p                          | 1/2 | 1.0659       | 1.08 <sup>g</sup>   |
| 9p                          | 3/2 | 1.0669       |   |
| 10p                         | 1/2 | 1.436        | 1.46 <sup>g</sup>   |
| 10p                         | 3/2 | 1.4373       |   |
| 11p                         | 1/2 | 1.8854       | 1.85 <sup>g</sup>   |
| 11p                         | 3/2 | 1.887        |   |
| 12p                         | 1/2 | 2.4219       | 2.45 <sup>g</sup>   |
| 12p                         | 3/2 | 2.4239       |   |
| 13p                         | 1/2 | 3.0532       | 2.96 <sup>g</sup>   |
| 13p                         | 3/2 | 3.0557       |   |
| 14p                         | 1/2 | 3.7873       | 3.92 <sup>g</sup>   |
| 14p                         | 3/2 | 3.7904       |   |
| 15p                         | 1/2 | 4.632        | 4.26 <sup>g</sup>   |
| 15p                         | 3/2 | 4.6357       |   |
| 16p                         | 1/2 | 5.5951       | 4.3 <sup>g</sup>  |
| 16p                         | 3/2 | 5.5996       |   |
| 17p                         | 1/2 | 6.6846       | 5.03 <sup>g</sup>   |
| 17p                         | 3/2 | 6.6899       |   |
| 18p                         | 1/2 | 7.9083       |   |
| 18p                         | 3/2 | 7.9145       |   |
| 19p                         | 1/2 | 9.2741       |   |
| 19p                         | 3/2 | 9.2814       |   |
| 20p                         | 1/2 | 10.7899      |   |
| 20p                         | 3/2 | 10.7983      |   |
| 21p                         | 1/2 | 12.4635      |   |
| 21p                         | 3/2 | 12.4731      |   |

| Term<br>1s <sup>2</sup> .nl | J   | This<br>work | Other works   |
|-----------------------------|-----|--------------|---|
| 22p                         | 1/2 | 14.3028      |   |
| 22p                         | 3/2 | 14.3138      |   |
| 23p                         | 1/2 | 16.3157      |   |
| 23p                         | 3/2 | 16.3282      |   |
| 24p                         | 1/2 | 18.5101      |   |
| 24p                         | 3/2 | 18.5242      |   |
| 25p                         | 1/2 | 20.8939      |   |
| 25p                         | 3/2 | 20.9098      |   |
| 26p                         | 1/2 | 23.4749      |   |
| 26p                         | 3/2 | 23.4927      |   |
| 27p                         | 1/2 | 26.2611      |   |
| 27p                         | 3/2 | 26.2809      |   |
| 28p                         | 1/2 | 29.26        |   |
| 28p                         | 3/2 | 29.2821      |   |
| 29p                         | 1/2 | 32.4742      |   |
| 29p                         | 3/2 | 32.5049      |   |
| 30p                         | 1/2 | 35.906       |   |
| 30p                         | 3/2 | 35.9494      |   |
| 3s                          | 1/2 | 0.11         | 0.10985 <sup>a</sup> , 0.121 <sup>c</sup> , 0.12 <sup>c</sup> , 0.11 <sup>g</sup> |
| 4s                          | 1/2 | 0.1733       | 0.5 <sup>a</sup> , 0.17212 <sup>c</sup> , 0.18 <sup>g</sup>                       |
| 5s                          | 1/2 | 0.2849       | 0.29 <sup>g</sup>   |
| 6s                          | 1/2 | 0.4486       | 0.45 <sup>g</sup>   |
| 7s                          | 1/2 | 0.6731       | 0.68 <sup>g</sup>   |
| 8s                          | 1/2 | 0.9678       | 0.98 <sup>g</sup>   |
| 9s                          | 1/2 | 1.3425       | 1.39 <sup>g</sup>   |
| 10s                         | 1/2 | 1.8069       | 1.84 <sup>g</sup>   |
| 11s                         | 1/2 | 2.3709       |   |
| 12s                         | 1/2 | 3.0444       |   |
| 13s                         | 1/2 | 3.8375       |   |
| 14s                         | 1/2 | 4.7599       |   |
| 15s                         | 1/2 | 5.8216       |   |
| 16s                         | 1/2 | 7.0327       |   |
| 17s                         | 1/2 | 8.403        |   |
| 18s                         | 1/2 | 9.9426       |   |
| 19s                         | 1/2 | 11.6614      |   |
| 20s                         | 1/2 | 13.5694      |   |
| 21s                         | 1/2 | 15.6766      |   |
| 22s                         | 1/2 | 17.993       |   |
| 23s                         | 1/2 | 20.5286      |   |
| 24s                         | 1/2 | 23.2933      |   |
| 25s                         | 1/2 | 26.2971      |   |
| 26s                         | 1/2 | 29.5501      |   |
| 27s                         | 1/2 | 33.0623      |   |
| 28s                         | 1/2 | 36.843       |   |
| 29s                         | 1/2 | 40.8972      |   |
| 30s                         | 1/2 | 45.1651      |   |

<sup>a</sup>[2], <sup>b</sup>[7], <sup>c</sup>[9], <sup>d</sup>[10], <sup>e</sup>[13], <sup>f</sup>[14], <sup>g</sup>[15]

The plot of lifetimes is smoothly increasing; the different polynomials J were fitted on the calculated lifetime's data of N V. Third-degree polynomial ( $\tau = a_0 + a_1n + a_2n^2 + a_3n^3$ ) fitting gives closer values of calculated lifetimes. Figure 1(i)-1(vii) gives plots of calculated lifetimes with fitted values for 1s<sup>2</sup> ns, 1s<sup>2</sup> np, 1s<sup>2</sup> nd, and 1s<sup>2</sup> nf levels of N V. In fitting the np series, the level 1s<sup>2</sup> 2p was not included as its lifetime does not follow the trend.



**Figure 1.** The plot of lifetimes calculated in this work with the fitted values for (i) ns ( $j = 0.5$ ), (ii) np ( $j = 0.5$ ), (iii) np ( $j = 1.5$ ), (iv) nd ( $j = 1.5$ ), (v) nd ( $j = 2.5$ ), (vi) nf ( $j = 2.5$ ), (vii) ng ( $j = 3.5$ ) levels of N V

Third-degree polynomial represents the lifetimes of various Rydberg series  $1s^2 ns$ ,  $1s^2 np$ ,  $1s^2 nd$ ,  $1s^2 nf$  of N V. The coefficients of the polynomials are given in Table 2. There are two polynomials for each of the series np, nd, and nf for two different possible values of angular momentum. nd series is the only series for which the coefficients for both angular momenta are approximately the same.

**Table 1.** Coefficients of the polynomial fitted for calculated lifetimes of N V for different series

| Term and J value | Coefficients of the Polynomial |          |          |          |
|------------------|--------------------------------|----------|----------|----------|
|                  | $a_0$                          | $a_1$    | $a_2$    | $a_3$    |
| ns (1/2)         | 0.053562                       | 0.009137 | 0.005815 | 0.00164  |
| np (1/2)         | 0.019362                       | 0.005954 | 0.000504 | 0.001306 |
| np (3/2)         | 0.017367                       | 0.006626 | 0.000449 | 0.001309 |
| nd (3/2)         | -0.0035                        | 0.001466 | 0.000128 | 0.000754 |
| nd (5/2)         | -0.0034                        | 0.001432 | 0.00013  | 0.000755 |
| nf (5/2)         | 0.018192                       | -0.00479 | 0.000773 | 0.001689 |
| nf (7/2)         | -0.00463                       | 0.001625 | 0.000275 | 0.001701 |

### CONCLUSIONS

The Rydberg levels series  $1s^2 ns$ ,  $1s^2 np$ ,  $1s^2 nd$ , and  $1s^2 nf$  of N V have been studied. The lifetimes of multiplets belonging to these levels have been calculated and compared with the available data. The agreement is excellent except for  $1s^2 2p$  levels, where a slight difference between calculated and experimental values of lifetimes was observed. The same can be observed in all Lithium like ions due to the cooper minimum. The cooper minimum was first observed for alkali atoms. It occurs due to overlapping the positive and negative amplitude of the wavefunctions of the levels taking part in the transition [16]. Due to this, the dipole matrix elements have zero or a minimum value for a particular set of principal quantum numbers of both levels. According to Fano and Cooper [19], the oscillator strength as a function of effective principal quantum number ( $n^*$ ) drops rapidly towards a minimum, where a reversal sign in the R integral occurs. QDT and NCA were used to calculate energy, quantum defects, and lifetimes of N V. The lifetimes of 196 multiplets were

calculated, but the available experimental data was insufficient. However, Biemont et al. theoretically determined the lifetimes of forty-eight levels; hence, 100 lifetimes presented in this paper are new. The plot of lifetimes against the principal quantum number follows a polynomial, so different polynomials were fitted, and in each series third-degree polynomial fits well on the calculated lifetimes of the N V Rydberg series.

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## СЕРЕДНИЙ ЧАС ЖИТТЯ РІВНІВ ns, np, nd та nf у N V

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Азот є одним із ключових елементів еволюції та формування зоряних об'єктів. Атмосфера Землі містить 21% кисню і 78% азоту; ці два гази викликають полярне сяйво, коли з ними стикаються іони сонячного вітру в іоносфері. Деякі аерозолі з азоту та кисню також знаходяться в атмосфері. Азот, водень, вуглець і кисень є головними причинами виникнення життя на Землі. Спектр іонів азоту (N V) досліджено за допомогою квантової теорії дефектів (КТД) і чисельного кулонівського наближення (НКА). N V має два електрони в ядрі з ядром і один електрон поза ядром. Це робить його подібним до водню або літію. У першій частині за допомогою QDT розраховано енергії ns, np, nd та nf до  $n < 30$ . У другій частині довжини хвиль були розраховані з використанням параметрів енергії та сили лінії за допомогою NCA. Доступно дуже мало експериментальних даних щодо тривалості життя та ймовірності переходу; однак Biemont et al. розрахували час життя 48 рівнів N V, використовуючи кулонівське наближення. У цьому дослідженні ми розрахували тривалість життя 196 мультиплетів N V. Результати порівнюються з доступними експериментальними та теоретичними тривалістю життя; У цій роботі було знайдено відмінну відповідність між відомими і розрахованими часами життя. Тривалість життя 100 мультиплетів представлена вперше. Часи життя кожного ряду Рідберга N V були підібрані поліномом третього ступеня яким представлено час життя кожного ряду.

**Ключові слова:** іон азоту (N V); терміни життя; квантова теорія дефектів; числова кулонівська апроксимація; літійподібні іони