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# THE EFFECT OF THERMAL STRATIFICATION ON FLOW PAST AN INFINITE VERTICAL PLATE IN PRESENCE OF CHEMICAL REACTION[†]

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This study examines how thermal stratification affect the movement of a fluid in presence of first order chemical reaction past an infinite vertical plate. To solve the non-dimensional governing equations in closed form for $Pr = 1$, the Laplace's transform system is applied. Significant findings resulting from thermal stratification are compared to the case of no stratification. The effects of many parameters, including $S, K, Gr, Gc, Sc$ and time on velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood number are explored and graphically displayed. It is shown that the steady state is attained at shorter times as a result of the application of stratification on the flow.

**Keywords:** *Thermal Stratification; Chemical Reaction; Heat and Mass Transfer; Vertical Plate; Schimdt Number*

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## 1. INTRODUCTION

The phenomena of thermal stratification is widely observed in natural systems like lakes and oceans. The dynamics of the flow can be further complicated by the existence of chemical reactions. In this article, we seek to figure out the effects of thermal stratification on flow dynamics and interactions with chemical reactions.There are numerous applications for this research. It can be utilized to create chemical reactors and heat exchangers that are more effective. It can also be used to investigate the impact of thermal stratification on the performance of cooling systems in electrical equipment.

The combined impact of thermal stratification and chemical reaction flow past an infinite vertical plate has never been studied before, and this study is the first to do so. [1, 2] and [3] investigated unsteady flows in a Stably Stratified Fluid, focusing on infinite plates. Furthermore, buoyancy-driven flows in a stratified fluid were examined by [4] and [5]. [6] came up with an analytical solution to describe how fluid would flow past an infinite vertical plate that had been affected chemically. In their research, [7] and [8] look at what happens when a chemical reaction is applied to an infinite vertical plate under various conditions. [9] and [10] investigate the combined Effects of Chemical Reaction and Thermal stratification on MHD flow for vertical stretching surfaces. Similarly, [11] researched the consequences of non-Newtonian fluid flow over a porous medium on both effects.

In this paper, we derived the special solutions for $Sc = 1$ and classical solutions for the case $S = 0$ (without stratification). These solutions are compared with the primary solutions, and graphs are used to demonstrate the differences. The impacts of physical parameters on velocity, temperature, and concentration profiles, including the thermal stratification parameter ($S$), thermal Grashof number ($Gr$), mass Grashof number ($Gc$), Schimdt number ($Sc$), Chemical Reaction Parameter ($K$), and time ($t$), are explored and presented in graphs. The results of this research have a wide range of applications in a variety of industries and chemical factories. Additionally, this research is to be helpful for chemical processing tasks like fibre drawing, crystal extraction from melts, and polymer manufacturing.

## 2. MATHEMATICAL ANALYSIS

We investigate the unsteady flow of a viscous in-compressible stratified fluid past an infinite vertical plate with chemical reaction effects. The diffusing species and the fluid are thought to be involved in a first-order chemical reaction. We use a coordinate system in which the $x'$ axis is taken vertically upward along the plate and the $y'$ axis is taken normal to the plate to explore the flow scenario as seen in Fig.1. The temperature $T'_\infty$ and concentration $C'_\infty$ of the plate and the fluid are initially the same. The plate temperature is raised to $T'_w$ and the concentration level is raised to $C'_w$ at time $t' > 0$. Since the plate has an endless length, all flow variables are independent of $x'$ and are solely affected by $y'$ and $t'$. As a result, we get a one-dimensional flow flow with only one non-zero vertical velocity component, $u'$. The equations for motion, energy, and concentration are then represented by Boussinesqs' approximation as follows:

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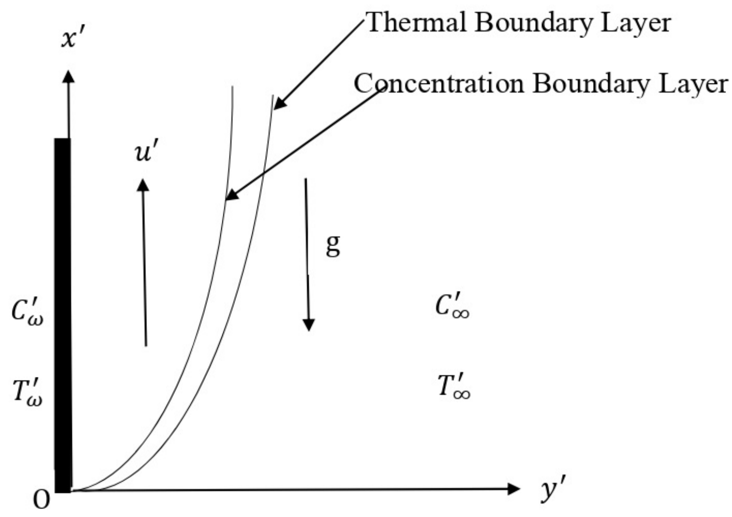


**Figure 1.** Physical Model and coordinate system

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \alpha\frac{\partial^2 T'}{\partial y'^2} - \gamma u' \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_1 C' \tag{3}$$

with the following initial and boundary Conditions:

$$\begin{array}{llll} u' = 0 & T' = T'_\infty & C' = C'_\infty & \forall y', t' \leq 0 \\ u' = 0 & T' = T'_w & C' = C'_w & \text{at } y' = 0, t' > 0 \\ u' = 0 & T' \to T'_\infty & C' \to C'_\infty & \text{as } y' \to \infty, t' > 0 \end{array}$$

where, $\gamma = \frac{dT'_\infty}{dx'} + \frac{g}{C_p}$ denotes the thermal stratification parameter and $\frac{dT'_\infty}{dx'}$ denotes the vertical temperature convection known as thermal stratification. In addition, $\frac{g}{C_p}$ represents the rate of reversible work done on fluid particles by compression, often known as work of compression. The variable ($\gamma$) will be referred to as the thermal stratification parameter in our research because the compression work is relatively minimal. For the purpose of testing computational methods, compression work is kept as an additive to thermal stratification.

Now, we express the reference velocity, length, and time as follows:

$$u_0 = \{g\beta\nu(T'_w - T'_\infty)\}^{1/3}, \quad y_0 = \frac{\nu^{2/3}}{\{g\beta(T'_w - T'_\infty)\}^{1/3}}, \quad t_0 = \frac{\nu^{1/3}}{\{g\beta(T'_w - T'_\infty)\}^{2/3}}$$

and we provide non-dimensional quantities in the following:

$$U = \frac{u'}{u_0}, \quad t = \frac{t'}{t_0}, \quad y = \frac{y'}{y_0}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{u_0^3}$$

$$Gc = \frac{g\beta^*\nu(C'_w - C'_\infty)}{u_0^3}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D}, \quad K = \frac{\nu K_1}{u_0^2}, \quad S = \frac{\gamma\nu}{u_0(T'_w - T'_\infty)}$$

The non-dimensional forms of the equations (1)-(3) are given by

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial y^2} \tag{4}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - SU \tag{5}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - KC \tag{6}$$

Non-dimensional form of initial and boundary Conditions are:

$$\begin{array}{llll} U=0 & \theta=0 & C=0 & \forall y, t\leq 0 \\ U=0 & \theta=1 & C=1 & \text{at } y=0, t>0 \\ U=0 & \theta\to 0 & C\to 0 & \text{as } y\to\infty, t>0 \end{array} \tag{7}$$

## 3. METHOD OF SOLUTION

The non-dimensional governing equations (4)- (6) with boundary conditions (7) are solved using Laplace's transform method for $Pr = 1$. Hence, the expressions for concentration, velocity and temperature with the help of [12] and [13] are given by

$$C = \frac{1}{2}\left[e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)+e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right] \tag{8}$$

$$\begin{aligned} U = {} & \frac{Gc}{2(Sc-1)}\left[C_1\left\{f_1(iA)+f_1(-iA)\right\}+(C_2-iC_3)\left\{f_2(iA,B+iB_1)+f_2(-iA,B+iB_1)\right\}\right. \\ & +(C_2+iC_3)\left\{f_2(iA,B-iB_1)+f_2(-iA,B-iB_1)\right\}\left.\right]+\frac{Gc}{2iA}\left[(D_1-1)\left\{f_1(iA)-f_1(-iA)\right\}\right. \\ & +(D_2+iD_3)\left\{f_2(iA,B+iB_1)-f_2(-iA,B+iB_1)\right\}+(D_2-iD_3)\left\{f_2(iA,B-iB_1)\right. \\ & \left.-f_2(-iA,B-iB_1)\right\}\left.\right]+\frac{iA}{2S}\left\{f_1(iA)-f_1(-iA)\right\} \\ & -\frac{Gc}{(Sc-1)}\left[\frac{C_1}{2}\left\{e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)+e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right\}\right. \\ & \left.+(C_2-iC_3)\left\{f_3(K,B+iB_1)\right\}+(C_2+iC_3)\left\{f_3(K,B-iB_1)\right\}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} \theta = {} & \frac{SGc}{2iA(Sc-1)}\left[C_1\left\{f_1(iA)-f_1(-iA)\right\}+(C_2-iC_3)\left\{f_2(iA,B+iB_1)-f_2(-iA,B+iB_1)\right\}\right. \\ & +(C_2+iC_3)\left\{f_2(iA,B-iB_1)-f_2(-iA,B-iB_1)\right\}\left.\right]+\frac{SGc}{2(Sc-1)^2}\left[E_1\left\{f_1(iA)+f_1(-iA)\right\}\right. \\ & +(E_2-iE_3)\left\{f_2(iA,B+iB_1)+f_2(-iA,B+iB_1)\right\}+(E_2+iE_3)\left\{f_2(iA,B-iB_1)\right. \\ & \left.+f_2(-iA,B-iB_1)\right\}\left.\right]+\frac{1}{2}\left\{f_1(iA)+f_1(-iA)\right\} \\ & -\frac{SGc}{(Sc-1)^2}\left[\frac{E_1}{2}\left\{e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)+e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right\}\right. \\ & \left.+(E_2-iE_3)f_3(K,B+iB_1)+(E_2+iE_3)f_3(K,B-iB_1)\right] \end{aligned} \tag{10}$$

where,

$$\eta=\frac{y}{2\sqrt{t}},\quad A=\sqrt{SGr},\quad B=\frac{ScK}{Sc-1},\quad B_1=\frac{A}{Sc-1}=\frac{\sqrt{SGr}}{Sc-1},\quad C_1=\frac{B}{(B^2+B_1^2)}$$

$$C_2=\frac{-B}{2(B^2+B_1^2)},\quad C_3=\frac{-B_1}{2(B^2+B_1^2)},\quad D_1=\frac{B^2}{(B^2+B_1^2)},\quad D_2=\frac{B_1^2}{2(B^2+B_1^2)}$$

$$D_3=\frac{BB_1}{2(B^2+B_1^2)},\quad E_1=\frac{1}{(B^2+B_1^2)},\quad E_2=\frac{-1}{2(B^2+B_1^2)},\quad E_3=\frac{B}{2B_1(B^2+B_1^2)}$$

Also, $f_i$'s are inverse Laplace's transforms given by

$$f_1(ip)=L^{-1}\left\{\frac{e^{-y\sqrt{s+ip}}}{s}\right\},\quad f_2(ip,q_1+iq_2)=L^{-1}\left\{\frac{e^{-y\sqrt{s+ip}}}{s+q_1+iq_2}\right\},\quad f_3(p,q_1+iq_2)=L^{-1}\left\{\frac{e^{-y\sqrt{Sc(s+p)}}}{s+q_1+iq_2}\right\}$$

We separate the complex arguments of the error function contained in the previous expressions into real and imaginary parts using the formulas provided by [13].

### 3.1. Special Case [For Sc=1]

We came up with answers for the special case where $Sc = 1$. Hence, the solutions for the special case are as follows:

$$C^* = \frac{1}{2}\left[e^{-2\eta\sqrt{Kt}}erfc\left(\eta-\sqrt{Kt}\right)+e^{2\eta\sqrt{Kt}}erfc\left(\eta+\sqrt{Kt}\right)\right] \tag{11}$$

$$\begin{aligned} U^* =& \frac{KGc}{2(K^2+A^2)}\left\{f_1(iA)+f_1(-iA)\right\}+\frac{iA}{2}\left(\frac{1}{S}+\frac{Gc}{K^2+A^2}\right)\left\{f_1(iA)-f_1(-iA)\right\} \\ &-\frac{KGc}{2(K^2+A^2)}\left[e^{-2\eta\sqrt{Kt}}erfc\left(\eta-\sqrt{Kt}\right)+e^{2\eta\sqrt{Kt}}erfc\left(\eta+\sqrt{Kt}\right)\right] \end{aligned} \tag{12}$$

$$\begin{aligned} \theta^* =& \frac{SKGc}{2iA(K^2+A^2)}\left\{f_1(iA)-f_1(-iA)\right\}+\frac{1}{2}\left(1+\frac{SGc}{K^2+A^2}\right)\left\{f_1(iA)+f_1(-iA)\right\} \\ &-\frac{SGc}{2(K^2+A^2)}\left\{e^{-2\eta\sqrt{Kt}}erfc\left(\eta-\sqrt{Kt}\right)+e^{2\eta\sqrt{Kt}}erfc\left(\eta+\sqrt{Kt}\right)\right\} \end{aligned} \tag{13}$$

### 3.2. Classical Case (S=0)

We derived solutions for the classical case of no thermal stratification ($S = 0$). We want to compare the results of the fluid with thermal stratification to the case with no stratification. Hence, the corresponding solutions for the classical case is given by :

$$\theta_c = = erfc(\eta) \tag{14}$$

$$\begin{aligned} U_c =& \frac{Gc}{2KSc}\Big[2erfc\left(\eta\right)-e^{-Bt}\left\{e^{-2\eta\sqrt{-Bt}}erfc\left(\eta-\sqrt{-Bt}\right)+e^{2\eta\sqrt{-Bt}}erfc\left(\eta+\sqrt{-Bt}\right)\right\} \\ &-\left\{e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)+e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right\} \\ &+e^{-Bt}\left\{e^{-2\eta\sqrt{Sc(K-B)t}}erfc\left(\eta\sqrt{Sc}-\sqrt{(K-B)t}\right)+e^{2\eta\sqrt{Sc(K-B)t}}erfc\left(\eta\sqrt{Sc}+\sqrt{(K-B)t}\right)\right\}\Big] \\ &+2t\eta Gr\left\{\frac{e^{-\eta^2}}{\sqrt{\pi}}-\eta\ erfc\left(\eta\right)\right\} \end{aligned} \tag{15}$$

### 3.3. Skin-Friction

The non-dimensional Skin-Friction, which is determined as shear stress on the surface, is obtained by

$$\tau = -\left.\frac{dU}{dy}\right|_{y=0}$$

The solution for the Skin-Friction is calculated from the solution of Velocity profile $U$, represented by (9), as follows:

$$\begin{aligned} \tau =& \frac{Gc}{Sc-1}\Bigg[C_1\left\{\frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}\left(r_1-r_2\right)-\sqrt{ScK}\ erf(\sqrt{Kt})-\sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}+2C_2\left\{\frac{\cos At}{\sqrt{\pi t}}-\sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\} \\ &+e^{-Bt}\left\{(C_2P_1+C_3Q_1)(r_3\cos B_1t+r_4\sin B_1t)+(C_3P_1-C_2Q_1)(r_4\cos B_1t-r_3\sin B_1t)\right\} \\ &+e^{-Bt}\left\{(C_2P_2-C_3Q_2)(r_5\cos B_1t-r_6\sin B_1t)-(C_3P_2+C_2Q_2)(r_6\cos B_1t+r_5\sin B_1t)\right\} \\ &-2e^{-Bt}\sqrt{Sc}\left\{(C_2P_3-C_3Q_3)(r_7\cos B_1t-r_8\sin B_1t)-(C_3P_3+C_2Q_3)(r_8\cos B_1t+r_7\sin B_1t)\right\}\Bigg] \\ &+\frac{A}{S}\left\{\frac{\sin At}{\sqrt{\pi t}}-\sqrt{\frac{A}{2}}\left(r_1+r_2\right)\right\}+\frac{Gc}{A}\Bigg[(D_1-1)\left\{\frac{-\sin At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}\left(r_1+r_2\right)\right\}-\frac{2D_2\sin At}{\sqrt{\pi t}} \\ &+e^{-Bt}\left\{(D_2P_1-D_3Q_1)(r_4\cos B_1t-r_3\sin B_1t)+(D_3P_1+D_2Q_1)(r_3\cos B_1t+r_4\sin B_1t)\right\} \\ &+e^{-Bt}\left\{(D_2P_2+D_3Q_2)(r_6\cos B_1t+r_5\sin B_1t)-(D_3P_2-D_2Q_2)(r_5\cos B_1t-r_6\sin B_1t)\right\}\Bigg] \end{aligned}$$

The solution for the Skin-Friction for the special case is given from the expression (12), which is represented by

$$\tau^* = \frac{KGc}{K^2+A^2}\left[\frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1-r_2)-\sqrt{K}\ erf(\sqrt{Kt})-\frac{e^{-Kt}}{\sqrt{\pi t}}\right] +A\left(\frac{1}{S}+\frac{Gc}{K^2+A^2}\right)\left[\frac{\sin At}{\sqrt{\pi t}}-\sqrt{\frac{A}{2}}(r_1+r_2)\right]$$

The solution for the Skin-Friction for the classical case is given from the expression (15), which is represented by

$$\tau_c = \sqrt{\frac{t}{\pi}}Gr+\frac{Gc}{KSc}\left[e^{-Bt}\left\{\sqrt{Sc(K-B)}\ erf(\sqrt{(K-B)t})-\sqrt{-B}\ erf(\sqrt{-Bt})\right\}-\sqrt{ScK}\ erf(\sqrt{Kt})\right]$$

### 3.4. Nusselt Number

The non-dimensional Nusselt number, which is determined as the rate of heat transfer, is obtained by

$$Nu = -\left.\frac{d\theta}{dy}\right|_{y=0}$$

The solution for the Nusselt number is calculated from the solution of Temperature profile $\theta$, represented by (10), as follows:

$$\begin{aligned} Nu = {} & \frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1-r_2)+\frac{SGc}{A(Sc-1)}\left[C_1\left\{\frac{-\sin At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1+r_2)\right\}-\frac{2C_2\sin At}{\sqrt{\pi t}}\right. \\ & +e^{-Bt}\{(C_2P_1+C_3Q_1)(r_4\cos B_1t-r_3\sin B_1t)-(C_3P_1-C_2Q_1)(r_3\cos B_1t+r_4\sin B_1t)\} \\ & \left.+e^{-Bt}\{(C_2P_2-C_3Q_2)(r_6\cos B_1t+r_5\sin B_1t)+(C_3P_2+C_2Q_2)(r_5\cos B_1t-r_6\sin B_1t)\}\right] \\ & +\frac{SGc}{(Sc-1)^2}\left[E_1\left\{\frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1-r_2)-\sqrt{ScK}\ erf(\sqrt{Kt})-\sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}\right. \\ & +2E_2\left\{\frac{cosAt}{\sqrt{\pi t}}-\sqrt{\frac{Sc}{\pi t}}e^{-Kt}\right\}+e^{-Bt}\{(E_2P_1+E_3Q_1)(r_3cosB_1t+r_4\sin B_1t)\} \\ & +(E_3P_1-E_2Q_1)(r_4cosB_1t-r_3\sin B_1t)+e^{-Bt}\{(E_2P_2-E_3Q_2)(r_5cosB_1t-r_6\sin B_1t) \\ & -(E_3P_2+E_2Q_2)(r_6cosB_1t+r_5\sin B_1t)\}-2e^{-Bt}\sqrt{Sc}\{(E_2P_3-E_3Q_3)(r_7cosB_1t-r_8\sin B_1t) \\ & \left.-(E_3P_3+E_2Q_3)(r_8cosB_1t+r_7\sin B_1t)\}\right] \end{aligned}$$

The solution for the Nusselt number for the special case is given from the expression (13), which is represented by

$$Nu^* = \frac{SKGc}{A(K^2+A^2)}\left[\frac{-\sin At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1+r_2)\right]+\left(1+\frac{SGc}{K^2+A^2}\right)\left[\frac{\cos At}{\sqrt{\pi t}}+\sqrt{\frac{A}{2}}(r_1-r_2)\right] -\frac{SGc}{K^2+A^2}\left[\sqrt{K}\ erf(\sqrt{Kt})+\frac{e^{-Kt}}{\sqrt{\pi t}}\right]$$

The solution for the Nusselt number for the classical case is given from the expression (14), which is represented by

$$Nu_c = \frac{1}{\sqrt{\pi t}}$$

### 3.5. Sherwood Number

The non-dimensional Sherwood number, which is determined as the rate of mass transfer, is obtained by

$$Sh = -\left.\frac{dC}{dy}\right|_{y=0}$$

The solution for the Sherwood number is calculated from the solution of Concentration profile $C$, represented by (8), as follows:

$$Sh = \sqrt{ScK}\ erf(\sqrt{Kt}) + \sqrt{\frac{Sc}{\pi t}}e^{-Kt}$$

The solution for the Sherwood number for the special case is given from the expression (11), which is represented by

$$Sh^* = \sqrt{K}\ erf(\sqrt{Kt}) + \frac{1}{\sqrt{\pi t}}e^{-Kt}$$

where,

$$B_2 = \sqrt{B^2 + (A - B_1)^2}, \quad B_3 = \sqrt{B^2 + (A + B_1)^2}, \quad B_4 = \sqrt{(K - B)^2 + B_1^2}, \quad P_1 = \sqrt{\frac{B_2 - B}{2}},$$

$$Q_1 = \sqrt{\frac{B_2 + B}{2}}, \quad P_2 = \sqrt{\frac{B_3 - B}{2}}, \quad Q_2 = \sqrt{\frac{B_3 + B}{2}}, \quad P_3 = \sqrt{\frac{B_4 - (K - B)}{2}}$$

$$Q_3 = \sqrt{\frac{B_4 + (K - B)}{2}}, \quad \sqrt{-B + i(A - B_1)} = P_1 + iQ_1, \quad \sqrt{-B + i(A + B_1)} = P_2 + iQ_2,$$

$$\sqrt{K - B + iB_1} = P_3 + iQ_3, \quad erf(\sqrt{iAt}) = r_1 + ir_2, \quad erf(P_1\sqrt{t} + iQ_1\sqrt{t}) = r_3 + ir_4,$$

$$erf(P_2\sqrt{t} + iQ_2\sqrt{t}) = r_5 + ir_6, \quad erf(P_3\sqrt{t} + iQ_3\sqrt{t}) = r_7 + ir_8$$

## 4. RESULT AND DISCUSSIONS

In order to better understand the physical significance of the problem, we calculated the velocity, temperature, concentration, Skin friction, Nusselt number, and Sherwood number using the solutions we found in the previous sections, for different values of the physical parameters $Gr, Gc, Sc, K, S$ and time $t$. Additionally, we represented them graphically in Figures 2-15.

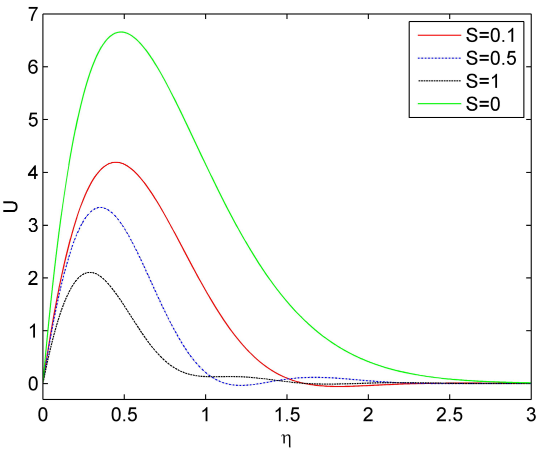


**Figure 2.** Effects of $S$ on Velocity Profile for $Gr = 5, Gc = 5, t = 1.6, Sc = 0.6, K = 0.2$

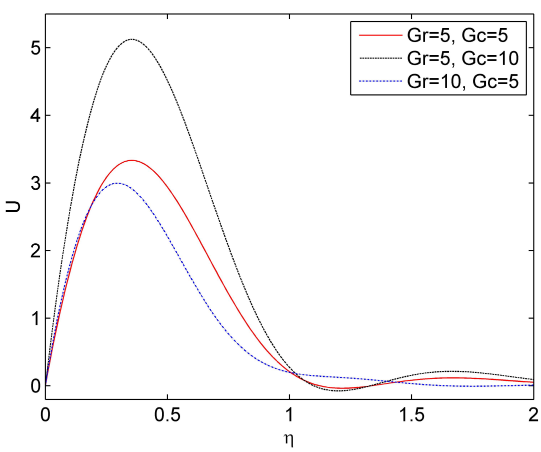


**Figure 3.** Effects of $Gr$ and $Gc$ on Velocity Profile for $S = 0.5, Sc = 0.6, t = 1.6, K = 0.2$

Figure 2 illustrates how the velocity profiles are affected by thermal stratification ($S$). It can be seen that the velocity is lowered due to thermal stratification. According to Figure 3, increasing $Gc$ results in an increase in velocity, whereas increasing $Gr$ results in a drop in velocity. The fluid's velocity was shown at various values of $Sc$ and $K$ in figures 4 and 5. As $Sc$ and $K$ values grow, the fluid velocity falls.

The influence of thermal stratification on fluid velocity and temperature is plotted against time in Figures 6 and 7. Without stratification, the velocity and temperature grow continuously over time; however, when stratification occurs, they eventually reach a stable condition. This study is more realistic than earlier ones with no stratification because it applies thermal stratification, which lowers velocity and temperature in comparison to the classical scenario ($S = 0$).

Time-varying velocity and temperature patterns are shown in Figures 8 and 9. The velocity is observed to increase with time and diminish to zero as the distance from the plate increases. However, temperatures fall over time and eventually reach absolute zero as one moves away from the plate.

The impact of thermal stratification on the temperature is seen in Figure 10. As the parameters of thermal stratification are raised, it is seen that the temperature drops. Figures 11, 12, and 13 demonstrate the impacts

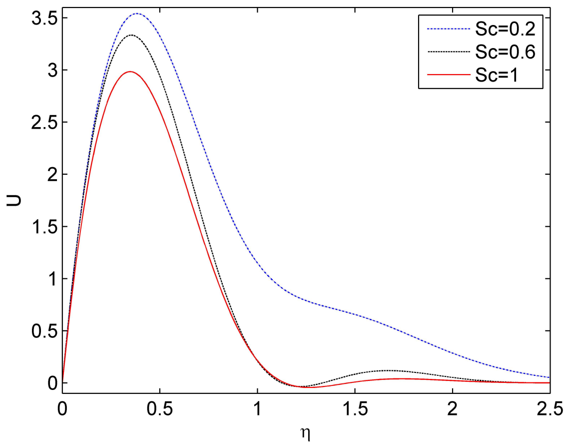


**Figure 4.** Effects of $Sc$ on Velocity Profile for $Gr = 5, Gc = 5, S = 0.5, t = 1.6, K = 0.2$

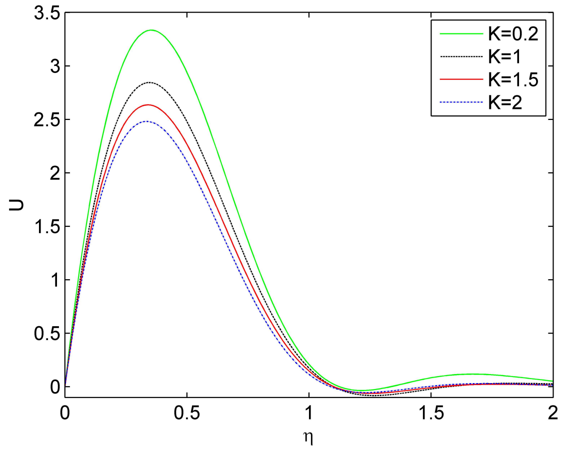


**Figure 5.** Effects of $K$ on Velocity Profile for $Gr = 5, Gc = 5, S = 0.5, Sc = 0.6, t = 1.6$

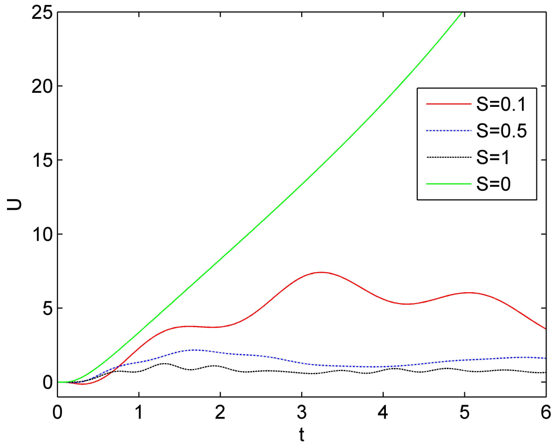


**Figure 6.** Effects of $S$ on Velocity Profile against time for $Gr = 5, Gc = 5, Sc = 0.6, y = 1.6, K = 0.2$

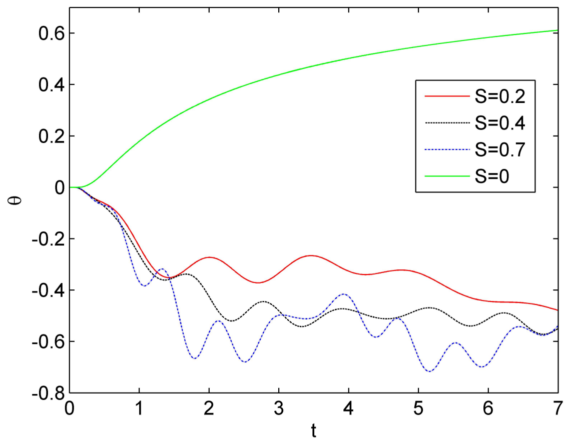


**Figure 7.** Effects of $S$ on Temperature Profile against time for $Gr = 5, Gc = 5, Sc = 0.6, y = 1.9, K = 0.2$

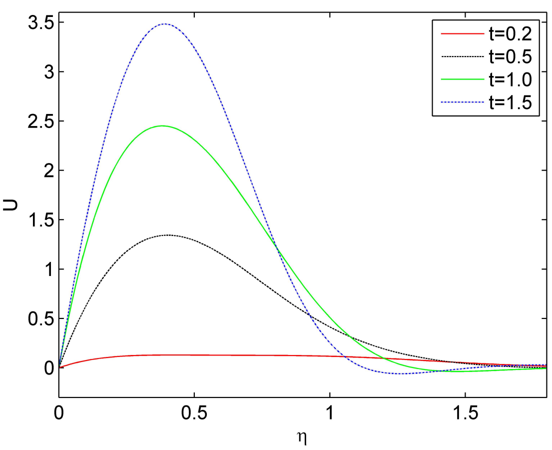


**Figure 8.** Velocity Profile at different time for $Gr = 5, Gc = 5, S = 0.5, Sc = 0.6, K = 0.2$

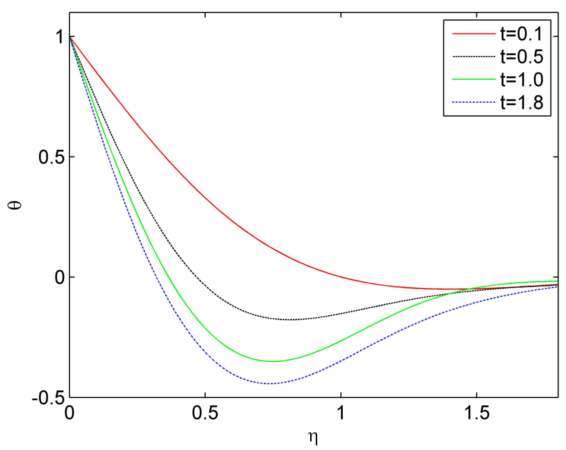


**Figure 9.** Temperature Profile at different time for $Gr = 5, Gc = 5, S = 0.5, Sc = 0.6, K = 0.2$

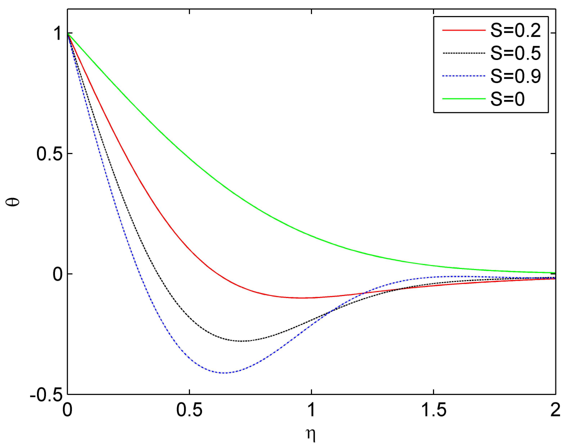


**Figure 10.** Effects of $S$ on Temperature Profile for $Gr = 5, Gc = 5, , Sc = 0.6, t = 0.6, K = 0.2$

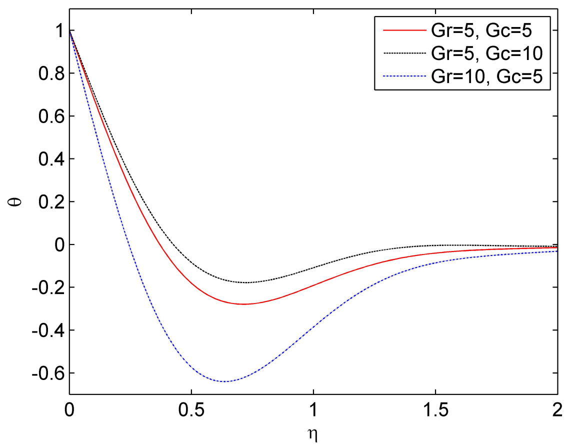


**Figure 11.** Effects of $Gr$ and $Gc$ on Temperature Profile for $S = 0.5, Sc = 0.6, t = 0.6, K = 0.2$

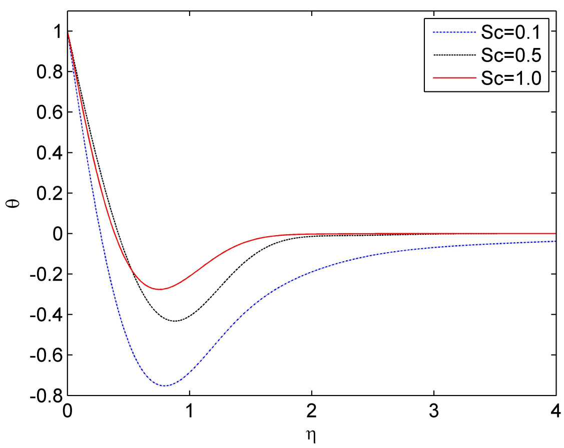


**Figure 12.** Effects of $Sc$ on Concentration Profile for $Gr = 5, Gc = 5, S = 0.5, t = 1.4, K = 0.2$

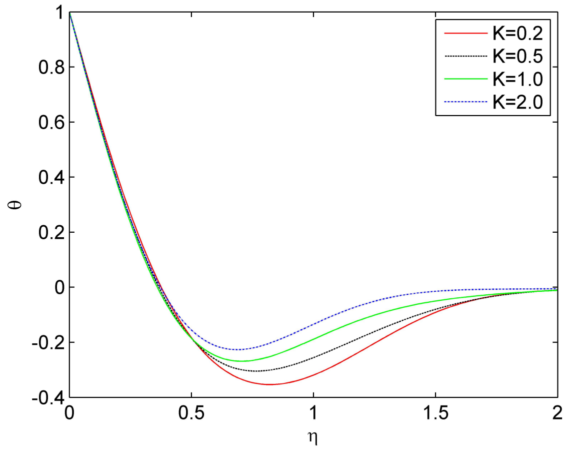


**Figure 13.** Effects of $K$ on Temperature Profile for $Gr = 5, Gc = 5, S = 0.5, Sc = 0.6, t = 1.4$

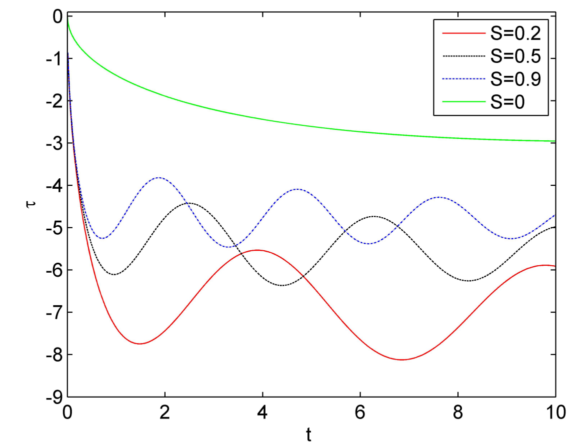


**Figure 14.** Effects of $S$ on Skin friction for $Gr = 5, Gc = 5, Sc = 0.1, K = 0.2$

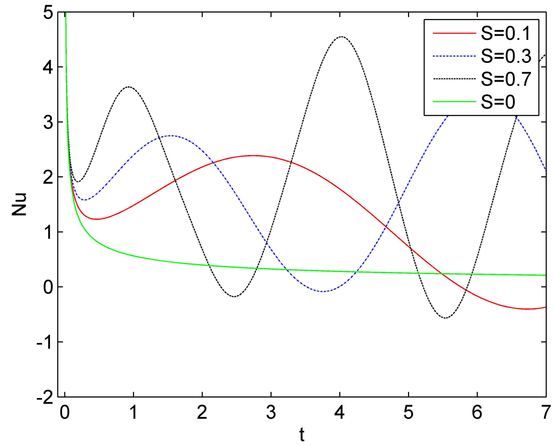


**Figure 15.** Effects of $S$ on Nusselt Number for $Gr = 5, Gc = 5, Sc = 0.1, K = 0.2$

of $Gr, Sc, K$ and $K$ ,on temperature profile respectively. The temperature drops for higher values of $Gc$ and lower values of $Gr, Sc, K$.

The effect of Thermal Stratification can be seen in Figures 14 and 15, which show skin friction and the Nusselt number, respectively. In the presence of stratification, they significantly increase in comparison to when there is no stratification.

## 5. CONCLUSION

We investigated at how chemical reactions affect the flow past an infinite vertical plate when there are different levels of temperature. The results found in the present study are compared to those found in the classical scenario, when stratification does not occur. The velocity of the fluid declines as $S, K, Sc$, and $Gr$ increase, but it goes up as $Gc$ increases. The application of thermal stratification, which reduces velocity and temperature relative to the classical case ($S = 0$), makes this study more feasible than previous ones. Temperature drops when $K, Sc$, and $Gc$ drop; conversely, it drops when $S, Gr$, and time rise. The temperature is maximum at the plate and drops to zero at greater distances from the plate, as demonstrated in research paper [2]. Thermal stratification makes the Nusselt number oscillate more frequently.

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# ВПЛИВ ТЕРМІЧНОЇ СТРАТИФІКАЦІЇ НА ПОТІК ПОВЗ НЕСКІНЧЕННУ ВЕРТИКАЛЬНУ ПЛАСТИНУ ЗА НАЯВНОСТІ ХІМІЧНОЇ РЕАКЦІЇ

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У цьому дослідженні досліджується, як теплове розшарування впливає на рух рідини за наявності хімічної реакції першого порядку повз нескінченну вертикальну пластину. Для розв'язування безрозмірних керуючих рівнянь у замкнутій формі при Pr = 1 застосовано систему перетворень Лапласа. Результати, отримані в для умов термічної стратифікації, порівнюються з випадком відсутності стратифікації. Вплив багатьох параметрів, включаючи S, K, Gr, Gc, Sc і часу, на швидкість, температуру, концентрацію, поверхневе тертя, число Нуссельта та число Шервуда досліджується та відображається графічно. Показано, що стаціонарний стан досягається за менший час в результаті застосування стратифікації потоку.

**Ключові слова:** *термічна стратифікація; хімічна реакція; тепло- та масообмін; вертикальна пластина; число Шмідта*