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ENERGY CONDITIONS WITH INTERACTING FIELD IN $f(R)$ GRAVITY[†]

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In the context of current scenario, it is crucial to look beyond Einstein's theory, which opens the door to certain modified theories of gravity. So, present study is devoted to investigate the various energy conditions, particularly, strong energy condition (SEC), weak energy condition (WEC), null energy condition (NEC) and dominant energy condition (DEC) corresponding to different functional forms of $f(R)$ gravity. We have studied for flat, isotropic and homogeneous FLRW cosmological model filled with interacting field i.e., perfect fluid is coupled with mass less scalar field for different models of modified $f(R)$ gravity in which R is the Ricci scalar. We have observed, the accelerated expansion of the Universe which exact match with recent observational evidences.

Keywords: *FLRW cosmological model; $f(R)$ gravity; Interacting field; Hubble's law*

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1. INTRODUCTION

During the past few decades, the astrophysical data from cosmic microwave background anisotropy, galaxy clustering and high red-shift supernovae [1, 2, 3] have disclosed large scale structure and accelerated expansion of the Universe caused by a dominated negative pressure fluid termed as dark energy (DE). According to experimental findings, the DE acquires almost 73%, dark matter occupied 23% and baryonic matter served 4% [4, 5, 6] of entire energy density. The vacuum energy (also known as the cosmological constant) with equation of state (EoS) parameter $w = -1$ are the most straightforward and theoretically plausible candidates to explain the existence of DE [7, 8]. But instead of introducing a dark energy it is more sensible to propose theories by modifying the Einstein-Hilbert action with Ricci scalar R and a general function along with matter-geometry coupling. Some of popular and important modified theories are $f(R)$ gravity [9], $f(T)$ gravity [10], $f(G)$ gravity [11], $f(R,T)$ gravity [12] etc.

The $f(R)$ gravity is one of the most powerful and interesting candidates to study the behaviour of DE and is a modification of general relativity that alters the Lagrangian density of the theory by incorporating a function $f(R)$ into the gravitational action [13]. To extend and generalize Einstein's theory of general relativity, there exist different approaches to describe modified gravity theories such as $f(R)$ gravity which approaches aim to expand the understanding of gravitational interactions and their implications. Recent research has focused on $f(R)$ gravity as a potential explanation for the accelerated expansion of the Universe. These theories have limitations such as instability, ghost-like degrees of freedom, and the fine-tuning problem. Despite these limitations, $f(R)$ gravity has been successful in explaining various phenomena, including the structure formation, the unification of cosmic epochs, and the late accelerated expansion of the Universe [14, 15, 16, 17]. Studies in $f(R)$ gravity have supported observations of Type Ia supernovae and WMAP data, which indicate an accelerated expansion of the Universe [18]. After a long period of time, the expansion will cease entirely and the Universe will attain isotropy i.e as $t \to \infty$, $\frac{\sigma^2}{\theta} \to 0$ [19].The Hubble parameter (H) and expansion scalar (θ) have been analyzed, revealing that the expansion rate of the Universe was higher during the Big Bang and has gradually decreased over time [20]. Researchers have also explored the solutions of Bianchi-type cosmology in metric $f(R)$ gravity and compared them with Lematre-Tolman cosmology, which describes inhomogeneity in the Universe on various scales[21]. The influence of magnetism in the early stages of the Universe's evolution has been observed in the investigation of Bianchi type I space-time in $f(R)$ gravity with the presence of string clouds and domain walls containing strange quark matter [22]. Both accelerating and decelerating phases of Bianchi type-I space-time have been studied in $f(R)$ gravity, with the Universe expanding in both phases. The initial expansion rate is much faster than the later rate [23]. The expansion of model begins with zero volume, finite string density as well pressure and continues to expand over time [24]. Fourth-order $f(R)$ extended theories of gravity have been

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examined [25], specifically the form $f(R) = R+R^2$, which is in line with theoretical constraints and cosmological data for static and spherically symmetric configurations. In contrast to the expansion scalar, shear scalar, and Hubble's parameter, the spatial volume in $f(R)$ gravity increases exponentially with time, indicating a constant rate of expansion that matches the present-day scenario [26]. Using a special form of the Hubble parameter, researchers [27] have investigated exact solutions of spatially homogeneous Bianchi type-I cosmological models in the context of $f(R)$ theory of gravity. The models depict an expanding Universe following a power law after the Big Bang singularity, with the presence of accelerating or decelerating phases depending on the parameters [28]. Furthermore, the solutions of field equations have been explored for higher dimensional Kaluza-Klein cosmological models using string models in $f(R)$ gravity, resulting in an accelerated expanding Universe [29].

The authors [30] employed a special case proposed by Schwinger and found solutions for the model FRW closed bulk viscous cosmological model with wet dark fluid was examined using the Nordtvedt general scalar-tensor theory. Another investigation focused on two fluid axially symmetric cosmological models in the $f(R,T)$ theory of gravitation. By imposing a supplementary constraint $H_3 = kH_1$, the authors [31] obtained a deterministic model that exhibited an expanding and shearing Universe. Some authors [32] presented a FRW cosmological model which satisfied the principle and considering the most recent research that suggested dark energy is responsible for the Universe's acceleration. The authors [33] selected a specific functional form and explored the cosmological behavior of FRW models in the $f(R,G)$ gravity. They discussed violations of the SEC (strong energy condition) and NEC (null energy condition), which are necessary for bouncing models. In the context of the Big Bang theory, researchers [34] investigated the cosmological behavior within the framework of the $f(R,T)$ gravity. They obtained a solution for the FRW cosmological model, which revealed that the Universe is accelerating and expanding according to current observations. By adopting the fractal parameter $\omega = 0$ and the relationship between p_m and ρ_m, i.e. $p_m = (\gamma - 1)\rho_m$, $1 \leq \gamma \leq 2$, [35] explored the flat FRW two-fluid cosmological models in fractal theory of gravity and discovered the answers.

By examining the plane-symmetric cosmological model in the presence of a source-free electromagnetic field coupled with mass less scalar field [36, 37] identified an expanding, shearing and non-rotating Universe. It is observed that the closed Universe correspond to quintessence while flat and open Universe corresponding to the phantom model of the Universe [38]. A realistic Universe dominated by matter has been obtained using two-fluid viscous dark energy models for an anisotropic general type of Bianchi space-time that is filled with barotropic and dark fluid in interacting and non-interacting modes with variable EoS parameter [39]. During investigations of the two-fluid Bianchi type-VI_0 anisotropic cosmological model coupled with the zero mass scalar field in Einstein's theory of gravitation, some authors [40] discovered that the cosmological constant decreased over time as the Universe expanded which supported by a recent finding from observations of type Ia supernova explosions (SNIa). Researchers [41] have explored modified $f(T)$ gravity two-fluid cosmological models and observed the expanding and shearing nature of the Universe. Some authors [42] studied both interacting and non-interacting cases while observing the matter and radiation fluids in the scale covariant theory of gravitation for Bianchi type I. Researchers [43] looked at a plane-symmetric cosmological model in $f(R,T)$ gravity with an interacting field as a source of energy and noticed that the pressure and density behaved differently in different models.

Energy conditions are fundamental for understanding concepts like black hole thermodynamics and the singularity theorem. The Raychaudhuri equations [44] provide an excellent model for describing the attractive nature of gravity and the positive energy present in the Universe. There are four different energy conditions: the null energy condition (NEC), weak energy condition (WEC), dominant energy condition (DEC), and strong energy condition (SEC). The weak energy condition states that the energy density measured by any observer should not be negative. It implies that the energy density must be greater than or equal to zero at all points in spacetime. On the other hand, the strong energy condition stipulates that the pressure observed by any observer should be less than or equal to the energy density, and the energy density itself should not be negative. In other words, the pressure should not counteract gravity. The dominant energy condition requires that the energy density measured by any observer must be positive, and the energy flux measurement must not be space-like. This means that the energy flow cannot exceed the speed of light, and the energy density at any point in spacetime must be greater than or equal to zero.

The article follows the following structure. Section II provides an introduction to the formalism of $f(R)$ gravity. Section III examines the field equations for the isotropic, flat, and homogeneous FLRW Universe. In Section IV, the solutions to the field equations for $f(R)$ gravity are obtained and calculations are performed for the Hubble parameter, average scale factor, spatial volume, and Scalar expansion. Section V discusses the energy conditions for various models of $f(R)$ gravity. Finally, Section VI presents a discussion of the findings and concludes the article.

2. FORMALISM OF $f(R)$ GRAVITY

A modification to the general theory of relativity is $f(R)$ gravity. The field equations of $f(R)$ gravity are obtained by varying Hilbert-Einstein action principle which is given by

$$S = \frac{1}{16\pi}\int f(R)\sqrt{-g}d^4x + \int S_m\sqrt{-g}d^4x \tag{1}$$

where $f(R)$ is a general function of Ricci scalar R and g is the determinant of g_{ij} and S_m is Lagrangian of matter.
The Ricci scalar R is obtained by contracting the Ricci tensor R_{ij} as

$$R = g^{ij}R_{ij} \tag{2}$$

Here the formulation of the Ricci tensor is given by

$$R_{ij} = \Gamma^p_{ij},_p - \Gamma^p_{ip},_j + \Gamma^p_{ij}\Gamma^q_{pq} - \Gamma^q_{ip}\Gamma^p_{jq} \tag{3}$$

where $\Gamma^\alpha_{\beta\gamma}$ represents the components of the renowned Levi-Civita connection as described by

$$\Gamma^i_{jk} = \frac{1}{2}g^{im}\left(g_{jm,k} + g_{km,j} - g_{jk,m}\right) \tag{4}$$

The corresponding field equations of $f(R)$ gravity are obtained by varying the action (1) with respect to g_{ij} as,

$$F(R)\,R_{ij} - \frac{1}{2}f(R)\,g_{ij} - \nabla_i\nabla_j F(R) + g_{ij}\Box F(R) = T_{ij} \tag{5}$$

Where $\Box = \nabla_k\nabla^k$ and $F(R) = \frac{df(R)}{dR}$
Here ∇_i is the covariant derivative and T_{ij} is energy momentum tensor derived from the Lagrangian S_m.
The energy momentum tensor T_{ij} is given by

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}S_m\right)}{\partial g^{ij}} \tag{6}$$

3. FLRW METRIC AND FIELD EQUATIONS

The observational data from Cosmic Microwave Background (CMB) [45, 46] point out that our Universe is spatially flat at late times, therefore we considered the isotropic, flat and homogeneous FLRW metric as

$$ds^2 = dt^2 - a^2(t)\sum_{i=1}^{3} dx_i^2 \tag{7}$$

Where, a termed as cosmic scale factors used to measure the expansion of Universe, is a functions of time t only.
from equation (3), we obtained the non-zero components of Ricci tensor as

$$R_{00} = -3\frac{\ddot{a}}{a}, \quad R_{11} = R_{22} = R_{33} = -\left(2\dot{a}^2 + a\dot{a}\right) \tag{8}$$

Therefore, the resulting Ricci scalar R for the line element (7) is

$$R = 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2}{a^2} = 6\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right] \tag{9}$$

Here $H = \frac{\dot{a}}{a}$ is a Hubble parameter.
We assume the gravitational field's energy source to be an interacting field with dark energy, which is coupling of a perfect fluid and a mass less scalar field for the metric (7) is given by

$$\overline{T_{ij}} = S_{ij} + T_{ij} \tag{10}$$

where S_{ij} represent an energy momentum tensor for perfect fluid distribution given by

$$S_{ij} = (p+\rho)u_iu_j - pg_{ij} \tag{11}$$

Here, p denotes the spatially isotropic pressure, ρ is the matter-energy density, and $u^i = (1, 0, 0, 0)$ represents the time-like four-velocity vector of the cosmic fluid satisfying $u^i u_i = 1$
and

$$T_{ij} = U_{,i}U_{,j} - \frac{1}{2}g_{ij}U_{,s}U^{,s} \tag{12}$$

The mass less scalar field (U) satisfies

$$g^{ij}U_{;ij} = \rho_c \tag{13}$$

where, U and ρ_c are mass less scalar field and charge density respectively depends on cosmic time (t) alone. Also semicolon (;) and comma (,) denotes covariant and partial derivative respectively
The Einstein field equations (5) for the cosmological model (7) using equation (10) are given by

$$\left[2\left(\frac{\dot{a}}{a}^2\right) + \frac{\ddot{a}}{a}\right]F(R) - \frac{1}{2}f(R) + 2\frac{\dot{a}}{a}\dot{F} + \ddot{F} = -p + \frac{\dot{U}^2}{2} \tag{14}$$

$$3\frac{\ddot{a}}{a}F(R) - \frac{1}{2}f(R) + 3\frac{\dot{a}}{a}\dot{F} = \rho - \frac{\dot{U}^2}{2} \tag{15}$$

An over-dot (.) denotes an ordinary derivative with respect to cosmic time t.

4. SOLUTIONS TO FIELD EQUATIONS

There are two linearly independent equations with four unknowns H, p, ρ and $\dot{U}$. Therefore to find above unknowns we have to use following plausible condition; The Hubble's parameter variation Bermann proposed in 1983, which provides the relationship between Hubble parameter and the average scale factor as,

$$H = ba^{-n} \tag{16}$$

where b is positive and $n \geq 0$ are constants.
from above equation we have

$$q = -a\frac{\ddot{a}}{\dot{a}^2} = n - 1 \tag{17}$$

where q is deceleration parameter.
Using equation (17) and equation (18) we obtained
Average scale factor as,

$$a(t) = (ct + d)^{\frac{1}{n}} \tag{18}$$

The Volume (V) is

$$V = a^3(t)$$

$$V = (ct + d)^{\frac{3}{n}} \tag{19}$$

The Hubble Parameter (H) is

$$H = \frac{\dot{a}}{a}$$

$$H = \frac{c}{n(ct + d)} \tag{20}$$

The Scalar expansion (θ) is

$$\theta = 3H$$

$$\theta = \frac{3c}{n(ct + d)} \tag{21}$$

where c and d are the constants of integration.
We have consider charge density $\rho_c = 0$
Therefore, the mass less scalar field is given by

$$U = c_1\frac{a^4}{4} + c_2 = c_1\frac{(ct + d)^{\frac{4}{n}}}{4} + c_2 \tag{22}$$

5. ENERGY CONDITIONS

To understanding geodesics of the Universe it is necessary to know the energy conditions (ECs), which are linear combinations of energy density and pressure. The well-known Raychaudhuri equations provide the foundation for energy conditions (ECs) which are of the form [43, 44, 45, 46]

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} - R_{ij}u^i u^j \tag{23}$$

$$\frac{d\theta}{d\tau} = -\frac{1}{2}\theta^2 - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} - R_{ij}\eta^i \eta^j \tag{24}$$

where θ is the expansion factor, n^i is the null vector and σ^{ij} and ω_{ij} are shear and rotation respectively for the vector field u^i.
For attractive gravity, equations (23) & (24) satisfy the following conditions

$$R_{ij}u^i u^j \geq 0 \tag{25}$$

$$R_{ij}\eta^i \eta^j \geq 0 \tag{26}$$

Hence, the energy conditions derived from standard GR for perfect fluid matter distribution are

- Strong Energy Conditions (SEC) : $p + 3\rho \geq 0$
- Weak Energy Conditions (WEC) : $\rho \geq 0$, $p + \rho \geq 0$
- Null Energy Conditions (NEC) : $p + \rho \geq 0$
- Dominant Energy Conditions (DEC) : If $\rho \geq 0$, $|p| \leq \rho$

We have discussed the viability of energy conditions in two model.

5.1. $f(R) = R$

To illustrate how the aforementioned conditions can be employed to establish limits on $f(R)$ theories, we will present a specific example. In this case, we will focus on a particular family of theories characterized by the form of $f(R)$

$$f(R) = R \tag{27}$$

With choice of above functional form of $f(R)$ gravity, equation (5) reduced to field equation of G. R.
Therefore, equation (14) & (15) are reduced to

$$p = -\frac{c^2}{n^2}\frac{(2n-3)}{(ct+d)^2} + \frac{c_1^2}{2}(ct+d)^{\frac{6}{n}} \tag{28}$$

$$\rho = \frac{1}{2}c_1^2(ct+d)^{\frac{6}{n}} - \frac{3c^2}{n^2}\frac{1}{(ct+d)^2} \tag{29}$$

SEC :

$$\rho + 3p = 2c_1^2(ct+d)^{\frac{6}{n}} + 6(1-n)\frac{c^2}{n^2}\frac{1}{(ct+d)^2} \geq 0 \tag{30}$$

WEC :

$$\rho + p = c_1^2(ct+d)^{\frac{6}{n}} - 2n\frac{c^2}{n^2}\frac{1}{(ct+d)^2} \geq 0$$

$$\rho = \frac{1}{2}c_1^2(ct+d)^{\frac{6}{n}} - \frac{3c^2}{n^2}\frac{1}{(ct+d)^2} \geq 0 \tag{31}$$

NEC :

$$\rho + p = c_1^2(ct+d)^{\frac{6}{n}} - 2n\frac{c^2}{n^2}\frac{1}{(ct+d)^2} \geq 0 \tag{32}$$

DEC :

$$\rho - p = 2(n-3)\frac{c^2}{n^2}\frac{1}{(ct+d)^2} \geq 0 \tag{33}$$

5.2. $f(R) = R + bR^m$

where m is an integer and b is a constant that can assume positive or negative values. To illustrate the energy conditions, we will assume in this subsection that the function $f(R)$ is an algebraic polynomial of R with the independent parameters b and m.
Using above functional form of $f(R)$ gravity, equation (14) & (15) yields,

$$\begin{aligned} p =& -\frac{c^2}{n^2}\frac{(2n-3)}{(ct+d)^2} + \frac{c_1^2}{2}(ct+d)^{\frac{6}{n}} \\ & - b(7-n)^{m-1}\left\{(2m-3)(2-n) + \frac{4c}{n(ct+d)}n^2 m(m-1)(2m-1)\right\}\left[\frac{c}{n(ct+d)}\right]^{2m} \end{aligned} \tag{34}$$

$$\rho = \frac{c^2}{n^2}\frac{3(1-n)}{(ct+d)^2} + \frac{c_1^2}{2}(ct+d)^{\frac{6}{n}} - 9mb6^{m-1}(2-n)^m(1+n+nm)\left[\frac{c}{n(ct+d)}\right]^{2(m+1)} \tag{35}$$

SEC :

$$\begin{aligned} \rho + 3p =& \frac{c^2}{n^2}\frac{3(4-3n)}{(ct+d)^2} + 2c_1^2(ct+d)^{\frac{6}{n}} - 9mb6^{m-1}(2-n)^m(1+n+nm)\left[\frac{c}{n(ct+d)}\right]^{2(m+1)} \\ & - 3b(7-n)^{m-1}\left\{(2m-3)(2-n) + \frac{4c}{n(ct+d)}n^2 m(m-1)(2m-1)\right\}\left[\frac{c}{n(ct+d)}\right]^{2m} \\ \geq& 0 \end{aligned} \tag{36}$$

WEC :

$$\begin{aligned} p + \rho =& \frac{c^2}{n^2}\frac{(6-5n)}{(ct+d)^2} + c_1^2(ct+d)^{\frac{6}{n}} - 9mb6^{m-1}(2-n)^m(1+n+nm)\left[\frac{c}{n(ct+d)}\right]^{2(m+1)} \\ & - b(7-n)^{m-1}\left\{(2m-3)(2-n) + \frac{4c}{n(ct+d)}n^2 m(m-1)(2m-1)\right\}\left[\frac{c}{n(ct+d)}\right]^{2m} \\ \geq& 0 \end{aligned}$$

$$\rho = \frac{c^2}{n^2}\frac{3(1-n)}{(ct+d)^2} + \frac{c_1^2}{2}(ct+d)^{\frac{6}{n}} - 9mb6^{m-1}(2-n)^m(1+n+nm)\left[\frac{c}{n(ct+d)}\right]^{2(m+1)} \geq 0 \tag{37}$$

NEC :

$$\begin{aligned} p + \rho =& \frac{c^2}{n^2}\frac{(6-5n)}{(ct+d)^2} + c_1^2(ct+d)^{\frac{6}{n}} - 9mb6^{m-1}(2-n)^m(1+n+nm)\left[\frac{c}{n(ct+d)}\right]^{2(m+1)} \\ & - b(7-n)^{m-1}\left\{(2m-3)(2-n) + \frac{4c}{n(ct+d)}n^2 m(m-1)(2m-1)\right\}\left[\frac{c}{n(ct+d)}\right]^{2m} \\ & \geq 0 \end{aligned} \tag{38}$$

DEC :

$$\begin{aligned} \rho - p =& \frac{-c^2}{n}\frac{1}{(ct+d)^2} + c_1^2(ct+d)^{\frac{6}{n}} - 9mb6^{m-1}(2-n)^m(1+n+nm)\left[\frac{c}{n(ct+d)}\right]^{2(m+1)} \\ & + b(7-n)^{m-1}\left\{(2m-3)(2-n) + \frac{4c}{n(ct+d)}n^2 m(m-1)(2m-1)\right\}\left[\frac{c}{n(ct+d)}\right]^{2m} \\ & \geq 0 \end{aligned} \tag{39}$$

From the figure we have observed that,

- The physical characteristics, such as the mean scale factor, spatial volume, Hubble parameter, expansion scalar, pressure, and energy density, exhibit a dependence on cosmic time. As we approach the limit of $t \to 0$, these parameters assume constant values without variation.
- In present described model, the spatial volume (V) undergoes a remarkable transformation, commencing with a small value at the inception of Universe and progressively stretching towards an immeasurable magnitude as shown in Figure 1.This conspicuous phenomenon unequivocally signifies the extraordinary expansion of the Universe.

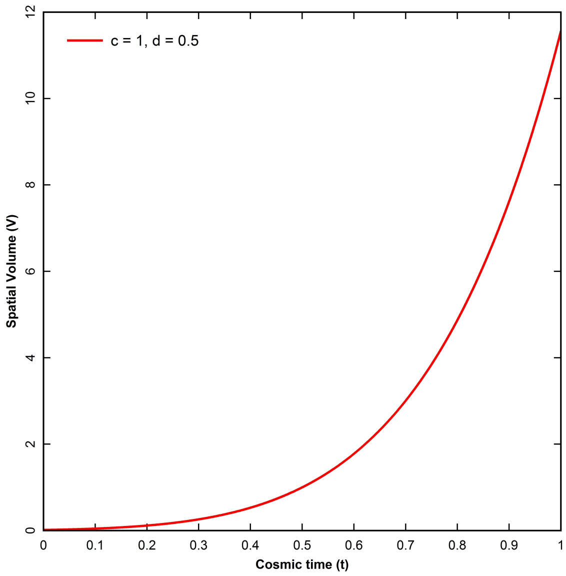

Figure 1. The behavior of Spatial Volume (V) against Cosmic Time (t) for the proper choice of constants: $c = 1$ & $d = 0.5$

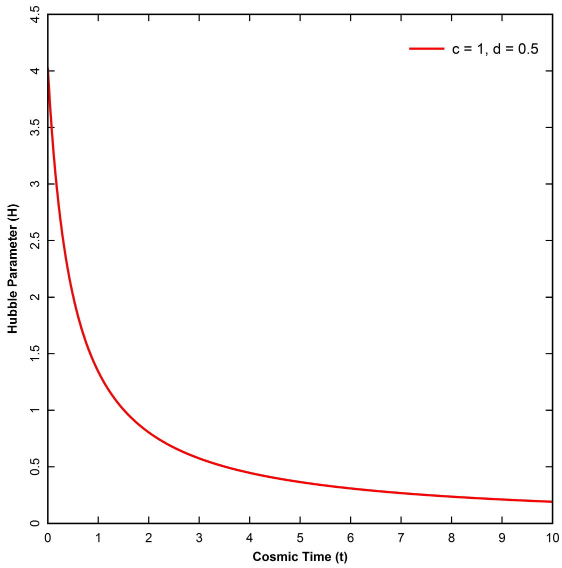

Figure 2. The behavior of Hubble parameter (H) against Cosmic Time (t) for the proper choice of constants: $c = 1$ & $d = 0.5$

- The Hubble parameter (H) represents the pace at which the Universe is expanding, signifying the fractional growth in its scale over a given unit of time. It possesses a positive value and exhibits a decreasing trend as time progresses. The expansion scalar offers insights into the expansion rate, revealing that it was more rapid during the initial stages and gradually decelerates in later phases, as depicted in Figure 2

- From Figure 3, it is observed that as time progresses, the mass less scalar field (U) also increases leading to expansion of the Universe. As time progresses, the scalar field gradually gains energy, resulting in an increasing mass which is responsible for the expansion rate of the Universe and overall cosmological evolution.

- The pressure exhibits a range of variation, shifting from significantly negative values to slightly negative values (as depicted in Figure 4 & 7). Furthermore, as time progresses towards infinity $(t \to \infty)$, the pressure tends to approach zero. These observations hold true for both functional forms of $f(R)$ gravity, namely $f(R) = R$ & $f(R) = R + bR^m$, respectively. The negative nature of the pressure serves as an indicates presence of dark energy and energy density for both functional form of $f(R)$ models are increasing

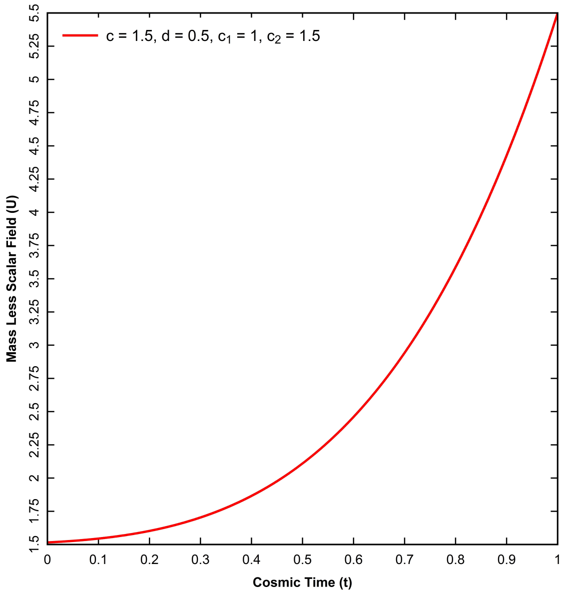

Figure 3. The behavior of Mass Less Scalar Field (U) against Cosmic Time (t) for the proper choice of constants: $c = 1, d = 0.5$, $c_1 = 1$ & $c_2 = 1.5$

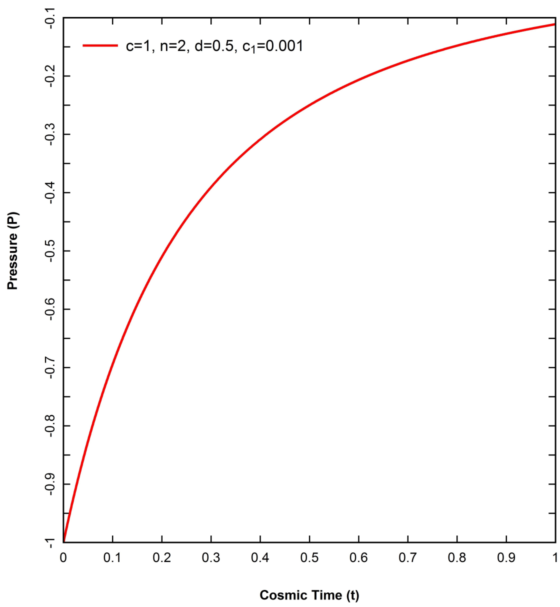

Figure 4. The behavior of energy pressure (p) for $f(R) = R$ against Cosmic Time (t) for the proper choice of constants: $c = 1, n = 2, d = 0.5$ & $c_1 = 0.01$

function of cosmic time (t) (Figure 5 & 8).

6. DISCUSSION & CONCLUSION

The $f(R)$ gravity offers an alternative explanation for the recent cosmic acceleration, by eliminating additional spatial dimensions or introducing exotic dark energy term. Also, different functional forms for $f(R)$ introduces a challenge in constraining the numerous theories of $f(R)$ gravity from a physical perspective.

In this this article we explored the FRW cosmological model in the presence of an interacting field. Specifically, we consider a scalar field that modifies the energy-matter content and influences the dynamics of Universe. We found that the expansion of Universe starts with a steady state and increases gradually. At a specific time, the Universe suddenly exploded and expanded to a large extent, which is consistent with the Big Bang scenario and agreed with recent observational data [1, 2]. It is observed that the Hubble parameter (H) initiates with a fixed value and gradually approaches zero as $t \to \infty$. Consequently, under such conditions, the Universe demon-

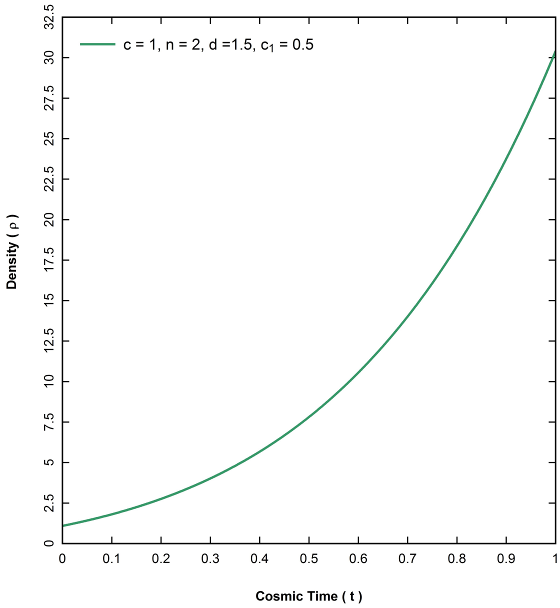

Figure 5. The behavior of energy Density (ρ) for for $f(R) = R$ against Cosmic Time (t) for the proper choice of constants: $c = 1, n = 2, d = 1$ & $c_1 = 0.5$

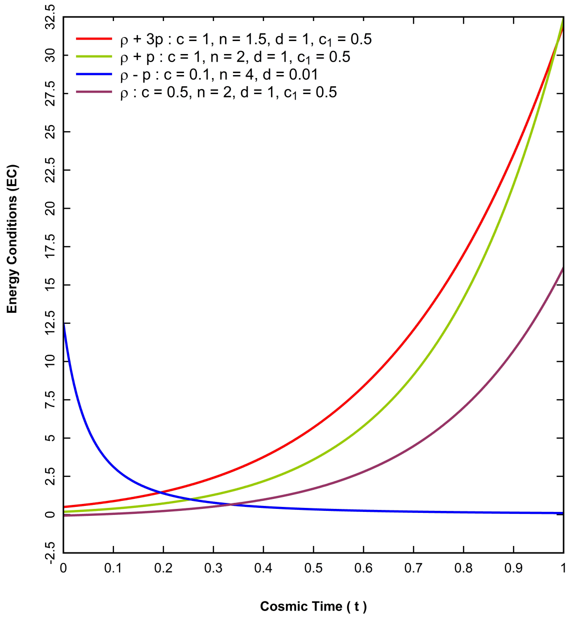

Figure 6. The behavior of Energy Conditions for $f(R) = R$ against Cosmic Time (t) for the proper choice of constants: c, n, d & c_1

strates a tendency towards de-Sitter space in an asymptotic manner. Concurrently, the deceleration parameter (q) remains constant, acquire negative value when $0 < n < 1$ which value signifies the accelerated expansion of the Universe. The energy density remains consistently positive and exhibits an upward trend as cosmic time progresses while the pressure associated with dark energy is increases negatively as time passes.Investigating energy conditions based on the behaviour of matter and energy under gravitational theories allows researchers to study the characteristics, evolution, and future of the Universe. These circumstances are strong tools for exploring basic physics ideas and understanding the intricate dynamics of our cosmos.

In this article, we have provided insights into a particular inquiry by exploring various limitations imposed on general $f(R)$ gravity, focusing on the energy conditions. By utilizing Raychaudhuri's equation and the essential criterion of gravitational attraction, we establish the SEC, WEC, NEC and DEC within the framework of $f(R)$ gravity. Our findings reveal that while these conditions bear resemblance to those derived in the general relativity.Based on our observations, in the present study, we find that the strong energy condition (SEC),

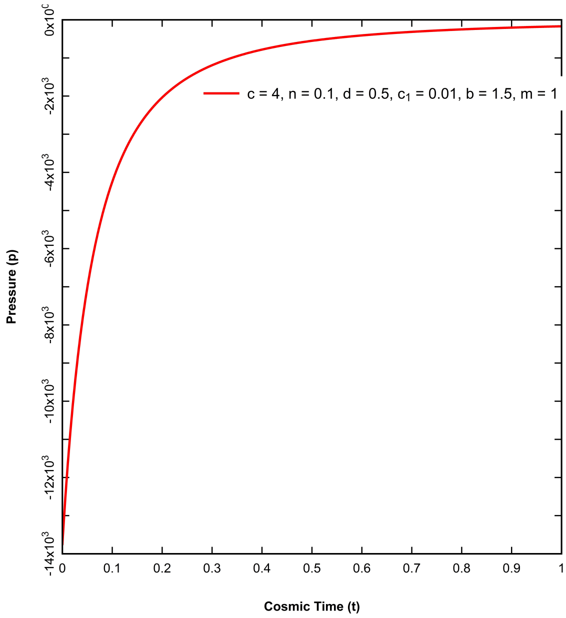

Figure 7. The behavior of energy pressure (p) for $f(R) = R + bR^m$ against Cosmic Time (t) for the proper choice of constants: $c = 4, n = 0.1, d = 0.5, c_1 = 0.01, b = 1.5$ & $m = 1$

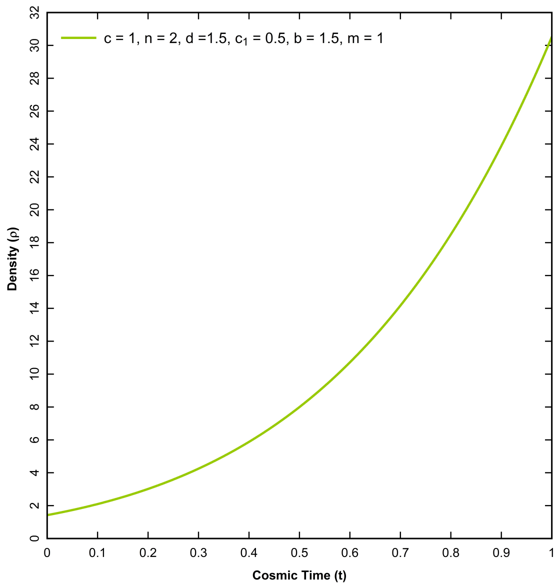

Figure 8. The behavior of energy Density (ρ) for $f(R) = R + bR^m$ against Cosmic Time (t) for the proper choice of constants: $c = 1, n = 2, d = 1.5, c_1 = 0.5, b = 1.5$ & $m = 1$

represented by the inequality $\rho + 3p \geq 0$, is satisfied for both the functional forms of $f(R)$. This result indicates that the model under consideration is non-singular, as the SEC imposes restrictions on the energy density ρ with the pressure p to ensure non-negative values.
Furthermore, our analysis reveals that the weak energy condition (WEC), given by the inequality $\rho \geq 0$, is also satisfied. This condition indicates that the energy density is non-negative, confirming the physical viability of the model. These conclusions can be observed from equations (29) and (35), which provide further evidence for the fulfillment of the WEC in the present study.

Moreover, we investigate the null energy condition (NEC), expressed as $\rho + p \geq 0$. Our findings demonstrate that the NEC is preserved for both functional forms of $f(R)$. This preservation of the NEC is evident from equations (32) and (37), reinforcing the consistency of the model with regard to the NEC. Additionally, our study reveals that the dominant energy condition (DEC), defined as $\rho - p \geq 0$, is also preserved. The preservation of the DEC, as indicated by equations (33) and (39), suggests an accelerated expansion of the Universe. In summary, our observations (from Figure 6 & Figure 9) shows that the present model satisfies the SEC, WEC,

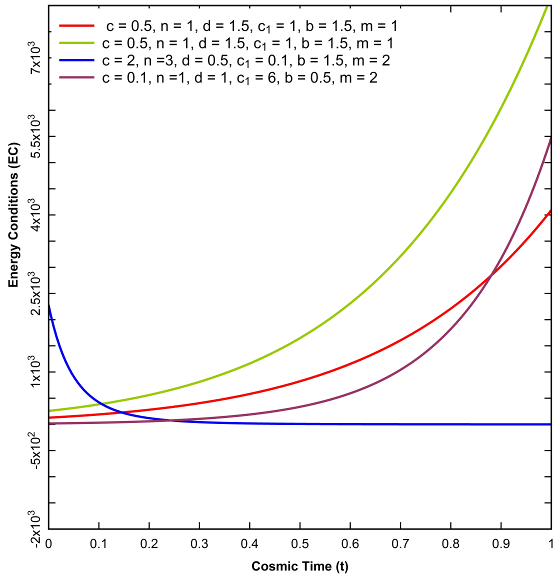

Figure 9. The behavior of Energy Conditions for $f(R) = R + bR^m$ against Cosmic Time (t) for the proper choice of constants: c, n, d, c_1, b & m

NEC and DEC also agreed with results of [47, 48, 49] for WEC, NEC and DEC . This indicates the non-singular nature of the model, ensures non-negative energy densities, confirms the preservation of the NEC, and suggests an accelerated expansion of the Universe. These results contribute to our understanding of the energy conditions in the context of $f(R)$ gravity and provide valuable insights into the behavior of matter and evolution of the Universe within this modified gravitational framework.

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ЕНЕРГЕТИЧНІ УМОВИ З ПОЛЕМ ВЗАЄМОДІЇ В f(R) ГРАВІТАЦІЇ

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У поточному контексті сценарію вкрай важливо дивитися за межі теорії Ейнштейна, яка відкриває двері до спеціально модифікованих теорій гравітації. Отже, дане дослідження присвячене дослідженню різних енергетичних умов, зокрема сильних енергетичних умов (SEC), слабких енергетичних умов (WEC), нульових енергетичних умов (NEC) і домінуючих енергетичних умов (DEC), що відповідають різним функціональним формам f(R) сила тяжіння. Ми досліджували плоскі, ізотропні та однорідні космологічні моделі FLRW, заповнені полем взаємодії, тобто; ідеальна рідина поєднується з безмасовим скалярним полем для різних моделей модифікованої f(R) гравітації, в якій R є скаляром Річчі. Ми спостерігали прискорене розширення Всесвіту, що відповідає останнім даним спостережень.

Ключові слова: *космологічна модель FLRW; гравітація f(R); взаємодіюче поле; закон Хаббла*