

## MULTIPARAMETER CONTROL OF ENERGY CHARACTERISTICS OF WAVEGUIDE-CAVITY RESONATOR-SLOT RADIATORS<sup>†</sup>

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The problem of connecting three electrodynamic volumes with ideally conducting walls through electrically narrow rectilinear connecting slots and a radiating slot is solved by the generalized method of induced magnetomotive forces (MMF). The solution is obtained in an analytical form, taking into account the finite thickness of the walls of the connected volumes. The volumes are an infinite rectangular waveguide excited by a fundamental wave, a rectangular cavity resonator, and a half-space above an infinite plane. The energy characteristics of this system have been comprehensively studied depending on the geometric parameters of the constituent elements of the structure under consideration.

**Keywords:** Radiation slot; Connecting slots; Rectangular waveguide; Cavity resonator; Electromagnetic waves

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Currently, in the antenna-waveguide technology of millimeter and centimeter wavelengths, slotted radiators in flat and spherical surfaces are widely used as feeds for highly directional mirror and lens antennas [1–5], elements of in-phase and scanning antenna arrays [6–19], as well as devices connections of electrodynamic volumes [20–22]. Single slotted radiators are characterized by a significant broadband, which in conditions, for example, of a complex electromagnetic environment, can lead to disruption of the operation of radio electronic systems (RES). In this regard, the problems of analysis, synthesis and control of the band characteristics of slot antennas are of undoubted interest for practice, in particular, from the point of view of ensuring the electromagnetic compatibility of various RES components.

The formation of the required frequency-energy characteristics of slot radiators and coupling holes can be ensured, for example, by using combined (with dipoles, dielectric inserts, etc.) radiators [21, 22 and references therein], or by placing in the supply waveguide channel through cavity resonators, which, in turn, are band-pass and band-stop filters [4, 16, 23]. There are also other constructive solutions to this problem, namely: placement between the radiating slot and the connection slot (slots) the cavity resonator [11, 18], and the presence of two closely spaced slots in the wall of the main waveguide makes it possible to significantly increase the value of the coupling coefficient in a narrow frequency band [14].

In this article, an electrodynamically rigorous mathematical model is constructed and the energy characteristics of the following radiating structure are studied: a system of two transverse slots in a wide wall of an infinite rectangular waveguide - a pass-through cavity resonator - a slot that radiates into a half-space above an infinite ideally conducting plane. On the basis of such structure, a linear or two-dimensional antenna array with new (compared to those known for similar structures) electrodynamic characteristics can be created.

### FORMULATION OF THE PROBLEM AND SOLUTION

The considered waveguide-resonator-slot structure and the designations adopted in the problem are shown in Fig. 1. Three electrodynamic volumes with ideally conducting walls, representing, respectively, an infinite rectangular waveguide with a cross section  $\{a \times b\}$  (index “Wg”), a rectangular resonator with dimensions  $\{a_R \times b_R \times H\}$  (index “R”), and a half-space above an unlimited screen (index “Hs”) are interconnected rectilinear slots  $S_1, S_2, S_3$  cut in infinitely thin common walls.

The geometric dimensions of all slots satisfy the following conditions

$$\frac{d_p}{2L_p} \ll 1, \quad \frac{d_p}{\lambda} \ll 1, \quad p=1,2,3, \quad (1)$$

where  $2L_p$  and  $d_p$  are the length and width of the slots, respectively, and  $\lambda$  is the wavelength in free space. In this case, the equivalent magnetic currents in the slots can be represented as ( $\vec{e}_{s_p}$  are the unit vectors,  $s_p$  and  $\xi_p$  are the local coordinates associated with slots,  $J_{0p}$  are the current amplitudes):

$$\vec{J}_p(s_p) = \vec{e}_{s_p} J_{0p} f_p(s_p) \chi_p(\xi_p). \quad (2)$$

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In this case, the functions  $f_p(s_p)$  must satisfy the boundary conditions  $f_p(\pm L_p) = 0$ , and the functions  $\chi_p(\xi_p)$  must satisfy the conditions on the edges of the slots and the normalization conditions:  $\int_{\xi_p} \chi_p(\xi_p) d\xi_p = 1$ .

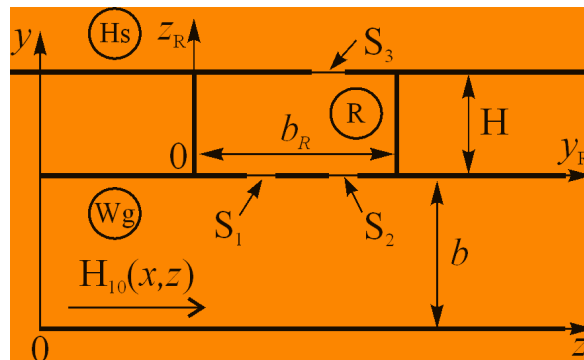


Figure 1. The structure geometry and notations

Let us choose as functional dependences  $f_p(s_p)$  on the longitudinal coordinates of the magnetic currents in the slots the functions obtained as a result of the approximate solution [15] of the integral equation for the current in a single transverse slot (symmetric with respect to the axis  $\{0x\}$ ), excited by a wave of the type  $H_{10}$  and connecting two rectangular waveguides ( $f_{1,2}(s_{1,2})$ ), and for the current in a slot in an infinite screen when a plane electromagnetic wave falls on it, the vector  $\vec{H}$  of which is parallel to the vector  $\vec{e}_{s_3}$  ( $f_3(s_3)$ ):

$$\begin{aligned} f_{1,2}(s_{1,2}) &= \left( \cos ks_{1,2} \cos \frac{\pi}{a} L_{1,2} - \cos kL_{1,2} \cos \frac{\pi}{a} s_{1,2} \right), \\ f_3(s_3) &= (\cos ks_3 - \cos kL_3). \end{aligned} \quad (3)$$

We note that such a choice of the function  $f_3(s_3)$  made it possible to obtain an expression for the external conductivity of the radiating slot in an analytical form.

Using the boundary conditions for the continuity of the tangential components of the magnetic field on the surfaces of slots and following the generalized method of induced MMF for a multi-slot structure [20], we obtain a system of algebraic equations for unknown current amplitudes  $J_{0p}$ :

$$\begin{cases} J_{01} (Y_{11}^{Wg} + Y_{11}^R) + J_{02} (Y_{12}^{Wg} + Y_{12}^R) + J_{03} Y_{13}^R = -\frac{i\omega}{2k} \int_{-L_1}^{L_1} f_1(s_1) H_{0s_1}(s_1) ds_1, \\ J_{02} (Y_{22}^{Wg} + Y_{22}^R) + J_{01} (Y_{21}^{Wg} + Y_{21}^R) + J_{03} Y_{23}^R = -\frac{i\omega}{2k} \int_{-L_2}^{L_2} f_2(s_2) H_{0s_2}(s_2) ds_2, \\ J_{03} (Y_{33}^R + Y_{33}^{Hs}) + J_{01} Y_{31}^R + J_{02} Y_{32}^R = 0. \end{cases} \quad (4)$$

Here

$$Y_{pp}^{Wg,R,Hs} = \frac{1}{2k} \int_{-L_p}^{L_p} f_p(s_p) \left[ \left( \frac{d^2}{ds_p^2} + k^2 \right) \int_{-L_p}^{L_p} f_p(s'_p) G_{s_p}^{Wg,R,Hs}(s_p, s'_p) ds'_p \right] ds_p \quad (5)$$

are the own conductivities of slots;

$$Y_{pq}^{Wg,R,Hs} = \frac{1}{2k} \int_{-L_{p,q}}^{L_{p,q}} f_{p,q}(s_{p,q}) \left[ \left( \frac{d^2}{ds_{p,q}^2} + k^2 \right) \int_{-L_{p,q}}^{L_{p,q}} f_{q,p}(s'_{q,p}) G_{s_{p,q}}^{Wg,R,Hs}(s_{p,q}, s'_{q,p}) ds'_{q,p} \right] ds_{p,q} \quad (6)$$

are the mutual conductivities of slots ( $q = 1, 2, 3$ );  $G_s^{Wg,R,Hs}$  are the  $s$ -components of quasi-one-dimensional ( $|\xi_p - \xi'_p| \approx d_p / 4$ ) Green's functions for the vector potential of the corresponding volumes [20];  $H_{0s_{1,2}}(s_{1,2})$  are the projections of the field of external sources on the axes of the first and second slots;  $\omega$  is the circular frequency;  $k = 2\pi / \lambda$ .

After substituting functions (3) and formulas for  $G_s^{Wg,R,Hs}$  [20] into relations (5), (6), we obtain the following expressions for the own and mutual conductivities of slots ( $p=1,2$ ):

$$\begin{aligned}
 Y_{pp}^{Wg} &= \frac{2\pi}{ab} \sum_{m=1,3,\dots} \sum_{n=0}^{\infty} \frac{\varepsilon_n(k^2 - k_x^2)}{kk_z} e^{-k_z \frac{d_p}{4}} I_{Wg}^2(kL_p) \\
 Y_{12(21)}^{Wg} &= \frac{2\pi}{ab} \sum_{m=1,3,\dots} \sum_{n=0}^{\infty} \frac{\varepsilon_n(k^2 - k_x^2)}{kk_z} e^{-k_z z_0} I_{Wg}(kL_{1(2)}) I_{Wg}(kL_{2(1)}) \\
 Y_{pp}^R &= \frac{4\pi}{a_R b_R} \sum_{m=1,3,\dots} \sum_{n=0}^{\infty} \frac{\varepsilon_n(k^2 - k_{xR}^2)}{kk_{zR}} \coth k_{zR} H \cos k_{yR} y_{0p} \cos k_{yR} \left( y_{0p} + \frac{d_p}{4} \right) I_{R}^2(kL_p) \\
 Y_{12(21)}^R &= \frac{4\pi}{a_R b_R} \sum_{m=1,3,\dots} \sum_{n=0}^{\infty} \frac{\varepsilon_n(k^2 - k_{xR}^2)}{kk_{zR}} \coth k_{zR} H \cos k_{yR} y_{01(02)} \cos k_{yR} \left( y_{02(01)} + \frac{d_{2(1)}}{4} \right) I_R(kL_{1(2)}) I_R(kL_{2(1)}) \\
 Y_{33}^R &= \frac{4\pi}{a_R b_R} \sum_{m=1,3,\dots} \sum_{n=0}^{\infty} \frac{\varepsilon_n(k^2 - k_{xR}^2)}{kk_{zR}} \coth k_{zR} H \cos k_{yR} y_{03} \cos k_{yR} \left( y_{03} + \frac{d_3}{4} \right) I_{R3}^2(kL_3) \\
 Y_{p3(3p)}^R &= \frac{4\pi}{a_R b_R} \sum_{m=1,3,\dots} \sum_{n=0}^{\infty} \frac{\varepsilon_n}{k_{zR} \operatorname{sh} k_{zR} H} \cos k_{yR} y_{0p(3)} \cos k_{yR} \left( y_{03(p)} + \frac{d_{3(p)}}{4} \right) I_R(kL_p) I_{R3}(kL_3) \\
 Y_{33}^{Hs} &= (\operatorname{Si} 4kL_3 - i \operatorname{Cin} 4kL_3) \\
 &\quad - 2 \cos kL_3 \left[ \frac{2(\sin kL_3 - kL_3 \cos kL_3) \left( \ln \frac{16L_3}{2d_3} - \operatorname{Cin} 2kL_3 - i \operatorname{Si} 2kL_3 \right)}{+ \sin 2kL_3 e^{-ikL_3}} \right].
 \end{aligned}$$

Here

$$\begin{aligned}
 I_{Wg(R)}(kL_p) &= 2 \left\{ \frac{k \sin(kL_p) \cos(k_{x(xR)} L_p) - k_{x(xR)} \cos(kL_p) \sin(k_{x(xR)} L_p)}{k^2 - k_{x(xR)}^2} \cos(k_c L_p) \right. \\
 &\quad \left. - \frac{k_c \sin(k_c L_p) \cos(k_{x(xR)} L_p) - k_{x(xR)} \cos(k_c L_p) \sin(k_{x(xR)} L_p)}{k_c^2 - k_{x(xR)}^2} \cos(kL_p) \right\}, \\
 I_{R3}(kL_3) &= 2 \frac{k_{xR} \sin(kL_3) \cos(k_{xR} L_3) - k \cos(kL_3) \sin(k_{xR} L_3)}{k_{xR}}
 \end{aligned}$$

$k_{x(xR)} = \frac{m\pi}{a(a_R)}$ ,  $k_{y(yR)} = \frac{n\pi}{b(b_R)}$ ,  $k_{z(zR)} = \sqrt{k_{x(xR)}^2 + k_{y(yR)}^2 - k^2}$ ,  $k_c = \pi/a$ ,  $\varepsilon_n = 1$  at  $n=0$ ,  $\varepsilon_n = 2$  at  $n \neq 0$ ,  $z_0$  is the distance between the axes of the slots  $S_1$  and  $S_2$ ,  $y_{0p}$  is the position of the  $p$ -th slot axis in the coordinate system associated with the resonator (Fig. 1), Si and Cin are the integral sine and cosine [24].

Solving the system of equations (4), taking into account the fact that for a wave of the type  $H_{10}$  in a rectangular waveguide  $H_{0s_1} = H_0 \cos k_c s_1$ ,  $H_{0s_2} = H_0 \cos k_c s_2 e^{-ik_g z_0}$  ( $H_0$  is the amplitude,  $k_g = \sqrt{k^2 - k_c^2}$  is the propagation constant of the  $H_{10}$ -wave), we find the currents in each of the slots and the reflection and transmission field coefficients  $S_{11}$  and  $S_{12}$ , as well as the power radiating coefficient  $|S_\Sigma|^2$ :

$$S_{11} = \frac{2\pi i k_c k_g}{bk^3} \left[ \tilde{J}_{01} F(kL_1) + e^{-ik_g z_0} \tilde{J}_{02} F(kL_2) \right] e^{2ik_g z}, \tag{7}$$

$$S_{12} = 1 + \frac{2\pi i k_c k_g}{bk^3} \left[ \tilde{J}_{01} F(kL_1) + e^{ik_g z_0} \tilde{J}_{02} F(kL_2) \right], \tag{8}$$

$$|S_\Sigma|^2 = 1 - |S_{11}|^2 - |S_{12}|^2. \tag{9}$$

In formulas (7)–(9)  $\tilde{J}_{0p} = J_{0p} / \left( -\frac{i\omega}{2k^2} H_0 \right)$  are the normalized amplitudes of currents in slots,

$$F(kL_p) = 2 \cos k_c L_p \frac{\sin kL_p \cos k_c L_p - (k_c / k) \cos kL_p \sin k_c L_p}{1 - (k_c / k)^2} - \cos kL_p \frac{\sin 2k_c L_p + 2k_c L_p}{(2k_c / k)}.$$

The normalized radiation pattern in the vector  $\vec{H}$  plane for the structure under consideration has the form ( $\theta$  is the angle measured from the axis  $\{0x\}$ , Fig. 1):

$$\bar{F}_H(\theta) = \frac{\sin kL_3 \cos(kL_3 \cos \theta) - \cos kL_3 [\sin(kL_3 \cos \theta) / \cos \theta]}{\sin \theta (\sin kL_3 - kL_3 \cos kL_3)}. \tag{10}$$

Accounting for the thickness  $h_p$  of metal walls, in which slots are located, can be made according to [20] by the following substitutions  $d_p \rightarrow d_{ep}(h_p)$  under conditions  $(h_p / \lambda) \ll 1$ :

$$d_{ep}(h_p) \cong d_p e^{\frac{\pi h_p}{2d_p}}, \tag{11}$$

where  $d_{ep}(h_p)$  is the “equivalent” width of the  $p$ -th slot.

### NUMERICAL RESULTS

On Figs. 2-5 are plots of dependences of the radiation coefficient  $|S_\Sigma|^2(\lambda)$  on the wavelength for the following parameters:  $a = 23.0$  mm,  $b = 10.0$  mm,  $H = a_R / 2$ ,  $2L_p = 14.0$  mm,  $d_p = h_p = 1.0$  mm. As can be seen from Fig. 2, the presence in the structure under study of a cavity resonator with one or two coupling slots makes it possible to form different frequency-energy characteristics of the entire system as a whole in comparison with a single transverse slot (1 slot) in the broad wall of the waveguide. With a certain mutual arrangement of slots relative to each other,  $|S_\Sigma|^2$  can reach a value of  $\sim 0.9$  in a narrow band of wavelengths or a value of  $\sim 0.5$  in a relatively large part of the range of the fundamental wave of the waveguide. A change in the transverse dimensions of the resonator ( $a_R, b_R$ ) has practically no effect on the position of the maximum  $|S_\Sigma|^2$  (Fig. 3). Moving the position of the radiating slot  $S_3$  within the resonator wall leads to a change in both the maximum (Fig. 4) and minimum (Fig. 5) values  $|S_\Sigma|^2$  at a certain wavelength, depending on the geometric dimensions of the slot and the thickness of the waveguide and resonator walls. Changing lengths of radiating and coupling slots at their fixed positions in the waveguide and resonator walls gives additional possibilities for the formation of characteristics required in the case of waveguide-resonator structures considered here (Fig. 6,  $a = 23.0$  mm,  $b = 10.0$  mm,  $a_R = 20.0$  mm,  $b_R = 10.0$  mm,  $H = a_R / 2$ ,  $d_p = h_p = 1.0$  mm,  $y_{01} = b_R / 8$ ,  $y_{02,03} = b_R / 4$ ).

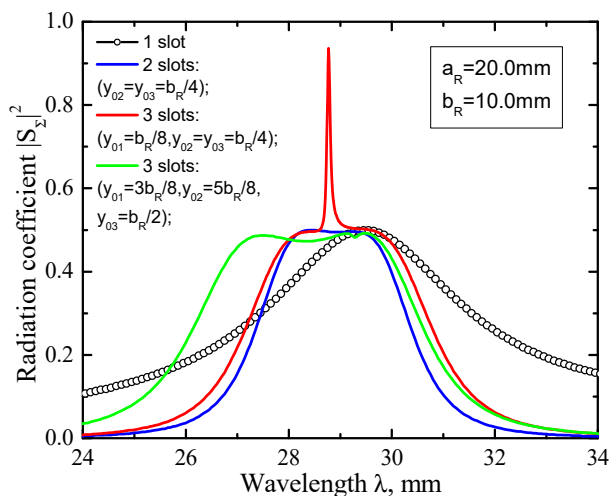


Figure 2. Wavelength dependence  $|S_\Sigma|^2(\lambda)$  for structures with one, two, and three slots

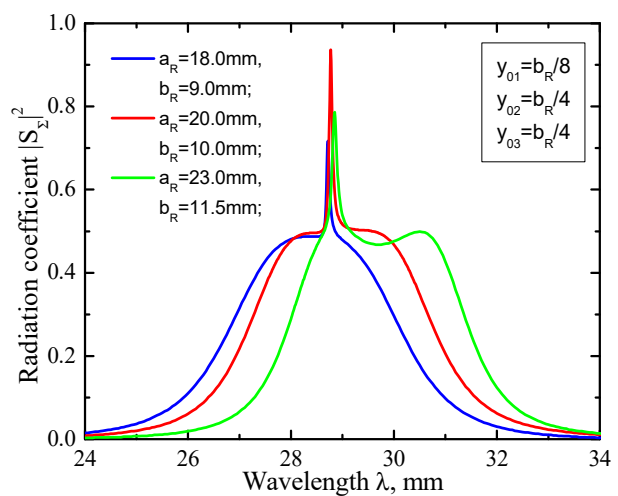
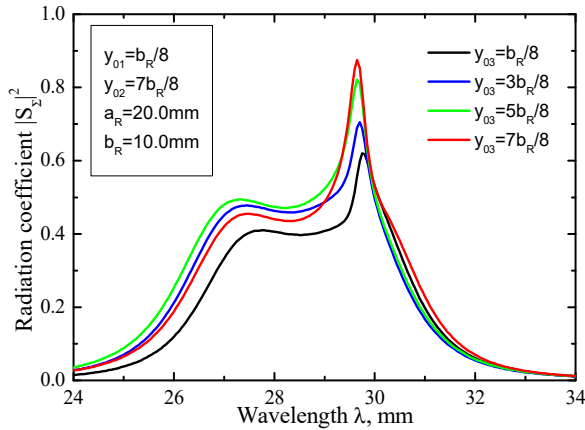
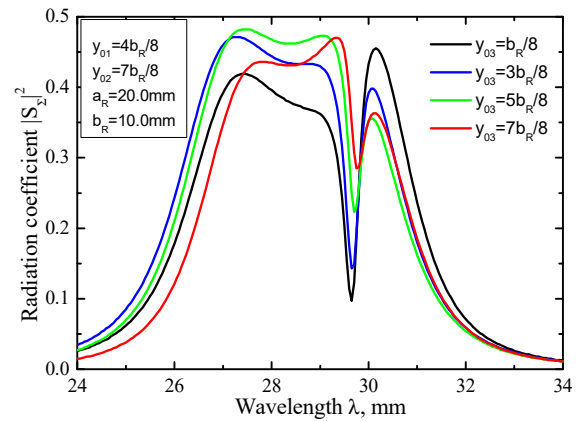


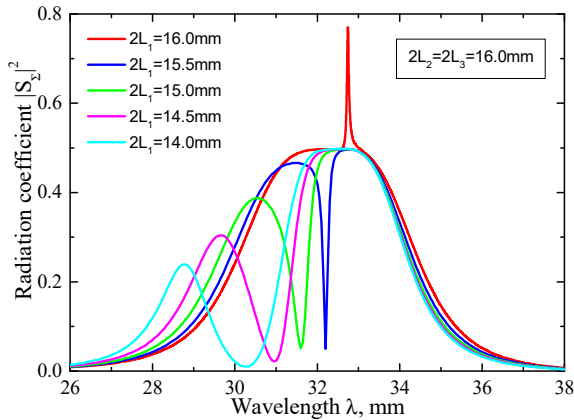
Figure 3. Wavelength dependence  $|S_\Sigma|^2(\lambda)$  for a structure with three slots for various resonator sizes



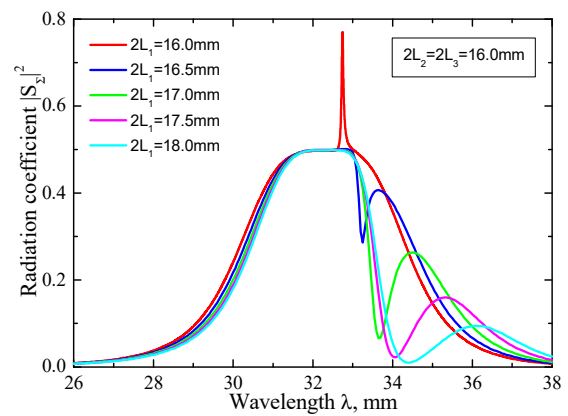
**Figure 4.** Wavelength dependence  $|S_z|^2(\lambda)$  for a structure with three slots at different positions of the radiating slot in the case of spaced coupling slots



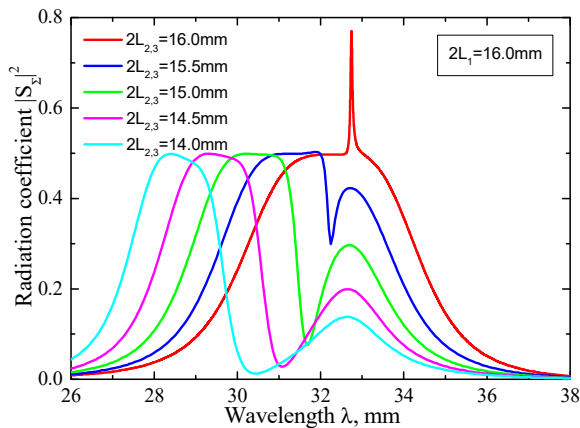
**Figure 5.** Wavelength dependence  $|S_z|^2(\lambda)$  for a structure with three slots at different positions of the radiating slot in the case of closely spaced coupling slots



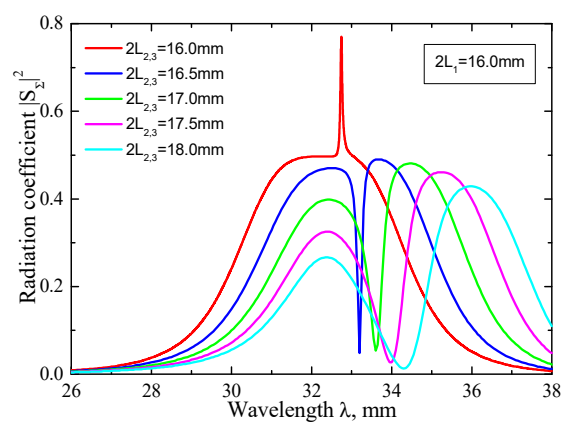
a)



b)



c)



d)

**Figure 6.** Wavelength dependence  $|S_z|^2(\lambda)$  for a structure with three slots at different lengths of the slots

### CONCLUSIONS

It is known from the classical theory of slot radiators and coupling holes [20] that a single slot in the side wall of a rectangular waveguide cannot radiate (transmit) more than half of the input power. An increase in the value of the radiation (transmission) coefficient can be carried out by placing other resonant elements in the waveguide, for example, vibrators (dipoles) [21] or dielectric inserts [22]. As shown by the calculations performed in the article, the placement of a rectangular resonator between the coupling slots and the radiating slot also leads to a significant increase the radiation coefficient value (up to  $\sim 0.9$ ) in a narrow wavelength (frequency) band. The multi-element waveguide-resonator-slot radiators studied in the article can be useful both in the development of new antenna transceiver systems of centimeter and millimeter wavelengths (including slotted antenna arrays), and for the modernization of existing ones based on the existing element base by means of insignificant design changes.

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**БАГАТОПАРАМЕТРИЧНЕ КЕРУВАННЯ ЕНЕРГЕТИЧНИМИ ХАРАКТЕРИСТИКАМИ  
ХВИЛЕВОДНО-РЕЗОНАТОРНО-ЩІЛИННИХ ВИПРОМІНЮВАЧІВ**

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Задача з'єднання трьох електродинамічних об'ємів з ідеально провідними стінками через електрично вузькі прямолінійні сполучні щілини та випромінювальну щілину вирішується узагальненим методом наведених магніторухливих сил (МРС). Розв'язок отримано в аналітичній формі з урахуванням кінцевої товщини стінок зв'язаних об'ємів. Об'єми являють собою нескінченний прямокутний хвилевід, який збуджено основною хвилею, резонатор прямокутної порожнини та півпростір над нескінченною площиною. Всебічно вивчено енергетичні характеристики цієї системи в залежності від геометричних параметрів складових елементів конструкції, що розглядається.

**Ключові слова:** випромінювальна щілина; сполучні щілини; прямокутний хвилевід; об'ємний резонатор; електромагнітні хвилі