The optical output of a semiconductor laser can fluctuate chaotically by modulating its direct current in limited conditions of the modulated current signal parameters in terms of modulation frequency and modulation index. In this work, single, double, and chaotic pulses of an InGaAsP laser with direct current modulation, are numerically presented through a bifurcation diagram. Numerically, the unidirectional optical coupling system realizes chaotic synchronization between two identical InGaAsP lasers with direct current modulation, as the transmitter/receiver configuration. The transmission time for transmitting light from the transmitted laser to the received laser is essential for controlling the quality of chaos synchronization. The transmission time applies on the order of nanoseconds. Chaos synchronization quality is estimated by a correlation plot and calculated by the cross-correlation coefficient. This study observed the best synchronization quality (complete chaos synchronization) when the two lasers are identical. On the other hand, the chaotic synchronization between two non-identical InGaAsP lasers was investigated. In this case, complete chaos synchronization is not found, and the quality of chaotic synchronization was observed to decrease as the mismatch between the parameters of the two lasers increased.

**Keywords:** InGaAsP laser; Chaos synchronization; Direct current modulation; Optical coupling; Cross-correlation coefficient

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1. **INTRODUCTION**

After the great success in the late last century in realizing chaos synchronization [1], chaos synchronization has been extensively investigated in various nonlinear dynamical systems [2-7]. Chaos synchronization represents the key to the success of chaotic communication systems, as the quality of the recovered message depends on the quality of synchronization between the transmitter and the receiver in chaotic communication systems [8-14]. Many studies have focused on the synchronization in chaotic systems of semiconductor lasers (SLs) with optical feedback [8,9,14-17], optical injection [10,11,18], and optoelectronic feedback [12,13,19,20]. In contrast, a few investigations have been published on the chaos synchronization and communication of SLs with direct current modulation [21,22]. The scant investigations of SLs with direct current modulation because the SLs have chaotic oscillations only under limited conditions. Also, it may be because the oscillation frequency of relaxation determines the modulation in direct current modulation, where the modulation efficiency decreases, and the intensity modulation becomes difficult as the modulation frequency is larger than the oscillation frequency of relaxation [23].

A solitary semiconductor laser (SL) is a very stable system. SL is easy to be destabilized by adding an extra degree of freedom when the SL is subjected to an external perturbation. Indeed, perturbed lasers show a chaotic oscillation. SL can be destabilized by the direct modulation of its current [24-28]. The direct current modulation exhibits doubled-period and chaos in some range of the modulation frequency and the modulation current [29,30]. The direct current modulation provides a third degree of freedom, which is responsible for making the nonlinear system non-autonomous [31].

In this paper, we numerically investigate the synchronization of a chaotic optical system consisting of two InGaAsP lasers with direct current modulation, which are unidirectionally coupled. Chaos synchronization can occur between two InGaAsP lasers (one the transmitter and the other the receiver) when a small proportion of the transmitter laser output power is optically coupled with the receiver laser. This synchronization method has been numerically investigated by Jones et al. They showed the synchronization of their optical model of self-pulsed semiconductor lasers with direct current modulation can be achieved by optical coupling [32]. In this study, the chaotic region of InGaAsP laser output with modulation index is shown by a bifurcation diagram. More importantly, the transmission time between the transmitter and the receiver was taken into account. The transmission time is executed on the order of nanoseconds. Also, the quality of the synchronization in this chaotic optical system is examined, where the cross-correlation coefficient is calculated between the output powers of the two identical and non-identical lasers.

2. **SYNCHRONIZATION MODEL**

The schematic diagram for the unidirectional optical coupling system, as transmitter/receiver configuration, is given in Fig. 1. The two semiconductor lasers are modulated with a sinusoidal current of GHz frequency. An optical isolator (OI) is ordinarily utilized to achieve one-way optical coupling between the two lasers. This optical coupling system can be modeled via the normalized rate equation of the photon density, S, and the carrier density, N, which is based on
Agrawal’s model of the InGaAsP laser with direct current modulation [30]. The rate equations of the optical coupling system are given by:

\[
\begin{align*}
\frac{dS_t}{dt} &= \frac{1}{\tau_{ph}} \left( \frac{N_s - \delta}{1 - \delta} (1 - eS_t)S_t - S_t + \beta N_t \right) S_t, \\
\frac{dS_r}{dt} &= \frac{1}{\tau_{ph}} \left( \frac{N_r - \delta}{1 - \delta} (1 - eS_r)S_r - S_r + \beta N_t \right) S_r + kS_t(t - T), \\
\frac{dN_{tr}}{dt} &= \frac{1}{\tau_c} \left( \frac{1}{\tau_c} - N_{tr} - \frac{\delta}{1 - \delta} S_r \right).
\end{align*}
\]

Where \(\tau_c\) and \(\tau_{ph}\) are the electron and photon lifetimes, \(\delta = n_{th}/n_0\) where \(n_{th}\) and \(n_0\) are the threshold carrier density and the carrier density for transparency respectively, \(\beta\) is the spontaneous emission factor, and \(\epsilon\) is the nonlinear gain reduction factor.

Figure 1. Schematic diagram for unidirectionally optical coupling system.

The modulating signal for the two lasers is given by [30]:

\[
I = I_{th} + I_m \sin(2\pi f_m t)
\]

Where \(I_0\) is the bias current, and \(m, f_m\) are the index and frequency of modulation, respectively. Here, \(t\) and \(r\) of the sub-position in the equations denote the transmitter and the receiver respectively. Eq.2 is written for the receiver only to effect synchronization between the two lasers by the term of delayed optical coupling from the transmitter. The coupling term is denoted in Eq.2 as \(kS_t(t - T)\), where \(S_t(t - T)\) is the photon density of the transmitter laser, which is optically injected into the receiver laser after the time-delayed for the transmission, \(T\) is the transmission time for transmitting light from the transmitted laser to the received laser, and \(k\) is the optical coupling given as [32]:

\[
k = x \frac{(1 - R)}{\tau_a \sqrt{R}}
\]

Where \(x\) is the portion of the transmitter optical power coupled into the receiver, is referred to as the optical coupling level, the mirror reflectivity of the receiver laser facing the transmitter laser, \(R = 0.32\), and the round-trip time inside the laser cavity, \(\tau_a = 2/\nu_g\), where the cavity length of InGaAsP laser, \(\nu = \ln(R) / (1/\tau_{ph} \nu_g - \alpha)\), the group velocity, \(\nu_g = c/n_e\), \(c\) is the speed of light in vacuum, the effective refractive index of InGaAsP laser, \(n_e = 4\), and for simplicity the losses factor of InGaAsP laser, \(\alpha = 0\) [23]. Matlab program is used to solve the model of the optical coupling system as a delay differential equations. The parameter values of InGaAsP lasers used in the numerical simulations of the chaotic optical coupling system are listed in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_{ph})</td>
<td>6 ps</td>
</tr>
<tr>
<td>(\tau_c)</td>
<td>3 ns</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.692</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>(10^{-4})</td>
</tr>
<tr>
<td>(\beta)</td>
<td>(5 \times 10^{-5})</td>
</tr>
<tr>
<td>(m)</td>
<td>0.57</td>
</tr>
<tr>
<td>(I_m)</td>
<td>(m I_0)</td>
</tr>
<tr>
<td>(I_b)</td>
<td>1.5(I_0)</td>
</tr>
<tr>
<td>(f_m)</td>
<td>0.8 GHz</td>
</tr>
</tbody>
</table>

3. RESULTS AND DISCUSSION

In the beginning, attention is paid to the generation of chaos in the InGaAsP laser. By the modulation frequency of the laser close to the relaxation oscillation frequency with large values of the modulation index, the dynamics of the laser becomes chaotic, as shown in the bifurcation diagram in Fig.2.
Figure 2, shows the bifurcation diagram of the peak series of laser pulses against the modulation index for a modulation frequency fixed at 0.8 GHz. It indicates the regions of single, doubled, and chaotic pulses which have been numerically published by Agrawal in Ref. 30.

To effect chaos synchronization between two InGaAsP lasers, the receiver laser is coupled to the transmitter laser by it is receiving a portion of the transmitter output power with the transmission time $T$. $T$ is executed in the order of nanoseconds (similar to the order of the modulation time $\tau_m = 1/f_m = 1.25$ ns). The time series of the chaotic outputs of the transmitter and the receiver lasers are plotted to highlight how well the chaotic outputs are synchronized, and the output power of the transmitter laser and the output power of the receiver laser are used to estimate the quality of chaos synchronization by the correlation plot.

Figure 2. Bifurcation diagram of the laser output power agains t modulation index. The laser bias current is $1.5I_{th}$, and the modulation frequency is 0.8 GHz.

The chaotic time series and the correlation plot of the output powers for the transmitter and the receiver lasers at the same modulation conditions for both lasers $I_{b(t,r)} = 1.5I_{th}$, $m_{t,r} = 0.57$, $f_m = 0.8$ GHz, and the optical coupling conditions are $\chi = 1\%$, and $T$ varies in the order of nanoseconds, are shown in Fig. 3, and Fig. 4, respectively.

Figure 3. Chaotic fluctuations time series of transmitter and receiver for different transmission times (a) $1\tau_m$ ns, (b) $2\tau_m$ ns, (c) $2\tau_m$ ns, (d) $4\tau_m$ ns, (e) $5\tau_m$ ns, (f) $6\tau_m$ ns, (g) $7\tau_m$ ns, (h) $8\tau_m$ ns, (i) $9\tau_m$ ns, (j) $10\tau_m$ ns, (k) $11\tau_m$ ns, (l) $12\tau_m$ ns, and (m) $13.4\tau_m$ ns. The optical coupling level is fixed at 1%. All modulation conditions and the parameters values of the two lasers are identical

In Fig. 3(a–i) when the transmission time is an odd integer number of the modulation time, the receiver's chaotic outputs are poor copies of the transmitter's chaotic outputs, as there is a large shifting between the chaotic fluctuations time series of the transmitter and receiver. This explains the poor quality estimation of synchronization as shown by the correlation plots in Fig. 4(a,c,e,g,i). While, the receiver's chaotic outputs are good copies of the transmitter's chaotic outputs. This is because the very small shifting between the chaotic fluctuations time series of the transmitter and receiver when the transmission time is an even integer number of the modulation time as shown in Fig. 3(a–i). So, the estimation of the good quality of synchronization as
shown by the correlation plots in Fig. 4(b,d,f,h). When the transmission time is increased, the shifting between the chaotic fluctuations time series of the transmitter and the receiver decreases, as shown in Fig. 3(j–l). Therefore, the receiver’s chaotic outputs are excellent copies of the transmitter’s chaotic outputs, and this explains the estimation of the excellent quality of synchronization as shown by the correlation plots in Fig. 4(j–l). The best case for synchronization is observed when the transmission time reaches $13.4\tau_{m}$ as shown in Fig. 3(m), where the receiver's chaotic output is an exact copy of the transmitter's chaotic output. Therefore, the distribution of all data points is a 45° diagonal line as shown by the correlation plot in Fig. 4(m), which indicates the receiver is completely driven by the transmitter.

**Figure 4.** Transmitter output power against receiver output power. All modulation conditions and the parameters values of the two lasers are the same as in Fig. 3

To calculate the chaos synchronization quality between the two lasers, a method similar to that used in Ref. 5. A cross-correlation coefficient, $C$, is calculated between the transmitter and the receiver output powers, which, for better synchronization, would give a large value of $|C|$ close to unity sometimes reaching unity. The cross-correlation coefficient against the transmission time for the case in Fig. 3, and Fig. 4, and for more optical coupling up to 20%, is shown in Fig. 4.

**Figure 5.** The cross-correlation coefficient against transmission time for optical coupling levels extends from 1% to 20%. Here, the transmission time on the order of nanoseconds. All modulation conditions and the parameters values of the two lasers are identical.

In Fig. 5, before the best chaos synchronization quality ($T = 13.4\tau_{m}$), the chaos synchronization quality decreases when the optical coupling level increases. Importantly, it can be seen that the chaos synchronization quality for each optical coupling level has a somewhat periodic behavior as a function of the transmission time. This behavior, in turn leads to synchronization quality being equal for each optical coupling level especially when all transmission times are an...
even integer of modulation time and also at some transmission times are an odd integer of modulation time (see the intersection points of the curves in the figure). Although the model is not described in terms of the electric field and phase of the two lasers, this behavior of the chaos synchronization quality and the intersection points of the curves are a consequence of the optical coherence nature between the two lasers, which is achieved because the transmission time is on the same order as the modulation time (in other words, the same order as the time series of the output oscillations of the two lasers). The intersection points of the curves are not observed when the transmission time is applied on the order of picoseconds (no the same order as the modulation time) as shown in Fig. 6, and this confirms that the intersection points in Fig. 5 are attributable to the optical coherence between the chaotic output of two lasers.

Moreover, for all-optical coupling levels, the best synchronization quality can be seen in Fig. 5 when the transmission time is equal to $T = 13.4\tau_m$ and beyond, where the cross-correlation coefficient is equal to $C = 1$. This confirms the estimation of the chaos synchronization quality of the correlation plot in Fig. 4(m). In this situation of synchronization quality is called complete synchronization.

In Fig. 6 for each optical coupling level and the transmission time is very small about (1ps-10ps), complete chaos synchronization is observed, where the cross-correlation coefficient is about $C = 1$. This result is similar to when the transmission time is neglected. It is not included here; the optical coupling system has been its carry out without taking into account the transmission time and applied coupling levels are the same as in Fig. 6. Here, the model of an optical coupling system is solved as ordinary differential equations.

Now, the synchronization between the two lasers for non-identical parameter values will be verified by changing the values of some parameters of one laser, such as changing the bias current and the modulation index, which are the two most experimentally changeable parameters. Figure 7, shows the chaotic time series and the correlation plot of the output powers for the transmitter and the receiver when 1% of the optical output of the transmitter is coupled to the receiver with a transmission time of $T = 13.4\tau_m$ for three different receiver bias currents. The transmitter bias current is fixed at $I_b(t) = 1.5I_{th}$, and the other modulation conditions for both lasers are $m_t = 0.57$, $f_m = 0.8$ GHz.

In Fig. 7(b-d) the small change in the bias current affects the relaxation oscillation frequency of the receiver laser which determines the modulation frequency of the laser subject to the direct current modulation, resulting in a frequency mismatch between the two lasers. Therefore, when the receiver's bias current increases and at the same time the transmitter’s bias current is kept constant, the receiver's chaotic outputs become poor copies of the transmitter's chaotic
outputs. This deteriorates the estimation of synchronization quality as shown by the correlation plots in Fig. 7(b1-d1). Figure 8, shows the chaotic time series and the correlation plot of the output powers for the transmitter and the receiver when 1% of the optical output of the transmitter is coupled to the receiver with a transmission time of $T = 13.4\tau_m$ for three different modulation indexes of the receiver. The modulation index of the transmitter is fixed at $m_t = 0.57$, and the other modulation conditions for both lasers are $I_{b(t,r)} = 1.5I_{th}$, and $f_m = 0.8$ GHz.

In Fig. 8(b-d) a slight change in the modulation index affects the amplitude of chaotic output with the keep the frequency of the receiver laser, resulting in a mismatch of the chaotic output amplitudes between the two lasers. Therefore, the increases of receiver's modulation index and at the same time the transmitter's modulation index is kept constant, the chaotic outputs of the receiver become poor copies of the transmitter's chaotic outputs. This deteriorates the estimation of synchronization quality as shown by the correlation plots in Fig. 8 (b1-d1). The cross-correlation coefficient against non-identical parameters for the case in Fig. 7, and Fig. 8, and for more values of non-identical parameters and optical coupling level up to 20%, are shown in Fig. 9.

From Figs. 9(a), and (b), for each optical coupling level, the chaos synchronization quality is calculated to be constant for each value of non-identical parameters. This is because the coupling between the two lasers was done optically (see Eq.2), and this does not affect the electrical non-identical parameters (bias current and modulation index, see Eq.4). Also, it’s evident, since the modulation frequency is not as sensitive to the modulation index as to the bias current, so the drop in synchronization in the case of mismatching modulation indexes is seen as less than in the case of mismatching bias currents between the two lasers.

4. CONCLUSION

Chaos synchronization is numerically investigated using two InGaAsP lasers with direct current modulation. The cross-correlation coefficient between the transmitter's output powers and the receiver's output powers is calculated when a few percent of the optical power of the transmitter is optically coupled to the receiver. The transmission time played an important role in the chaos synchronization quality. The transmission time is equal to $T = 13.4\tau_m$ that achieves the best quality of the chaos synchronization, complete synchronization, as the synchronization quality is calculated by the cross-correlation coefficient which is equal to $C = 1$. On the other hand, the quality of synchronization between two non-
identical lasers is investigated. Here, the quality of synchronization decreases as the mismatch between the parameters of the two lasers increases.

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**СИНХРОНИЗАЦИЯ ХАОСУ В InGaAsP ЛАЗЕРАХ**

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Оптичний вихідний сигнал напівпровідникового лазера може хаотично коливатись шляхом модуляції його постійного струму в обмежених умовах параметрів сигналу модульованого струму з точки зору частоти модуляції та індексу модуляції. У даній роботі коефіцієнти швидкість імпульсів лазера на InGaAsP з модуляцією постійного струму численно представлені через біфуркаційну діаграму. Чисельно, система одностороннього оптичного сполучення реалізує хаотичну синхронізацію між двома ідентичними лазерами InGaAsP з модуляцією постійного струму, як конфігурація передача/приймача. Час передачі світла від лазера є важливим для контролю якості синхронізації хаосу. У цьому випадку періодична хаотична синхронізація не виявлена, і спостерігалося зниження якості хаотичної синхронізації зі збільшенням невідповідності між параметрами двох лазерів.

**Ключові слова:** InGaAsP лазер: синхронізація хаосу; модуляція постійного струму; оптичний зв’язок; коефіцієнт крос-кореляції