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NUCLEAR ENERGY LEVELS SCHEME OF ^{46}Cr USING FPD6, FPY, AND KB3G INTERACTIONS†

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The ^{46}Cr isotope nuclear energy levels were studied using low-level FP-LS shell inside the shell model calculations**.** Nuclear energy levels have been calculated using FPD6, KB3G, and FPY interactions in the fp-shell model space and F742 and F7MBZ in the f7/2 model space. The results are compared to one another and to the experimental data that is already accessible and specific outcomes are clearly in agreement. In addition to having a strong arrangement in the reproduced values of the energy levels scheme, the used model space interactions are the two-body matrix element in the fp-shell model space that is best fitted. Particularly below 3 MeV, the general estimation of the replicated data is good. The wave vectors and analysis are modeled in diagrammatic notation, and all inscriptions are given in this style. Utilizing the oscillator's potential, a single particle vector is built, using ^{40}Ca as the core of the fp-shell and f7/2 model space. Results are obtained for all tested nuclei using the OXFORD BUENOS AIRES SHELL (OXBASH) model code.
Keywords: *Nuclear energy levels; FP Shell; FPY; Diagrammatic notation; FPD6; KB3G*
PACS: 21.10.-k, 21.60.-n, 21.60.Cs

1. INTRODUCTION

Many studies have been conducted to comprehend nuclear characteristics and internal composition. Because nuclei are so complicated, no comprehensive theory explains nuclear actions, characteristics, and structures [1]. The shell theory has many advantages and characteristics, such as model independence and applied physical N-N potential, in addition to the conventional Hamiltonian related to various types of eigenvectors and for a wide range of nuclei. The shell theory remains valid because it provides the primary theoretical techniques for achieving all measurable nuclei [1]. To obtain effective N-N matrix elements, excitation energies, binding energies, and spectroscopic factors were computed in the LS shell (1f7/2 1f5/2 2p3/2 2p1/2) [2]. Interactions between PN prompted the existence of an orbital distance at N = 32 in isotope-rich neutrons localized in the nearby magic nucleus ^{48}Ca [3]. Inspections of filled pf-LS shell of A=48 nuclei were conducted [4], and Kuo-Brown (KB) [10] to KB1 and KB3G were changed. The isobaric sequences A = 50, A = 51, and A = 52 were investigated [5] using KB3G, FPD6, and KB3G [6]. The shell theory created a critical technique for carrying out such research. Realistic potentials are established in this theory, and the basis vectors are indicated by precise quantum numbers of parity ($\boldsymbol{\pi}$), angular momentum (J), and isospin (T) [7]. Many studies [8] have been done to identify the distribution of Eigen functions and build the framework of the shell model [9].

The nuclear shell theory has been considered a significant theory for understanding nuclear structure [10]. Because of the extreme single-particle motion in spherical symmetry, only including powerful spin-orbit factors allowed the rethink of a wide variety of findings for isotopes near nuclear magic numbers [11]. Calculations were performed in model space of a complete fp-LS shell containing 1f7/2, 1f5/2, 2p3/2, 2p1/2 subshells, and ^{40}Ca as a core. There is no limit to the number of particles that can be excited. A comparison of results with that of his results would thus shed light on the role of intruder g9/2 orbital, appropriate choice of core, and the effect of truncation on the particles to be excited, in addition to testing the suitability of GXPF1A interaction in explaining the experimental data [12]. Within shell model computations, nuclear energy levels, total angular momenta, and even-even parity for nucleons present outside closed and no core for ^{46}Cr, which filled fp-shell (1f7/2, 1f5/2, 2p3/2, 2p1/2) were interesting. The nuclear energy spectrum of ^{46}Cr was calculated using four interactions. The associations of FPD6, FPY, F742, F7MBZ, and KB3G are compared to each other to access experimental data. Shell model calculations were interesting for (^{42}Ca, ^{44}Ca, ^{46}Ca, and ^{48}Ca), which occupied fp-shell (1f7/2, 1f5/2, 2p3/2, and 2p1/2). Calculating the nuclear energy spectra of ^{42}Ca, ^{44}Ca, ^{46}Ca, and ^{48}Ca involved four interactions. The outcomes of the interactions between the FPD6, GXPF1, and KB3G are contrasted with one another and with the existing experimental data. In order to produce model space wave vectors and simultaneously obtain the similar model space effective interaction that was chosen for this investigation, the code OXBASH was used [13].

Nuclear energy levels in ^{44}Sc [14] and ^{44}Ca [15] were studied through a nuclear shell model considering ^{40}Ca as an inert core, adopting FPD6, HW, and FPY as model space effective interactions, and comparing the reproduced data with experimental data. The code OXBASH was used to create model space wave vectors while also receiving the comparable model space effective interaction chosen for this research.

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A model space factor has been constructed using F7MBZ that considers the single-particle potential (harmonic oscillator) and elastic magnetic electron scattering form factors for ^{41}Ca with ^{40}Ca as the inert core. The entire theory is then examined using the nuclear shell theory [16] and calculating Ca quadrupole moments (41, 43, 45, and 47) using the shell model [17].

The theoretical level schemes of the selected states of each nucleus that we use for our calculation in the FP-space model for different effective interactions GX1A, KB3G, FPD6, and GX1A often include the spins and association energies of the many levels that have been experimentally identified in ^{52}Ca. Spin spectra of $^{52-54}$Ca and $^{56-58}$Ca are produced for effective interactions [18]. According to the influence of certain physical characteristics, such as the electromagnetic properties effects, such as elastic longitudinal form factors, electric quadrupole moments, and magnetic dipole moments, the nuclear structure of specific cobalt (Co) isotopes with mass number A=56-60 has been examined. By adopting the single-particle wave functions of the harmonic oscillator, calculations involving GXFP1 interaction are presented using the fp model space [19].

This work calculates nuclear energy levels in Cr isotopes using the inert core Ca, the fp shell FPD6, KB3G, and FBY, the 1f7/2 subshell orbit FZMBZ, and F742 [20].

2. THEORY

2.1. Interacting Particles in One and Two Active Orbits

The wave function of two particles may be expressed as a product of a spin and an isospin-dependent portion [21]:

$$\Phi JMTTz\ (1,2) = \Phi JM(j(1)j(2))\Theta TTz(t(1)t(2)). \tag{1}$$

where $j + j = J$, and $t + t = T$ with $T = 0$ or 1 since t = 1/2.
The spin component of Eq. (1) now has a diagrammatic notation, which may be written as [21]:

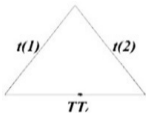
$$\Phi_J(j(1)j(2)) = \sum \langle jmjm'|JM\rangle \phi_{jm}(1)\phi_{jm'}(2) \equiv \tag{2}$$

$\Phi_{jm}(1)$ and $\Phi_{jm'}(2)$ are the single-particle states for particles 1 and 2 with their angular momenta j coupled to a total J. The coupling yields:

$$P_{12}\Phi_{JM}(j(1)j(2)) = (-1)^{J-2j}\Phi_{JM}(j(1)j(2)) = -(-1)^{J}\Phi_{JM}(j(1)j(2)) \tag{3}$$

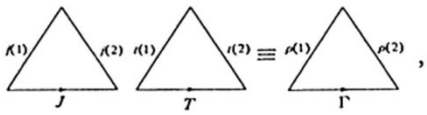
When P_{12}: interchanges operators So, the isospin-dependent part is [21]:

$$\Theta_{TT_Z}((1)t(2)) = \sum\langle tt_z tt'_z|TT_z\rangle\theta_{tt_Z}(1)\theta_{tt'_Z}(2) \equiv \tag{4}$$



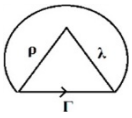
The notation has been condensed to include spin and isospin as (j, $\rho \equiv (j, t)$, and $\Gamma \equiv (J, T)$. So, Eq. (1) can be rewritten as:

$$\Phi_\Gamma(1,2) = \tag{5}$$



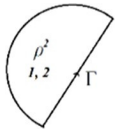
A circular arc represents the anti-symmetry of a wave function, and one obtains for two particles in two distinct orbits ρ and λ.

$$\Phi_\Gamma^{as}(1,2) \equiv \tag{6}$$

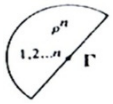


For two particles in the same Orbit, the notation can be extended as follows:

$$\Phi_\Gamma^{as}(1,2) \equiv \tag{7}$$

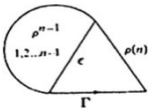


One can be extended easily to wave functions of more than two particles in one Orbit ρ as

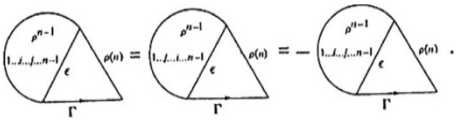
$$\Phi_{\Gamma}^{as}\ (1,2,\dots,n) \equiv$$  (8)

2.2 Coefficients of Fractional Parentage

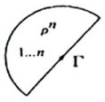
The n-particle function with all particles in one Orbit ρ is given as [20]:

$$\Phi_{\Gamma}(1,2,\dots,n) =$$  (9)

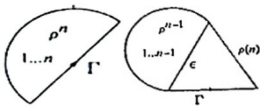
The group ρ^{n-1} is coupled to J_g, T_g, x_g, with x_g denoting all further quantum numbers needed to specify the state $|\rho^{n-1}\rangle_g$ uniquely. When the operator P_{ij} interchanges all coordinates of particles i and j, then one obtains for $i, j \leq n-1$ dueto the anti-symmetry:

P_{ij}  (10)

However, the result of the permutation Pij for i or j equal to n cannot be represented in general by a simple expression in terms of the original function, as in Eq (10).

$$\Phi_{\Gamma}^{a}(1,2,\dots,n) \equiv$$  (11)

The wave function of eq. (9) due to anti-symmetrization. Also, one can write:

 $= \sum\langle \rho^n \Gamma | \}^{n-1} \epsilon \rangle$ (12)

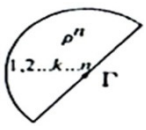
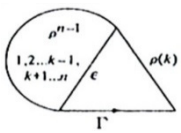
where $\langle \rho^n \Gamma |\ \}\ \rho^{n-1}\epsilon\rangle$ represented "coefficients of fractional parentage" or c.f.p. The normalization and orthogonality lead to the states $|\rho^n\rangle_{\Gamma x}$ being denoted by x as:

$$\sum_{\Gamma' x'} \langle \rho^n \Gamma x | \}^{-1} \Gamma' x' \rangle \langle \rho^n \Gamma x'' | \} \rho^{n-1} \Gamma' x' \rangle\ =\ \delta_{x x''}. \tag{13}$$

If the particle numbered k is willing to decouple, then the simple reordering based on equation (12) is as follows:

$$\Phi_{\Gamma}^{as}(1,2,\dots,k,\dots,n) =\ (-1)^{n-k} \Phi_{\Gamma}^{as}(1,2,\dots,n,k). \tag{14}$$

The completely antisymmetric wave function leads to expansion [21].

 $= (-1)^{-k} \sum_g \langle \rho^n \Gamma | \} \rho^{n-1} \epsilon \rangle$  (15)

It is useful to go over the derivation of c.f.p. in detail for the relatively simple case of three identical particles (maximum isospin) in one Orbit with j 7/2. Only for j 7/2 do three particles couple in a novel way to a given total spin J. Using the same diagrammatic representation as in [21], the coupling of three single-particle wave functions to a non-antisymmetrized function of total spin J can be obtained.

3. RESULTS AND DISCUSSION

For ^{46}Cr as illustrated in Fig. (1) the scheme is clear, experimental data has the values of ($0^+, 2^+, 4^+, 6^+, 8^+$) with the corresponding values (0.000, 0.892, 1.987, 3.226, 4.817) respectively, the reproduced data for the interaction (FPD6, FPY, F742, F7MBZ, and KB3G) are good in general and have the best sequence and slight differences from that of experimental data. All the ground states are the same, which is (0^+); the first excited state is 2^+, the second is 4^+, the third is 6^+ and the fourth is 8^+ and the order of sequences has coincided with all the reproduced results, but the

differences between interaction to another are obvious, the best results belong to FPD6 AND KB3G but the results of FPY are more shifted upward reflecting that the interaction in the same model space has different actions in the values of the study under respect. The energy levels in the range of E=0 -3 MeV are more interesting and the values higher than this range does not well reproduced because of the model adopted in our study and the interactions, in general do not reflect all the reactions inside the nucleus .shell theory and residual interaction do expect the term of two body matrix element and the fitting parameters, then the interaction succeeded or failed to reproduce the experimental data according to the term constructing the interaction and the range of consideration. Some interactions are best fitted to a range from A=42 to 48 and their fitting parameters are normalized on (A-42) terms which are taken from real (N –N) interaction where the Meson particles are the link between interacting nucleus.

Energy level density, as illustrated in Fig. (2), reveals that the states are condensed in the range of (5-6) MeV and decreased rapidly for all the interactions and the value of energy levels density varies from one interaction to another, but all of them reflect that there is a necessity to modify the analyzers in order to produce the energy level experimentally with hyperfine structure.

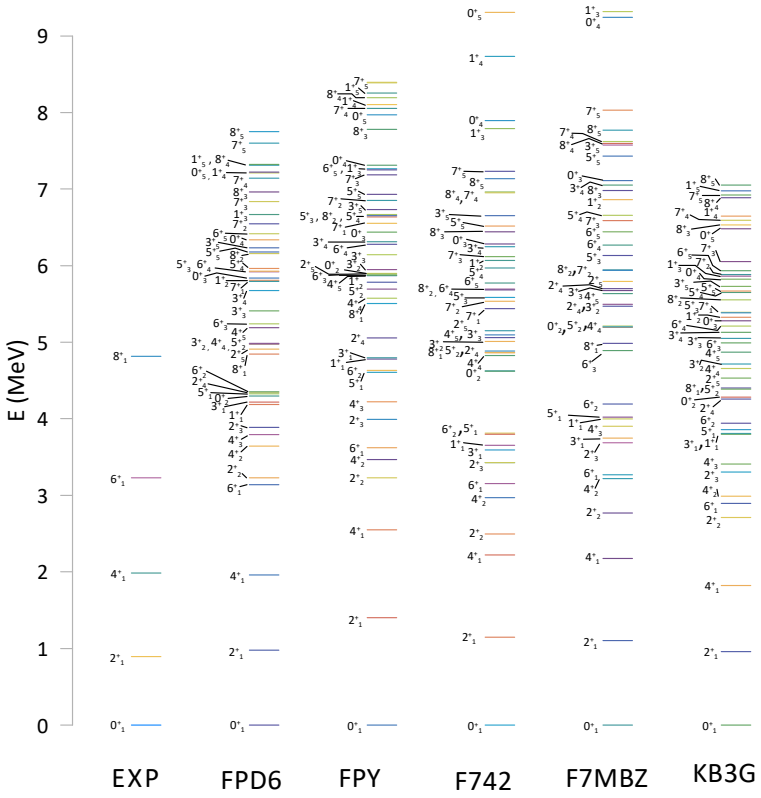


Figure 1. The energy levels scheme of ^{46}Cr by using FPD6, FPY, and KB3G interactions in fp shell model space and F742,and F7MBZ interactions in $1f_{7/2}$model space with close core ^{40}Ca, positive parity, and ten order.

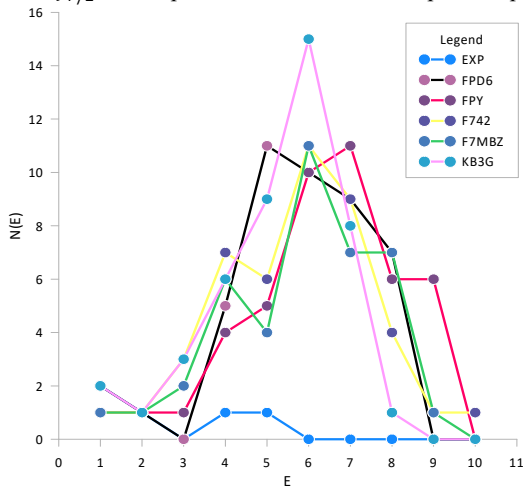


Figure 2. The density of states scheme of ^{46}Cr per 1MeV.

4. CONCLUSIONS

Modern society's effective interaction enhances energy levels and quality. The most suitable choice of interactions in some model spaces and for some isotopes needs to be determined by the values of binding energies and separation energies for both protons and neutrons reproduced by the chosen interaction. Then the interaction will succeed in the model space, according to shell theory. It will still be valid, and the obtained theoretical results will open a straight path for a physicist to modify the analyzers to measure experimental properties not yet reproduced.

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СХЕМА РІВНІВ ЯДЕРНОЇ ЕНЕРГІЇ ^{47}Ca З ВИКОРИСТАННЯМ ВЗАЄМОДІЙ FPD6, FPY ТА KB3G

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Рівні ядерної енергії ізотопу ^{46}Cr досліджувалися з використанням низькорівневої оболонки FP-LS в оболонковій моделі. Рівні ядерної енергії були розраховані з використанням взаємодій FPD6, KB3G і FPY у просторі моделі fp-оболонки та F742 і F7MBZ у просторі моделі f7/2. Результати порівнюються один з одним а також з доступними експериментальними даними, і конкретні результати добре зходяться. На додаток до сильного збігу у відтворених значеннях схеми енергетичних рівнів, використані взаємодії простору моделі є матричним елементом двох тіл у просторі моделі fp-оболонки, який підходить найкраще. Зокрема, нижче 3 МеВ, загальна оцінка відтворених даних добра. Хвильові вектори та аналіз змодельовані у вигляді діаграм, і всі написи наведені в цьому стилі. Використовуючи осциляторний потенціал, будується єдиний вектор частинок, використовуючи ^{40}Ca як ядро простору моделі fp-оболонки та f7/2. Результати отримані для всіх перевірених ядер з використанням коду оболонкової моделі Оксфорд-Буенос-Айрес.

Ключові слова: *рівні ядерної енергії; FP оболонка; FPY; матричне позначення; FPD6; KB3G*