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ENERGY CONDITIONS AND STATEFINDER DIAGNOSTIC OF COSMOLOGICAL MODEL WITH SPECIAL LAW OF HUBBLE PARAMETER IN *f*(*R*,*T*) GRAVITY[†]

V.R. Patil[a], P.A. Bolke[b]*, S.K. Waghmare[c], J.L. Pawde[d]
[a]Department of Mathematics, Arts, Science and Commerce College, Chikhaldara, Dist. Amravati-444807, India
[b]Department of Mathematics, Prof. Ram Meghe College of Engineering and Management,
New Express Highway Bandera, Dist. Amravati-444701, India
[c]Department of Mathematics, TGPCET Nagpur-441108, India
[d]Dadasaheb Gawai Vidyalaya, Malhara, Achalpur, Dist. Amravati-444806, India
**Corresponding Author: mrpravin.bolke@gmail.com*
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In this article, we examine the LRS Bianchi type-I cosmological model in the framework of *f*(*R*, *T*) gravity, where *R* is the Ricci scalar and *T* is the stress energy momentum tensor in the presence of Domain wall. we used the special law of variation of Hubble's parameter proposed by Berman (1983) to obtained the exact solution of field equation, corresponds to the model of the universe. The Energy conditions and physical behaviour of the universe has been obtained and their evolution has been discussed using some physical parameter and by means of their graphs. Also, we can use the Statefinder parameter for testing the validity of the model.
Keywords: *f*(*R*, *T*) *gravity; Statefinder parameters; LRS Bianchi type -I cosmological model; deceleration parameter*
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INTRODUCTION

There are three solutions to the equation of expanding universes, each predicting a different ultimate fate for the entire universe. The ultimate fate of the universe can be determined by measuring the rate at which it is expanding relative to the amount of matter it contains. Open, flat, and closed universes are the three forms of expanding universes that are theoretically possible. The universe would always grow if it were open. If the universe were flat, it would also continue to expand indefinitely, but the rate of growth would eventually reduce to zero. The universe would eventually stop growing and collapse in on itself if it were closed, possibly resulting in another big bang. in all three scenarios gravity is the reason for the slowed expansion.

The universe in which we exist must have gone through a phase of exponential accelerating expansion called inflation. Inflation not only explains the physics of the early universe, but also some conceptual problems in the big bang cosmology such as flatness problems, the horizon problem, etc. Guth [1,2] presented the revolutionized cosmic ideas on this expansion and the beginning of this universe as a result of the big bang. observational data from the Plank collaboration [3], Baryon Oscillation Spectroscopic Survey (BOSS) [4], and Atacama Cosmology Telescope Polarimeter (ACTPol) collaboration [5] provides relevant experimental evidence about the acceleration of our universe. Modern cosmology developed a fresh perspective in the previous century that allowed for significant improvements in the understanding of the universe's current, accelerating expansion. Observations from Supernovae cosmology project [6,7] high red-shift supernova [8], cosmic microwave background (CMB) fluctuations [9,10], large-scale structure (LSS) [11,12], cosmic microwave radiation (CMBR) [13] suggests that present universe is undergoing an accelerated expansion. To explain this acceleration Einstein introduced the simple proposition for Dark energy (DE) known as the cosmological constant to the gravitational field equation. However, this cosmological constant faces some theoretical difficulties such as fine-tuning problems, coincidence, etc. To avoid those cosmological difficulties researchers, approach two different ways to explain the cosmic acceleration. The first approach is to start working on quintessence [14, 15], phantom [16], k-essence [17], tachyons [18], holographic dark energy [19], Chaplygin gas [20], etc.

The second approach is to introduce modified gravity as an alternative to GR and it can describe the late time cosmology as well. In recent years, several theories of modified gravity have been developed corresponding to the modification in Einstein–Hilbert action by a polynomial function of R ($f(R)$ gravity) [21-23], a function of Ricci scalar, and a trace of energy-momentum tensor $f(R,T)$ gravity [24], a function of torsion scalar $f(\tau)$ gravity [25], Gauss-Bonnet scalar $f(G)$ gravity. Among the modified theories of gravitation, $f(R,T)$ gravity become most popular these days to the theoretical cosmologists and astrophysicts as it can explain many cosmological and astrophysical problems.

$f(R,T)$ gravity was proposed by Harko et al. [24] which is the extension of $f(R,T)$ gravity with trace energy momentum tensor T with the modification in Hilbert action principle. This of gravity takes into account the effects of minimal coupling between matter and geometry in the action of gravity. This idea gained popularity when it was first proposed, and researcher used it to explain a variety of other cosmological circumstances scenarios, including thermodynamics [26-28], gravitational waves [29-31], redshift drift [32], big bang nucleosynthesis and entropy evolution

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[33]. Nisha Godani [34] investigated FRW cosmology in f (R, T) gravity to study the age of universe, apparent magnitude and age of universe. Tiwari et al. [35] studied the quadratically varying parameter in $f(R,T)$ gravity. An anisotropic Tilted Marder's Cosmological Model is investigated by Pawar and Shahare [36] in the framework of $f(R,T)$ gravity and discussed the energy conditions of the model with the help of some physical parameters. Mishra et al [37] analysed cosmological models with an anisotropic variable parameter in $f(R,T)$ gravity. Sahoo et al. [38] discussed the energy conditions in non-minimally coupled in $f(R,T)$ gravity. late-time acceleration for the bulk-viscous fluid in $f(R,T)$ gravity is studied by Arora et al. [39].

The experimental and observations data of microwave background radiation suggest that our present universe is largely homogeneous and isotropic represented by FRW model. Bianchi space-time plays a significant role in modern cosmology to discuss and understand the early phases of the evolution of the universe due to spatial homogeneous and anisotropic behaviour as such cosmological models plays a significant role in describing the large structure and behaviour of the universe. Yadav et al. [40] investigated the existence of bulk viscous universe in $f(R,T)$ gravity. Patil et al. [41] discussed the Bianchi type IX cosmological model in Creation field. Koussour & Bennai [42] discussed Bianchi type I space-time with bulk viscosity in $f(R,T)$ gravity. Recently [43-46] investigated Bianchi type space-time in $f(R,T)$ gravity.

With this motivation, in this paper, we have emphasized to investigate the exact solution of LRS Bianchi type- I model in the framework of modified $f(R,T)$ theory of gravity with domain wall using special law of variation of Hubble's parameter. Also, we discuss energy conditions of the model. The article is organized as follows: In section **2** we briefly review $f(R,T)$ theories, and we present the field equation of gravity. Section **3** is used to find exact solution of LRS Bianchi type- I model, the energy conditions of the model are presented in section **4,** the Statefinder diagnostic are analysed in section **5.** Finally, the results are summarized and concluded in section **6**.

Gravitational field equations of *f (R,T)* gravity

The gravitational field equations of models in $f(R,T)$ theory of gravity is obtained from the Hilbert-Einstein action in variational principle.
The action for $f(R,T)$ modified gravity is given as,

$$S=\int\sqrt{-g}\left(\frac{f(R,T)}{16\pi G}+L_m\right)d^4x\ , \tag{1}$$

where, $f(R,T)$ is an arbitrary function of the Ricci scalar R and of the trace T of the stress-energy tensor of the matter $T_{\mu\nu}$ and L_m is the usual matter Lagrangian.
The stress-energy tensor of the matter $T_{\mu\nu}$ is defined as

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\,L_m\right)}{\delta g^{\mu\nu}}, \tag{2}$$

and its trace by $T=g^{\mu\nu}T_{\mu\nu}$
The corresponding field equations of $f(R,T)$ gravity by varying the action (1) with respect to the metric tensor $g_{\mu\nu}$ is given by

$$f_R(R,T)R_{\mu\nu}-\frac{1}{2}f(R,T)g_{\mu\nu}+\left(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right)f_R(R,T)=8\pi T_{\mu\nu}-f_T(R,T)T_{\mu\nu}-f_T(R,T)\Theta_{\mu\nu} \tag{3}$$

where,

$$f_R\equiv\frac{\partial f(R,T)}{\partial R},\qquad f_T\equiv\frac{\partial f(R,T)}{\partial T},\qquad \Box\equiv\nabla^i\nabla_j,$$

∇_i is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m.
By contracting equation (3) we get relation between the Ricci scalar R and trace T of stress-energy momentum tensor as,

$$f_R(R,T)R+3\Box f_R(R,T)-2f(R,T)=8\pi T-f_T(R,T)T-f_T(R,T)\Theta, \tag{4}$$

where, $\Theta=g^{\mu\nu}\Theta_{\mu\nu}$

In $f(R,T)$ gravity theory, the function $f(R,T)$ depends on the nature of matter source. Hence, one can obtain several theoretical models corresponding to the different matter models. Harko et al. [24] have considered the following three different classes of $f(R,T)$ gravity models:

$$f(R,T)=\begin{cases} R+2f(T) \\ f_1(R)+f_2(T) \\ f_1(R)+2f_2(R)f_3(T) \end{cases}. \tag{5}$$

In this paper, we assume $f(R,T)=R+2f(T)$, with the particular choice $f(T)=\lambda T$, where λ is constant.

The field equation in $f(R,T)$ theory with the function $f(R,T)=R+2f(T)$, where the matter source is perfect fluid are given by Harko et al. [24]

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=8\pi T_{\mu\nu}+2f'(T)T_{\mu\nu}+\left[2p\,f'(T)+f(T)\right]g_{\mu\nu}, \tag{6}$$

where prime denotes the differentiation with respect to argument.

The energy- momentum tensor of domain wall is taken as

$$T_{\mu\nu}=\rho\left(g_{\mu\nu}+w_iw_j\right)+pw_iw_j, \tag{7}$$

where ρ is the energy density , p is the pressure of domain wall and $w^{\mu}=(1,0,0,0)$ is the four velocity vector in the comoving coordinates which satisfies the condition $w^{\mu}w_{\mu}=-1$.

Then we have,

$$T_1^1=T_2^2=T_3^3=\rho, T_4^4=-p, T=3\rho-p. \tag{8}$$

Domain walls are cosmologically important due to their appearance in the phase transition of the early universe. Kibble [47] has highlighted that late time evolution of domain walls is governed by their surface tension and their interaction with matter. Different types of domain walls can occur with very different degrees of transparency, but in all cases the size of the overall structure increases over time.

Metric and field equations

We consider the LRS Bianchi type I space-time of the form

$$ds^2=dt^2-L(t)^2\,dx^2-M(t)^2\,(dy^2+dz^2), \tag{9}$$

where $L(t)$ and $M(t)$ are the scale factor and the function of the cosmic time t only.

The corresponding field equation (6) for the metric (9) for the function $f(R,T)=R+2f(T)$ with $f(T)=\lambda T$ can be written as

$$\left(\frac{\dot{M}}{M}\right)^2+2\frac{\ddot{M}}{M}=-(8\pi+5\lambda)\rho-\lambda p, \tag{10}$$

$$\frac{\ddot{L}}{L}+\frac{\ddot{M}}{M}+\frac{\dot{L}}{L}\frac{\dot{M}}{M}=-(8\pi+5\lambda)\rho-\lambda p, \tag{11}$$

$$\left(\frac{\dot{M}}{M}\right)^2+2\frac{\dot{L}}{L}\frac{\dot{M}}{M}=(8\pi+\lambda)p-3\lambda\rho, \tag{12}$$

where dot (.) represents the derivative with respect to t.

Equation (10), (11) and (12) are three linearly equations with four unknowns L, M, ρ and p. In order to solve the system completely we use variation law of Hubble's parameter between H and a proposed by Berman (1983).

$$H=la^{-n}=l(LM^2)^{-n/3}, \tag{13}$$

where $l>0$ and $n\geq 0$ are constants.

The spatial volume is given by

$$V=a^3=LM^2, \tag{14}$$

where a the mean scale factor.

The mean Hubble's parameter H for the metric (9) is given by

$$H=\frac{1}{3}\left(H_x+H_y+H_z\right)=\frac{1}{3}\left(\frac{\dot{L}}{L}+2\frac{\dot{M}}{M}\right)=\frac{\dot{a}}{a} \tag{15}$$

where $H_x = \frac{\dot{L}}{L}$ and $H_y = H_z = \frac{\dot{M}}{M}$ are the directional Hubble's parameter in the x, y, z respectively.
The volumetric deceleration parameter is

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -\frac{a\ddot{a}}{\dot{a}^2} = n - 1 \tag{16}$$

from equation (13) and (15), on integration we get

$$a = (nlt + k_1)^{\frac{1}{n}}, \quad n \neq 0 \tag{17}$$

Subtracting equation (11) from (10) we get,

$$\frac{d}{dt}\left(\frac{\dot{L}}{L} - \frac{\dot{M}}{M}\right) + \left(\frac{\dot{L}}{L} - \frac{\dot{M}}{M}\right)\left(\frac{\dot{L}}{L} + 2\frac{\dot{M}}{M}\right) = 0 \tag{18}$$

on integrating and using (14), (17) we get

$$L = (c_2)^{\frac{2}{3}} (nlt + k_1)^{\frac{1}{n}} \exp\left(\frac{2c_1}{3l(n-3)} (nlt + k_1)^{\frac{n-3}{n}}\right) \quad, n \neq 3 \tag{19}$$

$$M = (c_2)^{-\frac{1}{3}} (nlt + k_1)^{\frac{1}{n}} \exp\left(\frac{-c_1}{3l(n-3)} (nlt + k_1)^{\frac{n-3}{n}}\right) \quad, n \neq 3 \tag{20}$$

where c_1 and c_2 are the constant of integration.
using equation (11), (13), (19) and (20) we get,

$$\rho = \frac{1}{\left((8\pi + 3\lambda)^2 - \lambda^2\right)} \left\{ \frac{l^2 \left[8\pi(2n-3) + 2\lambda(n-3\right]}{(nlt + k_1)^2} - \frac{2c_1^{\,2}(12\pi - \lambda)}{9\left[(nlt + k_1)^{\frac{1}{n}}\right]^6} \right\}, \tag{21}$$

$$p = \frac{-1}{\left((8\pi + 3\lambda)^2 - \lambda^2\right)} \left\{ \frac{l^2 \left[-6\lambda(1+n) - 24\pi\right]}{(nlt + k_1)^2} + \frac{24c_1^{\,2}(\pi + \lambda)}{9\left[(nlt + k_1)^{\frac{1}{n}}\right]^6} \right\} \tag{22}$$

using equation (19) and (20) in equation (9), the model of the universe takes the form

$$ds^2 = dt^2 - (c_2)^{\frac{4}{3}} (nlt + k_1)^{\frac{2}{n}} \exp\left(\frac{4c_1}{3l(n-3)} (nlt + k_1)^{\frac{n-3}{n}}\right) dx^2 - (c_2)^{-\frac{2}{3}} (nlt + k_1)^{\frac{2}{n}} \exp\left(\frac{-2c_1}{3l(n-3)} (nlt + k_1)^{\frac{n-3}{n}}\right) (dy^2 + dz^2), \tag{23}$$

Definition for physical parameter such as , the spatial volume (V), directional Hubble parameter (H_i), mean Hubble's parameter (H), extension scalar (θ), shear scalar (σ) and anisotropy parameter (A_m), deceleration parameter (q) for the universe (23) are given by:

$$V = (nlt + k_1)^{\frac{3}{n}} \tag{24}$$

$$H_x = \frac{2c_1}{3(nlt + k_1)^{\frac{3}{n}}} + \frac{l}{(nlt + k_1)} \tag{25}$$

$$H_y = H_z = \frac{-c_1}{3(nlt + k_1)^{\frac{3}{n}}} + \frac{l}{(nlt + k_1)} \tag{26}$$

$$H = \frac{l}{(nlt + k_1)} \tag{27}$$

$$\theta = \frac{3l}{(nlt + k_1)} \tag{28}$$

$$A_m = \frac{2c_1^{\,2}}{9l^2}\left(nlt + k_1\right)^{\frac{2(n-3)}{n}} \tag{29}$$

$$\sigma^2 = \frac{c_1^{\,2}}{3\left(nlt + k_1\right)^{\frac{6}{n}}} \tag{30}$$

$$q = n - 1 \tag{31}$$

Fig. (1) shows the variation of Hubble parameter (H) with cosmic time (t). From figure we observe that H is decreasing function of cosmic time. Similarly, from **Fig. (4)** the expansion scalar (θ) is the decreasing function of time. Both Hubble's parameter (H) and expansion scalar (θ) starts from a positive value and approaches a small positive value for large value of time (t). Hence the expansion rate is faster at the beginning and slows at later stage. We plotted all the graphs by taking $n, l, k_1, c_1 = 1.5, 2, 1, 1$.

Fig. (2) and (3) shows the variation of spatial volume (V) and shear scalar (σ^2) with cosmic time (t). From the figure we observe that the shear scalar is decreasing positive value function of time and spatial volume of the model is increasing function of cosmic time. Hence the present model is expanding and shearing.

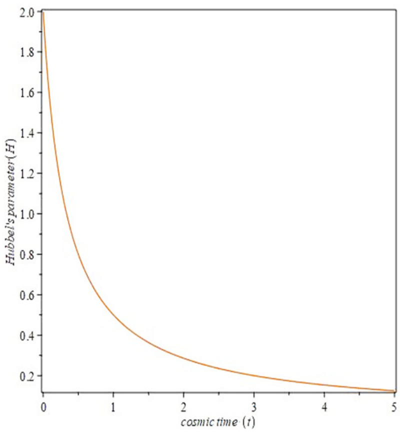

Figure 1. Plot of variation of Hubbel's parameter (H) vs cosmic time (t)

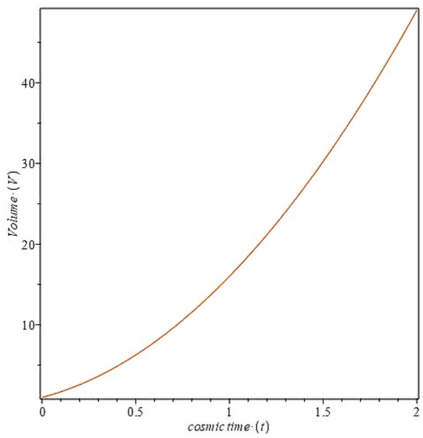

Figure 2. Plot of variation of volumr vs cosmic time (t)

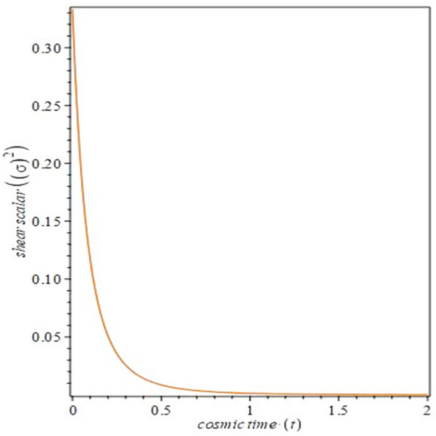

Figure 3. Plot of variation of shear scalar (σ^2) vs cosmic time (t)

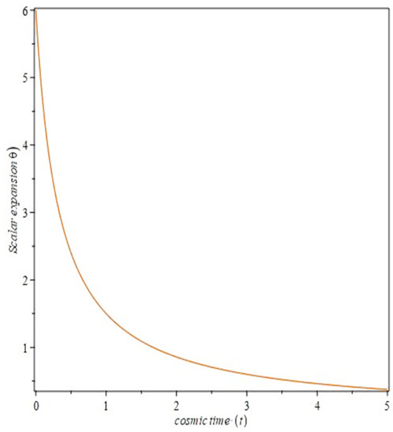

Figure 4. Plot of variation of shear scalar expansion (θ) vs cosmic time (t)

Fig. (5) and (6) represent the plot of pressure and energy density with cosmic time (t) for $\lambda = 0, -1 \text{and} -2$ by taking $n, l, k_1, c_1 = 3.2, 0.097, 0.1, 0.2$. The value of pressure approaches to a small negative value close to zero, for large value of time. From the recent observations data and accelerated cosmic expansion of the universe, it is assumed that the Universe is undergoing an accelerating expansion due to the negative pressure called as dark energy. Also, from the graph of energy density (ρ) vs. cosmic time (t), it is observed that, energy density is decreasing positive valued function of time and approaches zero at late time.

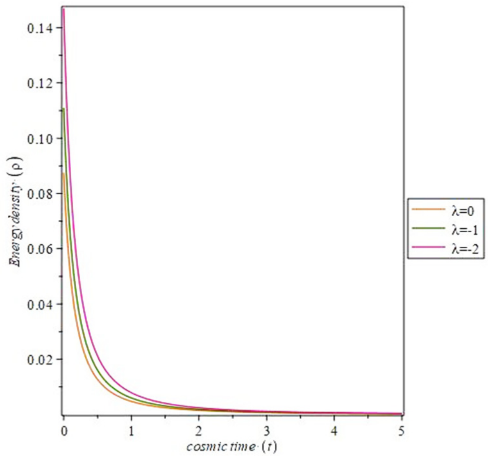

Figure 5. Plot of variation of Energy density (ρ) vs cosmic time (t)

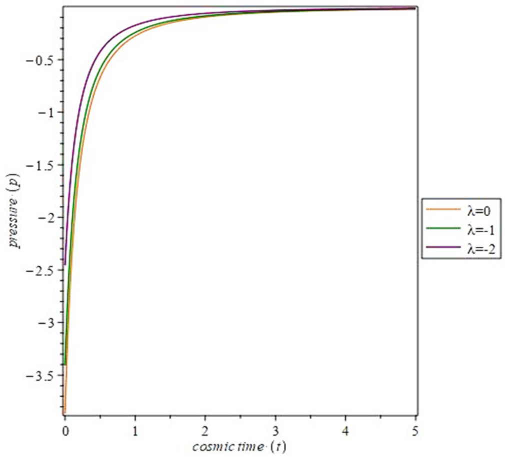

Figure 6. Plot of variation of pressure (p) vs cosmic time (t)

Energy conditions

In this section, we examine the energy condition for our constructed model. Additionally, we examine whether or not our model satisfies the energy conditions.

The energy conditions are:

i) Weak energy conditions (WEC): $\rho \geq 0$ and $\rho + p \geq 0$.

ii) Dominant energy conditions (DEC): $\rho - p \geq 0$.

iii) Strong energy condition (SEC): $\rho + 3p \geq 0$

$$\rho + p = \frac{1}{\left((8\pi + 3\lambda)^2 - \lambda^2\right)} \left\{ \frac{l^2 (16\pi n + 8\lambda n)}{(nlt + k_1)^2} - \frac{c_1^{\,2}(48\pi + 22\lambda)}{9\left[(nlt + k_1)^{\frac{1}{n}}\right]^6} \right\}, \tag{32}$$

$$\rho - p = \frac{1}{\left((8\pi + 3\lambda)^2 - \lambda^2\right)} \left\{ \frac{l^2 (16\pi n - 6\lambda n - 12\lambda - 48\pi)}{(nlt + k_1)^2} + \frac{26\lambda c_1^{\,2}}{9\left[(nlt + k_1)^{\frac{1}{n}}\right]^6} \right\}, \tag{33}$$

$$\rho + 3p = \frac{1}{\left((8\pi + 3\lambda)^2 - \lambda^2\right)} \left\{ \frac{l^2 (16\pi n + 20\lambda n + 12\lambda + 48\pi)}{(nlt + k_1)^2} - \frac{(70\lambda + 96\pi) c_1^{\,2}}{9\left[(nlt + k_1)^{\frac{1}{n}}\right]^6} \right\}, \tag{34}$$

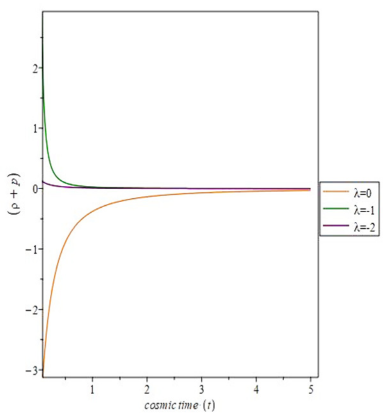

Figure 7. Plot of ($\rho + p$) vs cosmic time (t)

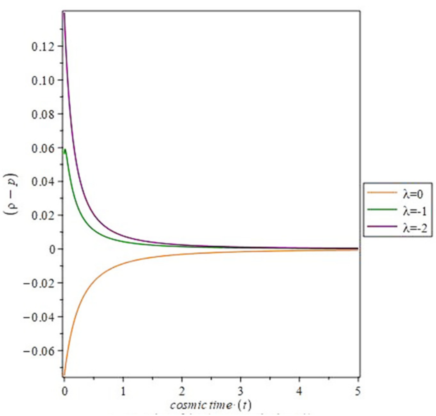

Figure 8. Plot of variation $\rho - p$ vs cosmic time (t)

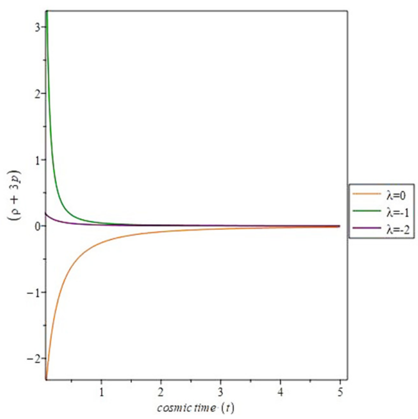

Figure 9. Plot of variation of $(\rho + 3p)$ vs cosmic time (t)

Fig. (7), (8), (9) represents the graph of energy conditions vs. cosmic time with $\lambda = 0, -1$ and -2. From these figures it is observed that all the energy conditions (EC) are satisfies by the model in $f(R,T)$ gravity.

State-finder diagnostic

A cosmological diagnostic parameter set $\{r,s\}$ called State-finder pair was introduced first by Sahni et al. [48] to study a geometric view of the dark energy models. The state finder parameters depend on the scale factor. The important property of state-finder pair is that, the fixed point $\{r,s\} = \{1,0\}$ characterizes the cold dark matter with Λ (ΛCDM) model, while the fixed point $\{r,s\} = \{1,1\}$ characterizes the standard cold dark matter (SCDM) model.
The state finder parameter r and s are defined as follows:

$$r = \frac{\dddot{a}}{aH^3} = (n-1)(2n-1), \tag{35}$$

$$s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} = \frac{2}{3}n. \tag{36}$$

From **Fig. (10)**, we can see that our model satisfies SCDM scenario of the universe for $n = 1.5$.

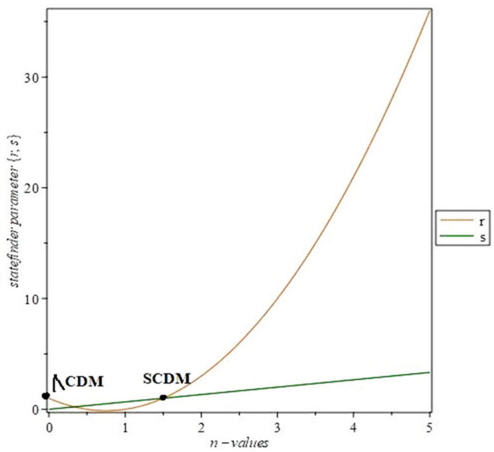

Figure 10. The state finder plane

DISCUSSION AND CONCLUSION

In this paper, we studied the LRS Bianchi -I cosmological model in the framework of $f(R,T)$ theory of gravity with $f(R,T) = R + 2f(T)$ by taking $f(T) = \lambda T$. Exact solution of the field equations is obtained with the help of special law of variation of Hubble's parameter proposed by Berman (1983). We have discussed the geometrical and kinematical properties of various parameters. Also we have studied the behaviour of the model according to graphs of physical parameters such as Hubble's parameter (H), the spatial volume (V), extension scalar (θ), shear scalar (σ), Energy density (ρ) and pressure (p). The Hubble's parameter, shear scalar and expansion scalar are constant as cosmic time tends to zero and this parameter diverges when cosmic time is $-k_1/(n \cdot l)$.

i) Hubble's parameter, shear scalar and expansion scalar are decreasing positive value function of cosmic time. This shows that at the beginning the expansion rate is faster and slows at later stage.
ii) The spatial volume admits constant value at early times of the universe (as cosmic time tends to zero), after that spatial volume start increasing with increase in cosmic time without showing any type of initially singularity and finally diverges to ∞ as $t \to \infty$. This indicates that initially, the evolution of universe starts at the big-bang singularity $t = -k_1/(n \cdot l)$ and then expands approaching to infinite volume.
iii) The positive sign of deceleration parameter (q) corresponds to standard decelerating model whereas the negative sign indicates acceleration. For $n > 1$, deceleration parameter $q > 0$, hence model represents the decelerating model whereas $0 < n < 1$, we get $-1 \le q < 0$, which describes an accelerating model of the universe.
iv) Energy density (ρ) is positive and decreasing function of cosmic time and also $\rho \to 0$ when $t \to \infty$.

v) It is worth to note that, $f(R,T)$ model reduces to general relativity (GR) for $\lambda = 0$. From **Figs. (7-9)**, it is observed that all energy conditions (EC) are satisfied in $f(R,T)$ model but these energy conditions are not satisfied in general relativity $(\lambda = 0)$ model.

vi) The Statefinder trajectory **(Fig. 10)**, showing that the model behaves as the SCDM model during the early universe and as the ΛCDM model during the late universe.

ORCID

V.R. Patil, https://orcid.org/0000-0002-0442-3962; **P.A. Bolke,** https://orcid.org/0000-0002-1212-5260
J.L. Pawde, https://orcid.org/0000-0001-8068-6265

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ЕНЕРГЕТИЧНІ УМОВИ ТА ДІАГНОСТИКА ВИЗНАЧНИКА СТАНУ КОСМОЛОГІЧНОЇ МОДЕЛІ ЗІ СПЕЦІАЛЬНИМ ЗАКОНОМ ПАРАМЕТРІВ ХАББЛА В $f(R,T)$ ГРАВІТАЦІЇ

В.Р. Патіл[a], П.А. Болке[b], С.К. Вагмаре[c], Дж.Л. Павде[d]
[a]*Департамент коледжа математики, мистецтв, науки та торгівлі, Чикхалдара, округ Амраваті-444807, Індія*
[b]*Кафедра математики, коледж інженерії та управління професора Рама Меге, Амраваті-444701, Індія*
[c]*Департамент математики, TGPCET Нагпур-441108, Індія*
[d]*Дадасахеб Гавай Відьялая, Малхара, Ачалпур, округ Амраваті-444806, Індія*

У цій статті ми розглядаємо космологічну модель LRS Bianchi типу I в рамках $f(R, T)$ гравітації, де R є скаляр Річчі, а T є тензор імпульсу енергії напруги за наявності доменної стінки. ми використали спеціальний закон зміни параметра Хаббла, запропонований Берманом (1983), щоб отримати точний розв’язок рівняння поля, що відповідає моделі Всесвіту. Були отримані енергетичні умови та фізична поведінка Всесвіту, а їх еволюція обговорювалася за допомогою деяких фізичних параметрів і за допомогою їхніх графіків. Крім того, ми можемо використовувати параметр визначника стану для перевірки валідності моделі.

Ключові слова: *$f(R, T)$ гравітація; параметри вимірювача стану; космологічна модель Б’янкі типу-I; параметр уповільнення*