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INVESTIGATION OF THERMAL RADIATIVE TANGENT HYPERBOLIC NANOFLUID FLOW DUE TO STRETCHED SHEET[†]

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The current study illuminates the enactment of tangent hyperbolic nanofluid past a bidirectional stretchable surface. The phenomena of heat and mass transfer with joule heating, chemical reaction and thermal radiation have been debated. For motivation of problem convective boundary conditions and heat source are part of this study. The modeled partial differential equations are mended into ordinary differential equations with the help of appropriate self-similarity transformations. Furthermore, the resulting system of ODEs is numerically handled by using well-established shooting scheme and acquired numerical outcomes are compared with ND Solve command of Mathematica. The Effects of prominent parameters on velocity, temperature and volumetric concentration distribution are inspected through graphs. The influence of emerging parameters involved in this study on flow and heat removal features are deliberated in detail. As we are increasing the values of power-law index n, Prandtl number Pr and Magnetic parameter M, outcomes increment in skin friction coefficient while decline in the Nusselt number is seen.
Keywords: *Shooting Method; Tangent hyperbolic nanofluid; MHD; Joule heating; chemical reaction*
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INTRODUCTION

Due to wider applications of non-Newtonian fluid, many researchers paid their attention, in the last few years. The applications of non-Newtonian fluids contain food products, personal protective equipment braking and damping devices, printing technology and drag reducing agents. Shampoo, melted butter, blood, paint, cornstarch, starch suspensions, toothpaste, custard and Ketchup are examples of such type of fluids [1].

One of the most significant kinds of non-Newtonian fluid is Tangent hyperbolic. Kumar et al. [2] reported that the Tangent hyperbolic fluid has capability to illustrate the behavior of shear thinning. Rehman et al. [3] examined the interplay between the stratification of tangent hyperbolic fluid and the combined effects of thermal radiation, and concluded that with the greater value of solar radiation, there are escalations in the fluid temperature. Shafiq et al. [4] discovered the magnetic flow of bio convective tangent hyperbolic fluid containing swimming microorganism with thermal effects and they reported that the temperature field surge for higher value of thermophoresis parameter. Salahuddin et al. [5] examined the tangent hyperbolic due to stretched cylinder across the stagnation point. Naseer et al. [6] exposed the concept of tangent fluid for buoyancy and thermal effects and reported that the temperature function decreased with increasing value of Prandtl number. Prabhakar et al. [7] scrutinized the flow tangent hyperbolic fluid with influence of inclined Lorentz forces due to stretched surface.

Nanofluid is an important category of non-Newtonian fluids having nanometer-sized particles. propylene glycol, water, ethylene glycol etc., are base fluids, to boost the thermal conductivity of base fluid's nanoparticles are used. Nanofluids are the homogeneous typical combination of nanoparticles are usually made by metals (Cu, Zn, Al), nonmetals (nanotubes, boron, carbon) and carbides (SiC, Fe_3C, CaC_2) with base fluids (glycol, water and ethylene glycol) [8]. The inclusion of a modest number of nanoparticles improved the thermal conductivity of heat transfer fluids, according to research by Choi et al. [9]. The concept of increasing the heat conductivity by using nanoparticles was first proposed by Choi and Eastman [10]. Izadi et al. [11] inspected the behavior of nanofluid with laminar forced flow. As noted by Wong et al. [12], they have numerous engineering and biomedical uses, including as in microelectronics, nuclear reactors, process industries, and cancer therapy. Nayak et al. [13] explored the influence of solar radiation on electrical conducting nanofluid dur to vertical sheet.

The study of squeezing flows in several commercial and practical applications in unusual domains including pharmaceutical manufacturing, energy production, polymer exclusion, energy in space technology, nuclear reactors, chemical reactions, and solar energy in space technology aim to slow down and employed other technologies. In industrial applications, squeezing flows are used in lubrication, bearings and motors. Lin et al. [14] exhibited the qualities of electrical conducting squeezed fluid flow among annular vertical plates. Thermal radiation and heat transfer has numerous uses in production procedures, semiconductors, chemical processes, engineering, and other several areas of technology. The micro convection thermal effect in two phase mixtures is, according to Sohn and Chen [15], more potent than it is in a single-phase

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fluid. Makinde [16] deliberated the radiative flow of heat transfer free convection in the presence of porous sheet. Hussain et al. [17] premeditated the heat transfer phenomena in squeezing flow with thermal radiation and bio convection between two equivalent plates. Hayat et al. [18] debated the influence of solar radiation in Jeffery squeezing fluid flow. Additional study on the influence of heat transfer process and thermal radiation in fluid flow can be considered in [19-21].

The study of electrically conducting fluids with magnetic field is known as magneto hydrodynamics MHD. Magneto-hydrodynamic flow has many presentations in numerous fields of science engineering like MHD accelerator, power generator, heat exchangers and cooling of reactors as deliberated by Hari et al. [22]. Rashidi et al. [23] determined the electrical conducting flow of nanofluid with nonlinear radiation due to vertical stretched surface. Zhang et al. [24] inspected the radiative flow of MHD nanofluid with heat flux, variable surface and chemical reaction.

The illustration of this investigation is to study the numerical outcomes of electrical conducting fluid flow of hyperbolic nanofluid fluid joule heating and chemical diffusion due to stretching sheet using convective boundary conditions. The couple of nonlinear PDEs of governing model are transformed into a set ODEs by use of appropriate revolution called similarity function. Operating the shooting scheme numerous mathematical outcomes are validated. Furthermore, impression of effective parameters is talk about through graphs in detail.

MATHEMATICAL ANALYSIS

Consider the laminar, 2d electrical conducting tangent hyperbolic nanofluid due to vertical stretched sheet. Moreover, the influence of Joule heating and thermal radiation are part of this investigation. Convective boundary conditions are applied for motivation of problem. Where u and v are component of flow of fluid. MHD effect having strength B_0 is considered perpendicular to the sheet. The governing equation including influence of thermal radiation, browning motion, chemical reaction and thermophoresis are described as [25-33]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=v(1-n)\frac{\partial^2 u}{\partial y^2}+\sqrt{2}vn\Gamma\left(\frac{\partial u}{\partial y}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0 u}{\rho_f}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\wedge\left[D_B\left(\frac{\partial C}{\partial y}\right)\left(\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\sigma {B_0}^2u^2}{\rho_f}-\frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y}+\frac{Q_0}{(\rho c)_f}(T-T_\infty), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right)-K_r(C-C_\infty). \tag{4}$$

The associated boundary conditions are given

$$\left.\begin{array}{c} u=u_w, v=0, -K\frac{\partial T}{\partial y}=h_f\left(T_f-T\right), D_B\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial y}=0, \text{at } y=0, \\ U\to U_\infty, T\to T_\infty, C\to C_\infty, as\ \ y\to\infty. \end{array}\right| \tag{5}$$

SIMILARITY TRANSFORMATION

The similarity variables are illustrated as:

$$\psi=\sqrt{avxf}(\eta),\quad \eta=\sqrt{\frac{a}{v}}y,\quad \phi(\eta)=\frac{C-C_\infty}{C_\infty},\quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}, u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}, \tag{6}$$

As a result, Eq. (1) is satisfied identically and equations (2-4) are transformed as:

$$(1-n)f'''+ff''-f'^2-Mf'+nWef'''f''=0, \tag{7}$$

$$\left(1+\epsilon\theta+\frac{4}{3}N_r\right)\theta''+P_rf\theta'+P_rMEcf'^2+P_rNb\,\phi'\theta'+P_rNt\theta'^2+P_r\Delta\theta=0, \tag{8}$$

$$\phi''+\frac{Nt}{Nb}\theta''+Sc\,f\phi'+Sck_1\phi=0, \tag{9}$$

The boundary conditions are converted as:

$$\left.\begin{array}{c} f(0)=0,\ f'(0)=1, \theta'(0)=Bi(\theta(0)-1), Nb\,\phi'(0)+Nt\theta'(0)=0, at\ \ \eta=0, \\ f'(\infty)\to 0,\ \theta(\infty)\to 0,\ \phi(\infty)\to 0,\ as\ \ \eta\to\infty. \end{array}\right| \tag{10}$$

Where

$$\begin{aligned} & Nt=\frac{(\rho c)_p D_T(T_f-T_\infty)}{(\rho c)_f v T_\infty},\ We=\frac{\sqrt{2}a^{\frac{2}{3}}x\Gamma}{\sqrt{v}},\ Nb=\frac{(\rho c)_p D_B(C_\infty)}{(\rho c)_f v},\ S_c=\frac{v}{D_B},\ \Delta=\frac{Q_0}{a(\rho c)_f}, \\ & Bi=\frac{h_f}{k}\sqrt{\frac{v}{a}},\ M=\frac{\sigma B^2{}_0}{\rho f a},\ Nt=\frac{(\rho c)_p D_T(T_f-T_\infty)}{(\rho c)_f v T_\infty},\ Pr=\frac{v}{\alpha}, Nr=\frac{4\sigma^* T^3{}_\infty}{k^* k}. \end{aligned} \tag{11}$$

PHYSICAL QUANTITIES OF INTEREST

The physical quantities are defined as:

$$C_f=\frac{\tau_w}{\rho u_w{}^2},\ Nu_x=\frac{xq_w}{k(T\to T_\infty)}, \tag{12}$$

where:

$$\begin{aligned} \tau_w &= \mu\left((1-n)\frac{\partial u}{\partial y}+\frac{n\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial x}\right)^2\right), \\ q_w &= -k\left(1+\frac{16\sigma^* T_\infty^3}{3k^* k}\right)\frac{\partial T}{\partial y}. \end{aligned} \tag{13}$$

The physical quantities in non-dimensional form:

$$\begin{aligned} C_f\sqrt{R}e_x &= \left((n-1)+\frac{n}{2}W_e f''(0)\right)f''(0), \\ Nu_x Re_x^{-1/2} &= -\left(1+\frac{4}{3}N_r\right)\theta'(0). \end{aligned} \tag{14}$$

Where $Re_x=\frac{ax^2}{v}$ is local Reynolds number.

NUMERICAL SOLUTION

Above equations (7-9) are coupled and highly nonlinear therefore exact solution is not applicable.

$$\begin{aligned} f''' &= \frac{1}{(1-n)+nW_e f''}\left[f'^2+Mf'-ff''\right], \\ \theta'' &= -\frac{Pr}{\left(1+\epsilon\theta+\frac{4}{3}N_r\right)}\left[f\theta'+Nb\phi'\theta'+Nt\theta'^2+MEcf'^2+\Delta\theta\right], \\ \phi'' &= \frac{Nt}{Nb}\theta''-fSc\phi'-Sck_1\phi. \end{aligned} \tag{15}$$

Boundary condition in dimensionless shape is:

$$\begin{aligned} & y_1(\eta)=0,\ y_2(\eta)=1,\ \ y_5(\eta)=Bi(y_4(\eta)-1), \\ & Nby_7(\eta)+Nty_5(\eta)=0,\ \ as\ \ \eta=0, \\ & y_2(\infty)\to 0,\ y_4(\infty)\to 0,\ \ y_6(\infty)\to 0,\ \ \ as\ \ \eta\to\infty \end{aligned} \tag{16}$$

Let us put $f=y_1,\ f'=y_2,\ f''=y_3,\ f'''=y_3{}',\theta=y_4,\theta'=y_5,\theta''=y_5{}',\phi=y_6,\ \phi'=y_7,\ \ \phi''=y_7{}'$. The equations (15) and (16) are written as:

$$\begin{aligned} y_1{}' &= y_2 \\ y_2{}' &= y_3 \\ y_3{}' &= \frac{y_2^2+My_2-y_1y_3}{(1-n)+nWey_3} \\ y_4{}' &= y_5 \\ y_5{}' &= \frac{-\Pr\left(y_1y_5+Nby_7y_5+\Delta y_4+Nty_5^2+2MEcy_2^2\right)}{\left(1+\epsilon\theta+\frac{4}{3}Nr\right)} \\ y_6{}' &= y_7 \\ y_7{}' &= -Scy_1y_7-Scky_7+\frac{Nt}{Nb}y_5{}' \end{aligned} \tag{17}$$

$$\begin{aligned} & y_1(0)=0,\ y_2(0)=1,\ \ y_3(0)=l\ \ \ \ y_4(0)=m \\ & y_5(0)=Bi(t-1),\ \ \ y_6(0)=n\ \ \ y_7(0)=-\frac{Nt}{Nb}Bi(t-1). \end{aligned} \tag{18}$$

RESULT AND DISCUSSION

The revelation of the section is to study the numerical outcomes exemplified in the form of graphs. The influence of emerging parameters like power-law index n, Magnetic parameter M, Prandtl number Pr, thermophoresis parameter Nt, thermal radiation parameter Nr, Weissenberg number We and Brownian motion parameter Nb on velocity, temperature and concentration field are discussed. The influence of the power-law index for non-dimensional velocity function is depicted in Figure 1. it is noted that velocity field is decreased as an increment in power-law index n. Because increased magnitude of n decreases fluid flow. The impression of Hartmann number M on velocity profile f' and temperature profile θ are presented in Figures (2-3).

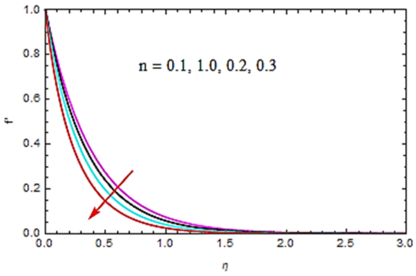

Figure 1. Distribution of f' for power-law index n

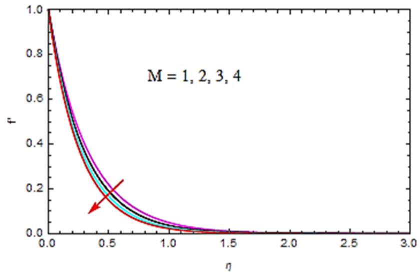

Figure 2. Distribution of f' for Hartmann number M

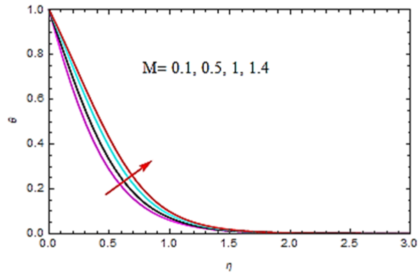

Figure 3. Distribution of θ for Hartmann number M

Figure 2 is plotted to imagine that the flow profile is diminished for improving values of Hartmann number M. In Figure 3, temperature field illustrates growing behavior for the increase of Hartmann number M. Physically, Magnetic field increases Lorentz forces, reducing fluid motion. The impact of Prandtl number Pr on temperature distribution θ and volumetric concentration distribution are exhibited in Figures (4-5).

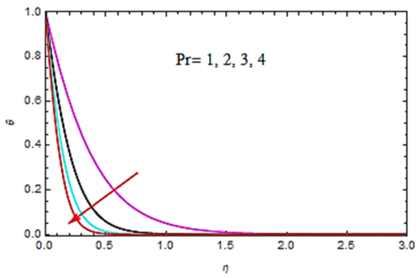

Figure 4. Distribution of θ for Prandtl number Pr

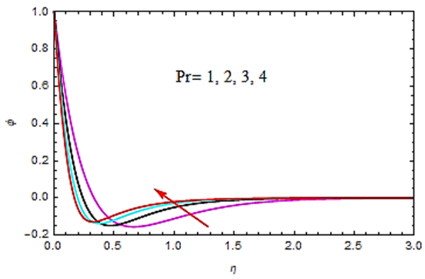

Figure 5. Distribution of ϕ for Prandtl number Pr

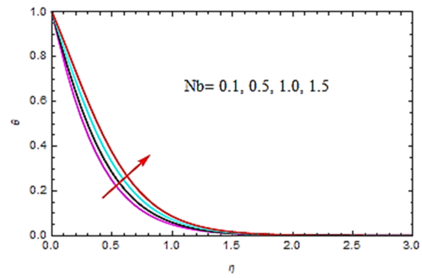

Figure 6. Distribution of θ for Brownian motion parameter Nb

Figure 4 is drawn to explore the influence of the Prandtl number Pr on energy field θ. It is cleared that for increasing value of Prandtl number Pr the energy distribution θ declines. Because of heat transfer rate decreases for increased Pr values. Figure 5, concentration field illustrates growing behavior for the increase of Prandtl number Pr. The impact of Brownian motion parameter Nb on temperature field θ and concentration field are exhibited in Figures (6-7).

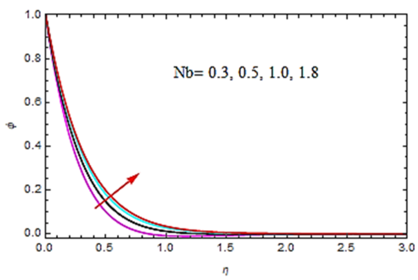

Figure 7. Distribution of ϕ for Brownian motion parameter Nb

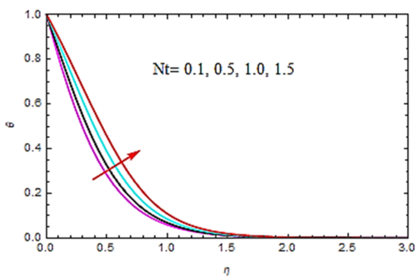

Figure 8. Distribution of θ for thermophoresis parameter Nt

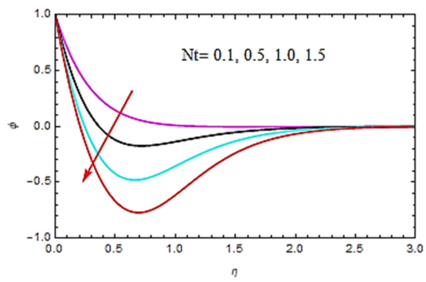

Figure 9. Distribution of ϕ for thermophoresis parameter Nt

Figure 6 is depicted to explore the manipulate of the Brownian motion parameter Nb on energy field θ. It is noted that for increasing value of Brownian motion parameter Nb the energy distribution θ increases. Because of heat transfer rate decreases for increased Nb values. Figure 7, concentration field illustrates growing behavior for the increase of Brownian motion parameter Nb. Figures (8-9) are communicated to imagine the influence of Nt on the energy and concentration distribution. In figures 8, it is observed that temperature function θ establish a growth for rising values of thermophoresis parameter. Figure 9 is drawn to explore the influence of the thermophoresis parameter Nt on concentration field ϕ. It is cleared that for increasing value of thermophoresis parameter Nt the concentration distribution

ϕ declines. Basically, heated particles move away from high temperatures, increasing fluid temperature. Figure 10 characterizes that by improving the values of (seen with the variable thermal conductivity) small parameter ϵ energy distribution θ also augmented. Figure 11 represents that by increasing the value of thermal radiation parameter Nr fluid temperature also increased. Physically, increase in thermal Radiation parameter increases energy flow to fluids). Figure 12 is pinched to investigate the stimulus of the Schmidt Sc on concentration field ϕ. It is evaporated that for increasing value of Schmidt Sc the concentration field ϕ declines.

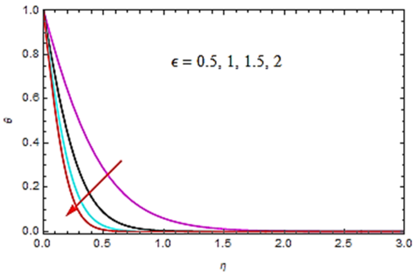

Figure 10. Distribution of θ for small parameter ϵ

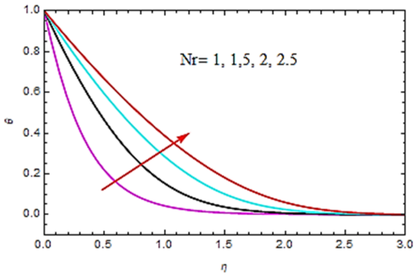

Figure 11. Distribution of θ for radiation parameter Nr

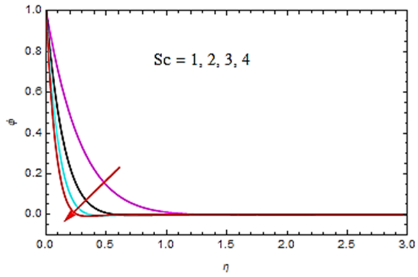

Figure 12. Distribution of ϕ for Schmidt number Sc

CONCLUSION

In this investigation, tangent hyperbolic flow of nanofluid due to stretched sheet with joule heating, and heat source is examined. Chemical diffusion and thermal radiation are part of this study. First of all, velocity, temperature and volumetric concentration equations are transformed into the set of ODEs by expending similarity variables. The resulting system of ODEs is numerically handled by using well-established shooting scheme and acquired numerical outcomes. These outcomes are useful in the field of engineering and technology due to heat transfer.

The key findings are listed below.

- By increasing the values of power-law index n and Hartmann number M velocity profile declines.
- By increasing values of thermal radiation Nr, thermophoresis parameter Nt and Brownian motion parameter Nb temperature field θ also increases.
- Strong abatement in temperature distribution is noted for raising values of Pr.
- Concentration and temperature and increase by increasing thermophoresis parameter Nt
- Concentration decreases by increasing Schmidt number Sc.

Conflict of interest: The authors announce that no conflict of curiosity exists.

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ДОСЛІДЖЕННЯ ТЕПЛОВИПРОМІНЮВАЛЬНОГО ДОТИЧНОГО ГІПЕРБОЛІЧНОГО ПОТОКУ НАНОРІДИНИ В УМОВАХ РОЗТЯГНЕННЯ ПОВЕРХНІ

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Поточне дослідження висвітлює потік дотичного гіперболічного нанофлюїду повз двонаправлену поверхню, що розтягується. Обговорювалися явища тепло- та масопереносу з джоулевим нагріванням, хімічними реакціями та тепловим випромінюванням. Для постановки задачі частиною цього дослідження були конвективні граничні умови та наявність джерела тепла. Змодельовані диференціальні рівняння в часткових похідних перетворюються на звичайні диференціальні рівняння за допомогою відповідних самоподібних перетворень. Крім того, отримана система ODE чисельно обробляється за допомогою добре налагодженого метода стрільби, а отримані числові результати порівнюються з командою ND Solve у Mathematica. Вплив основних параметрів на швидкість, температуру та розподіл об’ємної концентрації перевіряється за допомогою графіків. Детально розглянуто вплив нових параметрів, залучених до цього дослідження, на особливості потоку та відведення тепла. Зі збільшенням значень степеневого індексу n, числа Прандтля Pr і магнітного параметра M спостерігається збільшення коефіцієнта шкірного тертя при зниженні числа Нуссельта.

Ключові слова: *метод стрільби; дотична гіперболічна нанорідина; МГД; Джоулеве нагрівання; хімічна реакція*