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HOLE-HOLE COLLECTIVE EXCITATIONS IN $^{106,\,112,\,130}$Sn ISOTOPES†

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In this paper, energy-level schemes of neutron rich and reduced electric transition strengths of Tin isotopes ^{106}Sn, ^{112}Sn and ^{130}Sn were studied using collective models, i.e., hole-hole Tamm-Dancoff Approximation (*hh* TDA) and hole-hole Random Phase Approximation (*hh* RPA). According to these models, the excited states of closed core *A*-2 systems with multipolarity *J* and isospin *T* can be described as a linear combination of hole-hole (*hh*) pairs. Therefore, in our approach, the low-lying states of the investigated isotopes ^{106}Sn, ^{112}Sn and ^{130}Sn are obtained by acting two-hole operators on a correlated core ^{108}Sn, ^{114}Sn and ^{132}Sn, respectively. The Hamiltonian is diagonalized within the model space include {$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$ and $1h_{11/2}$} orbits, using the matrix elements of neutron-neutron (N-N) interaction and modified surface delta interaction (MSDI). The *hh* TDA and *hh* RPA are checked by using the resultant eigenvalues and eigenvectors to calculate the excitation energies and reduced electric transition strengths. A comparison had been made between our theoretical predictions and the recent available experimental data. Reasonable agreements were obtained from these comparisons.
Keywords: *energy-level schemes; collective excitations; hh RPA; hh TDA*
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1. INTRODUCTION

The study of the Sn isotopes is intriguing from a theoretical perspective because they provide a special testing ground for calculations of nuclear structure. In the literature, several theories have been proposed to explain the shell model calculations, for neutron-deficient Sn isotopes by Engeland et al. [1], Covello et al. [2] and Schubert et al. [3] taking ^{100}Sn as core, for ^{106,107,108,109}Sn isotopes using CD-Bonn and Nijmegen1 interactions by E. Dikman [4], for exotic $^{134.136.138,140}$Sn isotopes with a realistic effective interaction [5], for even $^{102\text{-}108}$Sn and odd $^{103-107}$Sn isotopes using different interactions by T. Trivedi, et.al. [6], for ^{104,106,108}Sn based on the CD-Bonn nucleon-nucleon interaction by Khalid S. Jassim [7], for even-even $^{100\text{-}108}$Sn isotopes by with the effective interactions Snet, SN100PN Delta interaction by Khalid H.H. Al-Attiah et.al. [8].

Fouad A. Majeed and Sarah M. calculated the energy levels, binding energy, and reduced transition probabilities B(E2;$0^+ \rightarrow 2^+$) for even-even $^{134,\,136}$Sn, and $^{134,\,136}$Te around doubly magic core ^{132}Sn by using shell model code Nushellx@MSU for Windows and employing the effective interactions jj56pna, jj56pnb, kh5082, cw5082, jj56cdb, and khhe [9]. Fatema H. Obeed and Baneen S. Abed, used the surface delta and modified surface delta interactions by applying the nuclear shell model to calculate values of excitation energies for isotopes of equal mass number containing two nucleons outside the closed core ^{114}Sn, these nuclei are; the isotope (Tin) ^{116}Sn contains two neutrons within the model space ($3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$) and the other isotope is ^{116}Te (Tellurium) contains two protons within the model space ($1g_{7/2}$, $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$) [10]. A. K. Hasan et al., investigated on the energy levels and transition probabilities B ($E2$; ↓) and B ($M1$; ↓) for ^{21,23}Na isotopes by using the (USDA and USDB) interactions in the (*sd*-shell) model space [11]. S. Akkoyun, investigated the nuclear structure properties of A=49 isobars, the double-magic ^{40}Ca nucleus was considered an inert core, and fp model space was taken into account for the valance nucleons [12]. In 2022, Mustafa M. Jabbar and Fatema H. Obeed, used the surface delta and modified surface delta interactions by applying the nuclear shell model to calculate values of excitation energies for ^{92}Nb and ^{92}Mo [13].

One of the significant and practical models that became a widely accepted theoretical model in the nuclear physics structure is the nuclear shell model [14, 10]. According to the Pauli principle, we fill the shells in this model in order of increasing energy, resulting in an inert core of filled shells and a certain number of valence nucleons. The model then presupposes that the valence nucleons are primarily responsible for determining nuclear properties [16].

The excited states of A-2 nuclei can be characterized as a linear combination of hole-hole (*hh*) pairs, such an approximation named Tamm-Dancoff Approximation *hh* TDA [17, 18]. When the ground states and excited states are treated more symmetrically, i.e., both the ground state and the excited states may be characterized as linear combinations of hole-hole states, both the ground state and the excited states are treated equally in this scenario, such an approximation is referred as the hole-hole Random Phase Approximation *hh* RPA [19-21].

In this paper, hole-hole excitations of some $^{106,\,112,\,130}$Sn (Z=50) isotopes were investigated using PPRPA code version 1, 2015 [22] in the presence of MSDI and N-N interactions. Our approach assumes that the low-lying states of ^{106}Sn, ^{112}Sn and ^{130}Sn by acting two-hole operators on a correlated core ^{108}Sn, ^{114}Sn and ^{132}Sn, respectively. A comparison had been made between our theoretical predictions and the recent available experimental data.

2. THEORY

A TDA generalization is the RPA. Bohm and Pines first presented this approach for investigating the plasma oscillations of the electron gas [20]. By using annihilation operators on the ground state $|A,0\rangle$ of the A-nucleon system, collective excited states of A-2 systems with multipolarity J and isospin T are produced [24],

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$$|A-2,\lambda,JT\rangle = \left(\sum_{i\le j} X_{ij}^{\lambda,JT} a_i a_j - \sum_{m\le n} Y_{mn}^{\lambda,JT} a_m a_n\right)|A,0\rangle \quad (1)$$

where a is annihilation operator, X and Y are amplitudes. indices ij and mn represent the quantum numbers of hole and particle stats, respectively. The compact matrix form of hh RPA equations has the following form [23, 24],

$$\begin{pmatrix} A_{iji'j'}^{\lambda,JT} & B_{ijmn}^{\lambda,JT} \\ B_{ijmn}^{\dagger,\lambda,JT} & C_{mnm'n'}^{\lambda,JT} \end{pmatrix} \begin{pmatrix} X_{ij}^{\lambda,JT} \\ Y_{mn}^{\lambda,JT} \end{pmatrix} = E_x \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X_{ij}^{\lambda,JT} \\ Y_{mn}^{\lambda,JT} \end{pmatrix} \quad (2)$$

with

$$A_{iji'j'}^{\lambda,JT} = -(\varepsilon_i + \varepsilon_j)\delta_{ii'}\delta_{jj'} + V_{iji'j'}$$

$$B_{ijmn}^{\lambda,JT} = -V_{ijmn}$$

$$C_{mnm'n'}^{\lambda,JT} = (\varepsilon_m + \varepsilon_n)\delta_{mm'}\delta_{nn'} + V_{mnm'n'} \quad (3)$$

where E_x is the excitation energy, $\varepsilon_i$is single particle energy and V_{ijmn} is antisymmetrized two-body matrix elements. A, B and C are submatrices of dimensions $n_h \times n_h, n_h \times n_p, n_p \times n_p$, respectively. If the sub-matrices C and B are vanished, the RPA equations will be reduced to TDA equation.

The antisymmetric matrix elements of the modified surface delta interaction (MSDI) used in this work has the form [25],

$$V_{ab,cd}^{JT} = \frac{1}{2}A_T(-1)^{n_a+n_b+n_c+n_d} \times \sqrt{\frac{(2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)}{(1+\delta_{ab})(1+\delta_{cd})}} \times \left\{(-1)^{j_a+j_b+j_c+j_d}\begin{pmatrix} j_a & j_b & J \\ \frac{1}{2} & \frac{-1}{2} & 0 \end{pmatrix} \times \begin{pmatrix} j_c & j_d & J \\ \frac{1}{2} & \frac{-1}{2} & 0 \end{pmatrix}[1 - (-1)^{J+T+l_c+l_d}] - \begin{pmatrix} j_a & j_b & J \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} j_c & j_d & J \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix}[1 + (-1)^T] + \{[2T(T+1) - 3]B + C\}\delta_{ac}\delta_{bd}\right\} \quad (4)$$

where A_0, A_1, B and C are the strength parameters of the (MSDI) and $A_0 = 0$ or 1 for $T = 0$ (isoscalar) or $T = 1$ (isovector), respectively.

In terms of RPA amplitudes X and Y, the radial transition density of state $|\tau, JT\rangle$ is [20,21],

$$\rho_{\tau,JT} = \sqrt{\frac{2T+1}{8\pi(2J_i+1)}}\, e(T) \sum_{T=0,1}(-)^{T_f - T_z}\begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \times \sum_{ab}\left[X_{ab}^{\omega,JT} + Y_{ab}^{\omega,JT}\right]\langle a\|Y_J\|b\rangle\, R_{n_a\ell_a}(r) R_{n_b\ell_b}(r) \quad (5)$$

Where $e(T) = e\left(\frac{1}{2}\right) + (-1)^T e\left(-\frac{1}{2}\right)$, $T_z = \frac{A-N}{2}$. The reduced matrix elements of spherical harmonic can be written as [20],

$$\langle a\|Y_J\|b\rangle = (-1)^{j_a+\frac{1}{2}}\sqrt{\frac{(2j_a+1)(2j_b+1)(2J+1)}{4\pi}} \times \begin{pmatrix} j_a & J & j_b \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}\frac{1}{2}\left[1 - (-1)^{\ell_a+J+\ell_b}\right] \quad (6)$$

and the radial wave function R of harmonic oscillator potential is given by [6],

$$R_{n\ell} = \frac{1}{(2\ell+1)!!}\sqrt{\frac{2^{\ell-n+3}(2n+2\ell-1)}{b^3\pi^{\frac{1}{2}}(n-1)!}}\left(\frac{r}{b}\right)^{\ell} e^{-\frac{r^2}{2b^2}} \times \sum_{k=0}^{n=1}(-1)^k \frac{(n-1)!2^k}{(n-k-1)!k!}\frac{(2\ell+1)!!}{(2\ell+2k+1)!!}\left(\frac{r}{b}\right)^{2k} \quad (7)$$

Thus, the reduced transition probability becomes [22],

$$B(EJ) = (2J+1)\left|\int r^{J+2}\,\rho_{\tau,JT}\,dr\right|^2 \quad (8)$$

3. RESULTS AND DISCUSSIONS

In the present study, the nuclear structures of Tin isotopes: ^{106}Sn, ^{112}Sn, ^{130}Sn are studied in the framework of *hh* TDA and *hh* RPA using PPRPA code version 1, 2015 [22]. The Hamiltonian is diagonalized within the model space include {$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$ and $1h_{11/2}$} orbits, using N-N interaction and modified surface delta interaction (MSDI), by choosing the coefficients values of the interaction strengths A_T by the values of {A_1 = 0.40 MeV, A_2=0.42 MeV, B = 0.5 MeV, C = 0 MeV} for ^{106}Sn, {A_1 = 0.20 MeV, A_2=0.35 MeV, B = 0.0 MeV, C = 0.0 MeV} for ^{112}Sn and {A_1 = 0.20 MeV, A_2=0.25 MeV, B = 0.0 MeV, C = 0.0 MeV} for ^{130}Sn , represented of the modified surface delta interaction. Our approach assumes that the low-lying states of ^{106}Sn are obtained by acting *hh* operator on a correlated ^{108}Sn core, of ^{112}Sn are obtained by acting *hh* operator on a correlated core of ^{114}Sn isotope, of ^{130}Sn are obtained by acting *hh* operator on a correlated core of ^{132}Sn isotope. The single particle energies are given in Table 1.

Table 1. Single particle energies used in different shell model approaches.

Orbits (*nlj*)	S.P. Energy (MeV)
$1g_{7/2}$	5.6014
$2d_{5/2}$	5.2819
$2d_{3/2}$	3.7090
$3s_{1/2}$	3.7077
$1h_{11/2}$	3.9843

The calculated eigenvalues and experimentally known energy levels for ^{106}Sn and ^{112}Sn are tabulated in Tables 2 and 3, and are also plotted in Figs. 1 and 2. The *hh* RPA results are plotted in columns (second and third) for both interactions MSDI and N-N respectively, while the *hh* TDA results are plotted in columns (fourth and fifth) for both interactions MSDI and N-N respectively. The calculated results are compared with the experimental data (first column). Both the interactions predict very well the ordering of levels.

Table 2. Our calculated low-lying states (MeV) of ^{106}Sn using MSDI and N-N interactions for the *hh* RPA and *hh* TDA in comparison with the experimental data [26].

J^π	EXP.	J^π	RPA, MSDI	J^π	RPA, N-N	J^π	TDA, MSDI	J^π	TDA, N-N
0^+	0	0^+	0	0^+	0	0^+	0	0^+	0
2^+	1.208	1^+	1.201	1^+	0.638	1^+	0.842	1^+	0.635
4^+	2.019	7^+	1.532	7^+	0.968	7^+	1.027	7^+	0.821
6^+	2.325	2^+	1.74	2^+	1.173	2^+	1.28	2^+	1.035
		3^+	1.768	3^+	1.205	3^+	1.285	3^+	1.079
		5^+	1.807	5^+	1.244	5^+	1.304	5^+	1.098
		4^+	1.979	4^+	1.617	4^+	1.484	4^+	1.472
		6^+	2.086	6^+	1.796	6^+	1.582	6^+	1.648

Table 3. Our calculated low-lying states (MeV) of ^{112}Sn using MSDI and N-N interactions for the *hh* RPA and *hh* TDA in comparison with the experimental data [26].

J^π	EXP.	J^π	RPA, MSDI	J^π	RPA, N-N	J^π	TDA MSDI	J^π	TDA, N-N
0^+	0	0^+	0	0^+	0	0^+	0	0^+	0
2^+	1.256	1^+	1.408	1^+	0.772	1^+	1.064	1^+	0.637
2^+	2.15	2^+	2.113	2^+	1.522	2^+	1.729	2^+	1.327
4^+	2.247	2^+	2.568	2^+	1.932	2^+	2.161	2^+	1.733
2^+	2.476	4^+	2.692	0^+	2.024	4^+	2.279	4^+	1.852
6^+	2.549	6^+	2.706	4^+	2.056	6^+	2.293	0^+	1.855
0^+	2.617	3^+	2.735	6^+	2.263	0^+	2.618	6^+	2.061
3^+	2.756	4^+	2.83	2^+	2.377	3^+	2.703	2^+	2.192
4^+	2.783	2^+	2.895	3^+	2.478	2^+	2.736	3^+	2.276
		0^+	3.025	4^+	2.592	4^+	2.802	4^+	2.391

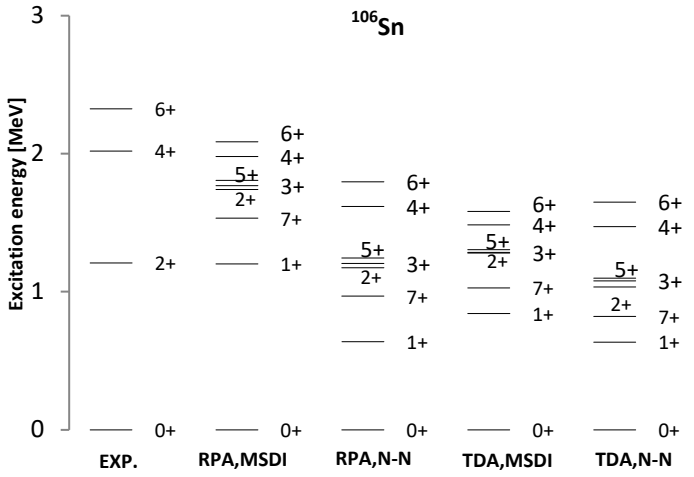

Figure 1. Energy level scheme of ^{106}Sn using MSDI and N-N interactions for the *hh* RPA and *hh* TDA in comparison with the experimental data [26].

For ^{106}Sn, Experimentally, only few excited states with $J^{\pi}= 2^{+}$, 4^{+} and 6^{+} are known, *hh* TDA and *hh* RPA calculations for MSDI and N-N predicted more states than those of experimental data. See Fig. 1.

For ^{112}Sn the obtained theoretical energy level for 1^{+} cannot be predicted by experiment. For example, the first experimental 2^{+} excited states occur at 1.256 MeV, while using *hh* RPA with the MSDI this state was found at 2.113 MeV, but with the N-N interaction the same state was found at 1.522 MeV, and using *hh* TDA with the MSDI this state was found at 1.729 MeV, but with the N-N interaction the same state was found at 1.327 MeV.

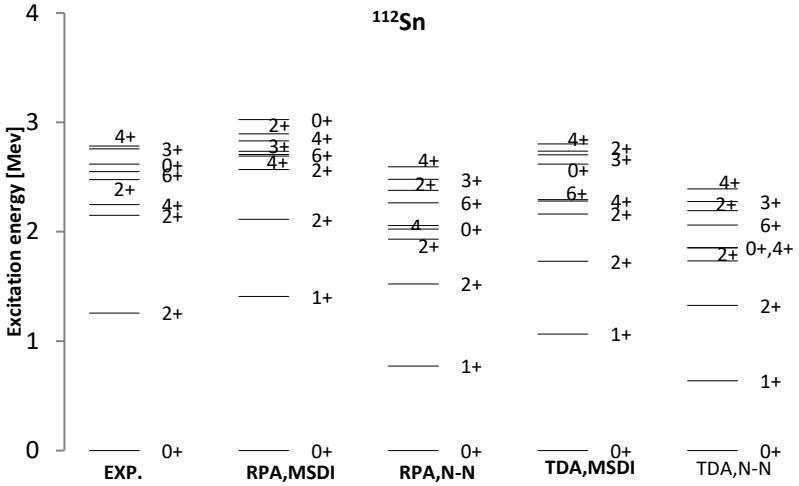

Figure 2. Energy level scheme of ^{112}Sn using MSDI and N-N interactions for the *hh* RPA and *hh* TDA in comparison with the experimental data [26].

In Table 4, we have tabulated all the calculated eigenvalues and experimentally known levels for ^{130}Sn and corresponding energy levels are also plotted in Fig. 3.

Table 4. Our calculated low-lying states (MeV) of ^{130}Sn using MSDI and N-N interactions for the *hh* TDA in comparison with the experimental data [26].

J^{π}	EXP.	J^{π}	TDA, MSDI	J^{π}	TDA, N-N
0^{+}	0	0^{+}	0	0^{+}	0
2^{+}	1.221	1^{+}	1.081	1^{+}	1.222
2^{+}	2.028	2^{+}	1.847	2^{+}	1.933
6^{+}	2.256	4^{+}	2.257	6^{+}	2.378
4^{+}	2.490	5^{+}	2.274	5^{+}	2.415
$4^{+},5^{+}$	2.492	2^{+}	2.298	2^{+}	2.438
3^{+}	3.167	6^{+}	2.367	4^{+}	2.442
4^{+}	3.425	4^{+}	2.544	4^{+}	2.684
2^{-}	4.119	3^{+}	2.986	3^{+}	3.126
9^{-}	4.206	4^{+}	3.145	4^{+}	3.242
3^{-}	4.224	9^{-}	3.811	2^{-}	3.633
4^{+}	4.405	2^{-}	4.184	9^{-}	3.951
4^{+}	4.463	3^{-}	4.222	3^{-}	4.226
4^{+}	5.262	4^{+}	4.459	4^{+}	4.275
		4^{+}	4.460	4^{+}	4.640
		4^{+}	5.688	4^{+}	5.768

The *hh* TDA results are plotted in columns (second and third) for both interactions MSDI and N-N respectively. The calculated results are compared with the experimental data [26] (first column).

For ^{130}Sn, one gets the ground state and low-lying excited states very nicely for both interactions (MSDI and N-N). But the obtained theoretical energy level for 1^{+} is not recorded by experiment.

Table 5 gives the calculated values of the reduced electric transition strengths B(E2) of transition $4^{+}\rightarrow 2^{+}$ for the investigated isotopes without and with the effective charge's $e_p = 1.5e$ and $e_n = 0.5e$. Good agreements were obtained.

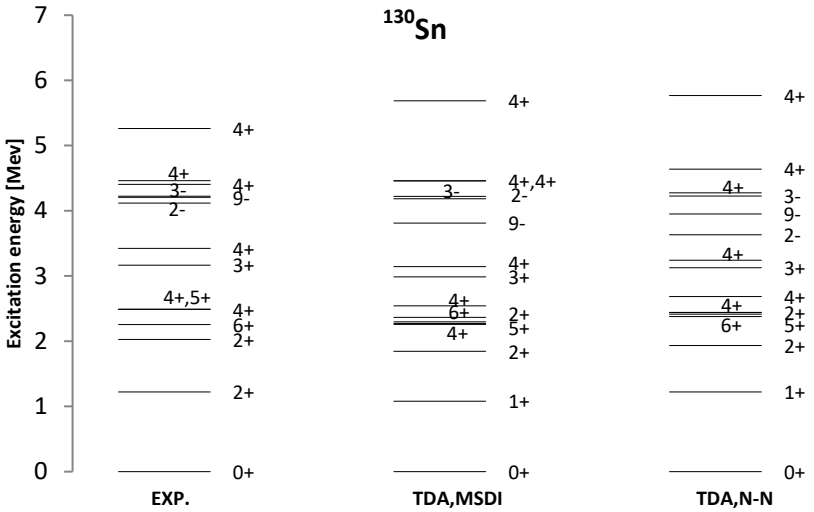

Figure 3. energy levels scheme of ^{130}Sn isotope using *hh* TDA with MSDI and N-N interactions in comparison with the experimental energies [26]

Table 5. reduced electric transition strengths B(E2) in units ($e^2.fm^4$) based on particle-particle calculations using different effective charge values.

Isotope	Exp.	TDA		RPA		Effective charges	
		MSDI	N-N	MSDI	N-N	e_p	e_n
^{106}Sn	811	470	470	450	480	1.0	0.0
		930	930	890	960	1.5	0.5
^{112}Sn	990	340	460	350	470	1.0	0.0
		670	910	690	950	1.5	0.5
	1264	450	480	440	510	1.0	0.0
		910	960	890	1000	1.5	0.5
^{130}Sn	774	500	600			1.0	0.0
		990	1200			1.5	0.5

4. CONCLUSIONS

When the Hamiltonian is diagonalized in the presence of N-N interactions and modifies surface delta interactions (MSDI), the calculated results of *hh* RPA and *hh* TDA are obtained in reasonable agreements to those of experimental data for the investigated Sn isotopes. Both the interactions predict very well the ordering of levels. But for ^{106}Sn, the N-N interaction have the same experimental 2^+ state. The ground states of ^{112}Sn and ^{130}Sn are very nicely predicted. The calculated electric transition strengths B(E2) for the investigated isotopes are well-obtained for the transition $4^+ \rightarrow 2^+$.

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ДІРКОВІ КОЛЕКТИВНІ ЗБУДЖЕННЯ В ІЗОТОПАХ ^{106,112,130}Sn

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У цій статті було вивчено схеми енергетичних рівнів нейтронно-насичених і знижених електричних переходів ізотопів олова ^{106}Sn, ^{112}Sn і ^{130}Sn з використанням колективних моделей, тобто hole-hole наближення Тамма-Данкова (*hh* TDA) і hole-hole апроксимації випадкової фази (*hh* RPA). Згідно з цими моделями, збуджені стани замкнутих ядер A-2 систем з мультиполярністю J і ізоспіном T можна описати як лінійну комбінацію пар hole-hole (hh). Тому в нашому підході низьколежачі стани досліджуваних ізотопів ^{106}Sn, ^{112}Sn і ^{130}Sn отримані шляхом дії hole-hole операторів на корельоване ядро ^{108}Sn, ^{114}Sn і ^{132}Sn відповідно. Гамільтоніан був діагоналізований у просторі моделі, що включає {$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$ and $1h_{11/2}$} орбіти, з використанням матричних елементів нейтрон-нейтронної (N-N) взаємодії та модифікованої поверхні дельта взаємодії (MSDI). hh TDA і hh RPA перевіряються за допомогою результуючих власних значень і власних векторів для розрахунку енергій збудження та зменшених сил електричних переходів. Було проведено порівняння між нашими теоретичними прогнозами та останніми доступними експериментальними даними. У результаті цих порівнянь були отримані розумні згоди.

Ключові слова: *енергетичні схеми; колективні збудження; hh RPA; hh TDA*