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STUDY OF FUSION REACTIONS OF LIGHT PROJECTILES ON LIGHT AND MEDIUM TARGETS†

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The fusion and breakup reactions of some light projectiles on light and medium targets using semi-classical and full quantum mechanical approaches were adopted to calculate the total cross section σ_{fus} and the distribution of the fusion barrier D_{fus} for the systems $^{12}C + ^{48}Ti$, $^{16}O+^{63}Cu$, $^{35}Cl+^{25}Mg$ and $^{35}Cl+^{27}Al$. The coupling between the channel's contribution from elastic and breakup channels were considered to show their importance in the calculations. The results compared with the measured data and shows reasonable matching, and it is shown that the coupling considered is very essential to be considered, especially below the Coulomb barrier V_b.
Keywords: *Coupled-channels; Fusion cross section; Fusion barrier distribution*
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1. INTRODUCTION

One of the most important modern researches fields in nuclear physics is studying the collision of weakly bound stable and radioactive nuclei, around the potential barrier [1]. The understanding of the processes associated with these reactions can be achieved throw adopting both theoretical and experimental offers in order to obtain the best agreement between them. many researches during the last years supported the strong relation between the different nuclear reaction modes starting by the elastic scattering and ending with the fusion reaction, so that will provide the researchers with a wide field of reaction modes to collect more information about the secrets of the nuclear structure and properties of our universe ingredients. The weakly bound systems collisions are very influenced by both transfer and breakup channels which have a very large cross-section according to their low breakup threshold. The projectile mass is one of the most effective factors on the reaction strength, so it is very important to study the reactions with medium mass projectile to examine the relationship between reaction modes and bombarding energy and nucleon number. The fusion of weakly bound colliding projectiles was very effected statically by their fusion barrier characteristics such as its long tail energy which cause a lower barrier and by the way will give an enhancement to the cross section of the fusion at the energies at the sub-barrier [2, 3]. Dynamically, fusion was affected by the different channels coupling, including the elastic, breakup, inelastic and transfer ones [4]. The kinetic energy of the projectile should exceed the corresponding Coulomb barrier for producing nuclear reaction [5].

All of the energy, mass number, momentum, charge, spin and parity are conserved during the nuclear reaction. The Q-value have a great effect on the fusion calculations because it represents the amount of energy emitted or absorbed during the reaction [6]. The two colliding nuclei in the fusion reaction are considered as objects with rigid spherical shape that interact with the potential barrier so the probability of nuclear fusion to accurse represents the ability of the system to penetrate the potential barrier [7], 8]. There are many important factors that have a major role in the experimental determination of the fusion cross section such as the colliding nuclei internal degrees of freedom relative motion, the particles transfer and the nuclear deformation [9, 10]. The collision process is very complex at low temperature, so to be understood it need to unify the description for the different reaction mechanism with a unique nuclear potential [11]. The fusion is very complex reaction because of the combination between the coulomb and nuclear interactions in addition to the effect of the flexible intrinsic synthesis during the reaction and the different reaction channels [12].

This study aims to study the fusion reaction of light projectiles on light targets for the systems $^{12}C + ^{48}Ti$, $^{16}O+^{63}Cu$, $^{35}Cl+^{25}Mg$ and $^{35}Cl + ^{27}Al$ by using semi-classically and full quantum mechanically methods where the coupling between the elastic and breakup channels will be considered.

2. THE SEMI-CLASSICAL TREATMENT

2.1. The single channel theory

In one-dimensional potential model, we need to use the semiclassical approach for the fusion cross-section determination by eliminating the degree of freedom by the relative motion between the colliding heavy ion only. [13,14,15]. Semi-classically, this can be treated by the assumption of energy and momentum independent Schrödinger equation;

$$[-\hbar^2\nabla^2/2u + V(r) - E]\,\psi(r) = 0, \qquad (1)$$

where μ and $V(r)$ are the reduced mass and total energy potential of the system respectively. The time dependence function can be used to determine the semiclassical amplitudes by evaluating the particle trajectory using classical dynamics, including all the potential types, as;

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$$V(r) = V_C(r) + V_N(r) + V_l(r). \tag{2}$$

In addition, the complex potential which represents the imaginary part of the nuclear potential, should be contained.

$$V_N(r) = U_N(r) - iW(r). \tag{3}$$

The above method can be used to study the effect of the nuclear potential with its real and imaginary parts on the interacting l waves [13,16,17]. According to the semi-classical theory, the fusion takes place when the nuclei be closer to the barrier, and the WKB approximation can be used to determine the penetration probability below the barrier [13,18,19,20].

$$P_{fus}^{WKB}(\ell, \mathrm{E}) = \frac{1}{1+e^{\left[2\int_{r_b^{(l)}}^{r_a^{(l)}} \mathrm{k}_l(\mathrm{r})dr\right]}}. \tag{4}$$

Then it can be simplified as:

$$P_{fus}^{WKB}(l, E) = \frac{1}{1+e^{\left[\frac{2\pi}{\hbar\Omega_l}(V_b(l)-E)\right]}}. \tag{5}$$

Where $r_b^{(l)}$ and $r_a^{(l)}$ represent the turning points of the fusion barrier potential for its inner and outer and $\kappa_l(r)$ is the wave number. If a parabolic function used as an approximation for the fusion barrier, then the Hill–Wheeler formula can be used to find the penetration probability above the barrier [13].

$$P_{fus}^{WH}(l, E) = \frac{1}{1+e^{\left[\frac{2\pi}{\hbar\Omega_l}(E-V_b(l))\right]}}. \tag{6}$$

E is the bombarding energy and $V_b(l)$ is the height parameter of the partial wave fusion barrier with curvature parameter Ω_l. The fusion cross-section can be determined by using the WKB approximations as [17, 21]:

$$\sigma_{fus}(\mathrm{E}) = \frac{\pi}{\mathrm{k}^2}\sum(2l+1)P_{fus}^{WKB}(\ell, \mathrm{E}), \tag{7}$$

$$P_{fus}^{\gamma}(\ell, \mathrm{E}) = \frac{4\mathrm{k}}{\mathrm{E}}\int dr\left|u_{\gamma l}(\mathrm{k}_\gamma, \mathrm{r})\right|^2 W_{fus}^{\gamma}(\mathrm{r}). \tag{8}$$

In the above equation, $u_{\gamma l}(k_\gamma, r)$ refers to the wave function of the radial part in γ channel, and the potential imaginary part denoted by $W_{fus}^{\gamma}(\mathrm{r})$.

The using of semiclassical theory to compute heavy ions fusion cross section by approximating the trajectory r, and the projectile intrinsic states (ξ) using the Coupled-Channel Continuum Discretized (CCCD) method with the helpful of Winther and Alder (AW) theory [21,22,23,24,25]. The Hamiltonian of the projectile is,

$$h = h_0(\xi) + V(\xi, \mathrm{r}), \tag{9}$$

where $\mathrm{h}_0(\xi)$ is the Hamiltonian fundamental states and $V(\xi, \mathrm{r})$ is the interaction potential that determine as;

$$V(\xi, \mathrm{r}) = V_N(\xi, \mathrm{r}) + V_C(\xi, \mathrm{r}).$$

The path of Rutherford transmits on the reaction energy,E, and the momentum, l. Classically, the potential can be solved as;

$$V(\mathrm{r}) = \langle\psi_0|\, V(\mathrm{r}, \xi)|\psi_0\rangle,$$

where Ψ_0 refers to the bounded state of the projectile. Therefore, time dependence Schrödinger equation have been satisfied inn both ξ-space $V_l(\xi.t) = V(\mathrm{r}_{l(t)}, \xi)$ and Hamiltonian for intrinsic eigenstates $|\psi_\gamma\rangle$ [26, 27],

$$h|\psi_\gamma\rangle = \varepsilon|\psi_\gamma\rangle. \tag{10}$$

The wavefunction expansion as a function of the intrinsic ground sate is,

$$\psi(\xi, t) = \sum a_\gamma(l, t)\psi_\gamma(\xi)e^{-i\varepsilon_\gamma t/\hbar}. \tag{11}$$

Then the AW equation can be written as;

$$i\hbar\, \dot{a}_\gamma(l, t) = \sum_\epsilon\langle\psi_\gamma|\, V(\xi, t)|\psi_\gamma\rangle\, e^{i(\varepsilon_\gamma-\varepsilon_\epsilon)t/\hbar}\gamma\epsilon(l, t). \tag{12}$$

The AW equations can be computed by the assumption of the ground state at initial conditions $a_\gamma(l, t\to-\infty) = \delta_{\gamma 0}$. The final population of γ-channel final population of the collision is $P_{fus}^{\gamma}(l, \mathrm{E}) = \left|a_\gamma(l, t\to-\infty)\right|^2$, where l is the angular momentum [17].

2.2. The Coupled Channel Description

The dynamics of the projectile-target can be described by using $\vec{r}$ and ξ in the projectile intrinsic Hamiltonian $H_0(\xi)$ and the interaction of the projectile-target $V(\vec{r},\xi)$ as; [28],

$$H = H_0(\xi) + V(\mathrm{r},\xi). \tag{13}$$

The eigenstates of $H_0(\xi)$ is [28],

$$H_0|\varphi_\beta\rangle = \varepsilon_\beta|\varphi_\beta\rangle, \tag{14}$$

ε_β is the internal motion energy.

There are two steps to consider the AW method. First, the evolution of time of the variable $\vec{r}$ has been considered classically. The energy E, and the momentum $\hbar\ell$ are the two effected parameters on the path with $V(\vec{r}) = \langle\varphi_0|V(\vec{r},\xi)|\varphi_0\rangle$, where $|\varphi_0\rangle$ represents the ground level. The coupling will be a time dependence and $V_\ell(\xi,t) \equiv V(\vec{r}_\ell(t),\xi)$. Second, the quantum mechanical time-dependent problem has been used to treat the dynamics in the intrinsic apace. Throw expanding the wavefunction as [29],

$$\psi(\xi,t) = \sum_\beta a_\beta(\ell,t)\varphi_\beta(\xi)e^{-i\varepsilon_\beta t/\hbar}, \tag{15}$$

the AW equations can be evaluated by substituting the above expansion into Schrodinger equation, we get [32],

$$i\hbar\dot{a}_\beta(\ell,t) = \sum_\alpha a_\gamma(\ell,t)\langle\varphi_\beta|V_\ell(\xi,t)|\varphi_\beta\rangle e^{-i(\varepsilon_\beta-\varepsilon_\gamma)t/\hbar}. \tag{16}$$

Under the initial conditions at the ground state the solution of the coupled differential equations can to be obtained by assuming $a_\beta(\ell,t\to-\infty) = \delta_{\beta 0}$, at $(t\to-\infty)$. The final population is $P_\ell^{(\beta)} = |a_\beta(\ell,t\to+\infty)|^2$, where ℓ is the angular momentum at β channel. the integration of the cross section gives [29,30],

$$\sigma_\beta = \frac{\pi}{k^2}\sum_\ell(2\ell+1)P_\ell^{(\beta)} . \tag{17}$$

For a simple determination of the fusion reaction cross section, the whole contribution channels can be assumed to be bound to zero spin. Using the expansion of the wave function for all contributions, leads to [18],

$$\sigma_F = \sum_\beta\left[\frac{\pi}{k^2}\sum_\ell(2\ell+1)P_\ell^F(\beta)\right], \tag{18}$$

with,

$$P_\ell^F(\beta) = \frac{4k}{E}\int W_\beta^F(r)\left|u_{\beta\ell}(k_\beta,r)\right|^2 dr. \tag{19}$$

Where W_β^F is the imaginary part of the optical potential in the channel β and $u_{\beta\ell}(k_\beta,r)$ its ℓth-partial wavefunction. The approximated formula can be adopted to find the cross section with the help full of AW, as [28],

$$P_\ell^F(\beta) \simeq \overline{P}_\ell^{(\beta)}T_\ell^{(\beta)}(E_\beta). \tag{20}$$

above, $\overline{P}_\ell^{(\beta)}$ represents the probability of β -channel for the system to be at classical trajectory, and $T_\ell^{(\beta)}(E_\beta)$ is the probability for the particle at $E_\beta = E - \varepsilon_\beta$ and reduced mass $\mu = M_P M_T/(M_P + M_T)$, referring to the masses of the projectile and target by M_P, M_T, respectively, [28].

By using loosely bound projectiles, the CF for some systems will be studied. For simplicity, the projectile ground state is considered to be the only bound one in which the breakup reaction achieved in two parts, F_1 and F_2 . therefore, it will be referred to the ground and breakup states by the labels $\beta = 0$ and $\beta \neq 0$, respectively. If the sequential contribution has been neglected, the CF can only contribute from the elastic channel. So, σ_{CF} is determined as,

$$\sigma_{CF} = \frac{\pi}{k^2}\sum_\ell(2\ell+1)P_\ell^{Surv}T_\ell^{(0)}(E) . \tag{21}$$

P_ℓ^{Surv} is the survival probability, which is given by; [18].

$$P_\ell^{Surv} \equiv \overline{P}_\ell^{(0)} = |a_0(\ell,t_{ca})|^2 . \tag{22}$$

2.3. Quantum Mechanical Approximation

The relative motion between the colliding nuclei in addition to the nuclear intrinsic degrees of freedom need to be studied quantum mechanically by assuming $\Psi(\mathbf{r},\xi)$ to be the entire wave function for the reaction with r represents the separation vector of the projectile and target while ξ refers to their intrinsic coordinates set. By the Hamiltonian, the reaction dynamics can be determined as [18],

$$H = H_0 + T + U(23).$$

In which H_0 represents the inherent Hamiltonian, T is the operator of the energy associated with the collide nuclei movement which given as $T \equiv -\hbar^2 \nabla^2/2\mu$, and the potential of the interaction $U \equiv U(\mathbf{r}, \xi)$. an intrinsic Hamiltonian with eigenstates $|\alpha\rangle$ can satisfy the Schrödinger equation as [13],

$$(e_\eta - H_0)|\alpha\rangle = 0. \tag{24}$$

With

$$\langle\alpha'|\alpha\rangle = \int d\xi\ \varphi^*_{\alpha'}(\xi)\ \varphi_\alpha(\xi) = \delta_{\eta\eta'}, \tag{25}$$

where the wave function $\varphi_\alpha\ (\xi)\ \left(\ \varphi_{\alpha'}\ (\xi)\right)$ is corresponding to $|\alpha\rangle$ ($|\alpha\rangle$) state in the ξ- space. The potential represented as,

$$U = U' + U'', \tag{26}$$

where U' is the channel space diagonal, such that [13],

$$U(\mathrm{r}) = \int d\xi\ |\varphi_\alpha(\xi)|^2\ U'(\mathrm{r}, \xi), \tag{27}$$

$$U''_{\alpha,\alpha'}(\mathrm{r}) = \int d\xi\ \varphi^*_{\alpha'}(\xi)\ U''(\mathrm{r}, \xi)\ \varphi_\alpha(\xi). \tag{28}$$

The potential U' is not random for the diagonal in channel space. It is appropriate to take U' in the case of U'' is not diagonal and $U'' = U - U'$ with [13],

$$U''_{\alpha,\alpha'}(\mathbf{r}) = \int d\xi\ \varphi^*_{\alpha'}(\xi)\ U''(\mathbf{r}, \xi)\ \varphi_\alpha(\xi) - \ \delta_{\alpha\alpha'}\ U'_\alpha(\mathbf{r}), \tag{29}$$

from the Schrödinger equation, the equation of the coupling is,

$$(E - H)\ |\Psi_\alpha(\alpha_0\ \mathbf{k}_0)\rangle = 0, \tag{30}$$

and the expansion,

$$|\Psi_\alpha(\alpha_0\ \mathbf{k}_0)\rangle = \textstyle\sum_\alpha |\psi_\alpha(\alpha_0\ \mathbf{k}_0)\rangle\ |\alpha\rangle, \tag{31}$$

where $|\Psi(\alpha_0\ \mathbf{k}_0)\rangle$ represents the collision initiating in channel α_0, $\mathbf{k}_0$ is the wave vector, the energy scale was chosen to be $e_{\alpha_0} = 0$. The Schrödinger equation solution components according to the off-diagonal part of the reaction are $|\Psi_\alpha(\alpha_0\ \mathbf{k}_0)\rangle$ for $\alpha = \alpha_0$ and $\alpha \neq \alpha_0$. The Hamiltonian written as [13],

$$H = H_0 + H' + U''. \tag{32}$$

We can get the coupled channel equations from Eqs. (24), (25) and (23) as, [13],

$$(E_\alpha - H'_\alpha)|\psi_\alpha(\alpha_0\ \mathbf{k}_0)\rangle = \textstyle\sum_{\beta'} U''_{\alpha,\alpha'}(\mathbf{r})\ |\psi_{\alpha'}(\alpha_0\ \mathbf{k}_0)\rangle. \tag{33}$$

With using $|\psi_\alpha(\alpha_0\ \mathbf{k}_0)\rangle \longrightarrow \psi_\alpha(\mathbf{r})$ in Eq. (25), we get,

$$U'_\alpha = V_\alpha + iW_\alpha. \tag{34}$$

The imaginary part W_α refers to the flux gain by the other channels from channel α. The non-Hermitian nature of H leads to break down of the continuity equation, while for the Hermitian U''_α in the coupled channel interaction, the continuity equation has the form [30].

$$\nabla \cdot \textstyle\sum_\alpha \mathbf{J}_\alpha = \frac{2}{\hbar} \sum_\alpha W_\alpha(\mathbf{r})\ |\psi_\alpha(\mathbf{r})|^2 \neq 0. \tag{35}$$

$\mathbf{J}_\alpha$ represents the probability density. By taking the integration of Eq. (39) in spherical region covering the interaction area and with the helpful of σ_α definition, we get [22],

$$\sigma_\alpha = \frac{k}{E} \textstyle\sum_\alpha \langle\psi_\alpha|W_\alpha|\psi_\alpha\rangle, \tag{36}$$

the potential absorption is given as;

$$W_\alpha = W^D_\alpha + W^F_\alpha, \tag{37}$$

where W^D_α refers to the lost flux and W^F_α refers to the fusion absorption in channel α, the total cross section represented as [22],

$$\sigma_F = \frac{k}{E} \textstyle\sum_\alpha \langle\psi_\alpha|W^F_\alpha|\psi_\alpha\rangle. \tag{38}$$

3. FUSION BARRIER DISTRIBUTION

The ability of the wave to cross a barrier is influenced by the fusion due to the opposite nuclear and Coulomb forces. One of the most important factors that should be taken into account is the fusion barrier distribution D_{fus} that can be divided into several to describe the coupling effect as [13, 18],

$$D_{fus}(E) = \frac{d^2F(E)}{dE^2}. \quad (39)$$

$F(E)$ in the above equation is given by;

$$F(E) = E\sigma_{fus}(E). \quad (40)$$

It is very important to understand the fusion reaction through collecting information on the collision coupling channels from the distribution of the fusion barrier, the most important progress of that understanding can be achieved from the experimental data of the reaction. From the above equation we have numerical uncertainties that appears from the barrier distribution data extraction [28, 29].

$$D_{fus}(E) \approx \frac{F(E+\Delta E)+F(E-\Delta E)-2F(E)}{\Delta E^2}, \quad (41)$$

where ΔE is the energy value between the measured total fusion reaction cross sections. There is a statistical error associated to the fusion barrier distribution that can be determined from Eqn. 24, as [30],

$$\delta D_{fus}^{stat}(E) \approx \frac{\sqrt{[\delta F(E+\Delta E)]^2+[\delta F(E-\Delta E)]^2+4[\delta F(E)]^2}}{(\Delta E)^2}, \quad (42)$$

where $\delta F(E)$ refers to the confidence $(E\sigma_f)$ product at a certain energy of the reaction. The uncertainty can be given as [18],

$$\delta D_{fus}^{stat}(E) \approx \frac{\sqrt{6}\delta F(E)}{(\Delta E)^2} \quad (43)$$

4. RESULT AND DISCUSSION

The effect of the breakup channel on fusion reaction have been studied by adopting the semi-classical theory performed using code SCF and the coupled channel with continuum discretized (CCCD) conducted by the code CC to obtain the fusion cross section (σ_{fus}) and the fusion barrier distribution (D_{fus}) for the systems ^{12}C + ^{48}Ti , ^{16}O+^{63}Cu , ^{35}Cl+^{25}Mg and ^{35}Cl + ^{27}Al . The WS potential parameters are tabularized in Table 1.

Table 1. The WS potential parameters for the studied systems

	Real parts			imaginary parts					
Systems	V_0 **(MeV)**	r_0 **(fm)**	a_0 **(fm)**	W_0 **(MeV)**	r_i **(fm)**	a_i **(fm)**	L_{min}	L_{max}	V_b **(MeV)**
^{12}C + ^{48}Ti	-33.9	0.99	0.52	-10.5	0.923	0.777	0	21	24.68
^{16}O+^{63}Cu	-91.9	1	0.9	-27.6	0.931	0.769	0	39	32.72
^{35}Cl+^{25}Mg	-100	1	0.8	-29.6	0.935	0.765	0	45	30.4
^{35}Cl +^{27}Al	-63.8	1.18	0.74	-18.8	0.937	0.763	0	44	30.7

4.1 ^{12}C + ^{48}Ti System

The obtained σ_{fus} and D_{fus} for ^{12}C + ^{48}Ti are drown in Figure 1 with its labels (a) and (b), respectively.

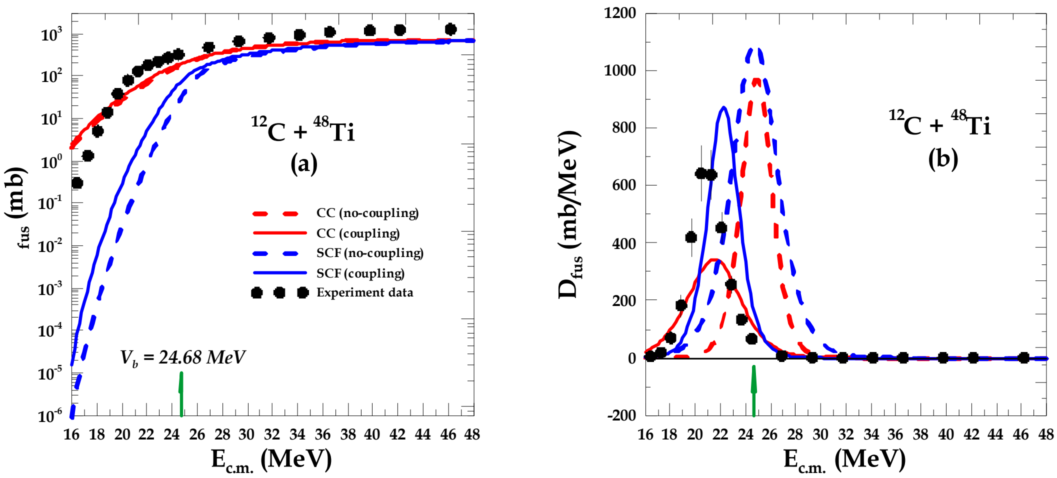

Figure 1. The semiclassical calculations with the blue colour and quantum mechanical calculations with the red colour for both σ_{fus} and D_{fus} in panels (a) and (b) respectively for the system **^{12}C + ^{48}Ti**

The semiclassical calculations are represented in blue colour curves, while quantum mechanical calculations are represented in red colour curves. The solid and dashed curves represent the calculations with and without the channel coupling respectively. Figure 1 show that the best obtained calculations for both σ_{fus} and D_{fus} under and above the Coulomb barrier V_b are those including the channel coupling in the quantum mechanical calculations.

4.2 ^{16}O+^{63}Cu System

The calculations for σ_{fus} are more accurate for those treated using the simiclassical treatment with channel coupling as shown in panel (a) of Figure 2, while the best calculations for D_{fus} are those treated using the quantum mechanical treatment with channel coupling as shown in panel (b) of Figure 2.

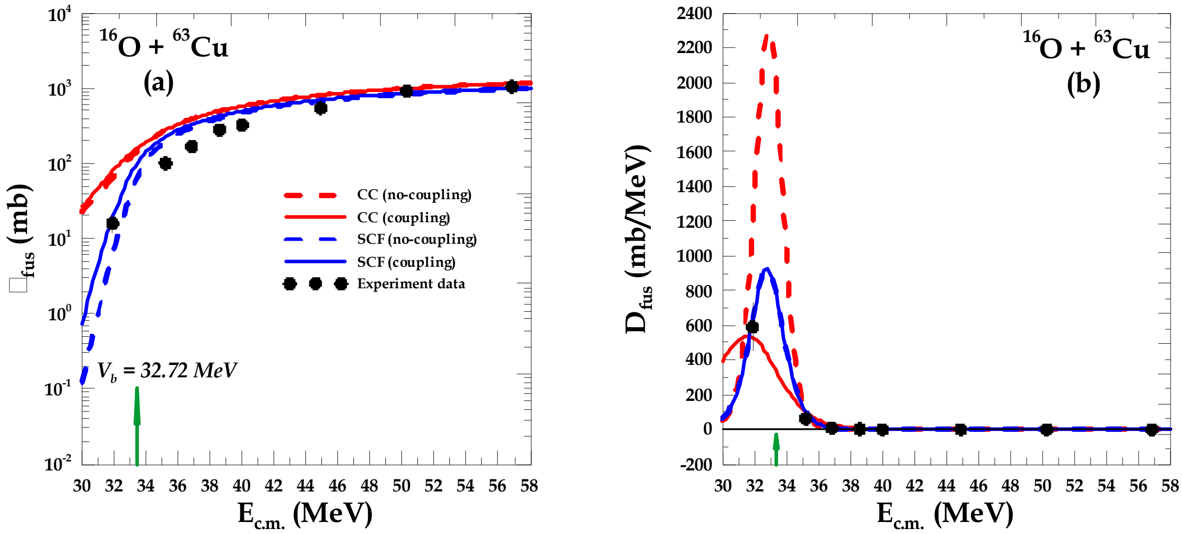

Figure 2. The semiclassical and quantum mechanical calculations for both σ_{fus} and D_{fus} in panels (a) and (b) respectively for the system **^{16}O+^{63}Cu**

4.3 ^{35}Cl+^{25}Mg System

The calculations for σ_{fus} are in more agreement with the experimental data for those treated using the semi-classical approach with channel coupling as shown in panel (a) of figure 3, while the best D_{fus} calculations are those treated using the channel coupling in quantum mechanical treatment as shown in panel (b) of the figure.

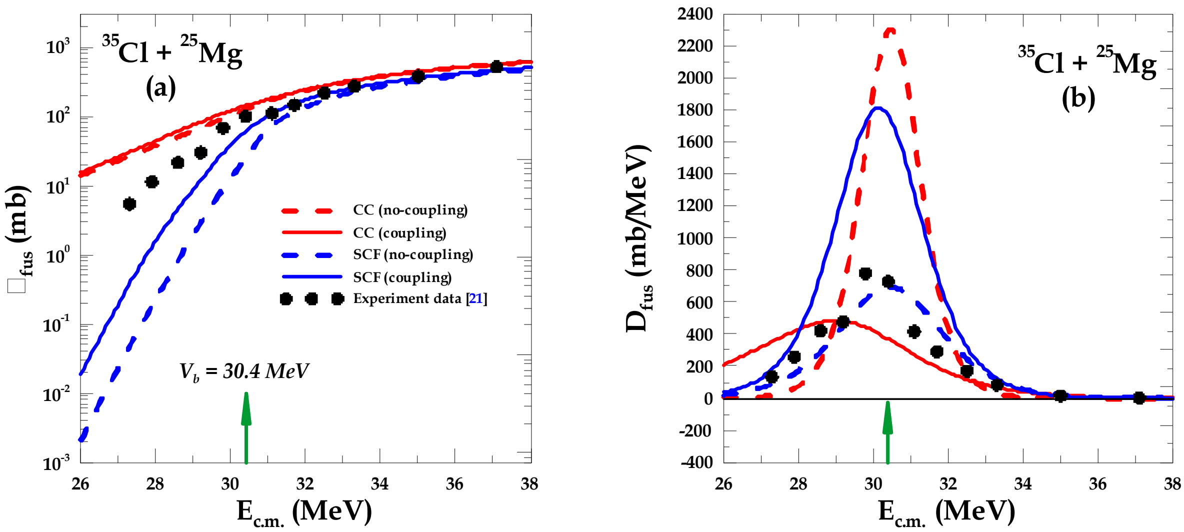

Figure 3. The semiclassical and quantum mechanical calculations for both σ_{fus} and D_{fus} in panels (a) and (b) respectively for the system **^{35}Cl+^{25}Mg**

4.4 ^{35}Cl +^{27}Al System

Panel (a) in figure 4 show in panel (a) that the best obtained calculations for σ_{fus} under and above the Coulomb barrier V_b are those calculated using semi-classical treatment with effect of coupled channel included, while the panel (b) show that the best calculations for D_{fus} are those obtained using the quantum mechanical treatment with the effect of channel coupling.

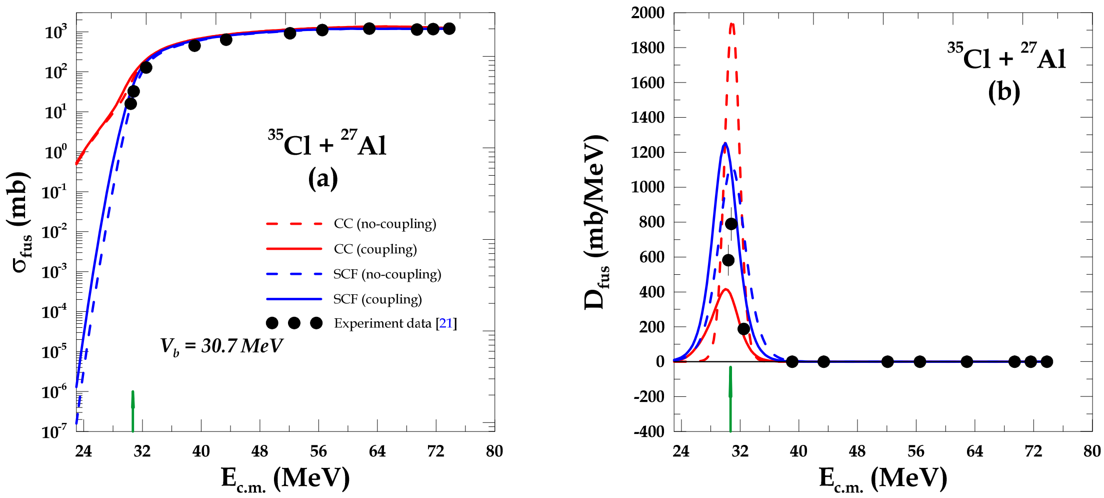

Figure 4. The semiclassical and quantum mechanical calculations for both σ_{fus} and D_{fus} in panels (a) and (b) respectively for the system ^{35}**Cl +**^{27}**Al**

5. CONCLUSION

The results for all the studied systems show a remarkable influence for the channel coupling on the calculations of σ_{fus} and D_{fus} for ^{12}C + ^{48}Ti, ^{16}O+^{63}Cu, ^{35}Cl+^{25}Mg and ^{35}Cl + ^{27}Al systems, also we conclude that quantum mechanical treatment was proved to be successful for the total cross section determination while the fusion barrier distribution. The semi-classical calculations succeeded in describing the measured data especially above the Coulomb barrier V_b.

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ДОСЛІДЖЕННЯ РЕАКЦІЙ ЗЛІТТЯ ЛЕГКИХ ЯДЕР ІЗ ЛЕГКИМИ І СЕРЕДНІМИ ЯДРАМИ

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Для розрахунку повного перерізу σ_{fus} та розподілу бар'єру синтезу D_{fus} для систем $^{12}C + ^{48}Ti$, $^{16}O+^{63}Cu$, $^{35}Cl+^{25}Mg$ і $^{35}Cl + ^{27}Al$ були використані реакції синтезу та розпаду деяких легких ядер на легких та середніх мішенях з використанням напівкласичного та повного квантово-механічного підходів. Було розглянуто зв'язок між вкладом каналу від пружних каналів та каналів розпаду, щоб показати їх важливість у розрахунках. Результати порівнюються з виміряними даними і показують розумний збіг, також показано, що зв'язок, що розглядається, дуже важливий, особливо нижче кулонівського бар'єру V_b.

Ключові слова: *пов'язані канали; переріз реакцій синтезу; розподіл бар'єру синтезу*