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ON ACCOUNTING FOR OWN FIELDS OF EMITTERS WHEN DESCRIBING GENERATION MODES[†]

Volodymyr M. Kuklin*, Eugen V. Poklonskiy
V.N. Karazin Kharkiv National University,
Svobody Sq. 4, Kharkiv, Ukraine, 61022
**Corresponding Author: v.m.kuklin@karazin.ua*
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The paper discusses three different modes of electromagnetic field generation by an ensemble of oscillators placed at the radiation wavelength in the one-dimensional case. The excitation of the resonator field is considered, which, as a rule, is determined by the geometry of the system, with and without taking into account the eigenfields of the emitters. The superradiance regime of the same ensemble of oscillators is also analyzed. In fact, superradiance is formed due to the emitters' own fields even in the absence of a resonator. It is noted that the maximum achievable amplitudes of induced fields both in the superradiance regime and in the regime of excitation of the resonator field are comparable. This makes us think about the role of the self-fields of emitters in electronic devices. It is shown that in describing the resonator excitation mode, in addition to the resonator field, it is also necessary to take into account the sum of the natural fields of the emitters in the active zone. Synchronization of emitters leads not only to an increase in the resonator field, but also, as in the superradiance regime, it significantly increases the amplitude of the sum of the oscillator fields. It is shown that in the practically interesting case of open systems (dissipative generation modes), taking into account the eigenfields of the emitters significantly reduces the characteristic time for the development of the generation process and increases the maximum achievable oscillation amplitude. This account also changes the conditions for achieving the maximum energy flow from the system. This can change the operating point of the generation process, which is determined by the requirement for the maximum rate of energy output from the system.
Keywords: *Ensemble of oscillators; Resonator field; Sum of oscillators eigenfields; Resonator field excitation mode; Superradiance excitation mode*
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Introduction

Field generation in waveguides and resonators. The traditional description of the process of generating or amplifying a high-frequency field in waveguides and resonators of electronic devices in a nonlinear mode actually began with work [1], based on the formalism presented there. A significant increase in interest in describing the excitation of oscillations by beams of charged particles and oscillators in waveguide systems has caused numerous publications. The bibliographies of works [2,3] contain material that is very useful for understanding such a description.

The modes with output or loss of energy. The need to take into account absorption, as well as the output of radiation energy from electronic devices, forced us to consider the so-called dissipative generation modes [4-7]. Such modes of generation and amplification in such open (for energy output) systems were of practical interest to the creators of electronic devices. In addition, the analysis of such regimes makes it possible to take into account the effect of different levels of energy extraction on the generation efficiency.

It was in such devices that it was possible to find operating points that ensure the maximum output of energy from the system, as was, in particular, presented in [6,7]. The considered problems were solved under conditions when oscillators (or emitters) in the active zone of waveguides and resonators interacted only with a waveguide or resonator field, the form of which was determined by the geometry of the system. The induced field of the resonator or waveguide synchronized the oscillators due to phase change (or the emitters due to bunching), which led to a significant self-consistent amplification of the field amplitude. Moreover, the interaction of oscillators (or emitters) with each other was usually neglected, excluding their total field from consideration.

Apparently, at first it was believed that due to the spread of phases emitted by particles and oscillators, the intensity of their integral radiation would remain insignificant. On the other hand, it was believed that if the radiation of particles and oscillators is spontaneous, then their total radiation will also remain spontaneous. In other words, this radiation will be much less than the intensity of the waveguide or resonator field, which is obviously induced field. There was also a point of view that the field of emitters in a waveguide or resonator would rather quickly be rebuilt into a set of eigenmodes of waveguides and resonators. It was believed that under these conditions, the self-field of the emitters can be ignored. The last consideration made sense in the case of a single act of radiation by a particle in a waveguide and resonator. Although in the continuous mode, even under conditions of partial rearrangement of the eigenfields of the emitters into waveguide and resonator modes, their total field does not go anywhere and should be taken into account.

At the same time, it was known that the process of phase synchronization of oscillators (or spatial synchronization of emitters), even in the absence of waveguide and resonator fields, led to the regime of superradiance, with induced fields [8].

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The superradiance field arises as a total field of initially spontaneously emitting oscillators even in the absence of a waveguide and a resonator. Obviously, the own field of each oscillator (or emitter) is spontaneous. However, under the influence of the total ensemble field external to each particle, the phase of radiation of this particle can change. If the amplitude of the external field is large compared to the value of the spontaneous field of a given particle, the phase of its own radiation changes. This is the nature of phase synchronization of oscillators (and spatial synchronization of emitters) and the appearance of superradiance (see, for example, [9]).

Recall that the superradiance regime discovered for a compact bunch of particles [10] manifested itself as the appearance of coherence in the study of most emitters and oscillators. This phenomenon was clearly manifested both in the quantum case and in the classical case. In distributed systems of electronics, due to rather large distances between particles, the interaction between them, as a rule, occurs only due to their own electromagnetic fields.

The mechanism of phase synchronization of radiation of an ensemble of classical oscillators was discussed in [9,11,12]. Let us show that this process of phase synchronization of emitters and oscillators in the volume of the active zone of the waveguide, in addition to generating waveguide and resonator fields, can also excite superradiance fields with an intensity comparable to that of the waveguide or resonator field [13, 14].

Accounting for fields of particles. Let us return to the discussion of the processes of excitation and amplification of oscillations by particles in waveguides and resonators. The development of high-current electronics required taking into account slowly changing currents and fields of charged particles in the active zone of the waveguide, which was first pointed out by Ya.B. Fainberg in his work “Plasma electronics”, published in the “Ukrainian journal of physics” in 1978. This was required at first to search for the stability of charged particle beams and systems of oscillators [15]. Later N.I. Karbusev in the early 1980s drew attention to the high-frequency radiation fields of individual emitters and generators.

The question arose about the role of the radiation of particles in the process of generation and amplification of oscillations in waveguides and resonators. For even in the absence of a waveguide and a resonator, the ensemble of these active elements is capable of generating superradiance fields comparable to waveguide and resonator fields [13]. That is, for a correct description of the process of excitation or amplification in such devices, in addition to the waveguide or resonator field, it is important to take into account these own fields of the particles of the active zone.

The purpose of this work is to compare the fields generated by open systems in the cases of 1) excitation of only the resonator field, 2) excitation of only the self-radiation fields of active oscillators, that is, the superradiance field, and 3) joint excitation of the resonator field, taking into account the intrinsic fields of oscillators. Let us show that, in addition to the resonator field, it is necessary to take into account the self-radiation field of oscillators, because it qualitatively affects all characteristics of the oscillation generation mode.

1. The Excitation of the Fields of Oscillators

We discuss the nature of the excitation of the field with a system of oscillators. Consider an oscillator whose charge (electron) moves along the OX axis, that is $\vec{r} = (x(t), 0, z_0)$, where $x(t) = i\cdot a\cdot\exp\{-i\omega t + i\psi\}$, at the same time whose speed (electron) $dx/dt = a\cdot\omega_0\cdot\exp\{-i\omega t + i\psi\}$ is along the OX axis, $\operatorname{Re} x = a\cdot\sin(\omega_0 t - \psi)$. The current can be recorded as $J_x = -edx/dt = -e\cdot a\cdot\omega_0\cdot\exp\{-i\omega t + i\psi\}$. The equation describing the excitation of the field with the current of the oscillator

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 D_x}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial J_x}{\partial t} = \frac{4\pi}{c^2}\cdot e\cdot a\cdot\omega_0^2\cdot i\cdot\exp\{-i\omega t + i\psi\}\cdot\delta(z - z_0), \tag{1}$$

The dielectric permittivity of the medium in the absence of oscillators is taken as equal to one $\varepsilon_0 = 1$. We will look for a solution for the electric field amplitude in the form $\vec{E} = (E\cdot\exp\{-i\omega t + ikz\}, 0, 0)$, i.e. $E_x = E\exp\{-i\omega t + ikz\}$, assuming a slow change of the complex amplitude $E_x(t,z)$:

$$\left|\frac{1}{E_x(t,z)}\frac{\partial}{\partial t}E_x(t,z)\right| << \omega,\ \left|\frac{1}{E_x(t,z)}\frac{\partial}{\partial z}E_x(t,z)\right| << k\ . \tag{2}$$

Below we consider the excitation of the field of the resonators ensemble, which in our one -dimensional case are evenly distributed in the active zone at the interval equal to the length of the radiation wave. We will discuss three generation regimes of fields by the oscillator ensemble.

The first mode of excitation of the resonator field (or waveguide) meets the traditional description of the generation (or amplification) in electronics devices, when all oscillators (emitters) interact only with the resonator field, which houses an active zone. The interaction of the oscillators in this case will exclude. That is, we do not take into account the own fields of emitters in the active zone. The conclusion of the equations describing this process is given in Appendix 1.

The second mode of excitation of the field with the emitters ensemble takes into account only the own fields of emitters located in the same interval of the active zone, and there is no resonator field in this case. It is not difficult to see that this process meets the superradiation regime. By the way, at the same time, a resonator or waveguide in this case may be completely absent. The withdrawal of equations to describe such a process is considered in Appendix 2.

And the third regime of excitation of oscillations with oscillators takes into account both the resonator (waveguide) field and the amount of its own fields of particles. It is this case that meets the correct description of the excitation of oscillations in resonators and waveguides.

In further calculations, the number of oscillators is chosen $N = 1000$. The initial amplitude of the external field in expressions (6) and (11) is equal 0.01, accounting for non -linearity (relativism) is determined by a coefficient $\alpha = 1$.

2. The Excitation of The Resonator Field with The Exclusion of The Interaction of The Oscillators To Each Other

In the traditional description of generation, all oscillators interact only with the resonator field, in which an active zone is located. Oscillators do not interact with each other. We use equations in a dimensionless form (see Appendix 1) that describe this process. The resonator field in this case can be represented in the form of the sum of two waves that spread in opposite directions

$$\mathrm{E}_{wg}(Z,\tau) = \mathrm{E}_{+}(\tau)e^{i2\pi Z} + \mathrm{E}_{-}(\tau)e^{-i2\pi Z}, \tag{3}$$

Moreover, the components of radiation in different directions can be recorded

$$\frac{\partial \mathrm{E}_{+}}{\partial \tau} + \Theta\,\mathrm{E}_{+} = \frac{1}{N}\sum_{s} \mathrm{A}_{s}\cdot e^{-i2\pi Z_{s}}, \tag{4}$$

$$\frac{\partial \mathrm{E}_{-}}{\partial \tau} + \Theta\,\mathrm{E}_{-} = \frac{1}{N}\sum_{s} \mathrm{A}_{s}\cdot e^{i2\pi Z_{s}}, \tag{5}$$

At the initial moment we should set their value

$$\mathrm{E}_{+}(0) = \mathrm{E}_{0+} \text{ and } \mathrm{E}_{-}(0) = \mathrm{E}_{0-}\,. \tag{6}$$

Movement equations for oscillators take the form

$$\frac{d\,\mathrm{A}_{j}}{dt} = \frac{i\alpha}{2}\cdot\left|\mathrm{A}_{j}\right|^{2}\mathrm{A}_{j} - \mathrm{E}_{wg}(Z_{j},\tau). \tag{7}$$

For this case of excitation of the resonator field, excluding the fields of the oscillators, the field intensity $\propto |E|^2$ (continuous line), the reverse time (increment) $\gamma = \left(\frac{1}{|E|^2}\frac{d|E|^2}{d\tau}\right)_{\max}$ of the process (stroke-dashed), and the rate of energy output $\Theta|E|^2$ (dashed) are presented in Fig. 1 (a). If you focus on the excitation regime of the resonator field without taking into account its fields of the oscillators, then the most effective output of energy would occur in the area of about $\Theta = 3$ (see also [6]).

3. The Excitation of The Same Oscillators Ensemble

In this section, we take into account only the own fields of oscillators located in the same interval of the active zone. The resonator field is absent, like the resonator itself. The dimensionless system of equations in this case (see Appendix 2) takes the following form. For fields are fair the form

$$\mathrm{E}_{sr}(Z,\tau) = \frac{1}{\Theta N}\sum_{s=1}^{N} \mathrm{A}_{s}\cdot e^{i2\pi|Z-Z_{s}|}, \tag{8}$$

And the equations of movement describing the dynamics of the oscillators can be recorded as

$$\frac{d\,\mathrm{A}_{j}}{dt} = \frac{i\alpha}{2}\cdot\left|\mathrm{A}_{j}\right|^{2}\mathrm{A}_{j} - \mathrm{E}(Z_{j},\tau), \tag{9}$$

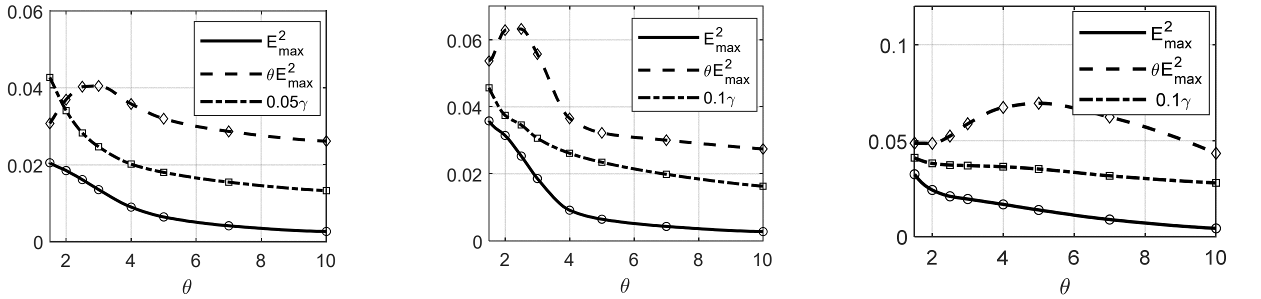
where for a common field the expression is true

$$\mathrm{E}(Z,\tau) = \mathrm{E}_{sr}(Z,\tau) + \mathrm{E}_{ex}(Z,\tau), \tag{10}$$

Moreover, the second term in (10) is an external field that is usually used to accelerate the process in the form.

$$\mathrm{E}_{ex}(Z,\tau) = \mathrm{E}_{0+}\, e^{i2\pi Z} + \mathrm{E}_{0-}\, e^{-i2\pi Z} \,. \tag{11}$$

For the field intensity $\propto |E|^2$ (continuous line), the reverse time $\gamma = \left(\dfrac{1}{|E|^2} \dfrac{d|E|^2}{d\tau} \right)_{\max}$ (increment) of the process (strokh- dashed), energy withdrawal $\Theta |E|^2$ (dashed) are presented in Fig. 1 (b). The most effective output of energy would be near the values $\Theta = 2.5\text{-}3$, and with an output one and a half times larger than in the case of an excitation only the resonator field.



a b c

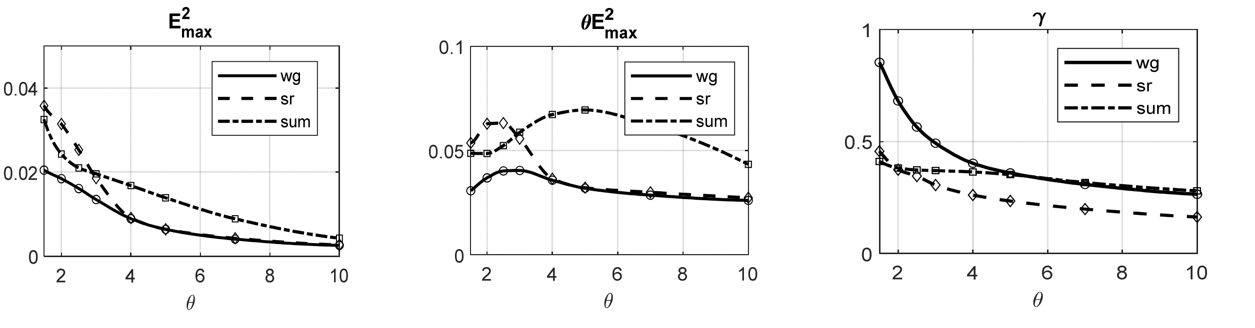
Figure 1. Field intensity $\propto |E|^2$ (continuous line), the reverse time (incremental) of the process γ (stroke-dashed), the rate of energy output $\Theta |E|^2$ (dashed) as function of energy loss Θ for cases: (a) of the generation of the resonator field, excluding the oscillator radiation field, (b) of the supperradiation mode, (c) the generation of the resonator field together with the radiation field of the oscillators.

4. The Excitation of the Resonator Field When Taking into Account the Interaction of Oscillators to Each Other

Using the entered variables and the expressions obtained in the applications for the fields, you can represent the system of equations for the excitation of the resonator, and we will additionally take into account the total field of oscillators. It is clear that thereby it is possible to take into account the direct interaction of the oscillators with each other. Here you cannot take into account the external initial field (11), the role of which will be assumed by the resonator field (4)-(5). The equation of motion for oscillators can be written in the form

$$\frac{d\,\mathrm{A}_j}{dt} = \frac{i\alpha}{2} \cdot \left|\mathrm{A}_j\right|^2 \mathrm{A}_j - \mathrm{E}(Z_j,\tau) \,. \tag{12}$$

Here, the resonator field and the total field of radiation of the oscillators are simultaneously taken into account $\mathrm{E}(Z,\tau) = \mathrm{E}_{sr}(Z,\tau) + \mathrm{E}_{wg}(Z,\tau)$, where $\mathrm{E}_{wg}(Z,\tau)$ it is described by expressions (3-5), and $\mathrm{E}_{sr}(Z,\tau)$ by expression (8). The initial conditions for the fields of the resonator will choose the same (6). The nature of the excitation of the resonator field, taking into account the own fields of the oscillators, the field intensity $\propto |E|^2$ (continuous line), the reverse time (increment) of the process $\gamma = \left(\dfrac{1}{|E|^2} \dfrac{d|E|^2}{d\tau} \right)_{\max}$ (stroke-dashed), the rate of energy output $\Theta |E|^2$ (dashed) are represented in Fig. 1 (c).



a) b) c)

Figure 2. Dependence on the level of energy loss Θ : (a) - the square of the field amplitude $\propto |E|^2$, (b) - the rate of energy output $\Theta |E|^2$, (c) - the inverse time (increment) of the process $\propto \left(\dfrac{1}{|E|^2} \dfrac{d|E|^2}{dt} \right)$. Here are cases: the generation of the resonator field, excluding the oscillator radiation field (continuous line), the superradiation mode (stroke-dashed), and the generation of the resonator field together with the radiation field of the oscillators (dashed).

It can be seen here that joint accounting for the excitation of the resonator field and the oscillator field leads to a noticeable increase in energy selection, and for large values about $\Theta = 5$.

5. Comparison of Three Modes

Consider three modes. A solid line on the graphs marks the resonator regime without taking into account the impact on the process of generating oscillators fields, The superradiation mode is represented by dashed lines on the graphs, and the strokh- dashed line on the graphs is responsible for the case of excitation of the resonator, taking into account the influence of the fields of the oscillators.

In the mode of excitation of the resonator, taking into account fields of emitters, the field intensity and output of energy are the highest. A slightly inferior to him is the superradiation mode. The working points (the rate of energy output from the system that meets the maximum), are realized for different values Θ. But the working point, for resonators, where we taking into account fields of oscillators is shifted in the large values of Θ.The characteristic times of the process near the highest energy output is half as much as for resonator which don't taking into account the of the own fields of the oscillators.

6. Conclusion

Three different generation modes of the electromagnetic field are discussed for the oscillators ensemble, placed on the length of the radiation wave. The consideration is carried out for an open system, which is characterized by the output of energy from the system. The parameter of the degree of openness is a value that is determined by the ratio of the attenuation to the value of the reverse characteristic time of the process (increment) in the absence of losses. The following generation modes are considered.

1. The generation of an open resonator, excluding own fields of oscillators - emitters.
2. The superradiation mode - the generation of the field in the open system of the same ensemble of the oscillators without resonator.
3. The excitation of an open resonator, taking into account the own fields of oscillators emitters.

The superradiation regime, which is of independent interest, demonstrates the nature of the synchronization of emitters (see Appendix 2). The superradiation, as you know, arises due to its own fields of emitters even in the absence of a resonator. As a result of the phase synchronization of fixed oscillators, an integral field is formed, which is not spontaneous, but induced field. It is this field that synchronizes the noticeable part of the emitters. It is important that the total field intensity of the is exceeds the intensity of the own field of emitter. Although the distributed system does not allow synchronizing all the oscillators, especially in areas where the total field of the ensemble is small.

Accounting for own radiation of the oscillators significantly reduces the characteristic time of the development of the generation process in resonators and waveguide, increases the most achievable amplitude of oscillations. At the same time, the conditions for achieving the maximum energy flow from the system also change. This is able to significantly shift the work point of the generation process, determined, for example, by the requirement of the maximum pace of energy output from the system.

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Appendix 1. The Field of Resonator

In the resonator (or waveguide), the field can be formed in such a way that the type of field will not depend on the radiation of individual oscillators. Note that such a field generally speaking should consist of running waves in two directions

$$E_x = E_+ \cdot \exp\{-i\omega t + ikz\} + E_- \cdot \exp\{-i\omega t - ikz\}, \tag{A1}$$

where the slowly changing complex amplitude of the waves has the form $E_\pm = |E_\pm| \cdot \exp\{i\varphi_\pm\}$.

The interaction of oscillators with these fields can be described by the equation [9,13]

$$2i\omega_0\left(\frac{\partial E_\pm}{\partial t} + \delta_D\right) = -e\omega^2{}_0 \frac{4\pi n_0}{N \cdot 2i} \int a_s dz \cdot \exp\{i\psi_s \mp ikz\} \cdot \delta(z - z_s), \tag{A2}$$

where added δ_D - the decrement of the absorption of the wave in the absence of sources, $A_j = a_j \exp(i\psi_j)$.

The equations of the movement will present in the form

$$\frac{d}{dt}\frac{v_i}{\sqrt{1 - \frac{|v_i|^2}{c^2}}} + \omega_0{}^2 x_i = -\frac{e}{m} E_x(z_i, t) \tag{A3}$$

where $x_i(t) = i \cdot a_i \cdot \exp\{-i\omega t + i\psi\} = iA \cdot \exp\{-i\omega t\}$, $v_i = \omega \cdot a_i \cdot \exp\{-i\omega t + i\psi\} = \omega A \cdot \exp\{-i\omega t\}$.

We further assume that the resonator has size b, equal to wavelength (without loss of generality the results are generalized to the case of several wavelengths), group velocity of radiation c, effective decrement of field damping equals $\delta_D = c/b$, number of oscillators equal to $M = b \cdot n_0$.

Equation (A2) for slowly varying amplitudes then takes the form

$$\frac{\partial E_{\pm}(t)}{\partial t} + \delta_D E_{\pm}(t) = \frac{2\pi \cdot e \cdot \omega \cdot M}{b} \frac{1}{N} \sum_s A_s \cdot \exp(\mp ikz_s) = 2\pi \cdot e \cdot \omega \cdot n_0 \cdot \frac{1}{N} \sum_s A_s \cdot \exp(\mp ikz_s). \quad \text{(A4)}$$

The equation of motion (A3) can be represented as

$$\frac{dA_j}{dt} = \frac{i\omega\alpha_1}{2} \cdot A_j(t) - \frac{e \cdot E_{wg}(t, z_j)}{2 \cdot \omega \cdot m}, \quad \text{(A5)}$$

$$E_{wg}(t, z) = E_+ \cdot \exp(ikz) + E_- \cdot \exp(-ikz). \quad \text{(A6)}$$

Here $E_{wg}(t, z)$ is the resonator field, the parameter $\alpha_1 = 3|A|^2 k^2 / 4$ takes into account the weak dependence of the relativistic mass of the particle on speed.

Choosing dimensionless variables and parameters

$$\mathrm{A} = A / a_0, \quad kz = 2\pi Z, \quad \gamma_0 = \omega_{pe} / 2, \quad \gamma_0^2 = \omega_{pe}^2 / 4 = \frac{\pi n e^2}{m}, \quad \gamma_0 t = \tau, \quad \beta = m / m_1, \quad kb = 2\pi \bar{b},$$

$$\delta = \frac{c}{b}, \quad \Theta = \delta / \gamma_0, \quad E_{01} = \frac{2m \cdot \gamma_0 \cdot \omega \cdot a_0}{e}, \quad \mathrm{E} = E / E_{01}, \quad \alpha = \frac{3\omega}{4\gamma_0}(ka_0)^2,$$

we get an expression for the resonator field an expression

$$\mathrm{E}_{wg}(Z, \tau) = \mathrm{E}_+(\tau) e^{i2\pi Z} + \mathrm{E}_-(\tau) e^{-i2\pi Z}. \quad \text{(A7)}$$

where the components corresponding to spreading of radiation in different directions can be written as

$$\frac{\partial \mathrm{E}_+}{\partial \tau} + \Theta \mathrm{E}_+ = \frac{1}{N} \sum_s \mathrm{A}_s \cdot e^{-i2\pi Z_s}, \quad \text{(A8)}$$

$$\frac{\partial \mathrm{E}_-}{\partial \tau} + \Theta \mathrm{E}_- = \frac{1}{N} \sum_s \mathrm{A}_s \cdot e^{i2\pi Z_s}, \quad \text{(A9)}$$

moreover, at the initial moment you should set their values

$$\mathrm{E}_+(0) = \mathrm{E}_{0+} \text{ and } \mathrm{E}_-(0) = \mathrm{E}_{0-}. \quad \text{(A10)}$$

Movement equations for oscillators take the form

$$\frac{d\,\mathrm{A}_j}{dt} = \frac{i\alpha}{2} \cdot |\mathrm{A}_j|^2 \mathrm{A}_j - \mathrm{E}_{wg}(Z_j, \tau). \quad \text{(A11)}$$

We can get the law of conservation of energy in the form

$$\left(\frac{\partial}{\partial \tau} + 2\theta\right)\left\{|\mathrm{E}_+|^2 + |\mathrm{E}_-|^2\right\} = 2\frac{\partial}{\partial \tau} \sum_{j=1}^{N} |\mathrm{A}_j|^2 \quad \text{(A12)}$$

Appendix 2. Description of the Superraiation Mode

Generally speaking, the field excited in the system of oscillators consists of the sum of all fields of individual oscillators. Consider the superradiation mode when the resonator field or a waveguid field is absent. It is also possible to determine the total field of radiation of oscillators in the same volume, as presented in Appendix 1. An important circumstance is the conditions for synchronization of the oscillators. It turns out, as noted, for example, in the work of Yu.A. Il'inskii, and N.S. Maslova, "Classical analog of superradiance in a system of interacting nonlinear oscillators" published in 1988, that only when the nonlinearity of oscillators is taken into account in this case, it becomes possible to ensure phase synchronization of the field and the oscillator (see also [11]).

The field of one oscillator. For the amplitude of the radiation field slowly changing in the space, the equation is fair

$$\frac{\partial E}{\partial z} = 2ea\omega^2 \frac{\pi}{c^2 k} \cdot \exp\{i\psi + ikz\} \cdot \delta(z - z_0) = \lambda \cdot \delta(z - z_0). \quad \text{(A13)}$$

Chose solution is form of $E = C + \lambda \cdot \theta(z - z_0)$ where $\theta(z<0)=0$, $\theta(z\geq 0)=1$. Since for the wave radiated by the oscillator the equation $D(\omega,k) \equiv (\omega^2\varepsilon_0 - k^2) = 0$, the roots of which $k_{1,2} = \pm(\omega_0 \operatorname{Re}\varepsilon_0 / c)(1 + i\operatorname{Im}\varepsilon_0 / \operatorname{Re}\varepsilon_0) \approx$ $\approx \pm(\omega_0 / c\varepsilon_0)(1 + i0)$, for the wave, which propagates in the direction $Z > Z_0$, the wave number is equal $k = k_1 > 0$ and the value of the constant C should be selected equal to zero in order to avoid unlimited growth of the field at infinity. For a wave which propagates in the direction $Z < Z_0$, the wave number is equal $k = k_2 < 0$, the value of constant C in the same reasons should be selected equal $-\lambda$. The amplitude of the electric field while

$$E_x = 2\pi e a\omega_0 M \cdot c^{-1} \exp\{-i\omega t + i\psi\}[\exp\{ik_0(z - z_0)\cdot\theta(z - z_0) + \exp\{-ik(z - z_0)\cdot\theta(z - z_0)\}\} \quad \text{(A14)}$$

For one particle in such a volume of a single section and the length of the resonator *b*, *M* it is numerically equal to one. The equation of motion for the oscillating electron has the form (A3).

The particle ensemble field. Using these designations, we will write particle ensemble field in the form

$$\frac{dA_j}{dt} - i\frac{|3A_j|^2\cdot\omega^3}{4c^2}A_j + i\frac{|3A_{j0}|^2\cdot\omega^3}{4c^2}A_{j0} = -\frac{\pi\cdot e^2\cdot M}{mc}\cdot\frac{1}{N}\sum_{s=1}^{N}A_s\left(e^{ik(z_j - z_s)}\cdot\theta(z_j - z_s) + e^{-ik(z_j - z_s)}\cdot\theta(z_s - z_j)\right). \quad \text{(A15)}$$

Let choose dimensionless variables and parameters

$$\mathrm{A} = A/a_0,\quad kz = 2\pi Z,\quad \gamma_0 = \omega_{pe}/2,\quad \gamma_0^2 = \omega_{pe}^2/4 = \frac{\pi n e^2}{m},\quad \gamma_0 t = \tau,\quad \beta = m/m_1,\quad kb = 2\pi\overline{b},$$

$$\delta = \frac{c}{b},\quad \Theta = \delta/\gamma_0,\quad E_{01} = \frac{2m\cdot\gamma_0\cdot\omega\cdot a_0}{e},\quad \mathrm{E} = E/E_{01},\quad \alpha = \frac{3\omega}{4\gamma_0}(ka_0)^2.$$

Let consider the excitation of the field in the resonator, the size of which b, equal to the wavelength (without losing community, the results are generalized in case of several wavelengths) group radiation speed *c*, and the effective decrement of the fading of the field is equal $\delta = \frac{c}{b}$. If you take into account radiation from the system (which, due to the small size of the system, can be considered distributed), then the increment of the process $\gamma \approx \gamma_0^2/\delta = \gamma_0/\Theta$, and $\delta > \gamma_0$. It should be noted that in all cases under discussion, due to the selected placement of the ensemble of the oscillators, the the increment of the regime without energy loss $\Theta = 0$ is equal γ_0 [13]. One gets the expression for the superradiation field

$$\mathrm{E}_{sr}(Z,\tau) = \frac{1}{\Theta N}\sum_{s=1}^{N}\mathrm{A}_s\cdot e^{i2\pi|Z - Z_s|}, \quad \text{(A16)}$$

moreover, the equation of movement describing the dynamics of the oscillators takes the form

$$\frac{d\,\mathrm{A}_j}{d\tau} = \frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2\mathrm{A}_j - \mathrm{E}\left(Z_j,\tau\right) \quad \text{(A17)}$$

where for a common field the expression is true

$$\mathrm{E}(Z,\tau) = \mathrm{E}_{sr}(Z,\tau) + \mathrm{E}_{ex}(Z,\tau). \quad \text{(A18)}$$

The second term in (A18) is an external stimulation field that is often necessary to accelerate the process can be recorded as

$$\mathrm{E}_{ex}(Z,\tau) = \mathrm{E}_{0+}e^{i2\pi Z} + \mathrm{E}_{0-}e^{-i2\pi Z}. \quad \text{(A19)}$$

The nature of the synchronization of the oscillators in this and other regimes is quite obvious [13]. Let return back to equation (A17), which can be recorded differently

$$\frac{d\left[\left|\mathrm{A}_j\right|\exp\left(i\psi_j\right)\right]}{dt} = \frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2\left|\mathrm{A}_j\right|\cdot\exp\left(i\psi_j\right) - \left|\mathrm{E}\left(Z_j,\tau\right)\right|\cdot\exp\left(i\varphi\right). \quad \text{(A20)}$$

Then the equation for the oscillator phase, which follows from (A20), takes the form

$$\frac{d\psi_j}{dt} - \frac{\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2 = -\left\{\left|\mathrm{E}\left(Z_j,\tau\right)\right| / \left|\mathrm{A}_j\right|\right\}\cdot\sin\left(\varphi-\psi_j\right). \qquad \text{(A21)}$$

You can pay attention to the fact that the right part of the last equation is quite large $\left|\mathrm{E}\left(Z_j,\tau\right)/\mathrm{A}_j\right| \gg 1$. This forces the phase of a separate oscillator to synchronize with the phase of the total field of the ensemble $\psi_j \to \varphi$. The value $0.5\cdot i\alpha\cdot\left|\mathrm{A}_j\right|^2$ gives regularisation, i.e. a certain spread of phase values.

ORCID IDs

Volodymyr Kuklin, https://orcid.org/0000-0002-0310-1582; **Eugen Poklonskiy,** https://orcid.org/0000-0001-5682-6694

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ПРО НЕОБХІДНІСТЬ ОБЛІКУ ВЛАСНИХ ПОЛІВ ВИПРОМІНЮВАЧІВ ПРИ ОПИСІ РЕЖИМІВ ГЕНЕРАЦІЇ

Володимир М. Куклін, Євген В. Поклонський

Харківський національний університет імені В. Н. Каразіна, Харків, Україна

У роботі обговорюються три різні режими генерації електромагнітного поля ансамблем осциляторів, розміщених на довжині хвилі випромінювання в одновимірному випадку. Розглянуто збудження резонаторного поля, яке, як правило, визначається геометрією системи, з урахуванням та без урахування власних полів випромінювачів. Аналізується також режим надвипромінювання такого ж ансамблю осциляторів. Фактично надвипромінювання формується за рахунок власних полів випромінювачів навіть без резонатора. Зазначається, що максимально можливі амплітуди індукованих полів як у режимі надвипромінювання, так і в режимі генерації поля резонатора можна порівняти. Це змушує подумати про роль власних полів випромінювачів у приладах електроніки. Наголошується на помилковості традиційного підходу до опису збудження резонаторів і хвилеводів, де облік суми власних полів випромінювачів не проводився. Показано, що при коректному описі режиму збудження резонатора, крім поля резонатора, необхідно враховувати також суму власних полів випромінювачів в активній зоні. Синхронізація випромінювачів також, як і в режимі надвипромінювання, приводить до появи значної амплітуди індукованого поля системи осциляторів. Показано, що в практично цікавому випадку відкритих систем (дисипативні режими генерації) цей облік помітно зменшує характерний час розвитку генерації, збільшує максимально можливу амплітуду коливань. Облік власних полів випромінювачів також змінює умови досягнення максимального потоку енергії із системи. Це здатне змінювати робочу точку процесу генерації, що визначається вимогою максимального темпу виведення енергії із системи.

Ключові слова: *ансамбль осциляторів; поле резонатора; сума власних полів випромінювачів; режим збудження поля в резонаторі; збудження поля в режимі надвипромінювання*