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A STUDY THE NUCLEAR POTENTIAL USING QUASI-ELASTIC SCATTERING CALCULATION FOR THE $^{9,10,11}Be+^{208}Pb$ REACTIONS†

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Specific systematic studies on the nuclear potential parameter for the heavy-ion reactions which includes the systems have been achieved by using large-angle quasi elastic scattering at deep sub-barrier energies close to the Coulomb barrier height.The single-channel (SC) and coupled-channels calculations have been carried out to elicit the nuclear potential. The chi-square method χ^2 has been used find the best value of the nuclear potential in comparison with the experimental data. The best values of the nuclear potential were found from the calculations of the coupled channels for an inert projectile and a vibrating target for systems: $^{9}Be+^{208}Pb$, $^{10}Be+^{208}Pb$, $^{11}Be+^{208}Pb$, which are equal to 45 MeV, 65 MeV, 53 MeV, respectively.

Keywords: *Coupled-channels calculations; Heavy-ion fusion reactions; quasi-elastic scattering; deep sub-barrier energies*

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1. INTRODUCTION

Knowing the nucleus-nucleus interaction potential is the key ingredient in nuclear reaction analysis [1] and it has played an important role in describing nucleus-nucleus collisions. It has been well recognized that heavy-ion collisions at energies about the Coulomb barrier are strongly influenced by the internal structure of colliding nuclei [2]. The couplings of the relative motion to the substantial degrees of freedom (such as collective inelastic excitation of the colliding nuclei and/or transfer processes) result in a single potential barrier being changed by many distributed barriers.The nucleus-nucleus potential is the cause of the interaction energy of colliding nuclei [5] it has been used to appreciate the cross sections of different nuclear reactions.too,in deformed nucleus interaction the nucleus-nucleus potential depends on the orientation angle of the deformed nucleus prorated to the beam direction. We can define the nucleus-nucleus potential as the sum of the nuclear potential $V_N(r)$ which is less defined and the Coulomb potential $V_C(r)$ which is well-known. By the specific description of the Coulomb or Rutherford scattering. The barrier height of the nucleus-nucleus reaction depends on the ratio between the nuclear and Coulomb potentials, which work at teeny distances between the surfaces of reactant nuclei [6]Ṡo,the nucleus-nucleus potential includes Coulomb and nuclear parts, so that long-range disharmony Coulomb potential acts between the protons in nuclei while the nuclear interaction between nucleons, the nuclear fraction is commonly expressed by the Woods-Saxon (WS) form, which is characterized by the deepness V_o, radius r_o and diffuseness a parameter [2]. The truth is that the WS form of a simple exponential had been exploited to study the surface-characteristic of nuclear potential. Quasi-elastic scattering can be defined as the sum of elastic scattering, inelastic scattering, and transfer reaction, it is very well equivalent to the fusion reaction, which is defined as a reaction where two discrete nuclei integrate to form a compound system [7]. Fusion and quasi-elastic scattering are both considered extensive operations that work in tandem. As a result,these interactions share the same potential and information about the mechanism of interaction, and both are sensitive to channel coupling impacts (due to collective in elastic excitements of colliding nuclei) at energies near the Coulomb barrier [8].

Experimentally, the measurement of quasi-elastic scattering is easier than that of fusion interaction, particularly at deep sub-barrier energies. As well as note that the scattering operation is sensitive fundamentally to the surface area of the nuclear potential, whilst the fusion reaction is also comparatively sensitive to the internal fraction [9]. The experimental measurement process for large-angle quasi-elastic scattering cross sections is more efficient and straight forward than the measurement process for fusion cross sections. At deep sub-barrier energies, the perversion of the rate of the quasi-elastic to the Rutherford cross sections from unity provides a clear way to set the account of the surface diffuseness parameter in the nucleus-nucleus potential [10].

As a result, diffuseness parameter can be defined as a landing of the nuclear potential and thus has a direct impact on the barrier width and coupling strong points,which to first order rely on the derivative of the potential. A coupling channel model is an ideal tool for simultaneously reproducing experimental data for

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several processes such as elastic and inelastic scattering,particle transfers, and fusion with in a unified framework [11].

The inter-nuclear potential is the most important component in coupled-channels calculations, with nuclear potential influencing barrier width and coupling strengths. The channel coupling is caused by the interaction of the internal degrees of freedom, which include transfer reactions and collective vibrational and rotational motions, with the relative motion of colliding nuclei [12].

The effect of coupling channels can be ignored in nucleus-nucleus collisions at deep sub-barrier energies near the Coulomb barrier because reflection probability is nearly unity at such energies; however, this analysis would be acceptable for spherical nuclei collisions. The use of coupling channel accounts does not play a significant role in determining the best value for the diffuseness parameters at deep sub-barrier energies, but their primary purpose is to achieve the effects of some calculation inputs on the resulting diffuseness parameters. The excitation states of colliding nuclei are critical for performing coupled-channel calculations.[13]
K. Washiyama et al. used large-angle quasi-elastic scattering at energies much less than the Coulomb barrier to investigate the surface characteristics of nucleus-nucleus potential in heavy-ion reactions.As a result, a single-channel potential model was appropriate for describing these energies [2, 14].

The goal of this study is to obtain the nuclear potential parameters for the systems $^{9,10,11}Be$ + ^{208}Pb by using large-angle quasi-elastic scattering at deep sub-barrier energies close to the Coulomb barrier height,and the single-channels and coupled-channels calculations were performed using the CQEL program, which includes all orders of coupling and is considered the most recent version of computer code CCFULL [15]. The chi-square χ^2 method was used to obtain the best-fitting values of the nuclear potential in comparison to the experimental data [9, 16].

2. THEORY

The nucleus-nucleus potential is divided into two parts nuclear part V_N, which can be well and reasonably described by the Woods-Saxon (WS)form given by [17]:

$$V_N(r) = -\frac{V_o}{1+\exp[\frac{r-R_o}{a}]} \tag{1}$$

where R_0 is a radius parameter of the system, V_o, a and r_o represent the potential depth,surface diffuseness parameter,and radius parameter,respectively, whilst r refers to the center–of–mass distance between the target nucleus of mass number A_T and the projectile nucleus of mass number A_P [18]. From another side,Coulomb part V_C between two spherical nuclei with regular charge density distributions and when they do not interfere is given by [18]:

$$V_c(r) = \frac{Z_P Z_T e^2}{4\pi\epsilon_o r} \tag{2}$$

where Z_p and Z_T represent the atomic number of the projectile and target,respectively, r the distance between the centers of mass of the colliding nuclei. When the nuclei interfere, then the Coulomb potential is given by [19]:

$$V_c(r) = \frac{Z_P Z_T e^2}{8\pi\epsilon_o R_c}\ [3-(\frac{r}{R_c})^2] \tag{3}$$

where R_c is the radius of the ball equivalent to the of the target and the projectile. The collision between two nuclei through the presence of coupling between the relative motion of the center of mass of the colliding nuclei $r \rightarrow = (r,\, \hat{r})$ and the nuclear intrinsic motion ξ. The Hamiltonian for the system is given by [1]:

$$H(\vec{r},\xi) = -\frac{h^2}{2\mu}\ \nabla^2 + V(r) + H_o(\xi) + V_{coup}(\vec{r}.\xi) \tag{4}$$

where r refers to the center of mass distance between the colliding nuclei, the reduced mass of the system while $V(r)$ is the naked potential in the absence of the coupling where $V(r) = V_N(r) + V_c(r)$,$H_0(\xi)$, represents the Hamiltonian for the intrinsic motion V_{coup} is the mentioned coupling. The Schrodinger equation for the total wave function would be given by [1]:

$$(-\frac{h^2}{2\mu}\ \nabla^2 + V(r) + H_o(\xi) + V_{coup}(\vec{r}.\xi))\psi(\vec{r}.\xi) = E(\vec{r}.\xi) \tag{5}$$

The internal degree of freedom ξ principally has a limited spin. We can write the coupling Hamiltonian in complications as [1]:

$$V_{coup}(\vec{r}.\xi) = \sum_{\lambda>0.\mu} f_{\lambda\mu}(r) Y_{\lambda\mu}(\hat{r}).T_{\lambda\mu}(\xi) \tag{6}$$

$Y_{\lambda\mu}(\hat{r})$refers to the spherical harmonics and $T_{\lambda\mu}(\xi)$refers to the spherical tensors,which are built from the internal coordinate. The sum is taken over all values of excluding for λ=0 since it is originally considered in$V(r)$ The expansion basis for the wave function in equation (5) for a fixed total angular momentum J and its z-component M is defined as [20]:

$$\langle \vec{r}\xi|(nlI)JM\rangle = \sum_{m2m1} \langle lm_I m_1|JM\rangle Y_{lm1}(\hat{r})\varphi_{n_1 m_1}(\xi) \tag{7}$$

where l refers to the orbital, Irepresent the internal angular momenta and represents the wave function for the internal motion which fulfills [16]:

$$H_o(\xi)\varphi_{n_1 n_2}(\xi) = \varepsilon_n \varphi_{n_1 m_1}(\xi) \tag{8}$$

The total wave function $\psi(r.\xi)$ has been expanded with this basis as [1]:

$$\psi((\vec{r}).\xi) = \sum_{n\cdot l\cdot I} \frac{u^J_{nlI}(r)}{r} \langle \vec{r}\xi|(nlI)JM\rangle \tag{9}$$

The Schrödinger equation [equation (2)] can then be written as a group of coupled equations for $u^J_{n_l I}$(r)[1].

$$[-\tfrac{h^2}{2\mu}\frac{d^2}{d(r)} + V(r) + \frac{l(l+1)h^2}{2^2} - E + \varepsilon_n]u^J_{n_l I}(r) + \sum_{n'\cdot l'\cdot I'} V^J_{(nlI,'n\ '.l'.I')}(r)u^J_{n'\cdot l'\cdot I'}(r) = 0 \tag{10}$$

Terms of the coupling matrix elements are given by:[1]

$$V_{(nlI,'n\ '.l'.I')}(r) = \langle JM(nlI)|V_{coup}(\vec{r}.\xi)|(n'.l'.I')JM\rangle = \sum_\lambda -1^{1-I'+l'+J} f_\lambda(r)\langle lI|\mathrm{Y}_\lambda|l'\rangle\langle nl|T_\lambda|n'l'\rangle$$

$$\times\ \sqrt{(2l+1)(2I+1)}\begin{bmatrix} I' & l' & J \\ l & I & \lambda \end{bmatrix}$$

(11)

The reduced matrix elements in equation (11) is defined by:[14]

$$\langle l_{ml}|Y_{\lambda\mu}|l'_{ml}{}'\rangle = \langle l'_{ml}{}'\lambda\mu|l_{ml}\rangle\langle l||Y_\lambda||l'\rangle \tag{12}$$

As can be observed in the equation,the coefficient has been suppressed since it is independent of the coefficient M. Coupled-channels equations are the name given to the equation. These equations are frequently solved using the incoming wave boundary conditions for heavy-ion fusion interactions [14].

$$u^J_{nlI}(r) \sim exp(-1\int_{r_{abs}}^{r} k_{nlI}(r')dr' \tag{13}$$

$$\to \frac{i}{2}(\begin{bmatrix} H_l^-(k_{nIr})\delta_{n,n\ i}\delta_{l,l\ i}\delta_{I,I\ i} \\ +\sqrt{\frac{(k_{nlr})}{k_{nl}}}S^I_{l\ I}H_l^+(k_{nIr}) \end{bmatrix}, \to \infty \tag{14}$$

$$k_{nIr} = \sqrt{2\mu(E-\epsilon_{nl})}/h^2, k_{nIi} = k = \sqrt{2\mu E/h^2} \tag{15}$$

The definition of the local wave number is [9]:

$$k_{nII}(r) = \sqrt{\frac{2\mu}{h^2}(E - \epsilon_{nl} - \frac{l(l+1)h^2}{2\mu r^2} - V(r) - V_{(nlI,'n\ '.l'.I')}(r)(16)}$$

The penetrability across the Coulomb barrier is calculated using the transmission coefficients and is given by:

$$P_l^J{}_{Ili}(E) = \sum_{n.l.I} \frac{k_{nlI}(r_{abs})}{k} |\tau_{nl}{}_I^l|^2 \quad (17)$$

is the wave number for the entrance channel.The fusion cross section for the unpolarized target is given by:

$$\sigma_{fus}(E) = \frac{\pi^2}{k} \sum_{J_{Mli}} \frac{2J+1}{2Ii+1} P_l^J{}_{Ili}(E) \quad (18)$$

Equation (17) therefore reads When the initial intrinsic spin=0,the initial angular momentum $=J$,with the coefficients and are suppressed in the penetrability [9]:

$$\sigma_{fus}(E) = \frac{\pi^2}{k} \sum_J 2J + 1P^J(E) \quad (19)$$

$$f_l^J{}_I(\theta.E) = i \sum_{J_I} \sqrt{\frac{\pi}{KK_{nl}}} i^{J-1}$$

$$\mathrm{e}^{i\sigma_J(E)+\sigma_l(e-\epsilon_{nl})} \sqrt{2J+1} Y_{l0}(\theta)(S_u^J - \sigma_{I,I\,2}\delta_{ll2}) + f_c(\theta.E)\delta_{I,I\,2}\delta_{ll2} (20)$$

σ_l is the Coulomb phase shift which is given by [9]:

$$\sigma_l = |\Gamma(l + 1 + i\eta)| \quad (21)$$

While f_c is the Coulomb scattering amplitude which is given by [9]:

$$f_c(\theta.E) = \frac{\eta}{2ksin^2(\frac{\theta}{2})} e^{[-i\eta|nsin^2(\frac{\theta}{2})+2i\sigma_o(E)]} \quad (22)$$

η is the Summerfield parameter,Equation (19) may be used to evaluate the differential cross-section, which is given by [9]:

$$\frac{d\sigma_{el}}{d\Omega}(\theta, E) = \sum_{J_{lI}} \frac{k_{nI}}{k} |f_l{}_I^J(\theta, E)|^2 \quad (23)$$

Equation (21) may be used to evaluate the Rutherford cross section [9]:

$$\frac{dR}{d\Omega}(\theta, E) = |f_c(\theta, E)|^2 = \frac{\eta}{4k^2} csc^4 \frac{\theta}{2} \quad (24)$$

3. PROCEDURE

The CQEL software,which is regarded as the most recent version of the computer code CCFULL,was used to do the computations for single-channel and coupled-channels. The Schrödinger equation and the linked equations are precisely solved by this program [15]. To prevent systematic mistakes in the current study, the chi-square approach was used as a normalizing factor between the theoretical computation and the experimental results. The nuclear potential, which has both real and fictitious components, was calculated using a WS form [21]. The research was done on the true potential parameters to find the one that suited the experimental data the best, allowing it to be replicated for all interactions [21].

The Woods-Saxon (WS) The radius parameter r_0 is taken to be $1.2fm$, while the values of potential depth V_0 depended on the diffuseness parameter are taken to be $[(45, 50, 60)MeV$, $(49, 55, 65)MeV$ and $(43, 53, 59)MeV$ for the ^9Be+^{208}Pb, ^{10}Be+^{208}Pb and ^{11}Be+^{208}Pb systems, respectively. The radius of the target was taken as $R_T = r_T A^{1/3}$ such that r_T= 1.16 fm while for the projectile R_p= $r_p A^1/3$ so r_p= $1.22fm$. The beam energies at the center of the reaction target were $88MeV$, $127MeV$ and 140 MeV for ^9Be, ^{10}Be, and ^{11}Be, respectively [21]. The experimental data for the quasi-elastic cross sections at deep sub-barrier energy for all systems were taken from the references [21]. To verify that the calculations are suitably compatible with the available experimental data, we analyze and display the calculated ratio of the quasi-elastic to the Rutherford cross sections as functions of the center of mass energies.

4. RESULT AND DISCUSSION

In the $^{9}Be+^{208}Pb$ system, the nuclear potential parameter has been discussed in four states, in the first state we considered the projectile ^{9}Be as well as target ^{208}Pb as inert nuclei (SC). As for the three cases,we assumed the target nucleus ^{20}Pb is vibrational coupling with deformation parameter β_0= 0.055 to the state $2^{+}(4.085MeV)$ and the projectile ^{9}Be was inert. We used single-quadruple phonon excitation for the projectile and target nuclei that were vibrationally excited. The values of the nuclear potential parameters (V_0) have been obtained from SC and CC analysis, as well as other parameters of WS potential(radius r_0 and diffuseness a_0) and the values of χ^2fitting between experimental and theoretical data for the $^{9}Be+^{208}Pb$ reaction were shown in Table (1).

By looking at the outcomes in Table (1), we find that the better suitable value nuclear potential parameter which has obtained from CC analysis (where the projectile ^{9}Be was inert and target ^{208}Pb nuclei was vibrational coupling) is $45MeV$ with χ^2=0.00242, this result considered very near for standard value and represented by the solid line in Fig. (1)(B), while the dashed line represents the single-channel accounts with the nuclear potential parameter, was drawn for the comparison, that our calculation outputs bred by all computational models are near one another and in consensus with the experimental outputs. The $\frac{d\sigma_{qel}}{d\sigma_R}$ at the best fitted nuclear potential parameter is $45MeV$, with χ^2=0.00242 using a coupled channel calculation at deep sub-barrier energies. In this reaction, we assumed that projectile ^{9}Be is inert whilst the target ^{208}Pb is vibrational coupling to the state 2^{+}. The Figure(C) represents a comparison between the best value of the single channel and the coupling channels, which was found using the chi-square χ^2 code. We concluded that CC and phonon excitation influences augment the calculated cross-sections at energies near the barrier district. It is observed that the influence of vibrational states for the spherical nuclei states for the deformed nuclei, is the effective couplings leading to big fusion cross-sections around the barrier regions.

Table 1. The parameters of the WS potential a_0,r_0,andV_0, as well as the values of χ^2 fitting between experimental and theoretical data for various types of reactions when the excited nuclei are in a vibrational excitation state with a single-quadruple phonon.

System	Case	r$_0(fm)$	a$_0(fm)$	$V_0(MeV)$	χ^2
$^{9}Be+^{208}Pb$	Single channel		0.63	45	0.00248
				50	0.00257
				60	0.00273
	Inert-Vib.		0.63	45	0.00242
	Inert-Vib.			50	0.00250
	Inert-Vib.			60	0.00257

In the $^{10}Be+^{208}Pb$ system, the nuclear potential parameter has been discussed in four states,in the first state we considered the projectile ^{10}Be as well as ^{208}Pb as inert nuclei,while in the three cases,we assumed the target nucleus ^{208}Pb is vibrational coupling with deformation parameter β_0=0.055 to the state $2^{+}(4.085MeV)$ and the projectile ^{10}Be was inert. We used single-quadruple phonon excitation for the projectile and target nuclei that were vibrationally excited. The values of the nuclear potential parameters (V_0) have been obtained from SC and CC analysis, as well as other parameters of WS potential (radius r_0 and diffuseness a_0) and the values of χ^2fitting between experimental and theoretical data for the $^{10}Be+^{208}Pb$ reaction were shown in Table (2).

By observing the results in Table(2),We find that the nuclear potential parameter's more appropriate value,as determined via CC analysis (where the projectile ^{10}Be was inert and target ^{208}Pb nuclei was vibrational coupling) is $65MeV$ with χ^2=0.00550,these results are perceived to be near the conventional value,and represented by the dash dot line in Fig.(2)(B), the Figure (C) represents a comparison between the best value of the single channel and the coupling channels, which was found using the chi-square χ^2 code. We have shown that in heavy ion fusion reactions, higher order couplings to nuclear surface vibrations play an important role.

In the $^{11}Be+^{208}Pb$ system, the nuclear potential parameter has been discussed in four states,in the first state we considered the projectile ^{11}Be as well as ^{208}Pb as inert nuclei, while in the As for the three cases, we assumed the target nucleus ^{208}Pb is vibrational coupling with deformation parameter β_0=0.055 to the state $2^{+}(4.085MeV)$ and the projectile ^{11}Be was inert. We used single-quadruple phonon excitation for the projectile and target nuclei were vibrationally excited. The values of the nuclear potential parameters (V_0) have been obtained from SC and CC analysis, as well as other parameters of WS potential (radius r_0 and diffuseness a_0) and the values of χ^2fitting between experimental and theoretical data for the $^{11}Be+^{208}Pb$ reaction were shown in Table (3).

By observing the results in Table(3), we find that the nuclear potential parameter's more appropriate value, as determined via CC analysis (where the projectile ^{11}Be was inert and target ^{208}Pb nuclei was vibrational

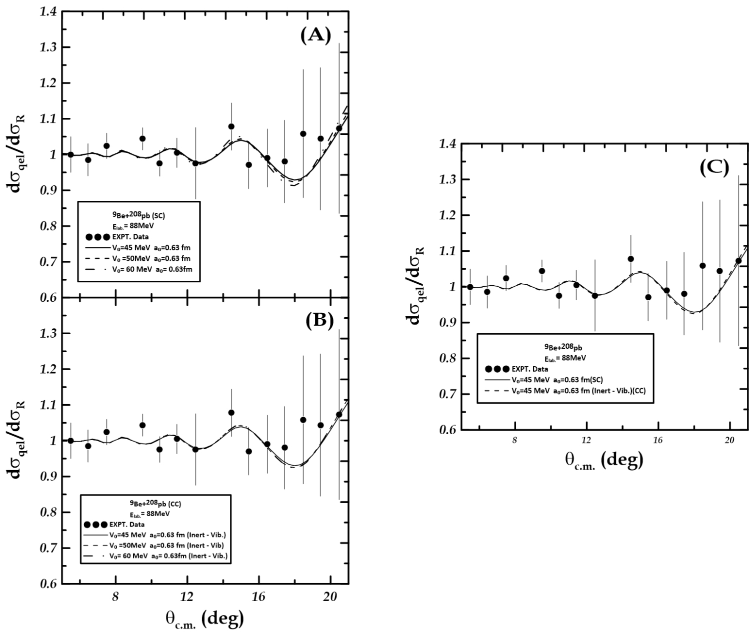

Figure 1. Comparison of accounts for single-channel and kinds of linked channels using experimental data[21]Referred to as points with error bars for the system.In the upper panel (a) the hard and dashed lines represent the results of SC analysis at $V_0 = 45MeV$,$V_0 = 50MeV$ and $V_0 = 60MeV$ respectively,while the hard, dashed and dot-dashed lines in the lower panel (b) represent the results of CC analysis at $V_0 = 45MeV$ (represents the better suitable value of the nuclear potential parameter)$V_0 = 50MeV$ and $V_0 = 60MeV$respectively (C) comparison between the best value of the single channel and the coupling channels.

Table 2. The parameters of the WS potential a_0,r_0,andV_0, as well as the values of χ^2 fitting between experimental and theoretical data for various types of reactions when the excited nuclei are in a vibrational excitation state with a single-quadruple phonon.

System	Case	r$_0(fm)$	a$_0(fm)$	$V_0(MeV)$	χ^2
				49	0.00630
^{10}Be+^{208}Pb	Single channel		0.63	55	0.00576
				65	0.00551
	Inert-Vib.			49	0.00636
	Inert-Vib.		0.63	55	0.00576
	Inert-Vib.			65	0.00550

coupling) is 53MeV with χ^2=0.00639, these results are perceived to be near the conventional value, and represented by the solid line in Fig.(3)(B), the figure (C) represents a comparison between the best value of the single channel and the coupling channels, which was found using the chi-square χ^2 code. We have shown that in heavy ion fusion reactions, higher order couplings to nuclear surface vibrations play an important role.

Table 3. The parameters of the WS potential a_0,r_0,andV_0,as well as the values of χ^2 fitting between experimental and theoretical data for various types of reactions when the excited nuclei are in a vibrational excitation state with a single-quadruple phonon.

System	Case	r$_0(fm)$	a$_0(fm)$	$V_0(MeV)$	χ^2
				43	0.00759
^{11}Be+^{208}Pb	Single channel		0.63	53	0.00644
				59	0.00647
	Inert-Vib.			43	0.00756
	Inert-Vib.		0.63	53	0.00639
	Inert-Vib.			59	0.00640

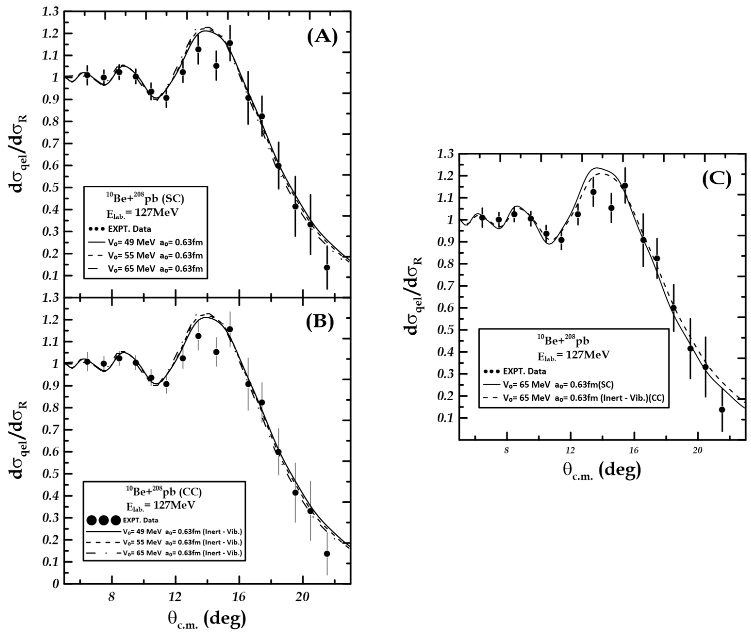

Figure 2. Comparison of accounts for single-channel and kinds of linked channels using experimental data[21]Referred to as points with error bars for the system.In the upper panel (a) the hard and dashed lines represent the results of SC analysis at V_0=49MeV ,V_0=55MeV and V_0=65MeV respectively,while the hard,dashed and dot-dashed lines in the lower panel (b) represent the results of CC analysis at V_0=49MeV ,V_0=55MeV and V_0=65MeV (represents the better suitable value of the nuclear potential parameter) respectively (C) comparison between the best value of the single channel and the coupling channels.

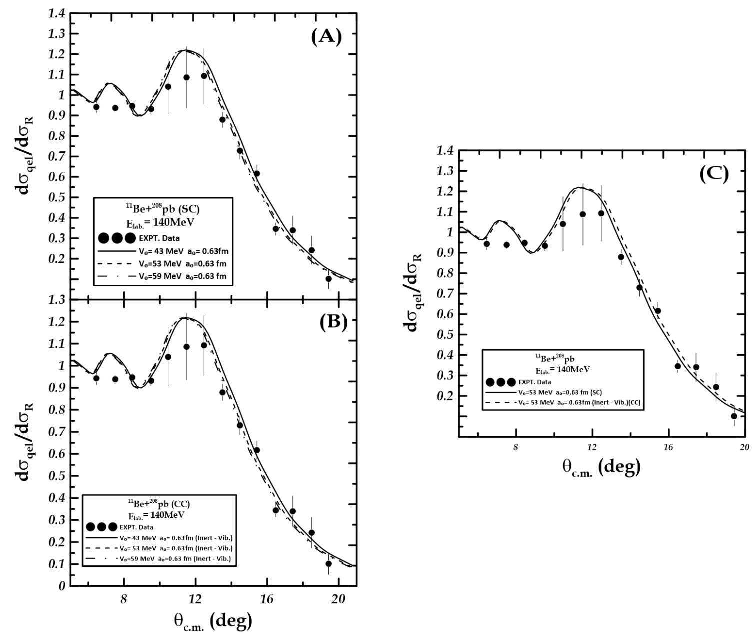

Figure 3. Comparison of accounts for single-channel and kinds of linked channels using experimental data [21]. Referred to as points with error bars for the system. In the upper panel (a) the hard and dashed lines represent the results of SC analysis at $V_0 = 43MeV$,$V_0 = 53MeV$ and $V_0 = 59MeV$ respectively, while the hard, dashed and dot-dashed lines in the lower panel (b) represent the results of CC analysis at $V_0 = 43MeV$,$V_0 = 53MeV$ (represents the better suitable value of the nuclear potential parameter) and $V_0 = 59MeV$ respectively (C) comparison between the best value of the single channel and the coupling channels.

5. CONCLUSION

We found, through micro methodology analyses of the data, that the method of large angle quasi-elastic scattering at deep sub-barrier energies near to the Coulomb barrier height is the perfect instrument for examining the surface property of Inter nucleus potential for the spherical systems discussed in this article. Single-channel analyses fit experimental data gives nuclear potential parameters for the systems ^{9}Be+^{208}Pb,^{10}Be+^{208}Pb and ^{11}Be+^{208}Pb respectively, does not differ substantially from the nuclear potential parameter's best fitted value, which was obtained using CC analysis (with an inert projectile and a vibrating target) and is exactly in line with the standard value. All coupling channel accounts produced values that were quite close to the nuclear potential parameter's standard value.

Declarations

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- Conflict of interest: No conflict of interest.
- Ethics approval: The Research is not involving the studies on human or their data.
- Consent to participate: Consent.
- Consent for publication: Consent.
- Availability of data and materials: Available
- Authors' contributions: All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by MAH and WHR. The first draft of the manuscript was written by MAH and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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REFERENCES

[1] R. Naik, PhD Thesis, "Studying fusion reactions for effect of P (CN) on heavy nucleus formation and for nuclear structure effects," Oregon State University, 2007.

[2] N.H. Hayef, and K.S. Jassim, "Coupled channels for quasi-elastic scattering of determining diffuseness parameters in Woods-Saxon potential for nuclear reaction," AIP Conference Proceedings, **2414**, 030012 (2023). https://doi.org/10.1063/5.0117002

[3] Z.M. Cinan, B. Erol, T. Baskan, and A.H. Yilmaz, "Heavy-Ion Fusion Reaction Calculations: Establishing the Theoretical Frameworks for 111" In Radionuclide over the Coupled Channel Model," Energies, **14**(24), 8594 (2021). https://doi.org/10.3390/en14248594

[4] F.A. Majeed, K.S. Jassim, and N.H. Abbas, "Role of Coupled-Channels in Heavy Ions Reactions at the Coulomb Barrier," Int. J. Sci. Res. **3**, 315-319 (2014). https://livedna.net/?dna=964.25272

[5] H. Timmers, J.R. Leigh, M. Dasgupta, D.J. Hinde, R.C. Lemmon, J.C. Mein, C.R. Morton, et al., "Probing fusion barrier distributions with quasi-elastic scattering," Nuclear Physics A, **584**(1), 190-204 (1995). https://doi.org/10.1016/0375-9474(94)00521-N

[6] K.S. Jassim, and Q.J. Tarbool, "A Study of the surface diffuseness of inter-nucleus potential with quasi-elastic scattering for the $^{32,34}{}_{16}$S+$^{208}{}_{82}$Pb reactions," Al-Bahir Journal for Engineering and Pure Sciences, **1**(1-2), 125-140 (2015). https://www.iasj.net/iasj/pdf/a338dda2a3e54bda

[7] H.K. Hagino, and N. Rowley, "Large-angle scattering and quasielastic barrier distributions," Phys. Rev. C, **69**, 054610 (2004). https://doi.org/10.1103/PhysRevC.69.054610

[8] M.M. Evers, Ph.D thesis, "Systematics of near-barrier nuclear reactions using quasi-elastic scattering", The Australian National University, (2010).

[9] Q.J. Tarbool, K.S. Jassim, and A.A. Abojassim, "Surface diffuseness parameter with quasi-elastic scattering for some heavy-ion systems," International Journal of Nuclear Energy Science and Technology, **13**(2), 2019. https://www.inderscienceonline.com/doi/pdf/10.1504/IJNEST.2019.100758

[10] N. Wang, and M. Liu, and Y.-X. Yang, "Heavy-ion fusion and scat- tering with Skyrme energy density functional," Sci. China Series G: Physics, Mechanics and Astronomy, 2009. **52**, 1554–1573 (2009). https://doi.org/10.1007/s11433-009-0205-z

[11] V.I. Zagrebaev, "Understanding the barrier distribution function derived from backward-angle quasi-elastic scattering," Phys. Rev. C, **78**, 047602 (2008), https://doi.org/10.1103/PhysRevC.78.047602

[12] M. Zadro, P. Figuera, A. Di Pietro, M. Fisichella, M. Lattuada, T. Lönnroth, M. Milin, et al., "Quasielastic backscattering and barrier distributions for the 6, 7 Li+ 64 Zn systems," Physical Review C, **87**, 054606 (2013). https://doi.org/10.1103/PhysRevC.87.054606

[13] P.R.S. Gomes, D.R. Otomar, T. Correa, L.F. Canto, J. Lubian, R. Linares, D.H. Luong, et al., "Complete fusion enhancement and suppression of weakly bound nuclei at near barrier energies," J. Phys. G: Nucl. Part. Phys. **39**, 115103 (2012). https://doi.org/10.1088/0954-3899/39/11/115103

[14] Y.Y. Yang, J.S. Wang, Q. Wang, D.Y. Pang, J.B. Ma, M.R. Huang, P. Ma, et al., "Quasi-elastic scattering of ^{10,11}C and ^{10}B from a ^{nat}Pb target," Phys. Rev. C, **90**, 014606 (2014). https://doi.org/10.1103/PhysRevC.90.014606

[15] K. Hagino, N. Rowley, and A.T. Kruppa, "A program for coupled-channel calculations with all order couplings for heavy-ion fusion reactions," Computer Physics Communications, **123**(1-3), 143-152 (1999). https://doi.org/10.1016/S0010-4655(99)00243-X

[16] M. Dasgupta, D.J. Hinde, J.O. Newton, and K. Hagino, "The nuclear potential in heavy-ion fusion," Progress of Theoretical Physics Supplement, **154**, 209–216 (2004). https://doi.org/10.1143/PTPS.154.209

[17] L.R. Gasques, M. Evers, D.J. Hinde, M. Dasgupta, P.R.S. Gomes, R.M. Anjos, M.L. Brown, et al., "Systematic study of the nuclear potential through high precision back-angle quasi-elastic scattering measurements," Phys. Rev. C, **76**, 024612 (2007). https://doi.org/10.1103/PhysRevC.76.024612

[18] M.V. Andres, N. Rowley, and M.A. Nagarajan, "Effect of deformation on the elastic and quasielastic scattering of heavy ions near the Coulomb barrier," Physics Letters B, **202**(3), 292-295 (1988). https://doi.org/10.1016/0370-2693(88)90473-X

[19] W. Reisdorf, "Heavy-ion reactions close to the Coulomb barrier," J. Phys. G: Nucl. Part. Phys. **20**, 1297 (1994). https://doi.org/10.1088/0954-3899/20/9/004

[20] K. Washiyama, K. Hagino, and M. Dasgupta, "Probing surface diffuseness of nucleus-nucleus potential with quasielastic scattering at deep sub-barrier energies," Phys. Rev. C, **73**, 034607 (2006). https://doi.org/10.1103/PhysRevC.73.034607

[21] RIBLL Collaboration, "Scattering of the halo nucleus 11Be from a lead target at 3.5 times the Coulomb barrier energy," Physics Letters B, **811**, 135942 (2020). https://doi.org/10.1016/j.physletb.2020.135942

ВИВЧЕННЯ ЯДЕРНОГО ПОТЕНЦІАЛУ ЗА ДОПОМОГОЮ РОЗРАХУНКУ КВАЗІПРУЖНОГО РОЗСІЮВАННЯ ДЛЯ РЕАКЦІЇ $^{9,10,11}Be+^{208}Pb$

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Конкретні систематичні дослідження параметра ядерного потенціалу для реакцій важких іонів, які включають системи, були досягнуті за допомогою великокутового квазіпружного розсіювання при глибоких підбар'єрних енергіях, близьких до висоти кулонівського бар'єру. Для визначення ядерного потенціалу були проведені розрахунки для одноканального (SC) варіанту та варіанту зв'язаних каналів (CC). Методом χ^2 було знайдено найкраще значення ядерного потенціалу в порівнянні з експериментальними даними. Найкращі значення ядерного потенціалу знайдено з розрахунків зв'язаних каналів для інертного налітаючого ядра та коливальної мішені для систем: ^{9}Be+^{208}Pb,^{10}Be+^{208}Pb, ^{11}Be+^{208}Pb, які дорівнюють 45 MeB, 65 MeB, 53 MeB відповідно.

Ключові слова: *розрахунки зв'язаних каналів; реакції синтезу важких іонів; квазіпружне розсіювання; глибокі підбар'єрні енергії*