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# ISOSCALAR GIANT OCTUPOLE RESONANCE ISGOR OF $^{116}$Cd USING SELF-CONSISTENT SKYRME QRPA[†]

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Collective models based on the random phase approximation (RPA) are widely used to accurately depict collective modes of response. They can quickly calculate the strength function for the entire nuclear mass range. The quasi-particle random phase approximation (QRPA), which considers the pairing effect, is an enhanced RPA model. It is anticipated that this effect will be significant for open-shell nuclei. In this work, the self-consistent Skyrme Hartree-Fock-Bardeen, Cooper, and Schrieffer (HF-BCS) and QRPA models have been used to study the isoscalar giant octupole resonance (ISGOR) in the $^{116}$Cd isotope. Ten Skyrme-type parameters are utilized in the computations since they may be identified by different values of the incompressibility modulus $K_{MN}$ in nuclear matter. The calculated strength distributions and centroid energy are compared with available experimental data. We saw that the strength distributions varied depending on the type of Skyrme-interaction, and we also observed a definite impact of the $K_{NM}$ values on the centroid energy.
**Keywords:** *Collective models; Isoscalar Giant Octupole Resonance (ISGOR); Skyrme force; Quasiparticle Random Phase Approximation (QRPA); Hartree-Fock (HF); Bardeen Cooper and Schrieffer (BCS)*
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## 1. INTRODUCTION

The Bardeen, Cooper, and Schrieffer (BCS) theory [1] provides both the quasi-particle energies and the occupation probabilities of the single particle levels to describe the ground state characteristics of even-even open shell nuclei. This method depends on a set of input single particle states. For each single particle state, the model gives partial occupation probabilities from using a pairing nucleon-nucleon interaction. The pairing effects on different ground state observables are determined using these probabilities. The description of the excited states must go farther because the HF+BCS cannot account for collective effects. The Random Phase Approximation (RPA) theory is frequently extended by the quasi-particle RPA (QRPA) [2,3], which was created to handle pairing and partial occupation probability of the single particle levels.

Giant resonances (GR) [4,5] serve as an example of the collective modes in atomic nuclei that occur at excitation energies between 10 and 30 MeV. These collective modes are related to the nucleons' collective motion inside the nucleus and are divided into different modes [6] based on their multipolarity L, spin S, and isospin T quantum numbers. According to theory, GR is influenced by the response's nucleon participation rate and transition amplitudes.

The E3 response is split into two branches, the 1ħω component has been referred to as the low energy octupole resonance, which is firstly observe by Moss et al. in 1976 utilizing inelastic scattering of alpha particles, while the higher (3ħω) component is referred to as the high energy octupole resonance [7].

The isotope $^{116}$Cd is one of the most promising 2β nuclei thanks to the favorable theoretical estimations of the decay probability [8,9], large energy release $Q_{2\beta}$ = 2813.50(13) keV [10], relatively high isotopic abundance δ = 7.49% [11] and a possibility of isotopic enrichment in a large amount.

In this work, the isoscalar giant octupole resonance (ISGOR) of isotope $^{116}$Cd was investigated within the framework of a self-consistent Hartree-Fock (HF)- Bardeen, Cooper, and Schrieffer (BCS) based on Quasi particle Random Phase Approximation (QPRPA) with 10 different sets of Skyrme effective nucleon-nucleon force: SkP [12], eMSL09 [13], MSL0 [14], T44 [15], BSK20 [16], Ska [17], SV [18], QMC2 [19], SII [20] and SGOI [17] of different incompressibility modulus [21] in nuclear matter $K_{NM}$ = 200.97, 229.6, 230.00, 230.01, 241.39, 263.16., 305.70, 330.10, 341.40, and 361.59 MeV, respectively. Having a large number of Skyrme-force parameterisations requires a continuous search for the best for describing the experimental data. In order to establish the best sets of Skyrme-force parameterizations for defining the experimental data, the strength function and centroid energy of the isoscalar ISGOR (J; T = 3-; 0) were compared with the available experimental data. It was also studied how the computed centroid energy changes with values of $K_{NM}$.

## 2. DESCRIPTION OF CALCULATIONS

The occupation probabilities of the single particle levels and the quasi-particle energies are provided by the Bardeen, Cooper, and Schrieffer (BCS) theory to describe the ground state characteristics of even-even open shell nuclei. When the standard BCS equations, which under spherical symmetry provide particle number n and gap equation $\Delta_a$, are coupled with the Hartree-Fock HF equations, the total HF-BCS energy can be determined.

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It is necessary to go beyond the HF+BCS, which is unable to account for collective effects, in order to describe the excited states. As a result, the Random Phase Approximation (RPA) theory has been extended to include quasi-particle RPA (QRPA), which is effective in describing collective states of open-shell nuclei [22,23]. Here is

$$\begin{pmatrix} A_{ab,cd} & B_{ab,cd} \\ -B^*_{ab,cd} & -A^*_{ab,cd} \end{pmatrix}\begin{pmatrix} X^{\nu}_{cd} \\ Y^{\nu}_{cd} \end{pmatrix} = E^{\nu}_x \begin{pmatrix} X^{\nu}_{ab} \\ Y^{\nu}_{ab} \end{pmatrix} (1) \quad (1)$$

with,

$$A_{ab,cd} = \frac{1}{\sqrt{1+\delta_{ab}}\sqrt{1+\delta_{cd}}}\big[(E_a + E_b)\delta_{ac}\delta_{bd} + (u_a u_b u_c u_d + v_a v_b v_c v_d)G_{abcd} + (u_a v_b u_c v_d + v_a u_b v_c u_d)F_{abcd} - (-1)^{j_c+j_d-J'}(u_a v_b v_c u_d + v_a u_b u_c v_d)F_{abdc}\big] \quad (2)$$

$$B_{ab,cd} = \frac{1}{\sqrt{1+\delta_{ab}}\sqrt{1+\delta_{cd}}}\big[-(u_a u_b v_c v_d + v_a v_b u_c u_d)G_{abdc} - (-1)^{j_c+j_d-J'}(u_a v_b u_c v_d + v_a u_b v_c u_d)F_{abdc} + (-1)^{j_a+j_b+j_c+j_d-J-j}(u_a v_b v_c u_d + v_a u_b u_c v_d)F_{abcd}\big], \quad (3)$$

where,

$$G_{abcd} = \sum_{m_a m_b m_c m_d}\langle j_a m_a j_b m_b|JM\rangle\,\langle j_c m_c j_d m_d|J'M'\rangle V^{pp}_{ab,cd}\,, \quad (4)$$

$$F = \sum_{m_a m_b m_c m_d}\langle j_a m_a j_b m_b|JM\rangle\,\langle j_c m_c j_d m_d|J'M'\rangle V^{ph}_{ab,cd}\,. \quad (5)$$

here $V^{pp}_{\alpha\beta\gamma\delta}$ and $V^{ph}_{\alpha\beta\gamma\delta}$ are the particle-particle and hole-hole matrix elements, respectively.

The QRPA states $|n\rangle$ with matching energy $E_n$ can be used to determine the strength or response function [24–26],

$$S(E) = \sum_n \left|\langle 0|\hat{F}|n\rangle\right|^2 \delta(E - E_n| \quad (6)$$

The energy moments can be calculate using,

$$m_k = \int E^k S(E) dE \quad (7)$$

## 3. RESULTS AND DISCUSSION

In this work, the response in $^{116}$Cd isotope has been studied in the framework of self-consistent QRPA+HFBCS method with Skyrme-type interactions. It is noteworthy to mention that 240 Skyrme interactions that were previously published in the literature underwent analysis by a separate team [27,28] to determine their ability to describe experimental data on nuclear matter, nuclei's properties and observational data of neutron stars, such as the binding energies, radii, effective mass, incompressibility coefficient, symmetry energy density, and fission barriers. The following 10 were chosen to be studied in this work: SkP [12], eMSL09 [13], MSL0 [14], T44 [15], BSK20 [16], Ska [17], SV [18], QMC2 [19], SII [20], and SGOI [17] of various incompressibility moduli in nuclear matter, with $K_{MN}$ = 200.97, 229.6, 230.00, 230.01, 241.39, 263.16., 305.70, 330.10, 341.40, and 361.59 MeV, respectively.

The E3 resonance is divided into low and high energy octupole resonance, as was indicated in the introduction section. The low energy octupole resonance includes 25% of the energy-weighted sum rules for electric E3 and the high energy octupole resonance contains 75% of the energy-weighted sum rules, according to the Harmonic Oscillator Shell Model's explanation of the Giant Resonances. When these modes are connected with the octupole residual reaction, a low energy octupole resonance with around 35% of the energy-weighted sum rules and a high energy octupole resonance with 65% of the energy-weighted sum rules are generated [29].

Our calculated fraction energy-weighted sum rules (EWSR/MeV) for ISGOR in $^{116}$Cd are displayed in Fig. 1. The low energy octupole resonance strength is obtained in the energy range from 5 and 15 MeV, and the high energy octupole resonance strength is located between 15 and 35 MeV. Our results were compared with available experimental data using two Lorenzian smearing widths of 3 and 5 MeV. The calculations utilizing emSL09 and QMC2 Skyrme interactions with ***Γ***= 5 MeV was the best in describing the practical results experimental data, as shown in Fig. 1.

Most of interactions work best and agree with data concerning centroid energy ($m_1/m_0$), widths and profiles of strength. The form of the calculated strength distribution for $^{116}$Cd are in good agreement with the experimental, but the calculated strength distribution peaks were 1-4 MeV higher than the experimental. In $^{116}$Cd, the form of the strength distribution is like Gaussian distribution in the low excitation region but with a large tailing on the high energy extending to 40 MeV.

It is not completely clear, from the experimental point of view, whether the constrained, centroid or scaling energies are more suitable to be compared with the experimental data. However, from what we have just concluded, it can be stated that the reasonable values of nuclear incompressibility that can be extracted from the present $^{116}$Cd data are either the one of SkP or emSL09, namely 200.97 or 229.6 MeV.

The formula of the centroid energy $E_{cen} = m_1/m_0$ is measured in a particular energy band close to the resonance peak. Our theoretical calculations of $m_1/m_0$ (MeV) for ISGOR in $^{116}$Cd and experimental values [30] are listed in Table 1.

The $m_1/m_0$ of the ISGOR as a function of the nuclear matter incompressibility coefficient $K_{NM}$ is presented in Table 1 and depicted in Fig. 2. The calculated centroid energies of the most Skyrme interactions are near and below of the experimental value. We found that emSL09 interaction with $K_{NM}$ = 229.6 MeV agree with data.

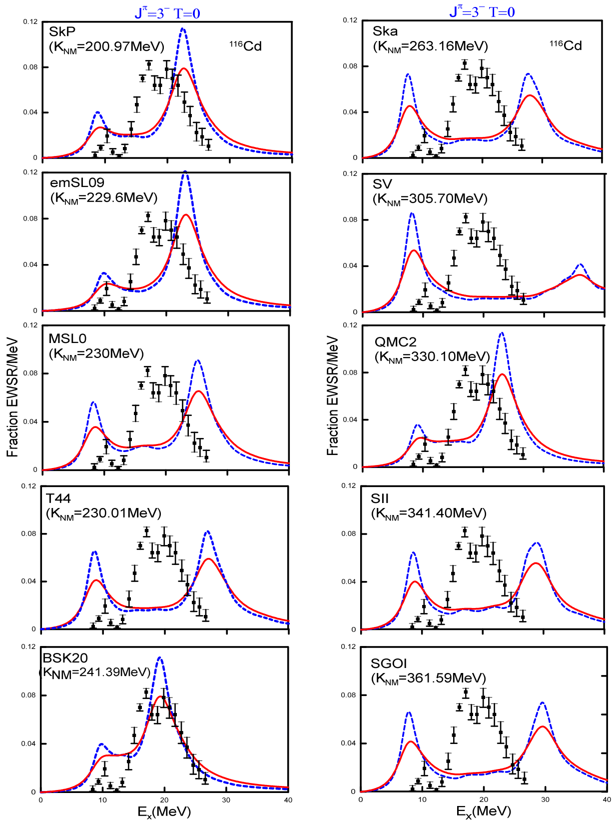

**Figure 1.** Our calculated fraction energy-weighted sum rules (EWSR/MeV) for ISGOR in $^{116}$Cd with Lorentzian smearing widths of 3 MeV (blue-dashed lines) and 5 MeV (red lines) were calculated using ten sets of the Skyrme force: SkP, eMSL09, MSL0, T44, BSK20, Ska, SV, QMC2, SII, and SGOI in comparison with the experimental data [30] (Black symbol).

In ISGOR, a high $K_{NM}$ causes the peak to shift to a lower energy while a low $K_{NM}$ causes the peak to shift to a higher energy. For example, an SGOI interaction with a high $K_{NM}$ of 361.59 MeV causes the centroid energy to be 14.964 MeV and a low $K_{NM}$ of 200.97 MeV causes the centroid energy to be 16.800 MeV. The experimental value of $m_1/m_0 = 18.28^{-0.09}_{+0.25}$ MeV [30] for $^{116}$Cd as shown in Table 2. Fig. 3, shows the significant change in the strength distribution with changing values of $K_{MN}$.

Whereas we can clearly confirm from the present results that the value of the nuclear matter incompressibility does play a key role in dictating the location of the ISGOR centroid energy, it is also true that the pairing interaction lowers the energy of the ISGOR to some extent, typically few hundreds keV. This qualitative conclusion is the same that was first found in Ref. [31]. Thus, the pairing interaction cannot be neglected if one aims to reproduce not only the ISGOR centroid energies in Cd isotopes, but also, more generally, in other open-shell nuclei.

For the low energy octupole resonance, the strength distributions are concentrated in a single peak around 10 MeV and for the high energy octupole resonance, the strength distributions are concentrated in a single peak around 25 MeV. However, the location of the peak found with each Skyrme interaction is slightly different. The SkP interaction predicts lowest peaks while the SGOI interaction gives peaks at the highest energies. As it is known from previous studies, the relative position of the peaks is governed by the nuclear matter incompressibility associated with each effective interaction.

**Table 1.** Our calculated ISGOR centroid energies, constrained energies, and scaling energies in $^{116}$Cd based on ten Skyrme-type interaction are compared with the experimental data [30].

| | $K_{NM}$ MeV | $E_{cen}=m_1/m_0$ MeV | $E_{con}=\sqrt{m_1/m_{-1}}$ MeV | $E_s=\sqrt{m_3/m_1}$ MeV |
|---|---|---|---|---|
| **Exp.** | | $\mathbf{18.28^{-0.09}_{+0.25}}$ | | |
| SkP | 200.97 | 16.80 | 15.207 | 20.737 |
| eMSL09 | 229.60 | 18.25 | 16.627 | 21.535 |
| MSL0 | 230.00 | 15.63 | 13.590 | 21.816 |
| T44 | 230.01 | 15.61 | 13.544 | 22.740 |
| BSK20 | 241.39 | 15.99 | 15.165 | 18.165 |
| Ska | 263.16 | 13.93 | 11.735 | 22.741 |
| SV | 305.70 | 14.19 | 11.907 | 27.423 |
| QMC2 | 330.10 | 17.45 | 15.917 | 21.191 |
| SII | 341.40 | 15.98 | 13.698 | 23.987 |
| SGOI | 361.59 | 14.96 | 12.503 | 24.395 |

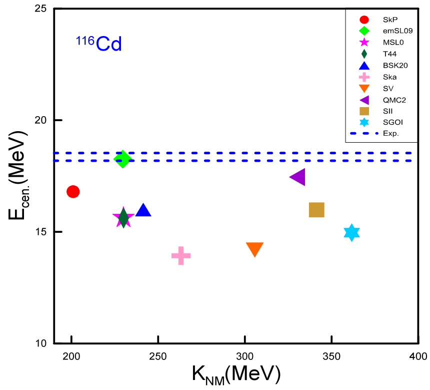


**Figure 2.** Our calculated ISGOR centroid energies (color symbols) as a function of $K_{NM}$ in $^{116}Cd$ based on 10 Skyrme-type interaction in comparison with the experimental data [30] (shown as the regions between the dashed blue lines).

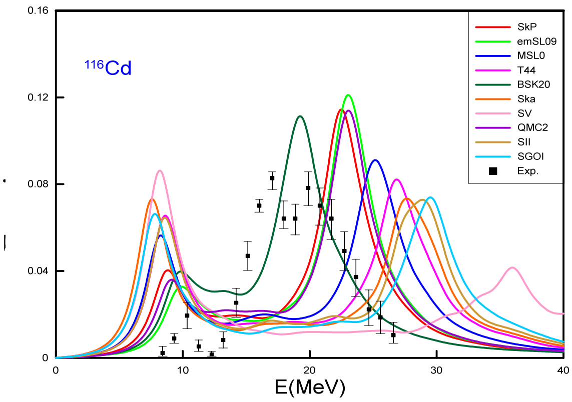


**Figure 3.** Using 10 Skyrme-type interaction and 3 MeV wide Lorenzian smearing, we estimated the fraction energy-weighted sum rules (EWSR/MeV) for ISGOR in $^{116}Cd$ and compared it to the experimental results [30].

## 4. CONCLUSIONS

The strength function of ISGOR has been subjected to self-consistent QRPA based on HF-BCS calculations. The low energy octupole resonance strength is identified between 5 and 15 MeV, and the high energy octupole resonance strength is focused between 15 and 40 MeV. The low energy octupole resonance strength is discovered between 5 and 15 MeV, while the high energy octupole resonance strength is concentrated between 15 and 40 MeV, putting the isoscalar E3 strength in the range of 5 to 40 MeV in $^{116}Cd$. By using emSL09 with $K_{NM}$ = 229.6 MeV. The centroid energy is pushed to a lower energy by high $K_{NM}$, whereas a higher energy is reached by low $K_{NM}$. For instance, the eMSL09 interaction with low $K_{NM}$ = 229.60 MeV produces the energy equivalent to 18.25 MeV in $^{116}Cd$ while the SGOI interaction with high $K_{NM}$ = 361.59 MeV does the same for $^{116}Cd$.

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## ІЗОСКАЛЯРНИЙ ГІГАНТСЬКИЙ ОКТУПОЛЬНИЙ РЕЗОНАНС ISGOR $^{116}$Cd З ВИКОРИСТАННЯМ САМОУЗГОДЖЕНОГО SKYRME QRPA

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Колективні моделі, засновані на апроксимації випадкової фази (RPA), широко використовуються для точного зображення колективних режимів реакції. Вони можуть швидко розрахувати функцію сили для всього діапазону ядерних мас. Квазічастинкове наближення випадкової фази (QRPA), яке враховує ефект спарювання, є розширеною моделлю RPA. Очікується, що цей ефект буде значним для ядер з відкритою оболонкою. У цій роботі використовуються самоузгоджені моделі Skyrme Hartree-Fock-Bardeen, Cooper, and Schrieffer (HF-BCS) і QRPA для дослідження ізоскалярного гігантського октупольного резонансу (ISGOR) в ізотопі $^{116}$Cd. У розрахунках використовуються десять параметрів типу Скірма, оскільки вони можуть бути ідентифіковані різними значеннями модуля нестисливості KMN в ядерній речовині. Розраховані розподіли міцності та енергія центроїда порівнюються з наявними експериментальними даними. Ми побачили, що розподіл сили змінювався залежно від типу взаємодії Скірма, і ми також спостерігали певний вплив значень KNM на енергію центроїда.

**Ключові слова:** *колективні моделі; ізоскалярний гігантський октупольний резонанс (ISGOR); сила Skyrme; квазічастинкове наближення випадкової фази (QRPA); Хартрі-Фок (HF); Бардін Купер і Шріффер (BCS)*