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OSCILLATORY POROUS MEDIUM FERROCONVECTION IN A VISCOELASTIC MAGNETIC FLUID WITH NON-CLASSICAL HEAT CONDUCTION[†]

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The classical stability analysis is used to examine the combined effect of viscoelasticity and the second sound on the onset of porous medium ferroconvection. The fluid and solid matrix are assumed to be in local thermal equilibrium. Considering the boundary conditions appropriate for an analytical approach, the critical values pertaining to both stationary and oscillatory instabilities are obtained by means of the normal mode analysis. It is observed that the oscillatory mode of instability is preferred to the stationary mode of instability. It is shown that the oscillatory porous medium ferroconvection is advanced through the magnetic forces, nonlinearity in magnetization, stress relaxation due to viscoelasticity, and the second sound. On the other hand, it is observed that the presence of strain retardation and porous medium delays the onset of oscillatory porous medium ferroconvection. The dual nature of the Prandtl number on the Rayleigh number with respect to the Cattaneo number is also delineated. The effect of various parameters on the size of the convection cell and the frequency of oscillations is also discussed. This problem may have possible implications for technological applications wherein viscoelastic magnetic fluids are involved.
Keywords: *Convection; Maxwell equations; Navier-Stokes equations for incompressible viscous fluids; Porous media; Viscoelastic fluids, Ferroconvection*
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1. INTRODUCTION

Ferroconvection is a transfer of heat from one place to another in ferromagnetic liquids and its importance is due to the applications suggested by several authors [1-3] and many more. Ferrofluids, also known as magnetic fluids, are colloidal suspensions of nanosized ferromagnetic particles stably dispersed in organic or non-organic carrier fluids such as kerosene, water, and hydrocarbon. When exposed to an external magnetic field, they behave paramagnetically with susceptibility usually large for magnetic liquids [4]. Ferrofluids have commercial applications like vacuum feed-throughs for manufacturing semi-conductors [5]. Ferrofluid is also used in taking the drug in a human body to a target site by applying a magnetic field [6]. However, we can find many applications in different fields [7]. Finlayson [8] studied the convective instability of ferromagnetic fluids due to Bénard in the presence of a uniform vertical magnetic field and explained the thermomechanical interaction concept of ferromagnetic fluids. Lalas and Carmi [9] studied the thermoconvective stability of ferrofluids in the absence of buoyancy effects. Non-Darcy ferroconvection problem with gravity modulation using regular perturbation has been addressed by Nisha Mary and Maruthamanikandan [10]. Darcy-Brinkman ferroconvection with temperature-dependent viscosity has been studied by Soya Mathew and Maruthamanikandan [11] and thermorheological and magnetorheological effects on Marangoni-ferroconvection with internal heat generation has been investigated by Maruthamanikandan et al. [12]. Effect of MFD viscosity on ferroconvection in a fluid saturated porous medium with variable gravity has been examined by Vidya Shree et al. [13].

A good amount of attention is honoured to Rayleigh–Bénard convection (RBC) problems in Newtonian liquids with respect to heat transfer and other engineering applications as referred above. On the other hand, at shallow depths of the reservoirs, oil sands contain waxy crude which are viscoelastic fluids. They exhibit both liquid and solid properties and have many applications to the nuclear, petroleum, and chemical industries. They also have applications in cooling electronic devices, crystal growth, and material processes. In the study of viscoelastic fluids, the rheological equation involves one or two relaxation times (Bird et al. [14] and Joseph [15]) and also oscillatory convection is witnessed which is not noticed in Newtonian fluids. The Oldroyd model [16] is used for describing the viscoelastic properties of dilute polymers. The fact that principle of exchange of stabilities is not valid was shown by Green [17]. Recently, the onset of Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer has been addressed studied by Swamy et al. [18].

The equation governing temperature (heat transport equation) in classical theory assumes a parabolic-type partial differential equation that admits thermal signals at an infinite speed, which is unrealistic. The new theories modified the classical Fourier's law of heat conduction and hence contain a hyperbolic-type heat transport equation that admits the thermal signals at a finite speed. As per this theory, heat propagates as a wave phenomenon rather than a diffusion phenomenon and the wavelike thermal disturbance is referred to as second sound (Chandrasekharaiah [19]). Gurtin and

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Pipkin [20] investigated a general principle of thermal conduction in nonlinear analysis, including memories, a concept having a finite propagation speed. Straughan and Franchi [21] addressed the Bénard advection problem when the Maxwell-Cattaneo heat flow law is utilized in place of the ordinary Fourier theory of thermal conductivity. Soya Mathew and Maruthamanikandan [22] investigated oscillatory porous medium ferroconvection with Maxwell-Cattaneo law of heat conduction where they showed that the oscillatory mode of convection is preferred to stationary mode for large values of Prandtl and second sound parameter.

Under these conditions, the present paper is dedicated to examining convective instability in a Cattaneo viscoelastic ferrofluid saturated sparsely packed porous medium. The influence of various parameters is explored that perhaps direct us to oscillatory convection.

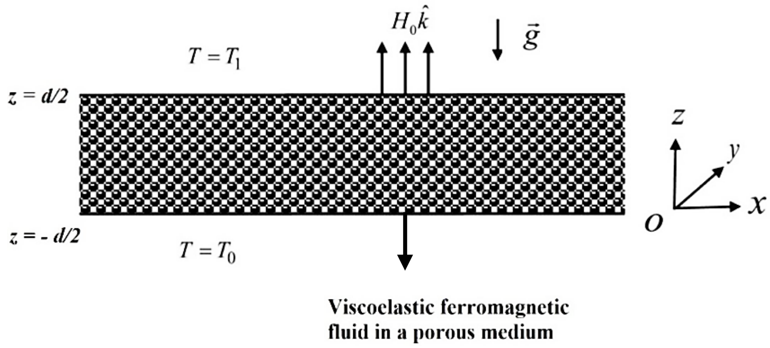

Figure 1. Physical Configuration

2. MATHEMATICAL FORMULATION

Let us consider an incompressible Cattaneo viscoelastic ferromagnetic fluid saturated porous medium confined between the two surfaces of non-finite length horizontally of finite thickness d. We consider Oldroyd's model to characterize the viscoelastic behaviour which is a non-Newtonian one. The lower surface at $z = -d/2$ and upper surface at $z = d/2$ are maintained at temperatures T_1 and T_0 respectively with $T_1 > T_0$ and $\Delta T = T_1 - T_0$ (see **Fig. 1**). It is assumed that at a quiescent state the temperature varies linearly across the depth. When the magnitude of ΔT become larger than the critical one, thermal convection will set in due to buoyancy force.

The fluid layer is exposed to a magnetic field $\vec{H}_0$ acting parallel to the vertical z-axis and the gravity force acting vertically downwards. We assume that Oldroyd's model is sufficient to characterize the viscoelastic behaviour which is simple enough to be tractable analytically. The governing equations supporting the Boussinesq approximation are written as follows.

$$\nabla \cdot \vec{q} = 0 \tag{2.1}$$

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \frac{\rho_0}{\varepsilon^2}\left(\vec{q}\cdot\nabla\right)\vec{q} + \nabla p - \rho\vec{g} - \nabla\cdot\left(\vec{H}\vec{B}\right)\right] = \left(1+\lambda_2\frac{\partial}{\partial t}\right)\left[-\frac{\mu_f}{k}\vec{q} + \overline{\mu_f}\,\nabla^2\vec{q}\right] \tag{2.2}$$

$$\begin{aligned}&\varepsilon\left[\rho_0 C_{V,H} - \mu_0 \vec{H}\cdot\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\right]\left[\frac{\partial T}{\partial t} + \vec{q}\cdot\nabla T\right] + (1-\varepsilon)\left(\rho_0 C\right)_s \frac{\partial T}{\partial t} \\ &+\mu_0 T\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\cdot\left[\frac{\partial \vec{H}}{\partial t} + \left(\vec{q}\cdot\nabla\right)\vec{H}\right] = -\nabla\cdot\vec{Q}\end{aligned} \tag{2.3}$$

$$\tau\left[\frac{\partial \vec{Q}}{\partial t} + \left(\vec{q}\cdot\nabla\right)\vec{Q} + \vec{\omega}\times\vec{Q}\right] = -\vec{Q} - k_1\nabla T \tag{2.4}$$

$$\rho = \rho_0\left[1 - \alpha\left(T - T_a\right)\right] \tag{2.5}$$

$$M = M_0 + \chi_m\left(H - H_0\right) - K_m\left(T - T_a\right) \tag{2.6}$$

where λ_1 is the stress relaxation time, λ_2 is the strain retardation time ($0 \le \lambda_2 < \lambda_1$), $\vec{q} = (u, v, w)$ is the fluid velocity, ρ_0 is the reference density, ε is the porosity, t is the time, p is the pressure, $\vec{g}$ is the acceleration due to gravity, ρ is the fluid density, μ_f is the dynamic viscosity, $\overline{\mu_f}$ is the effective viscosity, k is the permeability of the porous medium, $\vec{H}$ is the magnetic field, $\vec{B}$ is the magnetic induction, T is the temperature, μ_0 is the magnetic permeability, $\vec{M}$ is the magnetization, k_1 is the thermal conductivity, α is the thermal expansion coefficient, $C_{V,H}$ is the specific heat at constant volume and magnetic field, χ_m is the magnetic susceptibility, K_m is the pyromagnetic coefficient, $\vec{Q}$ is the heat flux, τ is a constant with the dimensions of time and $\vec{\omega} = \frac{1}{2}\nabla \times \vec{q}$.

Maxwell's equations for a non-conducting fluid with no displacement currents become (Finlayson [8])

$$\nabla \bullet \vec{B} = 0, \qquad \nabla \times \vec{H} = \vec{0}, \qquad \vec{B} = \mu_0\left(\vec{H} + \vec{M}\right). \tag{2.7}$$

Equations characterizing the basic state are introduced in the form

$$\left.\begin{aligned} &\frac{\partial}{\partial t} = 0,\ \vec{q}_b = (0,0,0),\ T = T_b(z), \\ &p = p_b(z),\ \rho = \rho_b(z),\ \vec{H} = H_b(z), \\ &\vec{M} = M_b(z),\ \vec{B} = B_b(z),\ \vec{Q} = \vec{Q}_b(0,0,k_1\beta) \end{aligned}\right\} \tag{2.8}$$

where $\beta = \frac{T_1 - T_0}{2}$. The solution pertaining to the basic state reads

$$\rho_b = \rho_0[1 + \alpha\beta z] \tag{2.9}$$

$$\vec{H}_b = \left[H_0 - \frac{K_m \beta z}{1+\chi_m}\right]\hat{k} \tag{2.10}$$

$$\vec{M} = \left[M_0 + \frac{K_m \beta z}{1+\chi_m}\right]\hat{k} \tag{2.11}$$

$$\vec{B} = \mu_0\left[\vec{H} + \vec{M}\right]\hat{k}\ . \tag{2.12}$$

3. STABILITY ANALYSIS

We shall obtain the dimensionless equations following the small perturbation stability analysis enveloping normal modes (Finlayson [8], Soya Mathew and Maruthamanikandan [11]). The perturbed state equations involving infinitesimally small perturbations are

$$\left.\begin{aligned} &\vec{q} = \vec{q}_b + \vec{q}\,',\ T = T_b + T',\ p = p_b + p', \\ &\rho = \rho_b + \rho',\ \vec{H} = \vec{H}_b + \vec{H}',\vec{M} = \vec{M}_b + \vec{M}', \\ &\vec{B} = \vec{B}_b + \vec{B}',\ \vec{Q} = \vec{Q}_b + \vec{Q}',\ \phi = \varphi_b + \phi' \end{aligned}\right\} \tag{3.1}$$

where the primes indicate perturbed quantities. The perturbed governing linearized equations take the form

$$\begin{aligned} &\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t}\left(\nabla^2 w'\right) - \alpha g \rho_0 \nabla_1^2 T' + \mu_0 K_m \beta \frac{\partial}{\partial z}\left(\nabla_1^2 \phi'\right) - \frac{\mu_0 K_m^2 \beta \nabla_1^2 T'}{1+\chi_m}\right] \\ &= \left(1+\lambda_2\frac{\partial}{\partial t}\right)\left[-\frac{\mu_f}{k}\nabla^2 w' + \overline{\mu_f}\,\nabla^4 w'\right] \end{aligned} \tag{3.2}$$

$$(\rho_0 C)_1 \frac{\partial T'}{\partial t} - \mu_0 T_a K_m \frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial z}\right) = -\nabla.\vec{Q}' + \left[(\rho_0 C)_2 - \frac{\mu_0 T_a K_m^2}{1+\chi_m}\right]\beta w' \tag{3.3}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\vec{Q}\,'=-\frac{\tau k_1\beta}{2}\left(\frac{\partial\,\vec{q}\,'}{\partial z}-\nabla w'\right)-k_1\,\nabla T' \tag{3.4}$$

$$(1+\chi_m)\frac{\partial^2\phi'}{\partial z^2}+\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\,\phi'-K_m\frac{\partial T'}{\partial z}=0\ . \tag{3.5}$$

We take divergence on both sides of equation (3.4) and substitute in equation (3.3) to eliminate $\vec{Q}\,'$ from equation (3.3). The resulting system of linearized perturbed equations are as follows

$$\begin{gathered}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t}\left(\nabla^2 w'\right)-\alpha g\rho_0\nabla_1^2\,T'+\mu_0K_m\beta\frac{\partial}{\partial z}\left(\nabla_1^2\,\phi'\right)-\frac{\mu_0K_m^2\beta\nabla_1^2\,T'}{1+\chi_m}\right]\\=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left[-\frac{\mu_f}{k}\nabla^2\,w'+\overline{\mu_f}\,\nabla^4\,w'\right]\end{gathered} \tag{3.6}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[(\rho_0\,C)_1\frac{\partial T'}{\partial t}-\mu_0\,T_aK_m\frac{\partial}{\partial t}\left(\frac{\partial\phi'}{\partial z}\right)\right]-\left[(\rho_0\,C)_2-\frac{\mu_0\,T_aK_m^2}{1+\chi_m}\right]\beta w'=-k_1\,\nabla^2T'-\frac{\tau k_1\beta}{2}\nabla^2w' \tag{3.7}$$

$$(1+\chi_m)\frac{\partial^2\phi'}{\partial z^2}+\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\,\phi'-K_m\frac{\partial T'}{\partial z}=0 \tag{3.8}$$

where

$(\rho_0\,C)_1=\varepsilon\rho_0C_{V,H}+\varepsilon\mu_0H_0K_m+(1-\varepsilon)(\rho_0\,C)_s$, $(\rho_0\,C)_2=\varepsilon\,\rho_0C_{V,H}+\varepsilon\,\mu_0H_0K_m$, $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, $\nabla^2=\nabla_1^2+\frac{\partial^2}{\partial z^2}$, $K_m=-\left(\frac{\partial M}{\partial t}\right)_{V,H}$ and $\chi_m=\left(\frac{\partial M}{\partial H}\right)_{H_0,T_a}$ with ϕ' being the magnetic potential.

The normal mode solution is accessible and the same has the form

$$\begin{bmatrix}w'\\T'\\\phi'\end{bmatrix}=\begin{bmatrix}W(z)\\\Theta(z)\\\Phi(z)\end{bmatrix}e^{i(l\,x+m\,y)+\sigma t} \tag{3.9}$$

where *l* and *m* are respectively the wave numbers in the *x* and *y* directions and σ is the growth rate. Substitution of (3.9) into equations (3.6) to (3.8) leads to

$$\begin{gathered}(1+\lambda_1\sigma)\left[\frac{\rho_0}{\varepsilon}\sigma\left(D^2-K_h^2\right)W+\alpha\rho_0gK_h^2\,\Theta-\mu_0K_m\beta K_h^2\,D\Phi\ +\frac{\mu_0K_m^2\beta K_h^2\,\Theta}{1+\chi_m}\right]\\=(1+\lambda_2\sigma)\left[-\frac{\mu_f}{k}\left(D^2-K_h^2\right)W+\overline{\mu_f}\left(D^2-K_h^2\right)^2W\right]\end{gathered} \tag{3.10}$$

$$\begin{gathered}(1+\tau\,\sigma)\left[(\rho_0\,C)_1\,\sigma\,\Theta-\mu_0\,T_aK_m\,\sigma\,D\,\Phi\right]\\-\left[(\rho_0\,C)_2-\frac{\mu_0\,T_aK_m^2}{1+\chi_m}\right]\beta\,W=\ k_1\left(D^2-K_h^2\right)\Theta-\frac{\tau k_1\beta}{2}\left(D^2-K_h^2\right)W\end{gathered} \tag{3.11}$$

$$(1+\chi_m)\,D^2\Phi-\left(1+\frac{M_0}{H_0}\right)K_h^2\,\Phi(z)-K_m\,D\,\Theta=0 \tag{3.12}$$

where $D=\frac{d}{dz}$ and $K_h^2=l^2+m^2$ is the overall horizontal wave number. Non-dimensionalizing equations (3.10) through (3.12) using the scaling

$$\left.\begin{aligned} &W^* = \frac{Wd}{\kappa},\ \Theta^* = \frac{\Theta}{\beta d},\ \Phi^* = \frac{\Phi}{\dfrac{K_m \beta d^2}{1+\chi_m}}, \\ &a = K_h d,\quad z^* = \frac{z}{d},\quad \sigma^* = \frac{\sigma}{\dfrac{\kappa}{d^2}} \end{aligned}\right\} \tag{3.13}$$

we obtain the following dimensionless equations (asterisks are neglected for simplicity)

$$(1+F_1\,\sigma)\left[\frac{\sigma}{\Pr}\left(D^2-a^2\right)W+(R+N)a^2\,\Theta-Na^2D\Phi\right]=(1+F_2\,\sigma)\left[-Da^{-1}\left(D^2-a^2\right)W+\Lambda\left(D^2-a^2\right)^2W\right] \tag{3.14}$$

$$(1+2G\sigma)\left[\lambda\sigma\Theta-M_2\,\sigma D\Phi-(1-M_2)\,W\right]=\left(D^2-a^2\right)\Theta-G\left(D^2-a^2\right)W \tag{3.15}$$

$$\left(D^2-M_3\,a^2\right)\Phi-D\Theta=0 \tag{3.16}$$

where $\lambda=\dfrac{(\rho_0 C)_1}{(\rho_0 C)_2}$ and $M_2=\dfrac{\mu_0 K_m^2 Ta}{(1+\chi_m)(\rho_0 C)_2}$. The parameter M_2 is neglected as it is of very small order (Finlayson [8]). When $\lambda=1$, we obtain the following equations

$$(1+F_1\,\sigma)\left[\frac{\sigma}{\Pr}\left(D^2-a^2\right)W+(R+N)a^2\,\Theta-Na^2D\Phi\right]=(1+F_2\sigma)\left[\Lambda\left(D^2-a^2\right)^2W-Da^{-1}\left(D^2-a^2\right)W\right] \tag{3.17}$$

$$(1+2G\sigma)(\sigma\,\Theta-W)-\left(D^2-a^2\right)\Theta+G\left(D^2-a^2\right)W=0 \tag{3.18}$$

$$\left(D^2-M_3\,a^2\right)\Phi-D\Theta=0 \tag{3.19}$$

where $F_1=\dfrac{\lambda_1\kappa}{d^2}$ is the non-dimensional stress relaxation parameter, $F_2=\dfrac{\lambda_2\kappa}{d^2}$ is the non-dimensional strain retardation parameter, $\Pr=\dfrac{\varepsilon\mu_f}{\rho_0\kappa}$ is the Prandtl number, $R=\dfrac{\rho_0\alpha g\beta d^4}{\mu_f\,\kappa}$ is the thermal Rayleigh number, $N=\dfrac{\mu_0K^2\beta^2d^4}{\mu_f\,(1+\chi_m)\kappa}$ is the magnetic Rayleigh number, $Da^{-1}=\dfrac{d^2}{k}$ is the inverse Darcy number, $\Lambda=\dfrac{\overline{\mu_f}}{\mu_f}$ is the Brinkman number, $G=\dfrac{\tau\kappa}{2d^2}$ is the Cattaneo number and $M_3=\left(\dfrac{1+\dfrac{M_0}{H_0}}{1+\chi_m}\right)$ is the non-buoyancy-magnetization parameter. The boundary conditions encompassing free and isothermal surfaces are $W=D^2W=\Theta=D\Phi=0$ at $z=\pm 1/2$ (Finlayson [8]).

3.1. Stationary Instability

As for the stationary mode, equations (3.17) - (3.19) turn out to be the following

$$\Lambda\left(D^2-a^2\right)^2W-Da^{-1}\left(D^2-a^2\right)W-(R+N)a^2\,\Theta+Na^2D\Phi=0 \tag{3.20}$$

$$\left[G\left(D^2-a^2\right)-1\right]W-\left(D^2-a^2\right)\Theta=0 \tag{3.21}$$

$$\left(D^2-M_3\,a^2\right)\Phi-D\Theta=0\ . \tag{3.22}$$

Equations (3.20) through (3.22) along with the boundary conditions embrace an eigenvalue problem with R being an eigenvalue. The straightforward solution $W = A_1 \cos(\pi z)$, $\Theta = A_2 \cos(\pi z)$, $\Phi = \frac{A_3}{\pi} \sin(\pi z)$, with A_1, A_2 and A_3 being constants, is taken into consideration. On applying the solvability condition, we obtain

$$R^{st} = \frac{\left(\pi^2 + a^2\right)^2 \left[Da^{-1} + \left(\pi^2 + a^2\right) \Lambda\right]}{a^2 \left[1 + G\left(\pi^2 + a^2\right)\right]} - \frac{N M_3 a^2}{\left(M_3 a^2 + \pi^2\right)} \tag{3.23}$$

where the superscript '*st*' stands for stationary convection. Equation (3.23) exactly coincides with that obtained by Soya Mathew and Maruthamanikandan [22] and Soya Mathew et al. [23] followed by the corresponding deductions.

3.2. Oscillatory Instability

The dimensionless equations concerning the overstable motion are

$$\left[\left(1+F_1\sigma\right)\frac{\sigma}{\Pr}+\left(1+F_2\sigma\right)\left(Da^{-1}+\Lambda\left(\pi^2+a^2\right)\right)\right]\left(\pi^2+a^2\right)A_1-\left(1+F_1\sigma\right)\left(R+N\right)a^2A_2+\left(1+F_1\sigma\right)Na^2A_3=0 \tag{3.24}$$

$$\left[1+2G\sigma+G\left(\pi^2+a^2\right)\right]A_1-\left[\left(\pi^2+a^2\right)+\left(1+2G\sigma\right)\sigma\right]A_2=0 \tag{3.25}$$

$$\pi^2 A_2-\left(\pi^2+M_3 a^2\right)A_3=0. \tag{3.26}$$

On applying the solvability condition, we obtain

$$R = \left\{\frac{p\left[\Pr\left(Da^{-1}+p\Lambda\right)\left(1+f_2\sigma\right)+\sigma+f_1\sigma^2\right]\left(p+\sigma+2g\sigma^2\right)}{a^2\Pr\left(1+f1\sigma\right)\left[1+g\left(p+2\sigma\right)\right]}\right\}-\frac{NM_3 a^2}{M_3 a^2+\pi^2} \tag{3.27}$$

where $p = \pi^2 + a^2$. If we let $\sigma = i\omega$ with ω being the frequency of oscillations, we obtain R as $R = R_1 + i R_2$. Both R_1 and R_2 are computed by means of the MATHEMATICA software mathematical package.

4. RESULTS AND DISCUSSION

The study is concerned with porous medium ferroconvection in a viscoelastic magnetic fluid with non-classical heat conduction. We have obtained the conditions for both stationary and oscillatory convection using linear theory, which is based on the normal mode technique. The thermal Rayleigh number R, characterising the stability of the system, is obtained as a function of the different parameters of the study. The eigenvalue expression and the associated critical numbers are determined by using MATHEMATICA software. As we can observe from the expression (3.23) stationary Rayleigh is independent of the viscoelastic parameters as obtained by Soya Mathew and Maruthamanikandan [22]. Also, if the Cattaneo number is taken below the threshold value, then only stationary convection occurs [23]. Hence, for stationary convection, viscoelastic fluid behaves same as Newtonian fluid. Rayleigh number for oscillatory mode is obtained as a function of Prandtl number, Cattaneo number, magnetic, viscoelastic and porous parameters.

In **Fig. 2** critical Rayleigh number R_c is expressed as a function of magnetic Rayleigh number N by keeping all other parameters as constant by fixing their values as $F_1 = 1.5$, $F_2 = 0.3$, $\Pr = 10$, $Da^{-1} = 5$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$. As N increases, R_c decreases and hence the system is destabilized. We observe that oscillatory convection is preferred to stationary convection as R_c^{osc} is less than R_c^{st} and hence the principle of exchange of instabilities is not valid. In **Fig. 3** critical Rayleigh number R_c is expressed as a function of the magnetic Rayleigh number N by varying F_1 and keeping all other parameters as constant by fixing their values as $F_2 = 0.3$, $\Pr = 10$, $Da^{-1} = 5$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$. We notice that, as F_1 increases, the R_c^{osc} value decreases which indicates that the stress relaxation parameter F_1 hastens the oscillatory ferroconvection. In **Fig. 4** critical Rayleigh number R_c is expressed as a function of the magnetic Rayleigh number N by varying F_2 and keeping all other parameters as constant by fixing their values as $F_1 = 1.5$, $\Pr = 10$, $Da^{-1} = 5$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$. As there is an increase in the values of F_2, we notice that there is an increase in R_c^{osc} which indicates that the strain retardation parameter F_2 slows down the onset of oscillatory ferroconvection.

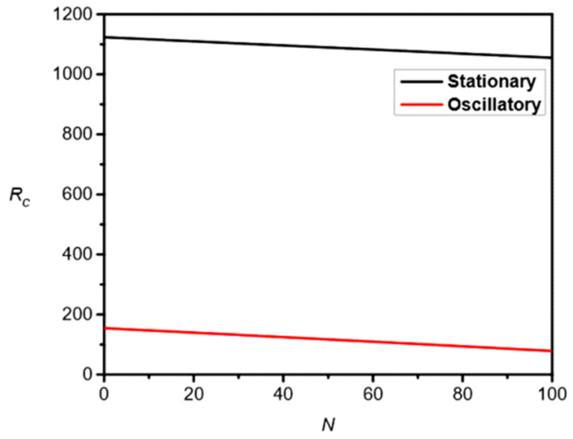

Figure 2. Plot of R_c versus N with $F_1 = 1.5$, $F_2 = 0.3$, $\Pr = 10$, $Da^{-1} = 5$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$.

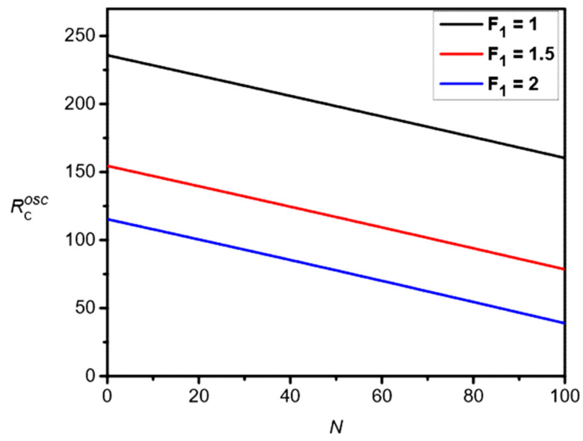

Figure 3. Plot of R_c^{osc} versus N with variation in F_1 with $F_2 = 0.3$, $\Pr = 10$, $Da^{-1} = 5$, $\Lambda = 3$, $G = 0.06$ $M_3 = 3$ and $Ta = 500$.

In **Fig. 5** critical Rayleigh number R_c is expressed as a function of N by varying G and keeping all other parameters as constant by fixing their values as $F_1 = 1.5$, $F_2 = 0.3$, $\Pr = 10$, $Da^{-1} = 5$, $\Lambda = 3$ and $M_3 = 3$. As G increases, there is a decrease in R_c^{osc}. As discussed by Straughan [24], the above threshold value of Cattaneo number G associated with oscillatory convection comes into picture. It destabilizes the system. In **Fig. 6** critical Rayleigh number R_c is expressed as a function of the magnetic Rayleigh number N by varying Pr and keeping all other parameters as constant by fixing their values as $F_1 = 1.5$, $F_2 = 0.3$, $Da^{-1} = 5$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$. As there is an increase in the values of Pr, we notice there is a decrease in R_c^{osc} due to the above threshold value of G and hence the system is destabilized. This is due to the hyperbolic nature instead of the parabolic one of the temperature equation.

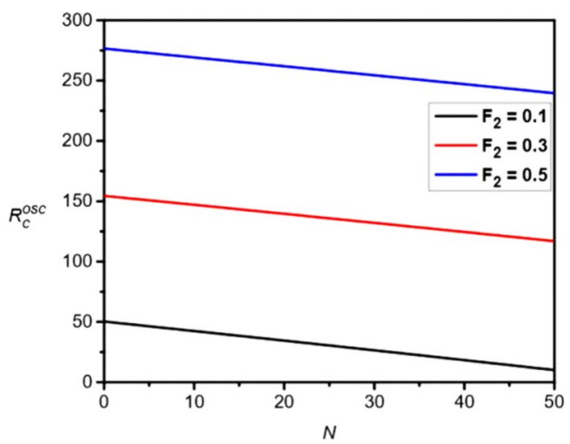

Figure 4. Plot of R_c^{osc} versus N with variation in F_2 with $F_1 = 1.5$, $\Pr = 10$, $Da^{-1} = 5$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$.

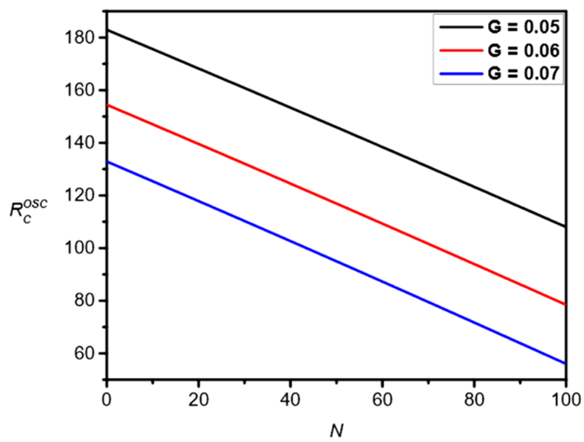

Figure 5. Plot of R_c^{osc} versus N with variation in G with $F_1 = 1.5$, $F_2 = 0.3$, $\Pr = 10$, $Da^{-1} = 5$, $\Lambda = 3$ and $M_3 = 3$.

In **Fig. 7** critical Rayleigh number R_c^{osc} is expressed as a function of magnetic Rayleigh number N by varying $\Pr$ and keeping all other parameters as constant by fixing their values as $F_1 = 1.5$, $F_2 = 0.3$, $Da^{-1} = 5$, $\Lambda = 3$, $G = 0$ and $M_3 = 3$. We notice that R_c^{osc} increases as $\Pr$ increases and hence system is stabilized. This is due to the absence of Cattaneo number.

From Figures 6 and 7, we witness the dual nature of the Prandtl number $\Pr$ depending on the Cattaneo number G. If the Cattaneo number G is above the threshold value, then on increasing $\Pr$ there is a decrease in R_c^{osc} as noticed in the work of Nagouda and Pranesh [25] and if the Cattaneo number G is below the threshold value, then on increasing Pr there is an increase in R_c^{osc} as noticed in the work of Swamy et al. [18].

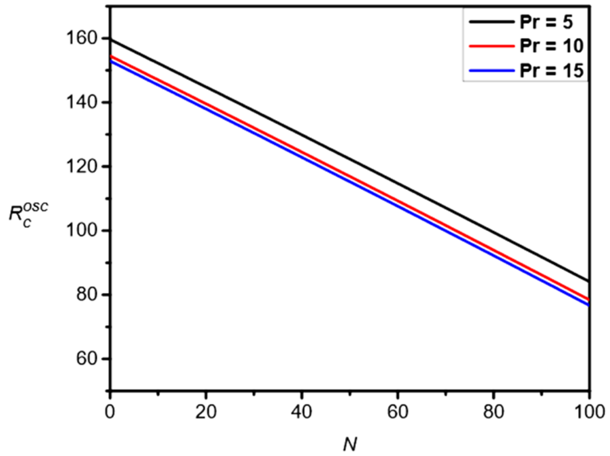

Figure 6. Plot of R_c^{osc} versus N with variation in $\Pr$ with $F_1 = 1.5,\ F_2 = 0.3, Da^{-1} = 5,\ \Lambda = 3,\ G = 0.06\ and\ M_3 = 3.$

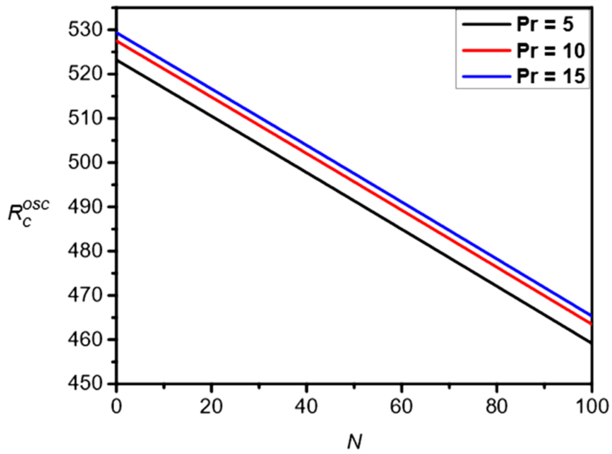

Figure 7. Plot of R_c^{osc} versus N with variation in $\Pr$ with $F_1 = 1.5,\ F_2 = 0.3,\ Da^{-1} = 5,\ \Lambda = 3,\ G = 0\ and\ M_3 = 3.$

Stationary vs Oscillatory Instability

Table 1. Critical values of the Rayleigh number and wave number with $F_1 = 1.5,\ F_2 = 0.3,\ \Pr = 10,\ Da^{-1} = 5,\ \Lambda = 3,\ G = 0.06$ and $M_3 = 3.$

N	Stationary		Oscillatory	
	R_c^{st}	α_c^{st}	R_c^{osc}	α_c^{osc}
0	1123.54	2.6486	154.486	3.05605
20	1109.91	2.65782	139.604	3.13009
40	1096.25	2.66701	124.546	3.20226
60	1082.56	2.67618	109.324	3.27246
80	1068.84	2.68533	93.9499	3.34063
100	1055.09	2.69446	78.4344	3.40676

Table 2. Critical values of the wave number varying with F_1 by fixing $F_2 = 0.3,\ \Pr = 10,\ Da^{-1} = 5,\ \Lambda = 3,\ G = 0.06$ and $M_3 = 3.$

N	$F_1 = 1$	$F_1 = 1.5$	$F_1 = 2$
	α_c	α_c	α_c
0	3.06861	3.05605	3.04212
20	3.11749	3.13009	3.14058
40	3.16559	3.20226	3.23566
60	3.21286	3.27246	3.32715
80	3.25927	3.34063	3.41501
100	3.30481	3.40676	3.49928

Table 3. Critical values of the wave number varying with F_2 by fixing $F_1 = 1.5,\ \Pr = 10,\ Da^{-1} = 5,\ \Lambda = 3,\ G = 0.06$ and $M_3 = 3.$

N	$F_2 = 0.1$	$F_2 = 0.3$	$F_2 = 0.5$
	α_c	α_c	α_c
0	3.4042	3.05605	2.99105
10	3.52012	3.09329	3.01222
20	3.63083	3.13009	3.03327
30	3.73634	3.16642	3.05418
40	3.83684	3.20226	3.07495
50	3.93259	3.23761	3.09558

Table 4. Critical values of the wave number varying with G by fixing F_1 =1.5, F_2 =0.3, Pr=10, Da^{-1}= 5, Λ= 3 and M_3= 3.

N	$G=0.05$	$G=0.06$	$G=0.07$
	α_c	α_c	α_c
0	2.98828	3.05605	3.10812
20	3.05035	3.13009	3.19439
40	3.11118	3.20226	3.27804
60	3.17068	3.27246	3.35892
80	3.2288	3.34063	3.437
100	3.28551	3.40676	3.5123

Table 5. Critical values of the wave number varying with Pr by fixing F_1 = 1.5, F_2 = 0.3, Da^{-1}= 5, Λ=3, G= 0.06 and M_3= 3.

N	$\Pr=5$	$\Pr=10$	$\Pr=15$
	α_c	α_c	α_c
0	3.01748	3.05605	3.06942
20	3.08869	3.13009	3.14436
40	3.15822	3.20226	3.21737
60	3.22595	3.27246	3.28835
80	3.29182	3.34063	3.35723
100	3.35581	3.40676	3.42402

G = 0 (In the absence of Second Sound)

Table 6. Critical values of the wave number varying with Pr by fixing F_1 = 1.5, F_2 = 0.3, Da^{-1}= 5, Λ= 3, G= 0 and M_3= 3.

N	$\Pr=5$	$\Pr=10$	$\Pr=15$
	α_c	α_c	α_c
0	2.38141	2.38512	2.38669
20	2.39726	2.40086	2.40238
40	2.41302	2.41651	2.418
60	2.42869	2.43208	2.43352
80	2.44427	2.44756	2.44895
100	2.45975	2.46293	2.46428

In **Fig. 8** critical Rayleigh number R_c^{osc} is expressed as a function of N by varying Pr and keeping all other parameters as constant by fixing their values as F_1 =0, F_2 = 0, Da^{-1} = 5, Λ= 3, G= 0.06 and M_3 = 3, i.e., in the absence of viscoelastic parameters. In this case also there is a decrease in R_c^{osc} as we increase Pr which again clearly suggests that the dual nature of Pr is only due to the presence of the Cattaneo number.

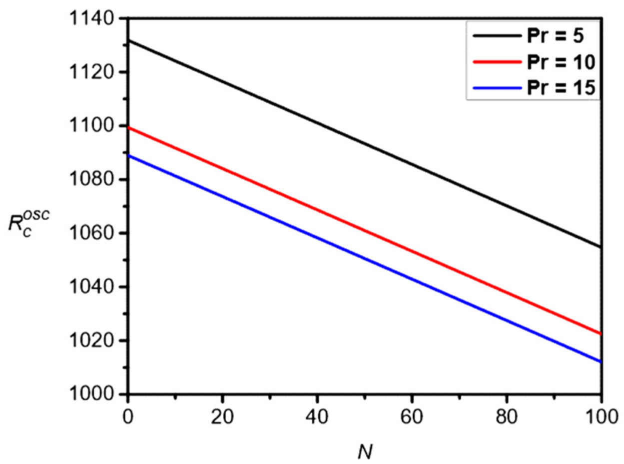

Figure 8. Plot of R_c^{osc} versus N with variation in Pr with F_1 = 0, F_2 = 0, Da^{-1}= 5, Λ= 3, G= 0.06 and M_3= 3.

For Newtonian ferromagnetic fluid $\left(i.e., F_1 = 0 \text{ and } F_2 = 0\right)$

Table 7. Critical values of the wave number α_c varying with Pr by fixing $F_1 = 0$, $F_2 = 0$, $Da^{-1} = 5$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$.

N	$\Pr = 5$	$\Pr = 10$	$\Pr = 15$
	α_c	α_c	α_c
0	3.30324	3.28459	3.27851
20	3.31363	3.29529	3.28931
40	3.324	3.30595	3.30008
60	3.33432	3.31658	3.3108
80	3.3446	3.32716	3.32148
100	3.35485	3.3377	3.33212

Table 8. Critical values of the wave number varying with Da^{-1} by fixing $F_1 = 1.5$, $F_2 = 0.3$, $\Pr = 10$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$.

N	$Da^{-1} = 0$	$Da^{-1} = 5$	$Da^{-1} = 10$
	α_c	α_c	α_c
0	2.94266	3.05605	3.15784
20	3.02243	3.13009	3.22714
40	3.10014	3.20226	3.29475
60	3.17561	3.27246	3.36059
80	3.24874	3.34063	3.42464
100	3.31951	3.40676	3.48689

In **Fig. 9** critical Rayleigh number R_c^{osc} is expressed as a function of magnetic Rayleigh number N by varying Da^{-1} and keeping all other parameters as constant by fixing their values as $F_1 = 1.5$, $F_2 = 0.3$, $\Pr = 10$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$. Oscillatory ferroconvection is delayed because as Da^{-1} is increased, there is an increase in the values of R_c^{osc}. The reason for this is the increase in Da^{-1} will decrease the porous medium permeability and hence the convective instability is impeded.

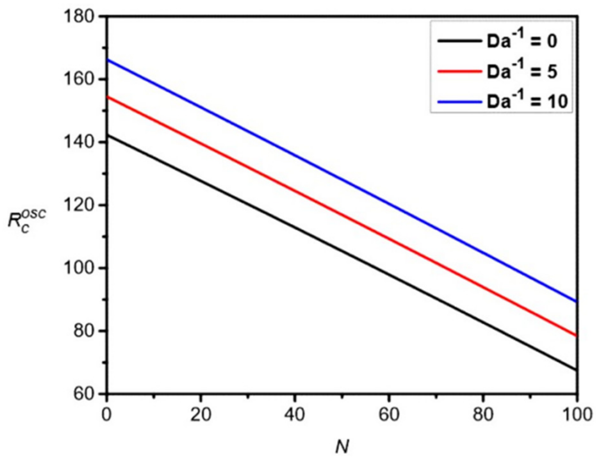

Figure 9. Plot of R_c^{osc} versus N with variation in Da^{-1} with $F_1 = 1.5$, $F_2 = 0.3$, $\Pr = 10$, $\Lambda = 3$, $G = 0.06$ and $M_3 = 3$.

In **Fig. 10** critical Rayleigh number R_c^{osc} is expressed as a function of the magnetic Rayleigh number N by varying the Brinkman number Λ and keeping all other parameters as constant by fixing their values as $F_1 = 1.5$, $F_2 = 0.3$, $\Pr = 10$, $Da^{-1} = 3$, $G = 0.06$ and $M_3 = 3$. As the Brinkman number Λ increases, R_c^{osc} also increases and therefore oscillatory ferroconvection is delayed. As the Brinkman model accounts for an effective viscosity $\overline{\mu_f}$ which is different from fluid viscosity μ_f and the ratio is assigned as the Brinkman number Λ. Hence viscous effect increases on increasing Λ and hence ferroconvective instability is hampered due to the presence of porous media.

Table 9. Critical values of the wave number varying with Λ by fixing $F_1=1.5$, $F_2=0.3$, $\Pr=10$, $Da^{-1}=5$, $G=0.06$ and $M_3=3$.

N	$\Lambda=1$	$\Lambda=3$	$\Lambda=5$
	α_c	α_c	α_c
0	3.16486	3.05605	3.02786
10	3.25535	3.09329	3.05126
20	3.3429	3.13009	3.07448
30	3.42738	3.16642	3.09754
40	3.50877	3.20226	3.12041
50	3.58714	3.23761	3.14309

Table 10. Critical values of the wave number varying with M_3 by fixing $F_1=1.5$, $F_2=0.3$, $\Pr=10$, $Da^{-1}=5$, $\Lambda=3$ and $G=0.06$.

N	$M_3=1$	$M_3=3$	$M_3=5$
	α_c	α_c	α_c
0	3.05605	3.05605	3.05605
20	3.15476	3.13009	3.11124
40	3.256	3.20226	3.16482
60	3.35878	3.27246	3.21682
80	3.46216	3.34063	3.26729
100	3.56526	3.40676	3.3163

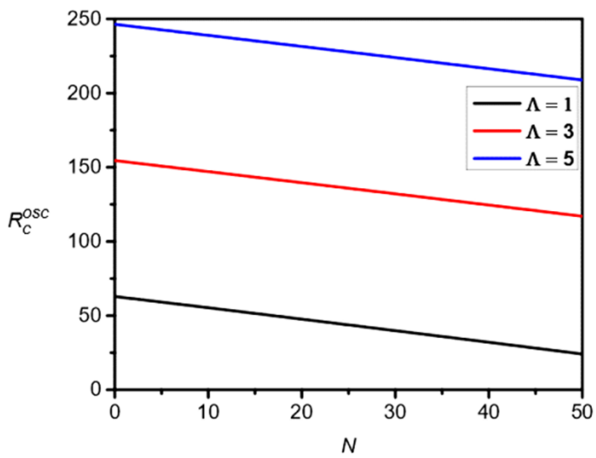

Figure 10. Plot of R_c^{osc} versus N with variation in Λ with $F_1=1.5$, $F_2=0.3$, $\Pr=10$, $Da^{-1}=5$, $G=0.06$ and $M_3=3$.

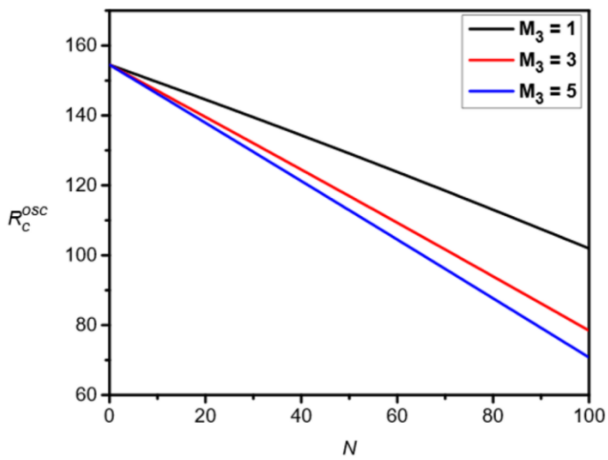

Figure 11. Plot of R_c^{osc} versus N with variation in M_3 with $F_1=1.5$, $F_2=0.3$, $\Pr=10$, $Da^{-1}=5$, $\Lambda=3$ and $G=0.06$.

In **Fig. 11** critical Rayleigh number R_c^{osc} is expressed as a function of the magnetic Rayleigh number N by varying M_3 and keeping all other parameters as constant by fixing their values as $F_1=1.5$, $F_2=0.3$, $\Pr=10$, $Da^{-1}=5$, $\Lambda=3$ and $G=0.06$. The linearity departure of magnetic equation is addressed by the parameter M_3. We notice from Fig. 11 that as M_3 increases, the R_c^{osc} monotonically decreases which implies that magnetic equation of state grows more and more to nonlinear state due to which ferroconvection is hastened.

From Figs. 12 through 17, one can observe that when all the respective parameters increase, ω_c^2 also increases, whereas from Figs. 18 through 20, as all relevant parameters increase, ω_c^2 also decreases. Hence, we can conclude that from Figs. 12 through 20 that the frequency ω_c of oscillatory ferroconvective instability is sensitive to all the parameters of the study. On the other hand, wave number depicts the size and shape of the convection cell. From Tables 2 through 10, it follows that convection cell size is also sensitive to the all the parameters of the study at hand. Indeed, the convection cell size is enlarged with an increase in F_2, Λ and M_3 and the opposite is found to be true with respect to an increase in the rest of the parameters.

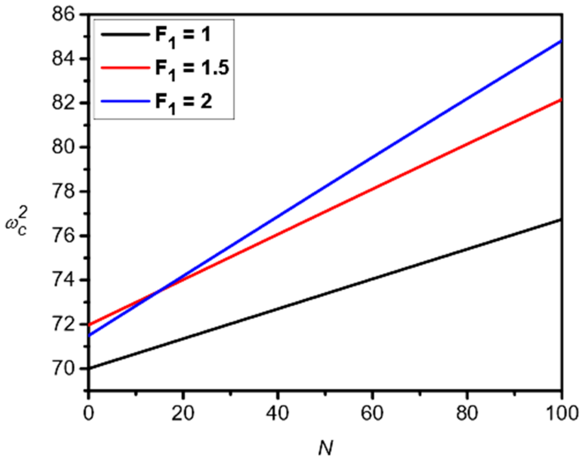

Figure 12. Plot of ω_c^2 versus N with variation in F_1 with F_2 =0.3, Pr=10, Da^{-1}=5, Λ=3, G=0.06 *and* M_3=3.

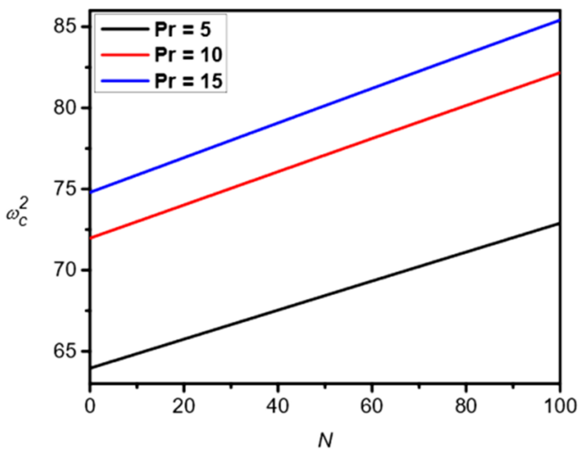

Figure 13. Plot of ω_c^2 versus N with variation in Pr with F_1 =1.5, F_2 =0.3, Da^{-1}=5, Λ=3, G=0.06 *and* M_3=3.

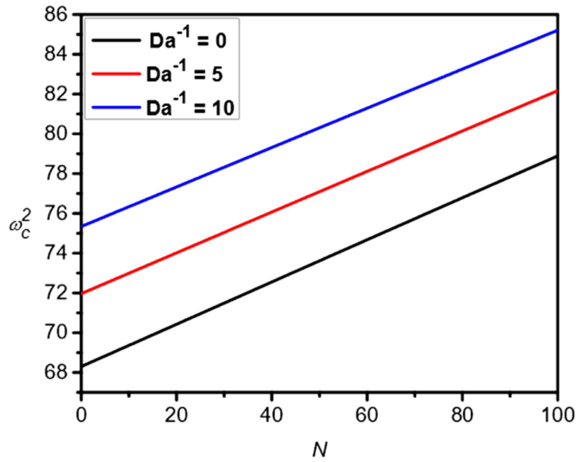

Figure 14. Plot of ω_c^2 versus N with variation in Da^{-1} with F_1 =1.5, F_2 =0.3, Pr=10, Λ=3, G=0.06 and M_3=3.

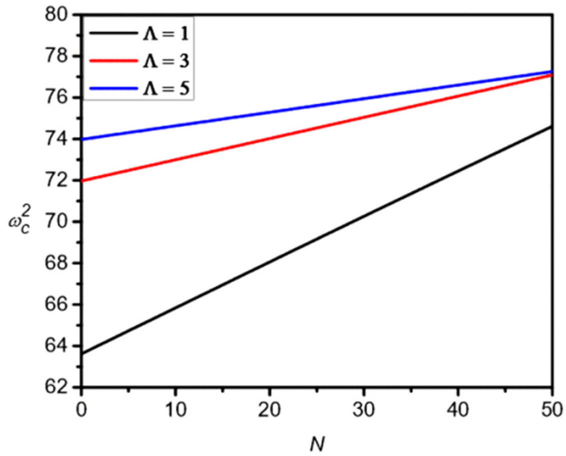

Figure 15. Plot of ω_c^2 versus N with variation in Λ with F_1 =1.5, F_2 =0.3, Pr=10, Da^{-1}=5, G=0.06 and M_3=3.

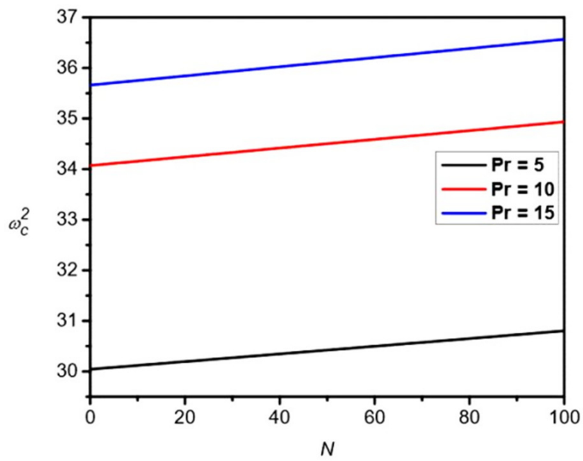

Figure 16. Plot of ω_c^2 versus N with variation in Pr with F_1 =1.5, F_2 =0.3, Da^{-1}=5, Λ=3, G=0 and M_3=3.

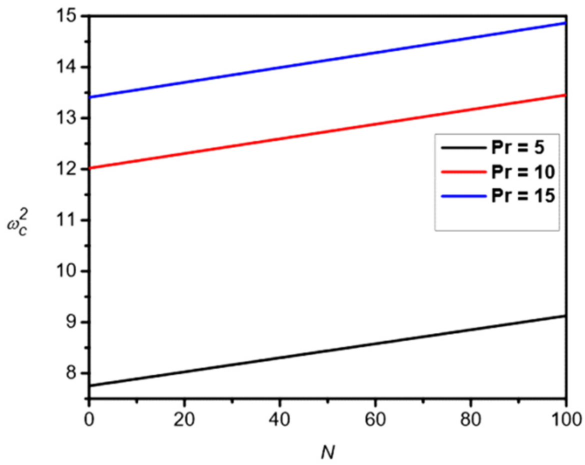

Figure 17. Plot of ω_c^2 versus N with variation in Pr with F_1 =0, F_2 =0, Da^{-1}=5, Λ=3, G=0.06 and M_3=3.

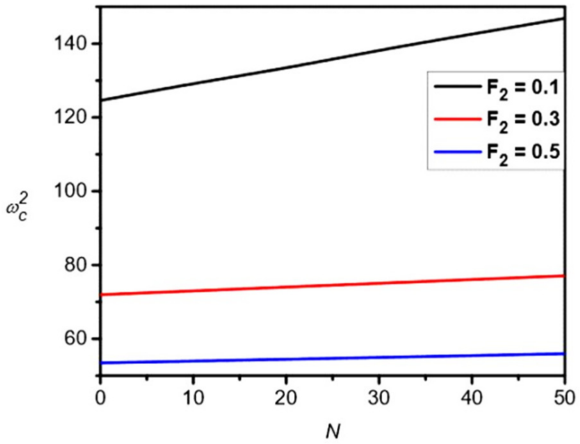

Figure 18. Plot of ω_c^2 versus N with variation in F_2 with F_1 =1.5, Pr=10, Da^{-1}=5, Λ=3, G=0.06 and M_3=3.

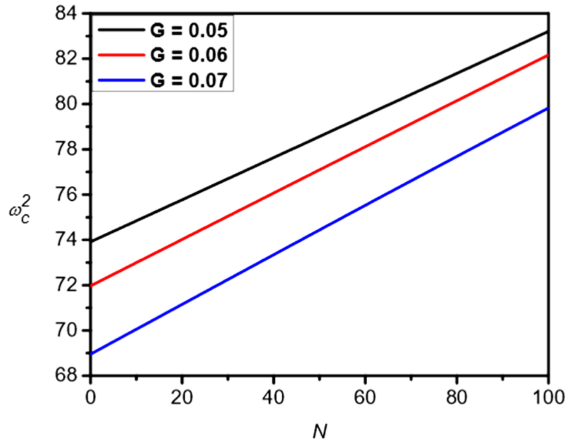

Figure 19. Plot of ω_c^2 versus N with variation in G with F_1 =1.5, F_2 =0.3, Pr=10, Da^{-1}=5, Λ=3 and M_3=3.

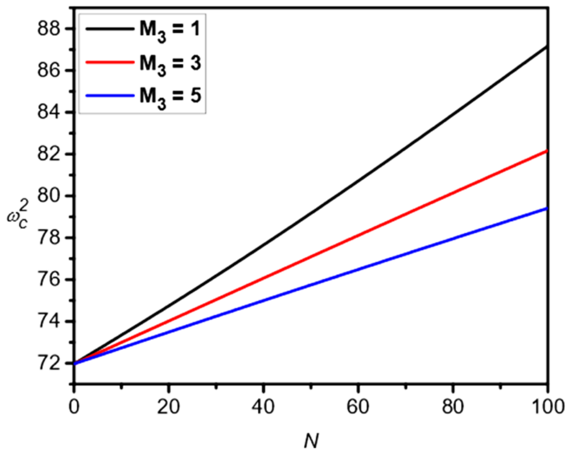

Figure 20. Plot of ω_c^2 versus N with variation in M_3 with F_1 =1.5, F_2 =0.3, Pr=10, Da^{-1}=5, Λ=3 and G=0.06.

It is worth mentioning that for Newtonian fluids only stationary convection is possible, but due to the presence of second sound, oscillatory instability is preferred to stationary stability as pointed out by Straughan [24].

CONCLUSIONS

1. The system is destabilized through the presence of magnetic forces caused by the magnetization of ferrofluids.
2. Nonlinearity in magnetization is shown to destabilize the system.
3. Viscoelastic relaxation and second sound are shown to destabilize the system.
4. Viscoelastic retardation, inverse Darcy number and Brinkman number are shown to stabilize the system.
5. Prandtl number destabilizes as well as stabilizes the system depending on the over and below threshold values of the Cattaneo number respectively.
6. Critical wavenumber and frequency of oscillatory motions are calculated as functions of all the parameters of the problem. They are shown to be sensitive to all the parameters of the problem.

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КОЛИВАЛЬНА ФЕРОКОНВЕКЦІЯ У ПОРИСТОМУ СЕРЕДОВИЩІ У В’ЯЗКОПРУЖНІЙ МАГНІТНІЙ РІДИНІ З НЕКЛАСИЧНОЮ ТЕПЛОПРОВІДНІСТЮ

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Використано класичний аналіз стабільності для вивчення комбінованого впливу в’язкопружності та другого звуку на початок фероконвекції у пористому середовищі. Вважається, що рідина і тверда матриця знаходяться в локальній тепловій рівновазі. Враховуючи граничні умови, відповідні для аналітичного підходу, критичні значення, що стосуються як стаціонарної, так і коливальної нестабільності, отримані за допомогою аналізу нормального режиму. Помічено, що коливальний режим нестабільності є кращим перед стаціонарним режимом нестабільності. Показано, що фероконвекція коливального пористого середовища розвивається через магнітні сили, нелінійність намагніченості, релаксацію напружень за рахунок в’язкопружності та другий звук. З іншого боку, спостерігається, що наявність затримки деформації та пористого середовища затримує початок осцилюючої фероконвекції у пористому середовищі. Також окреслено подвійну природу числа Прандтля на число Релея по відношенню до числа Каттанео. Також обговорюється вплив різних параметрів на розмір конвекційної комірки та частоту коливань. Ця проблема може мати можливі наслідки для технологічних застосувань, у яких використовуються в’язкопружні магнітні рідини.

Ключові слова: *конвекція; рівняння Максвелла; рівняння Нав'є-Стокса для нестисливих в'язких рідин; пористі середовища; в'язкопружні рідини, фероконвекція*