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DESCRIBE OF NUCLEAR STRUCTURE OF GERMANIUM ($^{66}_{32}Ge_{34}$) NUCLEUS USING NUCLEAR IBM-1, GVMI AND VMI MODELS[†]

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In this paper, the interacting bosons model-1 (IBM-1), a variable moment of inertia (VMI) and generalized moment of inertia (GVMI) models were used to calculate the energy levels of the positive parity and its gamma transitions as a function of the angular momentum of even-even ($^{66}_{32}Ge_{34}$) nucleus. To determine the dynamic symmetry of this nucleus, the ratios of the energy levels E_4^+/E_2^+, E_6^+/E_2^+, *and* E_8^+/E_2^+) were evaluated and compared with experimental energy values and the ideal scheme of the three dynamic symmetries SU(5), SU(3), and O(6). The current study showed that the dynamic symmetry of this nucleus is determined to be O(6) - SU(5). The intersection of the energy band and the phenomenon of back bending were also studied using the (VMI) and (GVMI) models. These consequences were compared with the experimental results, and the results obtained have been in good agreement.
Keywords: *Nuclear structure, $^{66}_{32}Ge_{34}$, (IBM-1, GVMI and VMI) Models*
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INTRODUCTION

The importance of the nuclear models lies in studying and explaining many nuclear properties. One of these models is the Interacting Bosons Model (IBM), as it is considered one of the important nuclear models that succeeded in finding most of the nuclear properties. It gives good theoretical values that are compared to practical values and its ability to explain the decay of excited nuclear levels that lead to the emission of gamma rays [1].

The first interacting boson model (IBM-1) treated the movement of nucleons inside the nucleus as the movement of a group of paired particles called bosons, which represent either pair of protons or neutrons or proton-neutrons nearest outside the outermost closed shell [2].

The simplest concept of the first model of interacting bosons, developed by Arima and Iachello, (1974) [3], assumed that the low-lying collective energy levels in even-even medium and heavy nuclei are far from closed shells, with magic numbers 2, 8, 20, 28, 50, 82 and 126 are the ones in which only valence protons and neutrons prevail, while the core of the shell is inactive, in addition to that, it is assumed that similar particles are interacting together in pairs with an angular momentum of $\boldsymbol{L} = \mathbf{0}$ called s-boson and $\boldsymbol{L} = \mathbf{2}$ are called d-boson[4]. This model can describe the nuclear levels with positive parity only, which have medium and heavy mass numbers, except for those with closed shells, because the number of bosons equals zero. This model depends on "Unitary group theory" in six components called U(6), that then produces three subsection symmetries; vibrational U(5), rotational SU(3), and γ-unstable O(6) [1,5]. Determining dynamic symmetry depends on the ground energy levels, as it explains many nuclear properties [6].

Several models were introduced for associating a large number of experimental data for energy levels bands of even-even nuclei, in particular, the variable moment of inertia (VMI) model [7]. This model proposes that moment of inertia is mutable, the energy of a level with angular momentum (L) comprises in addition to the usual rotational term, a potential energy term which depends on the change of the moment of inertia ($\boldsymbol{\vartheta}_L$) from that of the ground state ($\boldsymbol{\vartheta_o}$). It can be said that this model succeeded in describing the energy levels of the rotational and, to some extent, vibrational nuclei.

The (GMI) [8] model is easier and more comprehensive than the (VMI) model, although it consists of the basic building blocks upon which the (VMI) model is built. The energy levels, which are a function of spin (L), are reduced to two limits in the new model. This model gave results that are highly consistent with the experimental findings of nuclei that have limitations within the SU(3) and SU(5) regions [9].

The phenomenon of back bending was demonstrated using both the VMI model and the GVMI model. This phenomenon can be explained by using band crossing and Coriolis force effect. The phenomenon of back bending was discovered by Johnson and his group in 1971. This phenomenon is one of the characteristics of the moment of inertia of deformed nuclei [10].

BAND CROSSING

The phenomenon of bands crossing states that if an energy band, such as $(\gamma - band)$ with a moment of inertia ϑ_2 and the ground state band $(g - band)$ with a moment of inertia ϑ_1 so that $(\vartheta_2 > \vartheta_1)$, an intersection between the two bands at a certain angular momentum (L_{cross}) L_c will be occurred. what is meant by this intersection is that the high

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moment energy band will substitute energy by a low moment of inertia when the angular momentum is increased from L_c. At the intersection of two bands, all nucleons have a spin equal to zero under the influence of the pairing correlation interaction. In this case, the nucleus is considered super-soft and its self-energy is less than the double-bonding energy, and the nucleus has a small moment of inertia. The phenomenon of band crossing is an important characteristic of the moment of inertia [11].

CORIOLIS FORCE EFFECT

The force or effect of Coriolis is a physical term given to the visible and exposed deformation in the movement of objects when observed from a rotational frame of reference, and it was named after the French scientist Gustave Coriolis who described this deformation during 1835 AD. The Coriolis effect occurs due to the Coriolis force that appears in the equation of motion for a specific body in a rotational reference frame [12].

When the angular momentum of the nucleons is high, the effect of the Coriolis force begins to increase. This leads the boson located outside the closed shells to decoupling into two neutrons, where a new band called a *Two-quasiparticle* (2QP) band appears. While if the boson splits into a proton-neutron, here a *four-quasiparticle* (4QP) band appears. The intersection of the bands that appeared with $(g - band)$ at a certain angular momentum leads to the occurrence of the phenomenon of back bending [13].

SUMMARY OF THE FIRST INTERACTING BOSON MODEL (IBM-1)

The Hamiltonian operator for IBM-1 is given [14],

$$\hat{H} = \sum_{i=1}^{N} \varepsilon_i + \sum_{i<j}^{N} U_{ij} . \tag{1}$$

Where ε_i: represents the energy of the boson.

U_{ij}: The energy of interacting bosons.

N: It is the sum of the number of bosons of nucleons.

Thus

$$\hat{H} = \varepsilon_s(\hat{s}^+\hat{s}) + \varepsilon_d \sum_{\mu} \hat{d}_{\mu}^{+}\hat{\tilde{d}}_{\mu} + U \quad (\mu = 0, \pm 1, \pm 2, ...) . \tag{2}$$

Where

ε_s: The energy of s-type bosons.

ε_d: The energy of d-type bosons.

However, it is more mutual to write the Hamiltonian of the IBM-1 as a multipole extension, grouped into different boson-boson interactions [15]:

$$\hat{H} = \varepsilon(\hat{n}_d) + \alpha_0(\hat{P}.\hat{P}) + \alpha_1(\hat{L}.\hat{L}) + \alpha_2(\hat{Q}.\hat{Q}) + \alpha_3(\hat{T}_3.\hat{T}_3) + \alpha_4(\hat{T}_4.\hat{T}_4) . \tag{3}$$

Where, $\varepsilon = \varepsilon_d - \varepsilon_s$ is the energy of the bosons. And $(\hat{n}_d)$ is the boson number operator and the pairing bosons operator isoctupole operator, and the hexadecapole . The angular momentum operator, the quadruple operator, the $(\hat{P})$ operator are $(\hat{L})$ $(\hat{Q})$ $(\hat{T}_3)$ $(\hat{T}_4)$, respectively. And $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the phenomenological parameters.

Thus,

$$\left.\begin{aligned}
&\hat{n}_d = (\hat{d}^{\dagger}.\hat{\tilde{d}}) \\
&\hat{P} = 1/2(\hat{\tilde{d}}.\hat{\tilde{d}}) - 1/2(\hat{\tilde{S}}.\hat{\tilde{S}}) = \frac{1}{2}(\hat{\tilde{d}}^2 - \hat{\tilde{s}}^2) \\
&\hat{L} = \sqrt{10}[\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(1)} \\
&\hat{Q} = [(\hat{d}^{\dagger} \times \hat{\tilde{S}}) + (\hat{S}^{\dagger} \times d)]^{(2)} - CHI[\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(2)} \\
&\hat{T}_3 = [\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(3)} \\
&\hat{T}_4 = [\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(4)}
\end{aligned}\right\} \tag{4}$$

$CHI = -\frac{\sqrt{7}}{2}$ is for the rotational limitation and it equals zero for vibrational and γ-soft limitations.

The interacting boson model can be divided into three chains or three analytic solutions according to the eigenvalues problem of IBM-1 Hamiltonian. And these chains are [16],

$$U(6) \supset SU(5) \supset O(5) \supset O(3) \supset O(2) \tag{5}$$

Eq. 5 means anharmonic spherical vibrator.

$$U(6) \supset SU(3) \supset O(3) \supset O(2). \quad (6)$$

Eq. 6 means axially−deformed rotovibrator

$$U(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2). \quad (7)$$

Finally, Eq. 7 represents γ-unstable deformed rotovibrator.

The region of the transition $O(6) - SU(5)$

The region is meant to represent the joint characteristics of two dynamical symmetries $O(6) - SU(5)$. The Hamiltonian operator's function can be expressed as [17].

$$\hat{H} = \varepsilon(\hat{n}_d) + \alpha_0(\hat{P}.\hat{P}) + \alpha_1(\hat{L}.\hat{L}). \quad (8)$$

The ratio (ε/α) was necessary for solving the equation (8). When the ratio is high, the vibration dynamical symmetry SU(5) is under the control. While the gamma-unstable dynamical symmetry O(6) is in charge while the ratio is low.

Summary of the variable moment of inertia (VMI) model

In the VMI model, the energy of level E(L) with Angular momentum (L) can be written as [18],

$$E(L) = \frac{1}{2\vartheta(L)} L(L+1) + \frac{C}{2}[\vartheta(L) - \vartheta_o]^2. \quad (9)$$

Where, $\vartheta(L)$: the moment of inertia with angular momentum L, ϑ_o: the moment of inertia with ground state, C: the restoring force constant

According to the variable moment of inertia model, the nuclear softness coefficient δ can be written in the following form [7],

$$\delta = \frac{1}{2C\vartheta_o^3}. \quad (10)$$

The back bending phenomenon occurs due to an increase in the moment of inertia, which is accompanied by a decrease in the rotational energy at a certain angular momentum in some nuclei. The total angular momentum $L(L+1)$ can be separated into two parts $L(L+1)$ and $(L-2)(L-1)$. The square of the rotational energy $(\omega\hbar)^2$ and the moment of inertia $\frac{2\vartheta}{\hbar^2}$ can be written for (g-$band$) as [19],

$$(\omega\hbar)^2 = \left[\frac{E(L\to L-2)}{\sqrt{L(L+1)} - \sqrt{(L-2)(L-2+1)}}\right]^2 \quad (11)$$

$$\frac{2\vartheta}{\hbar^2} = \frac{4L-2}{E(L_i) - E(L_f)} = \frac{4L-2}{E_\gamma}. \quad (12)$$

For $\gamma - band$ as [18],

$$(\omega\hbar)^2 = \left[\frac{E(L\to L-1)}{\sqrt{L(L+1)} - \sqrt{(L-1)}}\right]^2. \quad (13)$$

$$\frac{2\vartheta}{\hbar^2} = \frac{L(L+1) - L(L-1)}{E(L\to L-1)} = \frac{2L}{E_\gamma}. \quad (14)$$

The transition energy E_γ of g-$band$ and any other band are the difference between the energy of the initial angular momentum $E(L_i)$ and the energy of the final momentum $E(L_f)$. Meaning that E_γ equals to $E(L) - E(L-2)$ regarding to g-$band$, while E_γ equals to $E(L) - E(L-1)$ following to γ-$band$.

Chi-square represents the suitability of VMI and GVMI models is given [20],

$$\chi^2 = \frac{(E_{cal} - E_{exp})^2}{(E_{exp})^2}. \quad (15)$$

Where, (E_{cal}) theoretically calculated energy and (E_{exp}) is the experimental error in the excitation energy.

The root mean square of the standard deviation was calculated from the following equation [20],

$$Standard\ Deviation = \sqrt{\left[\frac{1}{N}\sum_{i=1}^{N}(E_{cal} - E_{exp})^2\right]}. \quad (16)$$

where, N is the number of data points entering the fitting procedure.

CALCULATION PROCEDURES

Calculation of energy levels

- The energy levels of the nucleus under study were calculated using the IBM model by programming Equation 3 in the FN95-Plato program and through the *IBM program* code.
- The energy levels for the same core were calculated using the VMI model by programming Equation 9 in the same computer program through the code *VMI.For*.
- The energy levels of the studied nuclei were calculated using the GVMI model through the code *GVMI.For*.

The principle of matching between the values of the energy levels theoretically calculated in the IBM-1 model and its experimentally measured values is the first criterion adopted in our study. Table (1) shows the values of the parameters of the Hamiltonian function represented in Equation (3) were appropriated and adjusted.

Table 1. The parameters for which the best fit is chosen, which represent Equation 3. These parameters are the energies operator measured in (MeV).

Nuclei	N_π	N_υ	N	EPS (MeV)	$\hat{P}.\hat{P}$ (MeV)	$\hat{L}.\hat{L}$ (MeV)	$\hat{Q}.\hat{Q}$ (MeV)	$\hat{T}_3.\hat{T}_3$ (MeV)	$\hat{T}_4.\hat{T}_4$ (MeV)	CHI (MeV)	SO6 (MeV)
${}^{66}_{32}Ge_{34}$	2	3	5	0.5532	0.0269	0.0444	0.0365	0.0184	0.0071	0.0001	0.0001

While the energy levels were theoretically calculated using the (IVM) and (GVMI) models by applying Equation (9). These parameters were selected and fitted to obtain the smallest value of the chi-square (χ^2).

According to the best selected parameters: moment of inertia ($\vartheta_\circ/\hbar$), recovery force (C), band head energy (E_k), for the VMI model. As for the (GVMI) model, it depends on the three parameters in addition to the parameter (Y) is the constant parameter fitted with experimental data as shown in Table (2). The last three columns contain, respectively, the values of the nuclear ductility coefficient δ, standard deviation, and chi-squared χ^2.

Table 2. The consistent parameters of (VMI) and (GVMI) models for ${}^{66}_{32}Ge_{34}$ nucleus

Isotope	Band	Model	$\vartheta_\circ/\hbar$ $(MeV)^{-1}$	C $(MeV)^3$	E_k (MeV)	Y (MeV)	δ	Standard deviation	χ^2
${}^{66}_{32}Ge_{34}$	g	VMI	0.516370	0.036550	0.000000	-------	99.357262	0.203520	0.036356
		GVMI	2.034200	3.999000	0.000000	0.085224	0.024644	0.248871	0.052858
	γ	VMI	4.178000	0.040400	1.300000	-------	0.169701	0.182449	0.061080
		GVMI	1.000000	2.112000	0.100000	0.004224	0.469485	0.152464	0.038602

The values of the energy levels of the nucleus under study are shown in Figure (1). These results were compared according to the applied programs and the experimental values [22], and there was a good agreement between the experimental values and the theoretical values for these levels.

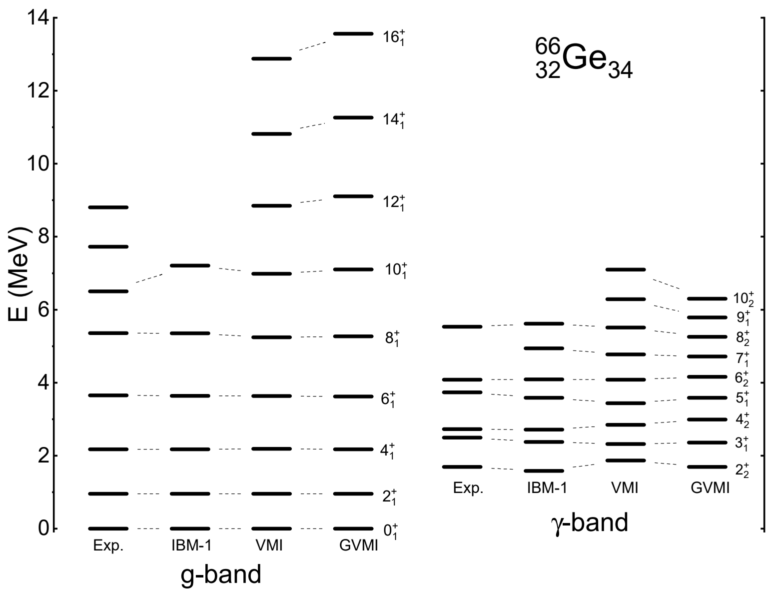

Figure 1. Experimental [22] and theoretical energy levels studied by (IBM-1) and (VMI) and (GVMI) models.

It is possible to identify and know the type of dynamic symmetry, which is considered one of the important nuclear properties to describe the behavior of the nucleus, and it can be reached by relying on the ratios for energy values calculated experimentally [23] and the theoretically calculated values of the nucleus under study and comparing them with the ideal values shown in Table (3), where practical and theoretical calculations indicate that this nucleus belongs to the two determinations $O(6) - SU(5)$ and according to the appearance of energy bands (g, γ). The presence of the $(\gamma - band)$ immediately after the $(g - band)$ confirms that the nucleus under study has O(6) dynamic symmetry.

Table 3. Perfect energy ratios for the three main limitations [23] compared with experimental [22] and theoretical values

Limit	$E4_1^+/E2_1^+$	$E6_1^+/E2_1^+$	$E8_1^+/E2_1^+$	Dynamicl symmetry
	2	3	4	SU(5)
Identical values [23]	3.33	7	12	SU(3)
	2.5	4.5	7	O(6)
Exp[22].	2.2712	3.8186	5.5997	O(6)-SU(5)
IBM-1	2.2735	3.8065	5.5999	O(6)-SU(5)
VMI	2.2905	3.7999	5.4801	O(6)-SU(5)
GVMI	2.2714	3.7823	5.5071	O(6)-SU(5)

The Energy Band Crossing

The importance of the energy band crossing phenomenon lies in explaining the back bending of deformed nuclei. Fig. (2) shows the band crossing of the nucleus under study using the (VMI) and (GVMI) models, depending on the energy equation (4). In both models, the (g-$band$) and (γ-$band$) intersect approximately at ($L_c = 6$).

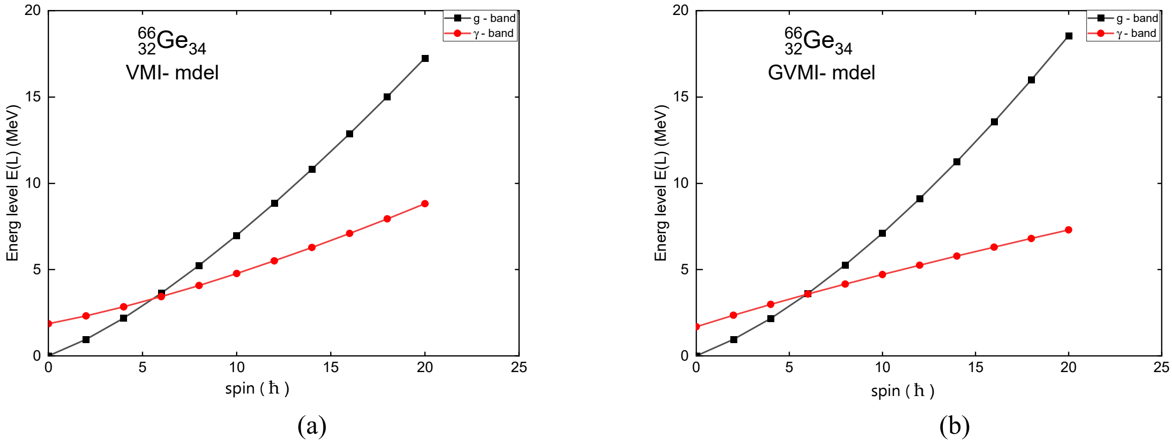

Figure 2. The energy levels calculated theoretically by means of a) the VMI model and b) the GVMI model for the nucleus under study**.**

The back bending phenomenon

In nuclear physics, the effect of the moment of inertia is an important topic in order to identify and study the phenomenon of back bending. To get acquainted with this phenomenon, both $(2\vartheta_\circ/\hbar)$ and $(\hbar\omega)^2$ were calculated by computing the equations (11, 12,13, and 14) using a computer simulation programs (*VMI.For*) and (*GVMI.Model*). Fig. (3) shows the relationship between the moment of inertia $(2\vartheta_\circ/\hbar)$ as a function of the rotational energy squared $(\hbar\omega)^2$ of the nucleus under study using the (VMI) and (GVMI) models. It was observed that there are backbends in the ground state band (g-$band$) and the (γ-$band$).

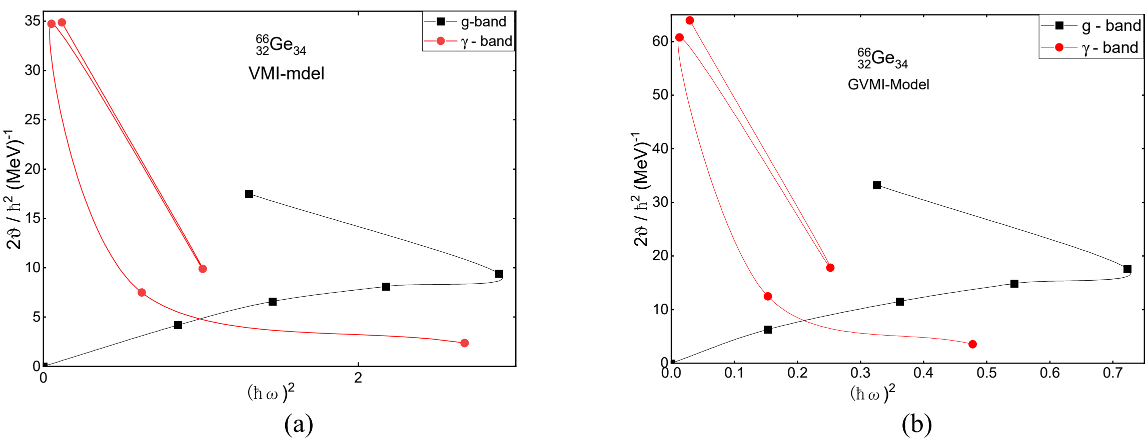

Figure 3. The relationship between moment of inertia $(2\vartheta/\hbar 2)$ and rotational energy $(\hbar\boldsymbol{\omega})^2$ of the nucleus under study within the scope of a) the (GVMI) model and b) the (VMI) model.

CONCLUSIONS

The three models (IBM-1, VMI and GVMI) achieved acceptable success in this study. In this study, it was found that the models used gave a good agreement in calculating the energy levels and excited energies as a function of angular momentum, compared with the experimental calculations.

It was concluded that the determination of dynamic symmetry by calculating the energy ratios for each of the three models and comparing them with the ideal values with the nucleus under study develops the determination of O(6) -SU(5).

With regard to the intersection of bands, it was found that each of the $g-band\ and\ the\ \gamma-band$, which were calculated through the VMI and GVMI models, intersect at angular momentum $L_c = 6$. This leads to the occurrence of the back bending phenomenon.

Through the theoretical results obtained, it was found that the models can give many explanations in the nuclear structure.

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ОПИС СТРУКТУРИ ЯДРА ГЕРМАНІЮ $^{66}_{32}$Ge$_{34}$ З ВИКОРИСТАННЯМ ЯДЕРНИХ МОДЕЛЕЙ IBM-1, GVMI ТА VMI

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У цій статті для розрахунку рівнів енергії позитивної парності та її гамма-переходів як функція кутового моменту парно-парного ($^{66}_{32}$Ge$_{34}$) ядра використовувалися модель взаємодіючих бозонів (IBM-1), модель змінного моменту інерції (VMI) і узагальненого моменту інерції (GVMI). Також були оцінені співвідношення рівнів енергій E_4^+/E_2^+, E_6^+/E_2^+, *and* E_8^+/E_2^+) та порівняні з експериментальними значеннями енергії та ідеальною схемою для трьох динамічних симетрій SU(5), SU(3) та O(6), щоб визначити динамічну симетрію цього ядра поточне дослідження показало, що при динамічна симетрія визначення O(6)-SU(5). Перетин енергетичної зони та явище вигину назад також вивчали за допомогою моделей (VMI) та (GVMI). Ці наслідки порівнювали з експериментальними результатами, і отримані результати добре узгоджувалися.

Ключові слова: *структура ядра;* $^{66}_{32}$Ge$_{34}$*; моделі (IBM-1, GVMI та VMI)*