COMPOSITE FERMIONS QED LAGRANGIAN DENSITY IN FRACTIONAL FORMULATION†

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Quantum electrodynamics (QED) is a highly precise and successful theory that describes the interaction between electrically charged particles and electromagnetic radiation. It is an integral part of the Standard Model of particle physics and provides a theoretical basis for explaining a wide range of physical phenomena, including the behavior of atoms, molecules, and materials. In this work, the Lagrangian density of Composite Fermions in QED has been expressed in a fractional form using the Riemann-Liouville fractional derivative. The fractional Euler-Lagrange and fractional Hamiltonian equations, derived from the fractional form of the Lagrangian density, were also obtained. When α is set to 1, the conventional mathematical equations are restored.

Keywords: Quantum Electrodynamics; Composite Fermions; Fractional derivative; Lagrangian density; Euler-Lagrange equations


1. INTRODUCTION

Composite Fermions are a theoretical concept in condensed matter physics that explains the behavior of electrons when subjected to a strong magnetic field [1-4]. These electrons can form composite particles with unique physical properties, such as those seen in fractional quantum Hall states [5-7]. Understanding composite fermions provides insights into the behavior of electrons in high magnetic fields and has applications in various fields, from technology development to the discovery of basic physical principles.

Fractional derivatives [8-12] have become a valuable tool in various fields because they provide the ability to accurately model physical phenomena that cannot be captured by ordinary derivatives. There has been a surge of research in fractional calculus, leading to its application in physics, engineering, and related areas [13-16]. The Maxwell equations have been expressed in fractional form [17-19], as have those in quantum mechanics, including the fractional Schrödinger equation [20, 21] and the fractional Dirac equation [22]. These advancements demonstrate the versatility of fractional calculus in describing a wide range of physical systems.

The main goal of this work is to examine the composite Fermions QED Lagrangian density and transform it into a fractional form using the Riemann-Liouville (RL) fractional derivative. The ultimate purpose is to derive the fractional Hamilton's equations and fractional Euler-Lagrange (EL) equations from this reformulation, thereby providing a fresh perspective on the dynamics of composite Fermions within a QED framework.

The structure of the paper is as follows: In Sec. 2, a brief explanation of RL fractional derivative is provided. The topic of the QED Lagrangian density is discussed in Sec. 3. In Sec. 4, the fractional form of the Lagrangian density and the fractional Euler-Lagrange equations are presented. The focus of Sec. 5 is on the Hamiltonian equations derived from the Lagrangian density. The paper concludes with a concise summary of the key points in Sec. 6.

2. PRELIMINARIES

This section provides essential definitions used in this study. For a more comprehensive understanding, readers can refer to reference [23]. The following are the definitions of the left and right RL fractional derivative.

The Left RL fractional derivative

\[ aD_x^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)} \left( \frac{d}{dx} \right)^n \int_0^x (x-\tau)^{n-\alpha-1} f(\tau) \, d\tau. \]  

(1)

The right RL fractional derivative

\[ bD_x^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)} \left( -\frac{d}{dx} \right)^n \int_0^x (\tau-x)^{n-\alpha-1} f(\tau) \, d\tau. \]  

(2)

The value of \( \alpha \) signifies the order of differentiation, where \( n-1 \leq \alpha < n \), with \( \Gamma \) symbolizing the gamma function. In cases where \( \alpha \) is an integer, the derivative is calculated using the conventional definition.

\[ \begin{cases} aD_x^\alpha f(x) = \left( \frac{d}{dx} \right)^n f(x) \\ bD_x^\alpha f(x) = \left( -\frac{d}{dx} \right)^n f(x) \end{cases} \]  

\( \alpha = 1, 2, ... \)  

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3. COMPOSITE FERMIONS QED LAGRANGIAN DENSITY

The Lagrangian density for composite fermions in QED, with the speed of light set to 1, has the following mathematical form [1]:

\[ \mathcal{L} = \overline{\psi} (i \gamma ^{\mu} \partial _{\mu} - m) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{e n v^2 eff \epsilon ^{\mu \nu \rho \sigma}}{2 \theta \phi _0} \partial _{\mu} A_{\nu} \partial _{\rho} A_{\sigma} - (1 + v_{eff}) e \overline{\psi} \gamma ^{\mu} \psi A_{\mu}. \] (4)

This equation involves the Levi-Civita symbol \( \epsilon ^{\mu \nu \rho \sigma} \), which is an antisymmetric tensor, as well as \( \psi \) which is a Dirac's spinor and made up of four complex parts, \( \overline{\psi} = \psi ^{\dagger} \gamma ^{\lambda} \), \( \gamma ^{\lambda} \) is Dirac matrix, \( e \) is the charge of an electron, \( \phi _0 \) is a unit of magnetic flux, it also includes a gauge parameter, \( \theta \), for the Chern-Simons fields. \( F_{\mu \nu} \) is the electromagnetic field tensor and the \( A_{\mu} \) are electromagnetic fields. The first term represents the fields that are associated with spinors, while the second term represents fields related to electromagnetism. The third term involves gauge Chern-Simons fields, and the final two terms describe the coupling of the spinor fields to both the electromagnetic fields and the Chern-Simons fields.

4. FRACTIONAL FORM OF COMPOSITE FERMIONS QED LAGRANGIAN DENSITY

The fractional Lagrangian density of (4) can be written as:

\[ \mathcal{L} = \overline{\psi} (i \gamma ^{\mu} a D_{\mu}^\alpha - m) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{e n v^2 eff \epsilon ^{\mu \nu \rho \sigma}}{2 \theta \phi _0} a D_{\mu}^\alpha A_{\nu} \partial _{\rho} A_{\sigma} - (1 + v_{eff}) e \overline{\psi} \gamma ^{\mu} \psi A_{\mu}. \] (5)

Begin by expanding the second term in the Lagrangian:

\[ F_{\mu \nu} F^{\mu \nu} = g^{\mu \rho} g^{\nu \lambda} \left( a D_{\mu}^\alpha A_{\nu} - a D_{\nu}^\alpha A_{\mu} \right) \left( a D_{\rho}^\alpha A_{\lambda} - a D_{\lambda}^\alpha A_{\rho} \right) + \frac{e n v^2 eff \epsilon ^{\mu \nu \rho \sigma}}{2 \theta \phi _0} a D_{\mu}^\alpha A_{\nu} - (1 + v_{eff}) e \overline{\psi} \gamma ^{\mu} \psi A_{\mu}. \] (6)

Hence, we have

\[ \mathcal{L} = \overline{\psi} (i \gamma ^{\mu} a D_{\mu}^\alpha - m) \psi - \frac{1}{4} g^{\mu \rho} g^{\nu \lambda} \left( a D_{\mu}^\alpha A_{\nu} - a D_{\nu}^\alpha A_{\mu} \right) \left( a D_{\rho}^\alpha A_{\lambda} - a D_{\lambda}^\alpha A_{\rho} \right) + \frac{e n v^2 eff \epsilon ^{\mu \nu \rho \sigma}}{2 \theta \phi _0} a D_{\mu}^\alpha A_{\nu} - (1 + v_{eff}) e \overline{\psi} \gamma ^{\mu} \psi A_{\mu}. \]

In the case of a Lagrangian involving multiple fields, there will be a separate equation for each field. The EL equation for the field \( A_{\beta} \) is expressed as follows:

\[ \frac{\partial \mathcal{L}}{\partial A_{\beta}} - a D_{\alpha}^\alpha \left[ \frac{\partial \mathcal{L}}{\partial (a D_{\alpha}^\alpha A_{\beta})} \right] = 0. \] (7)

Thus, the equation of motion is expressed as

\[ \frac{e n v^2 eff \epsilon ^{\mu \nu \rho \sigma}}{2 \theta \phi _0} a D_{\mu}^\alpha A_{\nu} - (1 + v_{eff}) e \overline{\psi} \gamma ^{\beta} \psi A_{\mu} = a D_{\alpha}^\alpha \left[ \frac{1}{4} g^{\mu \rho} g^{\nu \lambda} (\delta ^\alpha _\beta \delta ^\rho _\lambda - \delta ^\alpha _\lambda \delta ^\rho _\beta) \left( a D_{\mu}^\lambda A_{\nu} - a D_{\nu}^\lambda A_{\mu} \right) + \frac{e n v^2 eff \epsilon ^{\mu \nu \rho \sigma}}{2 \theta \phi _0} a D_{\mu}^\alpha A_{\nu} - (1 + v_{eff}) e \overline{\psi} \gamma ^{\mu} \psi A_{\mu} \right] = 0. \]

or alternatively,

\[ \frac{e n v^2 eff \epsilon ^{\mu \nu \rho \sigma}}{2 \theta \phi _0} e^{\mu \nu \rho \sigma} a D_{\mu}^\alpha A_{\nu} - e^{\nu \rho \sigma} a D_{\sigma}^\alpha A_{\nu} = (1 + v_{eff}) e \overline{\psi} \gamma ^{\beta} \psi - a D_{\alpha}^\alpha \left( a D_{\sigma}^\alpha A_{\beta} - a D_{\beta}^\alpha A_{\sigma} \right) = 0. \]

using \( F_{\alpha \beta} = (a D_{\alpha}^\alpha A_{\beta} - a D_{\beta}^\alpha A_{\alpha}) \), as a result of this

\[ \frac{e n v^2 eff \epsilon ^{\mu \nu \rho \sigma}}{2 \theta \phi _0} e^{\mu \nu \rho \sigma} a D_{\mu}^\alpha A_{\nu} - e^{\nu \rho \sigma} a D_{\sigma}^\alpha A_{\nu} = (1 + v_{eff}) e \overline{\psi} \gamma ^{\beta} \psi - a D_{\alpha}^\alpha F_{\alpha \beta} = 0. \]

By replacing the dummy indices \( \nu \) with \( \mu \) and \( \rho \) with \( \sigma \) in the first term on the left-hand side, we obtain

\[ \frac{e n v^2 eff \epsilon ^{\mu \nu \rho \sigma}}{2 \theta \phi _0} e^{\mu \nu \rho \sigma} a D_{\mu}^\alpha A_{\nu} = (1 + v_{eff}) e \overline{\psi} \gamma ^{\beta} \psi - a D_{\alpha}^\alpha F_{\alpha \beta} = 0. \] (8)
on the other hand, the EL equation for $\bar{\psi}$ reads as

$$\frac{\partial L}{\partial \bar{\psi}} - a D^\sigma_{x_\alpha} \left[ \frac{\partial L}{\partial \left( a D^\sigma_{x_\alpha} \bar{\psi} \right)} \right] = 0,$$

(9)

which becomes

$$\left( i \gamma^\mu a D^\sigma_{x_\mu} - m \right) \psi - (1 + \nu_{eff}) e \gamma^\mu \psi A_\mu = 0,$$

(10)

finally, the EL equation of the field $\psi$ is presented as follows:

$$\frac{\partial L}{\partial \psi} - a D^\sigma_{x_\alpha} \left[ \frac{\partial L}{\partial \left( a D^\sigma_{x_\alpha} \psi \right)} \right] = 0,$$

(11)

which can be written as

$$-m \bar{\psi} - (1 + \nu_{eff}) e \gamma^\mu \bar{\psi} A_\mu - a D^\sigma_{x_\alpha} \left( \bar{\psi} i \gamma^\mu \delta^\sigma_\mu \right) = 0,$$

(12)

$$m \psi + (1 + \nu_{eff}) e \gamma^\mu \psi A_\mu = -i \gamma^\sigma a D^\sigma_{x_\alpha} \bar{\psi}.$$  

5. FRACTIONAL HAMILTONIAN FORMULATION

In the following section, we will derive the fractional Hamiltonian equations using the RL fractional derivative approach, based on the fractional Lagrangian density. Let us consider the fractional Hamiltonian density as

$$\mathcal{H} = \left( \bar{\psi}, \psi, a D^\sigma_{x_\alpha} \psi, A_\mu, a D^\sigma_{x_\alpha} A_\mu, \bar{\pi}, \pi, \pi_\mu \right).$$

(13)

Now, taking the total differential of $\mathcal{H}$, we get:

$$d\mathcal{H} = \frac{\partial \mathcal{H}}{\partial \bar{\psi}} d\bar{\psi} + \frac{\partial \mathcal{H}}{\partial \psi} d\psi + \frac{\partial \mathcal{H}}{\partial A_\mu} dA_\mu + \frac{\partial \mathcal{H}}{\partial \left( a D^\sigma_{x_\alpha} \psi \right)} d\left( a D^\sigma_{x_\alpha} \psi \right) + \frac{\partial \mathcal{H}}{\partial \left( a D^\sigma_{x_\alpha} A_\mu \right)} d\left( a D^\sigma_{x_\alpha} A_\mu \right) + \frac{\partial \mathcal{H}}{\partial \bar{\pi}} d\bar{\pi} + \frac{\partial \mathcal{H}}{\partial \pi} d\pi + \frac{\partial \mathcal{H}}{\partial \pi_\mu} d\pi_\mu$$

(14)

The canonical momenta $\bar{\pi}, \pi, \pi_\mu$ are given as follows:

$$\bar{\pi} = \frac{\partial L}{\partial \left( a D^\sigma_{x_\alpha} \psi \right)}; \quad \bar{\pi} = \frac{\partial L}{\partial \left( a D^\sigma_{x_\alpha} A_\mu \right)}; \quad \pi_\mu = \frac{\partial L}{\partial \left( a D^\sigma_{x_\alpha} A_\mu \right)}.$$

(15)

In order to construct $\mathcal{H}$, we start by defining it in its general form as follows:

$$\mathcal{H} = \bar{\pi} a D^\sigma_{x_\alpha} \bar{\psi} + \pi a D^\sigma_{x_\alpha} \psi + \pi_\mu a D^\sigma_{x_\alpha} A_\mu - L \left( \psi, \bar{\psi}, a D^\sigma_{x_\alpha} \psi, A_\beta, a D^\sigma_{x_\alpha} A_\beta \right).$$

(16)

The total differential of $\mathcal{H}$ can also be defined as:

$$d\mathcal{H} = \bar{\pi} d\left( a D^\sigma_{x_\alpha} \bar{\psi} \right) + a D^\sigma_{x_\alpha} \bar{\psi} d\bar{\pi} + \pi d\left( a D^\sigma_{x_\alpha} \psi \right) + a D^\sigma_{x_\alpha} \psi d\pi + \pi_\mu d\left( a D^\sigma_{x_\alpha} A_\mu \right) + a D^\sigma_{x_\alpha} A_\mu d\pi_\mu - \frac{\partial L}{\partial \psi} d\psi -$$

$$- \frac{\partial L}{\partial \bar{\psi}} d\bar{\psi} - \frac{\partial L}{\partial \left( a D^\sigma_{x_\alpha} \psi \right)} d\left( a D^\sigma_{x_\alpha} \psi \right) - \frac{\partial L}{\partial \bar{\pi}} d\bar{\pi} + \frac{\partial L}{\partial \pi} d\pi + \frac{\partial L}{\partial \pi_\mu} d\pi_\mu.$$ 

(17)

but

$$\frac{\partial L}{\partial \psi} = a D^\sigma_{x_\alpha} \pi + \pi d\left( a D^\sigma_{x_\alpha} \psi \right),$$

(18)

$$\frac{\partial L}{\partial \bar{\psi}} = a D^\sigma_{x_\alpha} \bar{\pi} + \bar{\pi} d\left( a D^\sigma_{x_\alpha} \bar{\psi} \right),$$

(19)

and

$$\frac{\partial L}{\partial A_\mu} = a D^\sigma_{x_\alpha} \pi_\mu + \pi_\mu d\left( a D^\sigma_{x_\alpha} A_\mu \right).$$

(20)

Using Eqs. (18), (19), and (20) Eq. (17) can be written as
\[ d\mathcal{H} = \bar{\pi} d\left( aD_t\Psi \right) + aD_t^\alpha d\bar{\pi} + \pi d\left( aD_t^\alpha \psi \right) + aD_t^\alpha \psi d\pi + \pi d\left( aD_t^\alpha \mu \right) + aD_t^\alpha \mu d\pi - aD_t^\alpha (\pi) d\psi - aD_t^\alpha \left( \frac{\partial \mathcal{L}}{\partial (aD_t^\alpha \psi)} \right) d\psi - \frac{\partial \mathcal{L}}{\partial (aD_t^\alpha \mu)} d\left( aD_x^\alpha \psi \right) - aD_t^\alpha (\pi) dA_\beta - aD_t^\alpha \left( \frac{\partial \mathcal{L}}{\partial (aD_x^\alpha \mu)} \right) dA_\beta, \]

or

\[ d\mathcal{H} = aD_t^\alpha \Psi d\bar{\pi} + aD_t^\alpha \psi d\pi + aD_t^\alpha \mu d\pi - aD_t^\alpha (\pi) d\psi - aD_t^\alpha \left( \frac{\partial \mathcal{L}}{\partial (aD_t^\alpha \psi)} \right) d\psi - aD_t^\alpha (\pi) d\bar{\psi} - aD_t^\alpha \left( \frac{\partial \mathcal{L}}{\partial (aD_x^\alpha \psi)} \right) d\bar{\psi} - \frac{\partial \mathcal{L}}{\partial (aD_t^\alpha \mu)} d\left( aD_x^\alpha \psi \right) - aD_t^\alpha (\pi) dA_\beta - aD_t^\alpha \left( \frac{\partial \mathcal{L}}{\partial (aD_x^\alpha \mu)} \right) dA_\beta - \frac{\partial \mathcal{L}}{\partial (aD_x^\alpha \mu)} d\left( aD_x^\alpha A_\beta \right), \]

By comparing Eqs. (14) and (22), we obtain Hamilton’s equation of motion

\[ \frac{\partial \mathcal{H}}{\partial A_\mu} = -aD_t^\alpha (\pi_\mu) - aD_t^\alpha \left( \frac{\partial \mathcal{L}}{\partial (aD_t^\alpha A_\mu)} \right), \]

\[ \frac{\partial \mathcal{H}}{\partial \pi} = -aD_t^\alpha \pi = -aD_t^\alpha \left( \frac{\partial \mathcal{L}}{\partial (aD_t^\alpha \psi)} \right), \]

\[ \frac{\partial \mathcal{H}}{\partial \psi} = -aD_t^\alpha \pi - aD_t^\alpha \left( \frac{\partial \mathcal{L}}{\partial (aD_t^\alpha \psi)} \right), \]

\[ aD_t^\alpha \mu = \frac{\partial \mathcal{H}}{\partial (aD_x^\alpha A_\mu)} = \frac{\partial \mathcal{H}}{\partial (aD_x^\alpha \mu)}, \]

\[ \frac{\partial \mathcal{H}}{\partial \pi} = -\frac{\partial \mathcal{H}}{\partial (aD_x^\alpha \pi)} = \frac{\partial \mathcal{H}}{\partial (aD_x^\alpha \mu)} = -\frac{\partial \mathcal{H}}{\partial (aD_x^\alpha A_\mu)}. \]

Consider the Lagrangian density given in Eq. (5)

\[ \mathcal{L} = \bar{\Psi} \left( i\gamma^\mu aD_x^\mu - m \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{e\pi v_{\text{eff}}^2}{2\theta \phi_0} \left( \epsilon^{\mu\nu\rho\sigma} A_\mu dD_x^\rho A_\sigma - (1 + v_{\text{eff}}) \bar{\Psi} \gamma^\mu \psi A_\mu \right). \]

We can determine \( \bar{\pi}, \pi \) and \( \pi_i \) such that:

\[ \bar{\pi} = \frac{\partial \mathcal{L}}{\partial \left( aD_t^\alpha \pi \right)} = 0, \]

\[ \pi = \frac{\partial \mathcal{L}}{\partial \left( aD_x^\alpha \psi \right)} = i\gamma^\nu \bar{\Psi}, \]

\[ \left\{ \begin{array}{l}
\pi_i = \frac{\partial \mathcal{L}}{\partial \left( aD_x^\alpha A_i \right)} = -F_{ij} + \frac{e\pi v_{\text{eff}}^2}{2\theta \phi_0} \epsilon^{ij} A_j \\
\pi_0 = \frac{\partial \mathcal{L}}{\partial \left( aD_x^\alpha A_0 \right)} = 0
\end{array} \right. \]

Then we can write the fractional Hamiltonian density of the system as follows:

\[ \mathcal{H} = \bar{\pi} aD_t^\alpha \pi + \pi aD_t^\alpha \psi + \pi aD_t^\alpha A_\mu - \mathcal{L} \left( \psi, \bar{\Psi}, aD_x^\alpha \psi, A_\rho, aD_x^\alpha A_\beta \right). \]

By inserting Eqs. (5) and (28) into Eq. (29), we obtain

\[ \mathcal{H} = i\gamma^\nu \bar{\Psi} aD_t^\alpha \psi + \pi_i \left( -\pi_i + \frac{e\pi v_{\text{eff}}^2}{2\theta \phi_0} \epsilon^{ij} A_j + aD_x^\alpha A_0 \right) - i\gamma^\nu \bar{\Psi} aD_t^\alpha \psi - \bar{\Psi} \left( i\gamma^\nu aD_x^\alpha - m \right) \psi + \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \frac{e\pi v_{\text{eff}}^2}{2\theta \phi_0} \left( -\pi_i + \frac{e\pi v_{\text{eff}}^2}{2\theta \phi_0} \epsilon^{ij} A_j + aD_x^\alpha A_0 \right) - \frac{e\pi v_{\text{eff}}^2}{2\theta \phi_0} \epsilon^{ij} A_0, \]
Substituting for $F^{0i}$ from equation (28c), we get

$$H = -\frac{1}{2} \pi_i^2 + \pi_i aD_x^a A_0 - \overline{\psi} \left( i\gamma^j aD_s^a j - m \right) \psi - \frac{1}{4} \left( e_n v^2 \varepsilon_{ij} \right) A_j A_i^2 + \frac{1}{4} F_{ij} F^{ij} - \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu (\pi_i - aD_x^a A_0) - \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu aD_x^a A_\rho + (1 + v_{eff}) e\overline{\psi} \gamma^\mu \psi A_\mu. \right) \tag{30}$$

Now we will find the Hamiltonian equations of motion for the same system. Initially, the equation of motion for $A_\mu$

$$\frac{\partial H}{\partial A^\mu} - \frac{\partial L}{\partial (aD_s^a j)} - \left( \frac{\partial v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} \right)^2 A_\mu - \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} \left( -\pi_i + aD_x^a A_0 \right) - \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} \left( 1 + v_{eff} \right) e\overline{\psi} \gamma^\mu \psi A_\mu. \text{ (31)}$$

The substitution of $aD_s^a F^{\beta \rho} = aD_x^a F^{\beta \rho} + aD_s^a F^{\beta \rho}$ into Eq. (31) gives

$$- \left( \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} \right)^2 A_\mu - \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} \left( -\pi_i + aD_x^a A_0 \right) - \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} \left( -\pi_i + aD_x^a A_0 \right) - \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} \left( 1 + v_{eff} \right) e\overline{\psi} \gamma^\mu \psi + \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} aD_x^a A_\mu + \frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} aD_x^a A_\mu. \text{ (32)}$$

By substituting Eq. (28c) into Eq. (32) and undergoing mathematical manipulation, the following equation is obtained:

$$\frac{\pi v^2 \varepsilon_{ij} \varepsilon^{\mu \nu} A_\mu}{\partial \psi} aD_x^a A_\mu - (1 + v_{eff}) e\overline{\psi} \gamma^\rho \psi - aD_x^a F^{\beta \rho} = 0. \text{ (33)}$$

While the equation of motion for $\psi$ reads

$$\frac{\partial H}{\partial \psi} = - aD_x^a A_\mu - aD_s^a \left( \frac{\partial L}{\partial \psi} \right) \tag{34}$$

$m \overline{\psi} + (1 + v_{eff}) e \gamma^\rho \overline{\psi} A_\mu = -i\gamma^0 aD_x^a \overline{\psi} - aD_s^a \left( \overline{\psi} i\gamma^j \right),$

but

$$i\gamma^0 aD_x^a \overline{\psi} + i\gamma^j aD_s^a \overline{\psi} = i\gamma^\rho aD_s^a \overline{\psi} \tag{35}$$

Thus, Eq. (34) becomes

$$m \overline{\psi} + (1 + v_{eff}) e \gamma^\rho \overline{\psi} A_\mu = -i\gamma^\rho aD_x^a \overline{\psi}.$$

Similarly, the equation of motion for $\overline{\psi}$ is

$$\frac{\partial H}{\partial \overline{\psi}} = - aD_x^a \left( \overline{\psi} \right) - aD_s^a \left( \frac{\partial L}{\partial \overline{\psi}} \right)$$

or

$$\left( i\gamma^j aD_s^a - m \right) \psi - (1 + v_{eff}) e \gamma^\rho \psi A_\mu = 0. \tag{36}$$

The results from Eqs. (33), (35), and (36) are in full accordance with those derived from the fractional EL method.

6. CONCLUSION

The Riemann-Liouville fractional derivative were employed to reformulate the composite Fermions QED Lagrangian density. It was observed that the fractional Euler-Lagrange equations and the fractional Hamilton's equations of motion, both derived from the same Lagrangian density, produced the same outcomes. The fractional formulation was demonstrated to encompass the classical results as a specific case.

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ЩИЛЬНІСТЬ ЛАГРАНЖІАНА КОМПОЗИЦІЙНИХ ФЕРМІОНІВ QED У ДРОБОВОМУ ФОРМУЛОВАННІ

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Квантовая электродинамика (КЭД) – точна и узконаправленная теория, которая описывает взаимодействие меж электрически заряженными частицами и электромагнитным вихлопом. Квантовая электродинамика элементарных частиц имеет теоретическую основу, которая позволяет пояснить широкий спектр физических явлений, у которых выделенные атомы, молекулы и материалы. У этой работы изучается Лагранжевская кинематика фермийонов в КЭД носительным дробовым формулированием. В дробовой форме была введена обобщенная квантовая электродинамика, в которой были введены релевантные дробовые описания, как старые, так и новые, которые предшествовали ей.

Ключевые слова: квантовая электродинамика; композитные фермиони; дробовая похідна; щільність Лагранжіана; рівняння Ейлера-Лагранжі