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STRUCTURAL VARIATIONS OF DUST ACOUSTIC SOLITARY WAVES (DASWs) PROPAGATING IN AN INHOMOGENEOUS PLASMA[†]

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This paper presents our theoretical investigations on the structural variations of dust acoustic solitary waves (DASWs) in inhomogeneous unmagnetized plasmas. To study the structural variations of DASWs, we have considered collisionless, hot isothermal, and Boltzmannean distribution for electrons-ions with negatively charged dust grains in weakly inhomogeneous plasmas. We have used the reductive perturbation technique (RPT) in the governing equations of plasmas, derived the modified Korteweg-de-Vries (m-KdV) equation, and obtained the solitary wave solution. We have considered the appropriate stretched coordinates for space and time variables for the inhomogeneous plasma. This paper investigates the effects of dust particles on ion-acoustic solitary waves' propagation in the inhomogeneous plasma model. We have also included the effect of inhomogeneity parameters on the soliton structures.
Keywords: *Dusty plasma; reductive perturbation technique (RPT); isothermal electrons; inhomogeneous plasma*
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I. INTRODUCTION

Plasma and charged dusts or dust grains are the two fundamental ubiquities of the universe. Dusty plasma is a relatively new branch of the research area in plasma physics. In dusty plasmas, the interaction of dust grains and plasma particles are studied. Plasma particles consist of hot and electrically conductive electrons and ions, but the dusty plasma particles consist of both plasma particles and dust grains, i.e., dusty plasma particles are comprised of three fundamental ingredients ions, electrons, and charged dust particles or dust charged grains. The dust particles show more complex behaviours when added to the plasma particles. So, the dusty plasmas are also known as complex plasmas or multi-component plasmas. Investigations on these complex or multi-component plasmas are abundant throughout the universe. The majority of extant solid matter is thought to be made up of dust grains that are frequently contained by plasma particles. Planetary magnetospheres, cometary environments, nebulas, etc. [1], are some examples of dusty plasmas. These complex plasmas are enormous important to understand the theories of geophysics better, complete some space missions, advancements of knowledge in astrophysical environments like the formation of stars, galaxies, and nebulae, and manufacture some new materials the in the semiconductor industries, etc. Dusty plasmas are one of the significant fields for working in Controlled Thermonuclear Research (CTR). In 1982, Voyager spacecraft [2] discovered the radial spokes of the Saturn's B ring. Apart from the wide range of applications of dusty plasmas in the astrophysical problems, researchers have observed the wide use of dusty plasmas and their related issues during the use of dusty plasmas in the manufacturing of new materials in the semiconductor industries [3]. Later, a number of authors have worked in the both theoretical and experimental works on the dusty plasmas. In the early eighties of last century, the concept of dusty plasma could not be developed convincingly. However, to understand the fundamental properties of dusty plasmas, some devices are introduced in dusty plasma laboratories like the rotating drum system [4] and dust shaker systems [5]. Subsequently, in the review literature of Goertz [6] and Northrop [7], we have some details of works carried out in dusty plasma for astrophysical cases. Basically, for unmagnetized plasmas, dusty plasma waves are characterized by three different modes such as dust-acoustic waves [9] (DAW), dust ion-acoustic waves [8] (DIAW), and dust lattice waves [10] (DLW). Several theoretical [8-10] and experimental [11,12] studies have been done to achieve more understanding on the complex behaviours of DAWs, DIAWs, and DLWs. Apart from the above, large number of literatures could be found on the various nonlinear wave phenomena and instabilities for homogeneous dusty plasmas, inhomogeneous dusty plasmas [13-20] and nonlinear wave excitation in nonequilibrium plasmas. For studying the linear wave theories in plasmas, the nonlinearities for small amplitude waves are not considered, but in the case of large amplitude waves, the nonlinearities cannot be neglected. Due to the existence of nonlinearities in the plasma waves, the various physical parameters and their effects can be studied. The nonlinearities are also indicated in the experimental and theoretical behaviours of some nonlinear plasma wave structures such as solitons or solitary waves, supersolitons, rouge waves, shock waves, etc. In inhomogeneous plasmas, few more researchers have done their works on the properties and effects of dust ion-acoustic (DIA) solitary waves [21,22], dust acoustic (DA) solitary waves [8,27,28,32], shock waves [23-26], and dust lattice (DL) solitary waves [10,29], etc. Apart from the broad applications of complex or dusty plasmas in astrophysical systems and space science, the wide applications of dusty plasmas have also been seen in the fusion sciences and laboratory environments [30,31]. Gogoi and Deka also studied the propagation of dust acoustic (DA) solitary wave propagations in

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inhomogeneous plasmas [32]. Some more basic properties and the effects on dust-acoustic waves (DAWs) have been studied theoretically in the presence of both positively and negatively dust particles with negatively charged ion fluids and κ-distributed superthermal electrons [33]. Dusty plasmas are basically studied in inhomogeneous plasma systems to understand the advanced use of plasmas in astrophysical environments and in laboratory discharges [34]. The modulational instability and dust-cyclotron wave for DAWs in three-component plasmas have been explored in the presence of ions, electrons, and dust particles under the influence of a magnetic field [35]. In the presence of pair-ion fullerene plasma, magneto-acoustic wave propagation was also studied in both the linear and nonlinear plasma environments [36]. Pakzad and Nobahar [39] studied the properties of dust ion-acoustic (DIA) solitary waves in inhomogeneous unmagnetized plasmas in the presence of stationary dust grains, super thermal electrons, and inertial ions. They have also studied the behaviours of DIAWs propagating in the various astrophysical environments like Solar winds, Venus's ionosphere, and the Earth's atmosphere. Dehingia and Deka [41] have recently studied ion-acoustic solitons' variations in an inhomogeneous plasma. They have observed how the ion-acoustic solitary wave bends in some critical points due to the inhomogeneity present in the plasma system. Though many investigations on the effect of dust particles on plasma properties have been done till date, there are still many scopes to study the role of dust particles in affecting the nonlinear structures of plasmas, in particular for the case of inhomogeneous plasmas. In this paper, we will extend the investigations of Gogoi and Deka [32], to understand the structural variations of dust-acoustic solitary waves in inhomogeneous plasma under the following considered physical situations.

Here, we present our investigations on the structural variations of dust acoustic solitary waves in inhomogeneous plasma in the presence of hot isothermal electrons with Boltzmannean electron-ion distribution. This model consists of unmagnetized, collisionless, hot isothermal electrons and weakly inhomogeneous plasmas in the presence of negatively charged dust grains. Using the governing equations of plasmas and the reductive perturbation method or technique (RPT), we have derived the modified Korteweg-de-Vries (mKdV) equation with the help of appropriate stretched coordinates for space and time variables for the inhomogeneous plasmas. The solution of the above mKdV equation also indicates the various nonlinear effects of dust grains propagating in inhomogeneous, unmagnetized plasmas in the presence of negatively charged dust particles. We have also presented our results and investigated the effect of dust particles on ion-acoustic solitary waves' propagation due to the inhomogeneity parameters in the inhomogeneous above-considered plasma model.

II. GOVERNING EQUATIONS

We have considered an unmagnetized, collisionless, hot isothermal, Boltzmannean distributed electrons and ions, in the presence negatively charged dust particles in weakly inhomogeneous plasma. With the variable density gradient and along the x-direction only, the system is considered to be inhomogeneous. The set of dimensionless and nonlinear governing fluid equations for slowly moving dust acoustic waves along the x- direction is taken as follows:

Continuity equation:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0. \tag{1}$$

Momentum equation:

$$\frac{\partial v_d}{\partial t} + v_d\left(\frac{\partial v_d}{\partial x}\right) + q\left(\frac{\partial \phi}{\partial x}\right) = 0. \tag{2}$$

Poisson equation :

$$\frac{\partial^2 \phi}{\partial x^2} - n_e - q n_d + n_i = 0. \tag{3}$$

Boltzmannean distribution of electrons:

$$n_e = n_{e0} e^{k\phi}. \tag{4}$$

Boltzmannean distribution of ions:

$$n_i = n_{i0} e^{-\phi}. \tag{5}$$

In the above equations, n_d represents the number density of dust grains which is normalized by n_{d0} at an equilibrium condition. Here, v_d represents the fluid velocity of dust grains which is normalized by dust acoustic speed (DAS) $C_{ds} = \left(Z_{d0}\frac{T_i}{m_d}\right)^{1/2}$ where ion temperature is T_i and mass of the charged dust particles is m_d. Here, we assume ϕ as the electrostatic potential with the charge neutrality at equilibrium state $n_{i0} = n_{e0} + Z_{d0} n_{d0}$. The quantities n_d, v_d, ϕ, n_e, n_i are reduced to the dimensionless form with the help of Debye length $\lambda_D = \sqrt{\frac{T_e}{4\pi n_{i0} e^2}}$, DAS C_{ds} and thermal electrostatic potential $\phi = \frac{T_e}{e}$ where T_e is the electron temperature and e is the electron charge. For charged dust grains $q = Z_d e$, the balanced current equation is given by [6] $q_t + v_d q_x = I_e + I_i$ where q_t and q_x are dust currents, I_i and I_e are the ion and electron currents respectively.

Due to slow motion of the dust fluid its velocity v_{d0} is small at equilibrium state, the dust current is balanced by both the ion and electron currents together. Then we have the balanced current equation is given by [10] $I_e + I_i \approx 0$ where I_e and I_i are respectively given by [40]

$$\left.\begin{aligned} I_e &= -\pi r^2 e\left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_{e0} e^{k\phi} \\ I_i &= -\pi r^2 e\left(\frac{8T_i}{\pi m_i}\right)^{1/2} n_{i0}\left(1-\frac{e\phi}{T_i}\right) \end{aligned}\right\}, \tag{6}$$

where r denotes the radius of charged dust grains and $k = {T_i}/{T_e}$.

III. DERIVATION OF MODIFIED K-dV (m-KdV) EQUATION

To study the structural variations of dust acoustic (DA) solitary wave propagations in inhomogeneous plasmas, we use the reductive perturbation technique (RPT). To apply the RPT for some small amplitude wave, we use an appropriate one-dimensional space-time stretched coordinate which is given by [37]

$$\left.\begin{aligned} \xi &= \epsilon^{\frac{1}{2}}\left(\frac{x}{M}-t\right) \\ \tau &= \in^{\frac{3}{2}} x \end{aligned}\right\}, \tag{7}$$

where ϵ is a smallness parameter, M is the phase velocity of the DA soliton, normalized by C_{ds}.

Now, using Eq. (7) in Eqs. $(1)-(5)$ we get the following set of equations are as follows:

$$-\epsilon^{\frac{1}{2}}\frac{\partial n_d}{\partial \xi} + \frac{\epsilon^{\frac{1}{2}}}{M}\frac{\partial}{\partial \xi}(n_d v_d) + \in^{\frac{3}{2}}\frac{\partial}{\partial \tau}(n_d v_d) = 0, \tag{8}$$

$$-\epsilon^{\frac{1}{2}}\frac{\partial v_d}{\partial \xi} + \frac{\epsilon^{\frac{1}{2}} v_d}{M}\left(\frac{\partial v_d}{\partial \xi}\right) + \frac{\in^{\frac{3}{2}} v_d}{M}\left(\frac{\partial v_d}{\partial \tau}\right) + \frac{\epsilon^{\frac{1}{2}} Z_d}{M}\left(\frac{\partial \phi}{\partial \xi}\right) + \frac{\epsilon^{\frac{3}{2}} Z_d}{M}\left(\frac{\partial \phi}{\partial \tau}\right) = 0, \tag{9}$$

$$\frac{\epsilon}{M^2}\frac{\partial^2 \phi}{\partial \xi^2} + 2\frac{\in^2}{M}\frac{\partial^2 \phi}{\partial \xi \partial \tau} + \epsilon^3\frac{\partial^2 \phi}{\partial \tau^2} - n_{e0} e^{k\phi} n - n_d Z_d + n_{i0} e^{-\phi} = 0. \tag{10}$$

Expanding the dependent variables n_d, v_d, Z_d and ϕ about the equilibrium parts in terms of power series of ϵ as follows:

$$\begin{bmatrix} n_d \\ v_d \\ \phi \\ Z_d \end{bmatrix} = \begin{bmatrix} n_{d0} \\ 0 \\ 0 \\ Z_{d0} \end{bmatrix} + \epsilon \begin{bmatrix} n_{d1} \\ v_{d1} \\ \phi_1 \\ Z_{d1} \end{bmatrix} + \epsilon^2 \begin{bmatrix} n_{d2} \\ v_{d2} \\ \phi_2 \\ Z_{d2} \end{bmatrix} + \cdots. \tag{11}$$

Using the Eq. (11) in Eq. $(8), (9)$ and (10) respectively we get the following set of equations as follows: (Neglecting the higher terms having powers of ϵ more than $\frac{5}{2}$)

$$\left.\begin{aligned} -\epsilon^{\frac{1}{2}}\frac{\partial n_{d0}}{\partial \xi} + \epsilon^{\frac{3}{2}}\left[-\frac{\partial n_{d1}}{\partial \xi} + \frac{1}{M}\frac{\partial}{\partial \xi}(n_{d0} v_{d1})\right] \\ +\epsilon^{\frac{5}{2}}\left[-\frac{\partial n_{d2}}{\partial \xi} + \frac{1}{M}\frac{\partial}{\partial \xi}(n_{d0} v_{d2}) + \frac{1}{M}\frac{\partial}{\partial \xi}(n_{d1} v_{d1}) + \frac{1}{M}\frac{\partial}{\partial \tau}(n_{d0} v_{d1})\right] = 0 \end{aligned}\right\}, \tag{12}$$

$$\left.\begin{aligned} \epsilon^{\frac{3}{2}}\left[-\frac{\partial v_{d1}}{\partial \xi} + \frac{Z_{d0}}{M}\left(\frac{\partial \phi_1}{\partial \xi}\right)\right] \\ +\epsilon^{\frac{5}{2}}\left[-\frac{\partial v_{d2}}{\partial \xi} + \frac{v_{d1}}{M}\left(\frac{\partial v_1}{\partial \xi}\right) + \frac{Z_{d0}}{M}\left(\frac{\partial \phi_2}{\partial \xi}\right) + \frac{Z_{d1}}{M}\left(\frac{\partial \phi_1}{\partial \xi}\right) + Z_{d0}\left(\frac{\partial \phi_1}{\partial \tau}\right)\right] = 0 \end{aligned}\right\}, \tag{13}$$

$$\left.\begin{aligned} (n_{e0} + n_{d0} Z_{d0} - n_{i0}) + \epsilon\{Z_{d0} n_{d1} + n_{d0} Z_{d1} + (n_{e0} k + n_{i0})\phi_1\} \\ +\epsilon^2\left[\begin{matrix} \frac{1}{M^2}\frac{\partial^2 \phi_1}{\partial \xi^2} - (n_{e0} k + n_{i0})\phi_2 - \left(\frac{n_{e0}}{2} k^2 + \frac{n_{i0}}{2}\right){\phi_1}^2 \\ -(n_{d0} Z_{d2} + n_{d1} Z_{d1} + n_{d0} Z_{d2}) \end{matrix}\right] = 0 \end{aligned}\right\}. \tag{14}$$

Now we compare the coefficients of ϵ from lower to the highest powers in the Eq. $(12)-(14)$, we get
At $\in^{\frac{1}{2}}$, we get,

$$\frac{\partial n_{d0}}{\partial \xi} = 0. \tag{15}$$

At $\epsilon^{\frac{3}{2}}$, we get,

$$\frac{1}{M}\frac{\partial}{\partial \xi}(n_{d0}v_{d1}) = \frac{\partial n_{d1}}{\partial \xi}. \tag{16}$$

$$\frac{\partial v_{d1}}{\partial \xi} = \frac{Z_{d0}}{M}\left(\frac{\partial \phi_1}{\partial \xi}\right). \tag{17}$$

At ϵ, we get,

$$(n_{e0}k + n_{i0})\phi_1 + n_{d0}Z_{d1} + n_{d1}Z_{d0} = 0. \tag{18}$$

So, using the boundary conditions $n_{d1}, v_{d1}, \phi_1 \longrightarrow 0$ as $|\xi| \longrightarrow \infty$, we get from $(16) - (18)$

$$v_{d1} = \frac{Z_{d0}}{M}\phi_1, \tag{19}$$

$$n_{d1} = \frac{n_{d0}Z_{d0}}{M^2}\phi_1, \tag{20}$$

$$Z_{d1} = L\phi_1, \tag{21}$$

where

$$L = -\left[\frac{(n_{e0}k + n_{i0})}{n_{d0}} + \left(\frac{Z_{d0}}{M}\right)^2\right], \tag{22}$$

and

$$M = \sqrt{\frac{n_{d0}}{n_{e0}k + n_{i0}}}Z_{d0}, \tag{23}$$

where M determines the phase velocity of DAW.

At the highest order coefficients of ϵ we get,

$$-\frac{\partial n_{d2}}{\partial \xi} + \frac{1}{M}\frac{\partial}{\partial \xi}(n_{d0}v_{d2}) + 2\frac{n_{d0}Z_{d0}{}^2}{M^4}\phi_1\frac{\partial \phi_1}{\partial \xi} + \frac{1}{M}\frac{\partial}{\partial \tau}(n_{d0}v_{d1}\phi_1) = 0, \tag{24}$$

$$-\frac{\partial v_{d2}}{\partial \xi} + \frac{Z_{d0}}{M}\left(\frac{\partial \phi_2}{\partial \xi}\right) + \frac{Z_{d0}{}^2}{M^3}\phi_1\left(\frac{\partial \phi_1}{\partial \xi}\right) + \frac{Z_{d1}}{M}\left(\frac{\partial \phi_1}{\partial \xi}\right) + Z_{d0}\left(\frac{\partial \phi_1}{\partial \tau}\right) = 0, \tag{25}$$

$$\frac{1}{M^2}\left(\frac{\partial^2 \phi_1}{\partial \xi^2}\right) - (n_{e0}k + n_{i0})\phi_2 - \left(\frac{n_{e0}}{2}k^2 - \frac{n_{i0}}{2}\right)\phi_1{}^2 - (n_{d0}Z_{d2} + n_{d1}Z_{d1} + n_{d0}Z_{d2}) = 0. \tag{26}$$

Now, differentiating Eq. (26) w.r.t. ξ we get,

$$\frac{1}{M^2}\left(\frac{\partial^3 \phi_1}{\partial \xi^3}\right) - (n_{e0}k + n_{i0})\left(\frac{\partial \phi_2}{\partial \xi}\right) - (n_{e0}k^2 - n_{i0})\phi_1\left(\frac{\partial \phi_1}{\partial \xi}\right) - n_{d0}\left(\frac{\partial Z_{d2}}{\partial \xi}\right)$$

$$-n_{d1}\left(\frac{\partial Z_{d1}}{\partial \xi}\right) - Z_{d1}\left(\frac{\partial n_{d1}}{\partial \xi}\right) - Z_{d0}\left(\frac{\partial n_{d2}}{\partial \xi}\right) = 0 \tag{27}$$

Now, eliminating all the 2nd order quantities from Eqs. from (24), (25) and (27) and adding all of them we get a nonlinear PDE with variable coefficient is of the form:

$$\left(\frac{\partial \phi_1}{\partial \tau}\right) + A\phi_1\left(\frac{\partial \phi_1}{\partial \xi}\right) + B\left(\frac{\partial^3 \phi_1}{\partial \xi^3}\right) + C\phi_1\left(\frac{\partial n_{d0}}{\partial \tau}\right) = 0, \tag{28}$$

where

$$A = \frac{M}{(Z_{d0}M + Z_{d0}n_{d0})}\left[\frac{Z_{d0}{}^2}{M} - \frac{1}{M}\left\{\left(\frac{Z_{d0}}{M}\right)^2 + \left(\frac{n_{e0}k + n_{i0}}{n_{d0}}\right)\right\} + 2\frac{Z_{d0}n_{d0}}{M^4} + (n_{e0}k - n_{i0})\right]$$

$$+\frac{M}{(Z_{d0}M + Z_{d0}n_{d0})}\left[\left(\frac{n_{e0}k + n_{i0}}{n_{d0}}\right) - 2\frac{Z_{d0}n_{d0}}{M^2}\left\{\left(\frac{Z_{d0}}{M}\right)^2\right\}\right],$$

$$B = \frac{-1}{M^3 Z_{d0}(n_{d0} + M)},$$

$$C = \left[\frac{1}{Z_{d0}} + \frac{R^2}{MZ_{d0}}\right].$$

The above Eq. (28) is in the new form of the KdV equation known as mKdV equation, as there is an additional term due to the plasma inhomogeneity of number density. The solution of the above modified K-dV (mKdV) equation i.e., Eq. (28) represents the structural variations of nonlinear dust acoustic (DA) solitary waves propagating in inhomogeneous plasma. The nonlinear constant coefficient A and the dispersion coefficient B depend on the inhomogeneous number density. The extra term in the above considered inhomogeneous plasma model appears with the coefficient C due to the inhomogeneous number density gradient.

IV. SOLUTION OF MODIFIED K-dV (mKdV) EQUATION

We have considered the transformation[38] $\phi_1 = \mu e^{-Cn_{do}}$ to get the solitary wave solution of the Eq. (28) for the KdV equation is of the form

$$\frac{\partial \mu}{\partial \tau} + P\mu \frac{\partial \mu}{\partial \xi} + Q\frac{\partial^3 \mu}{\partial \xi^3} = 0, \quad (29)$$

where $P = Ae^{-Cn_{do}}$ and $Q = B$.

The above nonlinear coefficients P and Q functionally depends on the environment of the chosen plasma system. To reduce the complexities of the calculations, the variations are considered relatively small compared to the locally constants parameters. Now, we have considered a new frame of reference $U = \xi - V\tau$ w.r.t. velocity V to solve the Eq. (29). After using this new frame of reference and the Kodama - Taniuti method [38], we have obtained the solution of Eq. (29) is as follows:

$$\mu = A_m \left[sech^2\left(\frac{U}{W}\right)\right], \quad (30)$$

where $W = \sqrt{\frac{4Q}{V}}$ and $A_m = \frac{3V}{P}$ are the width and amplitude of the DA solitary wave respectively.

V. RESULTS AND DISCUSSION

In this paper, we have studied the nonlinear DA solitary waves to understand the structural variations of DA solitary wave while propagating in inhomogeneous, unmagnetized plasmas consisting of negatively charged dust grains. We have used the RPT method to derive an mKdV equation with the variable coefficients. We have considered appropriate stretched coordinates for both space and time to employ the RPT method. Then the Kodama-Taniuti method [38] is applied to get the DA solitary wave solution, and the numerical results for DA solitary wave propagation are obtained in Eq. (30). We have focused on the issues of structural variations of DA solitary waves in the above-considered plasma environment.

It is clear from the Eq.(23), the phase velocity M depends on the various choices of n_{e0}, n_{i0}, Z_{d0}, and k. Figures **1** to **4** shows the dependency of the phase velocity M with the inhomogeneous number density n_{d0} and with the various choices of parameters n_{e0}, Z_{d0}, k, and n_{i0}. Here, we have introduced the various figures of phase velocity depending on the various choices of parameters n_{e0}, Z_{d0}, k, and n_{i0} respectively. From Fig. **1**, we have seen that the phase velocity of the DA solitary wave i.e., M increases with the increase in the number density n_{d0}, with the less significance of n_{e0}. But in Fig. **2,** the phase velocity M increases uniformly with the increasing values of Z_{d0} and n_{d0}. Similarly, Figs. **3** and **4** show the expanding rate of M w.r.t. n_{d0}, where the phase velocity M decreases with the increase in k. Also Figs. **3** and **4** also indicates the rapid decrease of the phase velocity with the increasing values of n_{i0}.

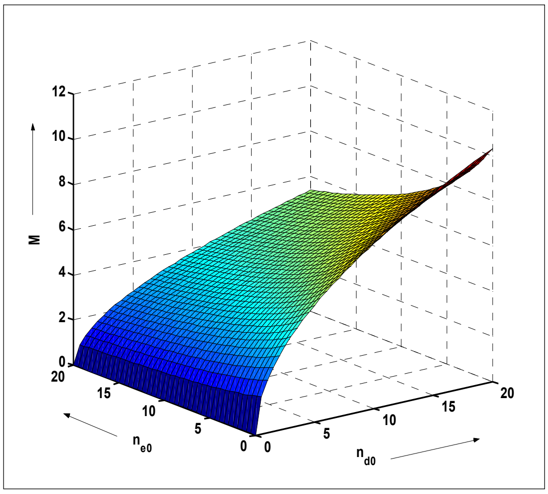

Figure 1. Dependency of phase velocity M with n_{d0} and n_{e0} and Z_{d0} at $n_{i0} = 0.75, Z_{d0} = 2$, and $k = 0.1$

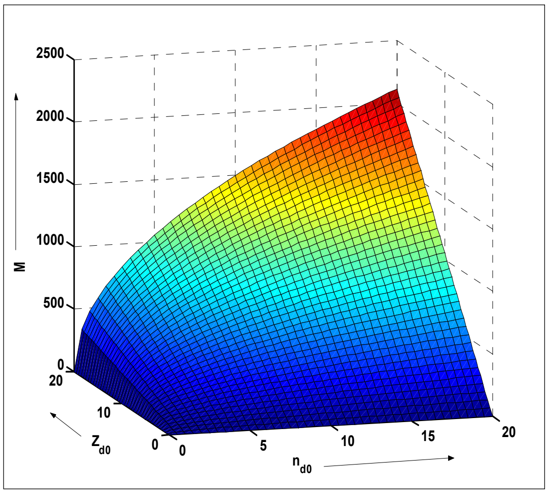

Figure 2. Dependency of phase velocity M at $n_{e0} = 2$, $n_{i0} = 0.008$and $k = 1.5$

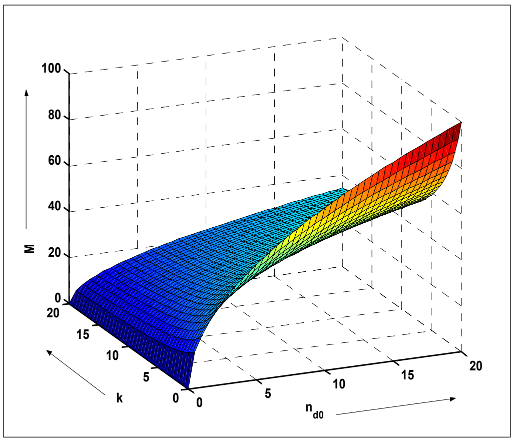

Figure 3. Dependency of phase velocity M with n_{d0} and k at $n_{i0} = 0.008, n_{e0} = 0.003$, and $Z_{d0} = 2$

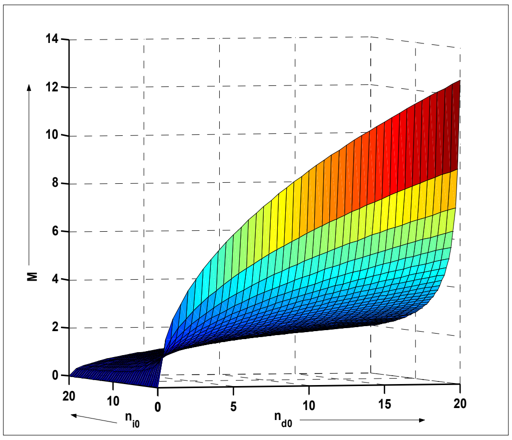

Figure 4. Dependency of phase velocity M with $k = 1, n_{e0} = 0.03$, and $Z_{d0} = 2$

We have observed the coefficients A and P in the above Eqs. (28) and (29), which indicates the dependency of the amplitude A_m of the DA solitary waves on the parameters $n_{e0}, n_{i0}, n_{d0}, Z_{d0}$, and k. Depending on the various choices of the values of $n_{e0}, n_{i0}, n_{d0}, Z_{d0}$, and k, the structural variations in the amplitude of the DA solitary wave A_m w.r.t. the density gradient n_{d0} will be seen in Figs. **5**, **6** and **7**. Here, the Figs. **6** and **7** shows the variations in the amplitude of DA solitary wave A_m w.r.t. n_{d0}, which is increasing with the increasing values of k and n_{e0}, respectively.

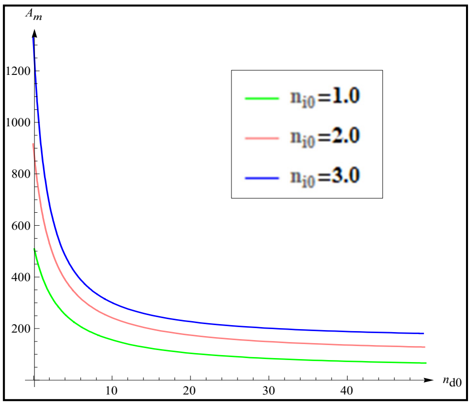

Figure 5. Structural variations of soliton amplitude A_m w.r.t n_{d0} for various choices of $n_{i0} = 1,2$ and 3 with $n_{e0} = 2,\ Z_{d0} = 1.0$ and $k = 0.2$

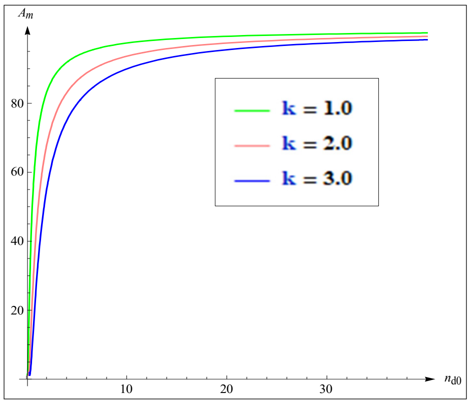

Figure 6. Structural variations of soliton amplitude A_m w.r.t n_{d0} for various choices of $k = 1,2$ and 3 with $n_{e0} = 0.1, Z_{d0} = 1.0$ and $n_{i0} = 0.001$

Based on the above results, it can be observed that with the increasing/decreasing of values of k and n_{e0}, the shape variations of the soliton amplitude will also be seen. With the increasing/decreasing of values of k and n_{e0}, the solitary wave amplitude will be increased/decreased, while on the other hand, in Fig. **5**, the amplitude of the solitary wave will be decreased/increased with the increasing/decreasing values of n_{i0} simultaneously.

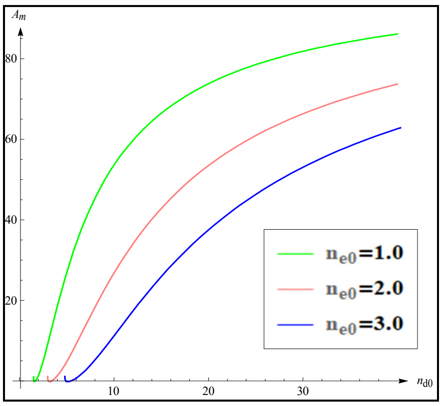

Figure 7. Structural variations of soliton amplitude A_m w.r.t n_{d0} for various choices of $n_{e0} = 1,2$ and 3 with $n_{i0} = 0.01,\ Z_{d0} = 1.0$ and $k = 1.5$

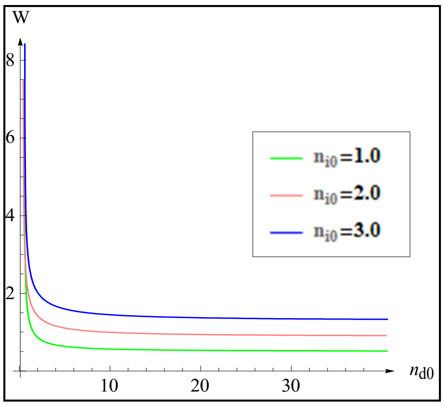

Figure 8. Structural variations of soliton width W w.r.t n_{d0} for various choices of $n_{i0} = 1,2$ and 3 with $n_{e0} = 0.01,\ Z_{d0} = 1.0$ and $k = 1.5$

We have also observed the coefficients $B = Q$ in the above Eqs. (28) and (29), which indicates the dependency of the DA soliton width W on the parameters $n_{e0}, n_{i0}, n_{d0}, Z_{d0}$, and k. Figs. **8**, **9** , and **10** shows the shape variations of DA solitons and their width with inhomogeneous density gradient n_{d0} for the various choices of the parameters n_{i0} (Fig. **8**), n_{e0} (Fig. **9**) and k (Fig. **10**) respectively. So, from Figs. **9** and **10**, it can be observed that the width of the solitary wave W increases with the increasing values of n_{e0}, and k, respectively, while the fig. **8** indicates the decreasing of soliton width with the increase of n_{i0}. Thus, based on the above results and discussions, it can be observed that phase velocity M of the DA solitary wave depends on the various choices of the parameters n_{e0}, n_{i0}, Z_{d0}, and k. Figures **1** to **4** show the dependency of the phase velocity M with the inhomogeneous number density n_{d0} and the other parameters n_{e0}, Z_{d0}, k, and n_{i0}. It can be ensured from figures **1** to **4** that due to the increase in the dust number density n_{d0}, the phase velocity of the solitary wave becomes larger during the propagation of the solitary waves in the plasma.

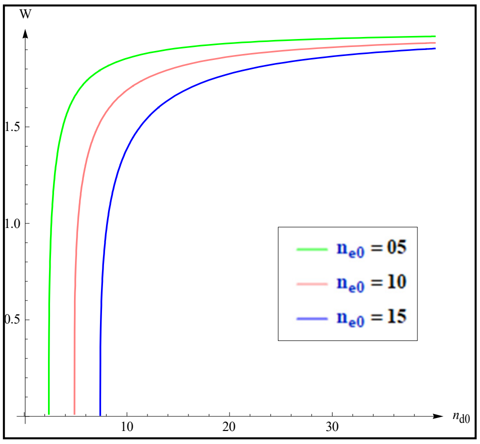

Figure 9. Structural variations of soliton width W w.r.t n_{d0} for various choices of $n_{e0} = 5, 10$ and 15 with $k = 0.5$, and $n_{i0} = 2$

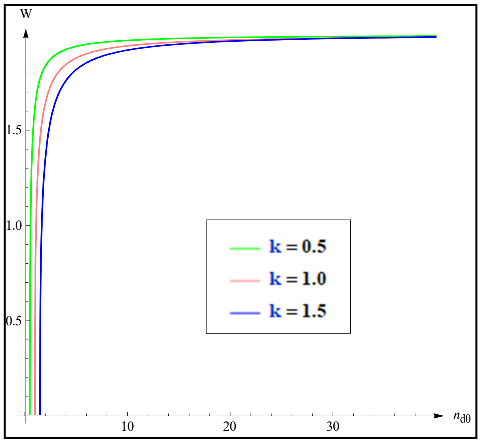

Figure 10. Structural variations of soliton width W w.r.t n_{d0} for various choices of $k = 0.5, 1.0$ and 1.5 with $n_{e0} = 0.01$, and $n_{i0} = 0.1$

Also, the solitary waves experience more deformation due to the faster compression and vibrations of intermolecular interactions in ions, electrons, and dust particles in the plasma system. So, the DA solitary waves produce variations in the soliton structures while propagating in an inhomogeneous plasma. The variations of structures in the amplitude A_m and width W of the DA solitary waves also occur with the increasing/decreasing of number density n_{d0}. While the DA soliton propagates in inhomogeneous plasma, it gets deflected due to plasma of inhomogeneity i.e., the number density n_{d0}. From the above figures (Fig. **5**), it can be observed that with the increase in number density and ion density profile, the smaller the soliton amplitude. Also, the higher the number density, electron density, and ratio of temperature difference, the higher the soliton amplitude which is shown in Fig (**6, 7**). Similarly, when the number density and ion density increase, the width of the solitary wave decreases (Fig. **8**) Also, when the number density and electron density increase, the width of the solitary wave is increased Fig (**9, 10**). Thus, it can be ensured that the structure of the solitary waves varies with the variations of amplitude and width of the DA solitary waves. From the above observations, it is observed that the amplitude with the width of the solitary waves slightly deforms during the propagations in inhomogeneous plasma. So, it ensures the shape conservation of solitary structures as the first principle of K-dV soliton.

VI. CONCLUSION

In this work, theoretically, we have studied variations in the structures of DA solitary wave propagations in inhomogeneous unmagnetized dusty plasma. We have considered collisionless, hot isothermal and Boltzmannean distributed ions and electrons, with negatively charged dust grains in weakly inhomogeneous plasmas. In our problem, the basic governing fluid equations are considered and the reductive perturbation technique (RPT) is employed to solve the modified KdV (mKdV) equation. We have used an appropriate set of stretched variable to use the RPT in the governing fluid equations of plasmas. Due to inhomogeneity in the plasma system, an extra term arises in the modified KdV (m-KdV) equation associated with charged dust particles and the inhomogeneous density gradient. During the investigations, we have also studied the structural variations of amplitude and width of the DA solitary waves depending on the various choices of the parameters i.e., number density of the dust grains (n_{d0}), electron number density (n_{e0}), temperature ratio of ion to electron (k), and the ion number density (n_{i0}).

In this paper, primarily, we have studied the variations in soliton structures of DA solitary wave propagation in inhomogeneous plasmas. But during the investigations, we have also observed and established a relation between amplitude A_m and width W of the dust acoustic (DA) solitons shown in the eq. (30). The Eq. (30) implies that if the plasma inhomogeneity is neglected, the soliton amplitude A_m will increase with the decrease in soliton width W. But based on the various choices of the parameters n_{e0} and n_{i0}, the relation between the amplitude A_m and width W, shown in the eq. (30), has been changed captiously. Also, from the above results, it is clear that A_m and W increase with the increase in n_{e0} (Figs. **7** and **9**). Similarly, A_m and W decrease with the increase in n_{d0} (Figs. **5** and **8**). Thus, from all the results and discussion, it can be concluded that the variations in the soliton structures are modified proportionately due to the presence of plasma inhomogeneity. Features like reflection, refraction, transmission, etc., i.e., variations in soliton structure, are also vital and relevant features in the inhomogeneous plasma systems.

Data Availability. There is no data associated with it.

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СТРУКТУРНІ ВАРІАЦІЇ ПИЛОВИХ АКУСТИЧНИХ СОЛІТОННИХ ХВИЛЬ (ПАСХ), ЯКІ ПОШИРЮЮТЬСЯ В НЕОДНОРІДНІЙ ПЛАЗМІ

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Ця стаття представляє наші теоретичні дослідження структурних варіацій пилових акустичних солітонних хвиль (DASW) у неоднорідній ненамагніченій плазмі. Для вивчення структурних варіацій DASW ми розглянули беззіткнівний, гарячий ізотермічний і больцманівський розподіл електронів-іонів з негативно зарядженими порошинками в слабо неоднорідній плазмі. Ми використали метод редуктивних збурень (*RPT*) у керуючих рівняннях плазми, вивели модифіковане рівняння Кортевега-де-Фріза (m-KdV) і отримали розв'язок із самотньою хвилею. Ми розглянули відповідні розтягнуті координати для просторових і часових змінних для неоднорідної плазми. У цій статті досліджено вплив частинок пилу на поширення іонно-акустичних солітонних хвиль у моделі неоднорідної плазми. Ми також включили вплив параметрів неоднорідності на солітонні структури.

Ключові слова: *пилова плазма; метод редуктивних збурень (RPT); ізотермічні електрони; неоднорідна плазма*