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BIANCHI TYPE V UNIVERSE WITH TIME VARYING COSMOLOGICAL CONSTANT AND QUADRATIC EQUATION OF STATE IN $f(R,T)$ THEORY OF GRAVITY[†]

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In recent years, modified theories of gravity have been extensively studied because of the discovery and confirmation of the current phase of accelerated expansion of the universe. The $f(R,T)$ theory of gravity is one such theory, proposed by Harko *et al*. in 2011, in which R is the Ricci scalar and T is the trace of the stress-energy tensor. In this paper, we study Bianchi type V universe in $f(R,T)$ theory of gravity with time varying cosmological constant and a quadratic equation of state $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ is a constant. We obtain exact solutions of the field equations for two cases: one with a volumetric expansion law and the other with an exponential expansion law. The physical features of the two models are discussed by examining the behavior of some important cosmological parameters such as the Hubble parameter, the deceleration parameter etc. We find that the models have initial singularity and the physical parameters diverge at the initial epoch. The model 1, corresponding to the volumetric expansion law does not resemble ΛCDM model while the model 2, corresponding to the exponential expansion law, resembles ΛCDM model. The energy conditions of the models are also examined and found to be consistent with recent cosmological observations.
Keywords: *Bianchi type V universe; $f(R,T)$ theory of gravity; Equation of state; ΛCDM model*
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1. INTRODUCTION

Various astrophysical and cosmological observations like type Ia supernovae [1-3], Cosmic Microwave Background (CMB) [4, 5], Large Scale Structure (LSS) [6, 7] and other improved measurements of supernovae conforms the discovery of the late-time cosmic acceleration although it is yet to be ascertained what led to the start of this acceleration. According to the recent Planck collaboration results [8], it is found that about 95% of the total constituent of the universe is mysterious. Within the framework of General Relativity, the observed cosmic acceleration can be attributed to an exotic component of the universe with large negative pressure which contributes nearly 68% of the total energy content of the universe. This unknown energy fluid, supposed to be responsible for the late-time cosmic acceleration, is given the name dark energy. In literature, several dark energy candidates like quintessence [9,10], k-essence [11], tachyon [12], phantom [13], Chaplygin gas [14], Holographic dark energy [15] etc. have been proposed and studied in various cosmological background. It is seen that even though the hypothetical dark energy can smoothly explain the accelerated expansion of the universe, many dark energy models encounter with problems when tested by some old red-shift objects [16, 17]. Therefore, the other way considered to explain the cosmic acceleration is modifications of Einstein's theory of gravitation. Some of the most studied modifications of Einstein's General theory of Relativity are the $f(R)$ theory of gravity [18, 19], $f(\mathrm{T})$ gravity [20], $f(R,T)$ theory of gravity [21], $f(G)$ gravity [22] etc. In the $f(R,T)$ theory of gravity, the gravitational Lagrangian in the Einstein-Hilbert action is modified by replacing the Ricci scalar R by an arbitrary function $f(R,T)$ of R and the trace T of the stress-energy tensor. Harko *et al*. [21] have derived the gravitational field equations of this theory in the metric formalism, as well as the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor. They have also presented the field equations corresponding to the homogeneous and isotropic FRW metric and provided a number of specific cosmological models that correspond to some explicit forms of the function $f(R,T)$ such as $f(R,T) = R + 2f(T)$, $f(R,T) = f_1(R) + f_2(T)$, $f(R,T) = f_1(R) + f_2(R)f_3(T)$. Since then many researchers have studied various isotropic and anisotropic cosmological models in different contexts within this framework of modified theory of gravity.

In literature, various homogeneous and anisotropic cosmological models such as the Bianchi type models are studied in the context of dark energy as well as in alternative or modified theories of gravity. Homogeneous and anisotropic models of the universe are becoming more and more popular because of the anomalies found in the observations like Cosmic Microwave Background (CMB) and Large-Scale Structure [23, 24]. Also, models that are spatially homogeneous and anisotropic are helpful in describing the evolution of the early stages of the universe. Bianchi type V models are significant because they include the space of constant negative curvature as a special case.

In this paper, we study a spatially homogeneous and anisotropic Bianchi type V universe with a time dependent cosmological constant Λ and a quadratic equation of state $p = \alpha\rho^2 - \rho$ [25], where $\alpha \neq 0$ is a constant within the

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framework of $f(R,T)$ theory of gravity. In Sect.2, we provide basic field equations of the $f(R,T)$ theory of gravity for the functional form $f(R,T)=R+2f(T)$. In Sect. 3, we obtain explicit field equations corresponding to Bianchi type V metric for $f(R,T)=R+2f(T)=R+2\lambda T$, where λ is a constant. The expressions for the directional scale factors A, B, C in terms of the average scale factor a are also obtained. In Sect. 4, we find exact solutions of the field equations for two cases: one with a volumetric expansion law and the other with an exponential expansion law. Evolutions of some relevant cosmological parameters are investigated in Sect. 5, and physical and geometrical properties of the models are discussed. We conclude the paper in Sect. 6.

2. BASIC FIELD EQUATIONS OF THE $f(R,T)$ THEORY OF GRAVITY

The gravitational Lagrangian in $f(R,T)$ theory of gravity, proposed by Harko *et al.* [21], is given by an arbitrary function $f(R,T)$ of the Ricci scalar R and of the trace T of the stress- energy tensor T_{ij}. The field equations of this theory are derived by varying the action

$$S=\frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x+\int L_m\sqrt{-g}d^4x, \tag{1}$$

with respect to the metric tensor g^{ij}, where L_m is the matter Lagrangian density.

The stress-energy tensor of matter is defined as

$$T_{ij}=\frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}. \tag{2}$$

Assuming the matter Lagrangian density L_m to depend only on the metric tensor components g_{ij}, and not on its derivatives, T_{ij} can be obtained as

$$T_{ij}=g_{ij}L_m-2\frac{\partial L_m}{\partial g^{ij}}. \tag{3}$$

Hence, the variation of (1) with respect to the metric tensor g^{ij} provides the field equations of the $f(R,T)$ theory of gravity as

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+\left(g_{ij}\nabla_k\nabla^k-\nabla_i\nabla_j\right)f_R(R,T)=8\pi T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\Theta_{ij}, \tag{4}$$

where $f_R(R,T)=\frac{\partial f(R,T)}{\partial R}$, $f_T(R,T)=\frac{\partial f(R,T)}{\partial T}$, ∇_i is the covariant derivative with respect to the symmetric connection Γ associated to the metric g and

$$\Theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lk}}. \tag{5}$$

Since there is no unique definition of the matter Lagrangian density L_m, therefore, by assuming the stress-energy tensor of matter to be given by the stress-energy tensor of a perfect fluid of density ρ and pressure p in the form

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij}, \tag{6}$$

where the four velocity u_i satisfies the conditions $u^i\nabla_j u_i=0$ and $u^iu_i=1$, the matter Lagrangian density can be taken as $L_m=-p$. Then from Eq. (5), we obtain

$$\Theta_{ij}=-2T_{ij}-pg_{ij}. \tag{7}$$

And for the functional form

$$f(R,T)=R+2f(T), \tag{8}$$

where $f(T)$ is an arbitrary function of the trace T of the stress-energy tensor of matter, the gravitational field equations, from Eq (4) are obtained as

$$R_{ij}-\frac{1}{2}Rg_{ij}=8\pi T_{ij}-2f'(T)T_{ij}-2f'(T)\Theta_{ij}+f(T)g_{ij}, \tag{9}$$

where the prime denotes differentiation with respect to the argument.
In view of Eq. (6), the Eq. (9) becomes

$$R_{ij}-\frac{1}{2}Rg_{ij}=8\pi T_{ij}+2f'(T)T_{ij}+[2pf'(T)+f(T)]g_{ij}. \tag{10}$$

3. METRIC AND FIELD EQUATIONS

We consider a spatially homogeneous and anisotropic Bianchi type V metric in the form

$$ds^2=dt^2-A^2dx^2-e^{2x}(B^2dy^2+C^2dz^2), \tag{11}$$

where A, B, C are functions of the cosmic time t only.

Using comoving coordinates the field equations (10) for the metric (11) with a time dependent cosmological constant Λ and the functional,

$$f(R,T) = R + 2f(T) = R + 2\lambda T$$

where λ is a constant, are obtained as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{1}{A^2} = -(8\pi + 3\lambda)p + \lambda\rho - \Lambda \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} = -(8\pi + 3\lambda)p + \lambda\rho - \Lambda \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = -(8\pi + 3\lambda)p + \lambda\rho - \Lambda \tag{14}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \frac{3}{A^2} = (8\pi + 3\lambda)\rho - \lambda p - \Lambda \tag{15}$$

$$-2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 0 \tag{16}$$

where an overhead dot indicates differentiation with respect to the cosmic time t.

For the Bianchi type V metric given in Eq. (11), the various parameters of cosmological importance are:
The spatial volume,

$$V = ABC \tag{17}$$

The average scale factor,

$$a = V^{\frac{1}{3}} = (ABC)^{\frac{1}{3}} \tag{18}$$

The mean Hubble parameter,

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{19}$$

The deceleration parameter,

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{20}$$

The expansion scalar,

$$\theta = 3H = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{21}$$

The shear scalar,

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} {H_i}^2 - 3H^2\right) \tag{22}$$

The anisotropy parameter,

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{23}$$

where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters.

4. SOLUTIONS OF THE FIELD EQUATIONS

From equation (16), on integration, we get

$$A^2 = BC \tag{24}$$

From (12), (13) and (14), we obtain

$$A = l_1 a \exp\left(m_1 \int \frac{dt}{a^3}\right) \tag{25}$$

$$B = l_2 a \exp\left(m_2 \int \frac{dt}{a^3}\right) \tag{26}$$

$$C = l_3 a \exp\left(m_3 \int \frac{dt}{a^3}\right) \tag{27}$$

where the constants satisfy the relations $m_1 + m_2 + m_3 = 0$ and $l_1 l_2 l_3 = 1$.

Using (25), (26), (27) in Eq. (24), we get

$$l_1 = \exp\left(-m_1 \int \frac{dt}{a^3}\right)$$

Now, since l_1 is a constant, so we may assume that $m_1 = 0$ so that $l_1 = 1$ and consequently $l_2 l_3 = 1$ and $m_2 + m_3 = 0$. Without loss of generality, we take

$$l_2 = {l_3}^{-1} = c_1 \text{ and } m_2 = -m_3 = c_2$$

where c_1 and c_2 are non-zero constants.
Then from (25)-(27), we obtain the directional scale factors as

$$A = a \tag{28}$$

$$B = c_1 a \exp\left(c_2 \int \frac{dt}{a^3}\right) \tag{29}$$

$$C = \frac{1}{c_1} a \exp\left(-c_2 \int \frac{dt}{a^3}\right) \tag{30}$$

Now, to find exact solution of the field equations, we need one extra condition for which we consider a volumetric expansion law. We also find another exact solution by using the exponential expansion law.

For volumetric expansion law, we consider

$$V = V_0 t^{3n} \tag{31}$$

where $V = ABC = a^3$, and V_0 and n are non-zero constants.
Then from (28), (29) and (30), we get

$$A = V_0^{\frac{1}{3}} t^n \tag{32}$$

$$B = c_1 V_0^{\frac{1}{3}} t^n \exp\left[-\frac{c_2 t^{-3n+1}}{V_0(3n-1)}\right] \tag{33}$$

$$C = \frac{1}{c_1} V_0^{\frac{1}{3}} t^n \exp\left[\frac{c_2 t^{-3n+1}}{V_0(3n-1)}\right] \tag{34}$$

For exponential expansion law, we consider

$$V = V_0 e^{3nt}$$

where $V = ABC = a^3$, and V_0 and n are non-zero constants.
Then from (28), (29) and (30), we get

$$A = V_0^{\frac{1}{3}} e^{nt} \tag{35}$$

$$B = c_1 V_0^{\frac{1}{3}} e^{nt} \exp\left\{\frac{-c_2 e^{-3nt}}{3nV_0}\right\} \tag{36}$$

$$C = \frac{1}{c_1} V_0^{\frac{1}{3}} e^{nt} \exp\left\{\frac{c_2 e^{-3nt}}{3nV_0}\right\} \tag{37}$$

5. PHYSICAL AND GEOMETRICAL PROPERTIES OF THE MODELS

Model 1

The average Hubble parameter H, the expansion scalar θ, the deceleration parameter q and the shear scalar σ and the anisotropy parameter A_m for the model corresponding to the volumetric expansion law are obtained as

$$H = \frac{n}{t} \tag{38}$$

$$\theta = 3H = \frac{3n}{t} \tag{39}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{1}{n} \tag{40}$$

$$\sigma^2 = \frac{{c_2}^2}{{V_0}^2 t^{6n}} \tag{41}$$

$$A_m = \frac{2}{3} \frac{{c_2}^2}{{V_0}^2 n^2 t^{6n-2}} \tag{42}$$

From equation (40), we see that the cosmic expansion accelerates for $n > 1$.
Now, adding equations (14) and (15) and using quadratic equation of state
$p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ constant, we get

$$\rho^2 = \frac{1}{(4\pi+\lambda)\alpha}\left[\frac{n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} - \frac{1}{V_0^{\frac{2}{3}} t^{2n}}\right] \tag{43}$$

$$p = \frac{1}{(4\pi+\lambda)}\left[\frac{n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} - \frac{1}{V_0^{\frac{2}{3}} t^{2n}}\right] - \sqrt{\frac{1}{(4\pi+\lambda)\alpha}\left[\frac{n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} - \frac{1}{V_0^{\frac{2}{3}} t^{2n}}\right]} \tag{44}$$

Using p and ρ in (12), we obtain

$$\Lambda = \frac{(8\pi+3\lambda)}{(4\pi+\lambda)}\left[\frac{{c_2}^2}{{V_0}^2 t^{6n}} + \frac{1}{V_0^{\frac{2}{3}} t^{2n}} - \frac{n}{t^2}\right] + 4(2\pi+\lambda)\sqrt{\frac{1}{(4\pi+\lambda)\alpha}\left[\frac{n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} - \frac{1}{V_0^{\frac{2}{3}} t^{2n}}\right]} - \frac{3n^2}{t^2} + \frac{2n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} + \frac{1}{V_0^{\frac{2}{3}} t^{2n}} \tag{45}$$

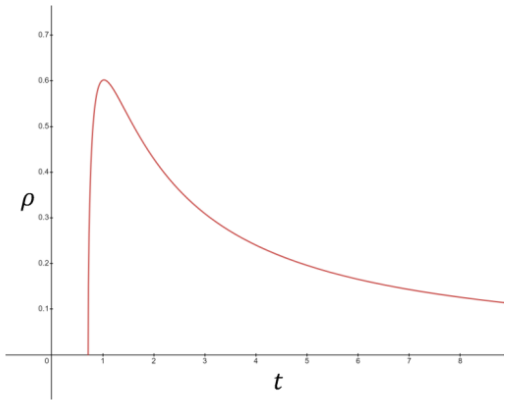

Figure 1. The plot of energy density ρ vs. cosmic time t graph with $\alpha = 0.1,\ c_2 = 0.1, V_0 = 1, n = 1.5, \lambda = 1$

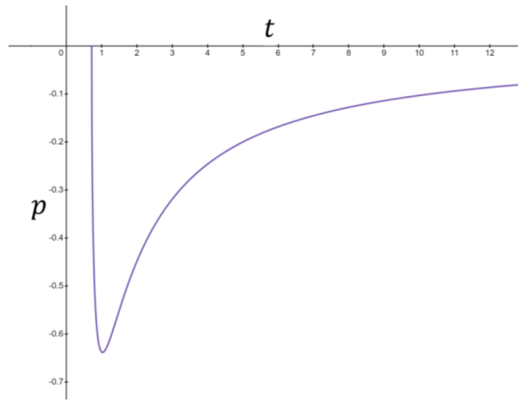

Figure 2. The plot of pressure p vs. cosmic time t graph with $\alpha = 0.1, c_2 = 0.1, V_0 = 1, n = 1.5, \lambda = 1$

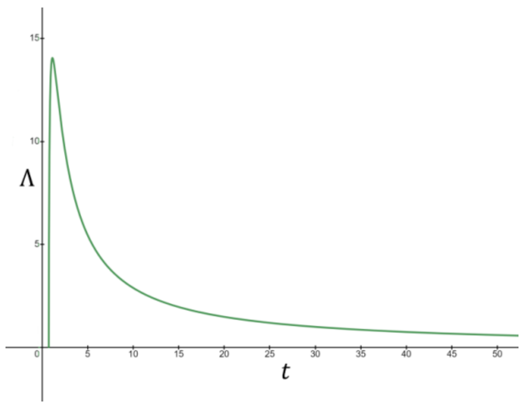

Figure 3. The plot of the cosmological constant Λ vs. cosmic time t graph with $\alpha = 0.1,\ c_2 = 0.1,\ V_0 = 1, n = 1.5, \lambda = 1$

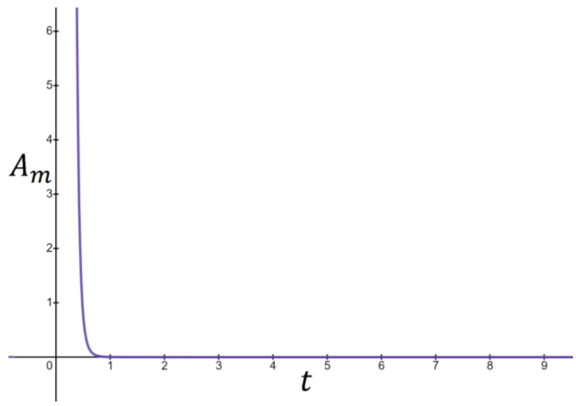

Figure 4. The plot of anisotropy parameter $A_m$vs. cosmic time t graph with $c_2 = 0.1, V_0 = 1, n = 1.5$

From the graphs we observe that the energy densityρ is a decreasing function of cosmic time, pressure$p$ is negative throughout the evolution of the universe and the cosmological constant Λ decreases rapidly and tend to zero. The figure 4 shows that the universe is highly anisotropic at its early stage and the anisotropy dies out in the course of evolution.

The Cosmic Jerk Parameter.
The cosmic jerk parameter is defined as

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a} \tag{46}$$

The equation (46) can be written in terms of the deceleration and the Hubble parameter as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} \tag{47}$$

From equations (38) and (40), using (47), we get the cosmic jerk parameter for this model as

$$j(t) = \frac{(n-1)(n-2)}{n^2} \tag{48}$$

At late times, the value of the cosmic jerk parameter is 1 for ΛCDM model. For this model, $j(t) = 1$ for $n = \frac{2}{3}$. But we have a restriction $n > 1$. Hence, this model does not resemble with ΛCDM model.

Energy Conditions

Weak Energy Condition (WEC), Null Energy Condition (NEC), Dominant Energy Condition (DEC) and Strong Energy Condition (SEC) are given by

WEC : $\rho \geq 0$

NEC : $\rho + p \geq 0$

DEC : $\rho - p \geq 0$

SEC :$\rho + 3p \geq 0$

For this model, we have

$$\rho + p = \frac{1}{(4\pi + \lambda)}\left[\frac{n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} - \frac{1}{V_0^{\frac{2}{3}} t^{2n}}\right]$$

$$\rho - p = 2\sqrt{\frac{1}{(4\pi + \lambda)\alpha}\left[\frac{n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} - \frac{1}{V_0^{\frac{2}{3}} t^{2n}}\right]} - \frac{1}{(4\pi + \lambda)}\left[\frac{n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} - \frac{1}{V_0^{\frac{2}{3}} t^{2n}}\right]$$

$$\rho + 3p = \frac{3}{(4\pi + \lambda)}\left[\frac{n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} - \frac{1}{V_0^{\frac{2}{3}} t^{2n}}\right] - 2\sqrt{\frac{1}{(4\pi + \lambda)\alpha}\left[\frac{n}{t^2} - \frac{{c_2}^2}{{V_0}^2 t^{6n}} - \frac{1}{V_0^{\frac{2}{3}} t^{2n}}\right]}$$

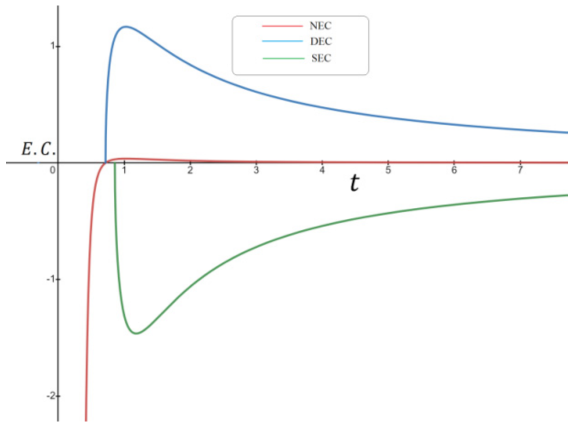

Figure 5. The plot of left hand sides of energy conditions vs. cosmic time t graph with $\alpha = 0.1, c_2 = 0.1, V_0 = 1, n = 1.5, \lambda = 1$

From figure 1 and figure 5, we see that the WEC and DEC are satisfied. NEC is satisfied only at late times while the SEC is violated for this model.

Model 2

The average Hubble parameter H, the expansion scalar θ, the deceleration parameter q and the shear scalar σ and the anisotropy parameter A_m for the model corresponding to the exponential expansion are obtained as

$$H = n \tag{49}$$

$$\theta = 3n \tag{50}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 \tag{51}$$

$$\sigma^2 = \frac{{c_2}^2}{V_0^2 e^{6nt}} \tag{52}$$

$$A_m = \frac{2{c_2}^2}{3n^2 V_0^2 e^{6nt}} \tag{53}$$

From the expression for the deceleration parameter q, we see that the expansion of the universe is decelerating throughout the evolution and does not depend on n.

Now, adding (12) and (15) and using quadratic equation of state $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$ is a constant, we get

$$\rho^2 = \frac{-1}{\alpha(4\pi+\lambda)}\left(\frac{c_2{}^2}{V_0{}^2 e^{6nt}} + \frac{1}{V_0^{\frac{2}{3}} e^{2nt}}\right) \tag{54}$$

$$p = \frac{-1}{4\pi+\lambda}\left(\frac{c_2{}^2}{V_0{}^2 e^{6nt}} + \frac{1}{V_0^{\frac{2}{3}} e^{2nt}}\right) - \sqrt{\frac{-1}{\alpha(4\pi+\lambda)}\left(\frac{c_2{}^2}{V_0{}^2 e^{6nt}} + \frac{1}{V_0^{\frac{2}{3}} e^{2nt}}\right)} \tag{55}$$

Using p and ρ in (12), we obtain

$$\Lambda = 2\left(\frac{2\pi+\lambda}{4\pi+\lambda}\right)\frac{c_2{}^2}{V_0{}^2 e^{6nt}} + 4\left(\frac{3\pi+\lambda}{4\pi+\lambda}\right)\frac{1}{V_0^{\frac{2}{3}} e^{2nt}} - 3n^2 + 4(2\pi+\lambda)\sqrt{\frac{-1}{\alpha(4\pi+\lambda)}\left(\frac{c_2{}^2}{V_0{}^2 e^{6nt}} + \frac{1}{V_0^{\frac{2}{3}} e^{2nt}}\right)} \tag{56}$$

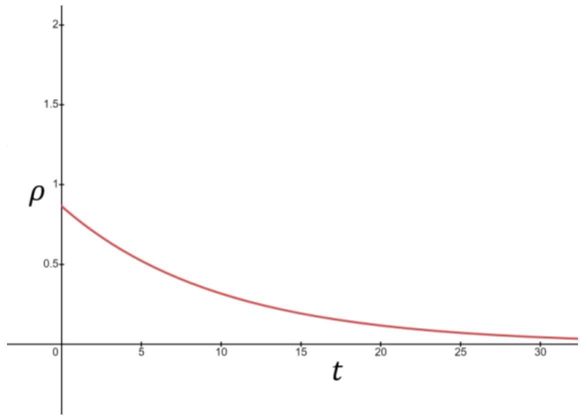

Figure 6. The plot of energy density ρ vs. cosmic time t graph with $\alpha = -0.1, c_2 = 0.1, V_0 = 1, n = 0.1, \lambda = 1$

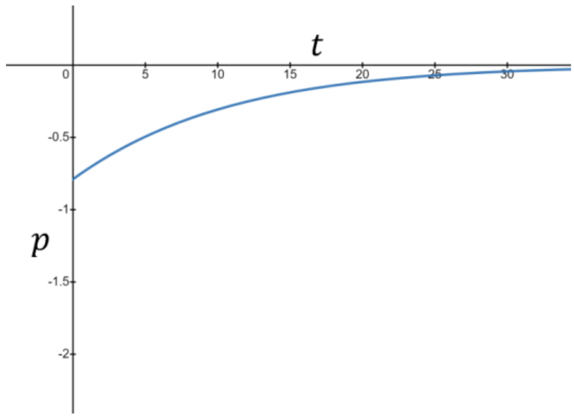

Figure 7. The plot of pressure p vs. cosmic time t graph with $\alpha = -0.1, c_2 = 0.1, V_0 = 1, n = 0.1, \lambda = 1$

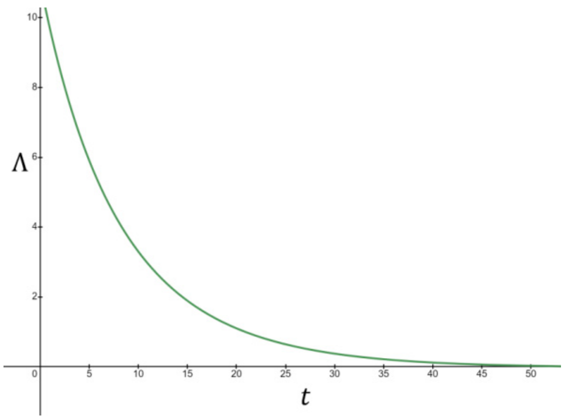

Figure 8. The plot of cosmological constant Λ vs. cosmic time t graph with $\alpha = -0.1, c_2 = 0.1, V_0 = 1, n = 0.1, \lambda = 1$

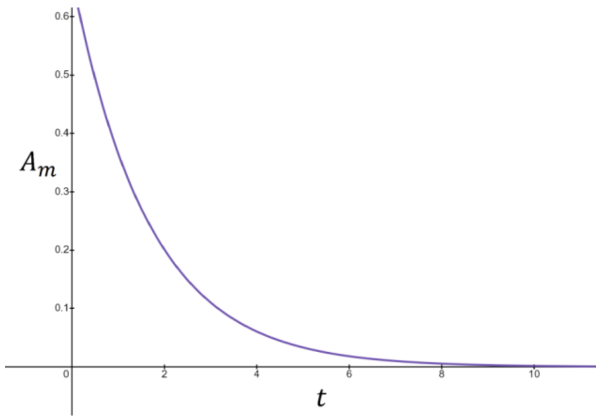

Figure 9. The plot of anisotropy parameter A_m vs. cosmic time t graph with $c_2 = 0.1, n = 0.1, V_0 = 1$

From the figures 6, 7, 8 and 9, we see that the behavior of the energy density, pressure, cosmological constant and anisotropy parameter satisfies the present cosmological observations. However, in this case, the constant α should assume negative values

The Cosmic Jerk Parameter:

From equations (49) and (51), using (47), we obtain the cosmic jerk parameter for this model as

$$j(t) = 1$$

This shows that this model resembles ΛCDM for any value of n.

Energy Conditions

For this model, the energy conditions are obtained as

$$\rho + p = -\frac{1}{4\pi + \lambda}\left(\frac{{c_2}^2}{{V_0}^2 e^{6nt}} + \frac{1}{V_0^{\frac{2}{3}} e^{2nt}}\right)$$

$$\rho - p = 2\sqrt{\frac{-1}{\alpha(4\pi + \lambda)}\left(\frac{{c_2}^2}{{V_0}^2 e^{6nt}} + \frac{1}{V_0^{\frac{2}{3}} e^{2nt}}\right)} + \frac{1}{4\pi + \lambda}\left(\frac{{c_2}^2}{{V_0}^2 e^{6nt}} + \frac{1}{V_0^{\frac{2}{3}} e^{2nt}}\right)$$

$$\rho + 3p = -\frac{3}{4\pi + \lambda}\left(\frac{{c_2}^2}{{V_0}^2 e^{6nt}} + \frac{1}{V_0^{\frac{2}{3}} e^{2nt}}\right) - 2\sqrt{\frac{-1}{\alpha(4\pi + \lambda)}\left(\frac{{c_2}^2}{{V_0}^2 e^{6nt}} + \frac{1}{V_0^{\frac{2}{3}} e^{2nt}}\right)}$$

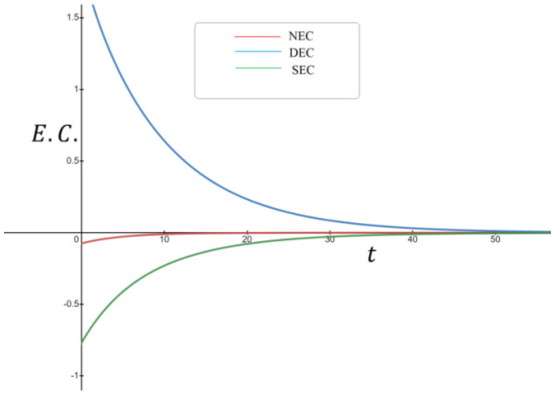

Figure 10. The plot of left-hand side of energy conditions vs. cosmic time t graph with $\alpha = -0.1, c_2 = 0.1, V_0 = 1, n = 0.1, \lambda = 1$

From figures 6 and 10, we see that for this model, the WEC and DEC are satisfied and NEC and SEC are violated.

6. CONCLUSION

In this paper, we study a spatially homogeneous and anisotropic Bianchi type V universe with time varying cosmological constant and a quadratic equation of state in $f(R,T)$ theory of gravity for the functional form $(R,T) = R + 2\lambda T$, where λ is a constant. We construct two cosmological models corresponding to a volumetric power law expansion (Model 1) and an exponential expansion (Model 2). We find that

- Both the models have initial singularity as the metric coefficients A, B and C vanish at the initial moment.
- The physical parameters H, θ, σ^2 for both the models diverge at the initial epoch and for large t, these parameters tend to 0. Also, the volume of the universe is zero at $t = 0$ and increases exponentially with time t. Hence, both the models start with the big bang singularity at $t = 0$ and then expand throughout the evolution.
- The energy density of the model 1 increases at the beginning but it decreases in the course of evolution and tends to 0 at late time. The energy density of the model 2 decreases from the evolution of the universe and tends to 0 as time goes on.
- For both the models, the cosmological constant is a decreasing function of the cosmic time and tends to 0 at late time.
- The model 1 exhibits accelerated expansion for $n > 1$, while for model 2, it happens for any values of n.
- The model 1 never approaches ΛCDM model while the model 2 resembles ΛCDM model for any values of n.
- The model 1 satisfies present cosmological observations for positive values of α while the model 2 satisfies the same for negative values of α.
- For both the models, the energy conditions WEC and DEC are satisfied and NEC and SEC are violated. The violation of SEC shows that the universe has anti-gravitating effect which results accelerating expansion of the universe.

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REFERENCES

[1] S. Perlmutter, G. Aldering, M. Della Valle, S. Deustua, R.S. Ellis, S. Fabbro, and A. Fruchter, et al, "Discovery of supernovae explosion at half the age of the universe", Nature (London), **391**, 51 (1998). https://doi.org/10.1038/34124

[2] A.G. Riess, A.V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P.M. Garnavich, R.L. Gilliland, et al, "Observational evidence from supernovae for an accelerating universe and a cosmological constant", Astron. J. **116**, 1009 (1998). https://doi.org/10.1086/300499

[3] S. Perlmutter, G. Aldering, G. Goldhaber, R.A. Knop, P. Nugent, P.G. Castro, S. Deustua, et al, “Measurement of Ω and *Λ* from 42 High-Red shift Supenovae”, Astrophys. J. **517**, 565 (1999). https://doi.org/10.1086/307221
[4] C.L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, M. Limon, S.S. Meyer, et al, “First-Year Wilkinson Microwave Anisotropy Probe (WMAP)* Observations: Preliminary Maps and Basic Results”, Astrophys. J. Suppl. Ser. **148**, 1 (2003). https://doi.org/10.1086/377253
[5] D.N. Spergel, L. Verde, H.V. Peiris, E. Komatsu, M.R. Nolta, C.L. Bennett, M. Halpern, et al, “First-Year Wilkinson Microwave Anisotropy Probe (WMAP)* Observations: Determination of Cosmological Parameters”, Astrophys. J. Suppl. Ser. **148**, 175 (2003). https://doi.org/0.1086/377226
[6] Ed. Hawkins, S. Maddox, S. Cole, O. Lahav, D.S. Madgwick, P. Norberg, J.A. Peacock, et al, “The 2dF Galaxy Redshift Survey: correlation functions, peculiar velocities and the matter density of the Universe”, Mon. Not. Roy. Astron. Soc. **346**, 78 (2003). https://doi.org/10.1046/j.1365-2966.2003.07063.x
[7] K. Abazajian, J.K. Adelman-McCarthy, M.A. Agüeros, S.S. Allam, K.S.J. Anderson, S.F. Anderson, J. Annis, et al, “The Second Data Release of the Sloan Digital Sky Survey”, Astron. J. **128**, 502 (2004). https://doi.org/10.1086/421365
[8] N. Aghanim, et al, [Planck Collaboration], (2018). https://doi.org/10.48550/arXiv.1807.06209
[9] T. Barreiro, E.J. Copeland, and N.J. Nunes, “Quintessence arising from exponential potentials”, Phys. Rev. D, **61**, 127301 (2000). https://doi.org/10.1103/PhysRevD.61.127301
[10] V. Sahni, and A.A. Starobinsky, “The Case for a Positive Cosmological Lambda-term”, Int. J. Mod. Phys. D, **9**(4), 373 (2000). https://doi.org/10.1142/S0218271800000542
[11] C. Armendariz-Picon, V. Mukhanov, and P.J. Steinhardt, “Essentials of k-essence”, Phys. Rev. D, **63**, 103510 (2001). https://doi.org/10.1103/PhysRevD.63.103510
[12] A. Sen, “Rolling Tachyon”, J. High Energy Phys. **2002**, JHEP04 (2002). https://doi.org/10.1088/1126-6708/2002/04/048
[13] R.R. Caldwell, M. Kamionkowski, and N.N. Weinberg, “Phantom Energy: Dark Energy with $\omega < -1$ Causes a Cosmic Doomsday”, Phys. Rev. Lett. **91**, 071301 (2003). https://doi.org/10.1103/PhysRevLett.91.071301
[14] A. Kamenshchik, U. Moschella, and V. Pasquier, “An alternative to quintessence”, Phys. Lett. B, **511**, 265 (2001). https://doi.org/10.1016/S0370-2693(01)00571-8
[15] M. Li, “A Model of Holographic Dark Energy”, Phys. Lett. B, **603**, 1 (2004). https://doi.org/10.1016/j.physletb.2004.10.014
[16] A.C.S. Friaça, J.S. Alcaniz, and J.A.S. Lima, “An old quasar in a young dark energy-dominated universe?”, Mon. Not. Roy. Astron. Soc. **362**, 1295 (2005). https://doi.org/10.1111/j.1365-2966.2005.09401.x
[17] S. Wang, X.-D. Li, and M. Li “Revisit of cosmic age problem”, Phys. Rev. D, **82**, 103006 (2010). https://doi.org/10.1103/PhysRevD.82.103006
[18] S.M. Carroll, V. Duvvuri, M. Trodden, and M.S. Turner, “Is cosmic speed-up due to new gravitational physics?”, Phys. Rev. D, **70**, 043528 (2004). https://doi.org/10.1103/PhysRevD.70.043528
[19] S. Nojiri, and S.D. Odintsov, “Modified f(R)gravity consistent with realistic cosmology: From a matter dominated epoch to a dark energy universe”, Phys. Rev. D, **74**, 086005 (2006). https://doi.org/10.1103/PhysRevD.74.086005
[20] B. Li, T.P. Sotiriou, and J.D. Barrow, “$f(T)$gravity and local Lorentz invariance”, Phys. Rev. D, **83**(6), 064035 (2011). https://doi.org/10.1103/PhysRevD.83.064035
[21] Harko T. *et al*., “$f(R,T)$gravity”, Phys. Rev. D, **84**(2), 024020 (2011).
[22] N.M. García, F.S.N. Lobo, J.P. Mimoso, and T. Harko, “$f(G)$modified gravity and the energy Conditions”, Journal of Physics: Conference Series, **314**, 012056 (2011). https://doi.org/10.1088/1742-6596/314/1/012056
[23] C.J. Copi, D. Huterer, D.J. Schwarz, and G.D. Starkman, “Large-Angle Anomalies in the CMB”, Advances in Astronomy, **2010**, 847541 (2010). https://doi.org/10.1155/2010/847541
[24] I. Agullo, D. Kranas, and V. Sreenath, “Anomalies in the Cosmic Microwave Background and Their Non-Gaussian Origin in Loop Quantum Cosmology”, Front. Astron. Space Sci. **8**, 703845 (2021). https://doi.org/10.3389/fspas.2021.703845
[25] G.P. Singh, and B.K. Bishi, “Bianchi type-I transit Universe in $f(R,T)$modified gravity with quadratic equation of state and Λ”, Astrophys. Space Sci. **360**, 34 (2015). https://doi.org/10.1007/s10509-015-2495-0

ВСЕСВІТ Б’ЯНЧІ ТИПУ V ЗІ ЗМІННОЮ В ЧАСІ КОСМОЛОГІЧНОЮ СТАЛОЮ ТА КВАДРАТИЧНИМ РІВНЯННЯМ СТАНУ В ТЕОРІЇ ГРАВІТАЦІЇ *f(R,T)*

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В останні роки модифіковані теорії гравітації широко вивчалися через відкриття та підтвердження поточної фази прискореного розширення Всесвіту. Теорія гравітації *f(R,T)* є однією з таких теорій, що запропонована Харко та ін. у 2011 році, де R є скаляром Річчі, а T є слідом тензора енергії напруги. У цій статті ми вивчаємо Всесвіт типу Б’янчі V в теорії гравітації *f(R,T)* із змінною в часі космологічною константою та квадратним рівнянням стану $p = \alpha\rho^2 - \rho$, де $\alpha \neq 0$ є константою. Отримано точні розв’язки рівнянь поля для двох випадків: один з законом об’ємного розширення, а інший – з експоненціальним законом розширення. Фізичні характеристики двох моделей обговорюються шляхом вивчення поведінки деяких важливих космологічних параметрів, таких як параметр Хабла, параметр уповільнення тощо. Ми виявили, що моделі мають початкову сингулярність, а фізичні параметри розходяться в початкову епоху. Модель 1, що відповідає закону об’ємного розширення, не нагадує модель ΛCDM, а модель 2, яка відповідає закону експоненціального розширення, нагадує модель ΛCDM. Енергетичні умови моделей також досліджуються та виявляються узгодженими з нещодавніми космологічними спостереженнями.

Ключові слова: *Всесвіт типу Б’янчі V; теорія гравітації f(R,T); рівняння стану; модель ΛCDM*